Mathematical Modeling of Manufacturing Processes
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Lecture 10
Force and Velocity Diagram - 2

If you observe the force diagram also and what we were discussing about this Merchant's force diagram.

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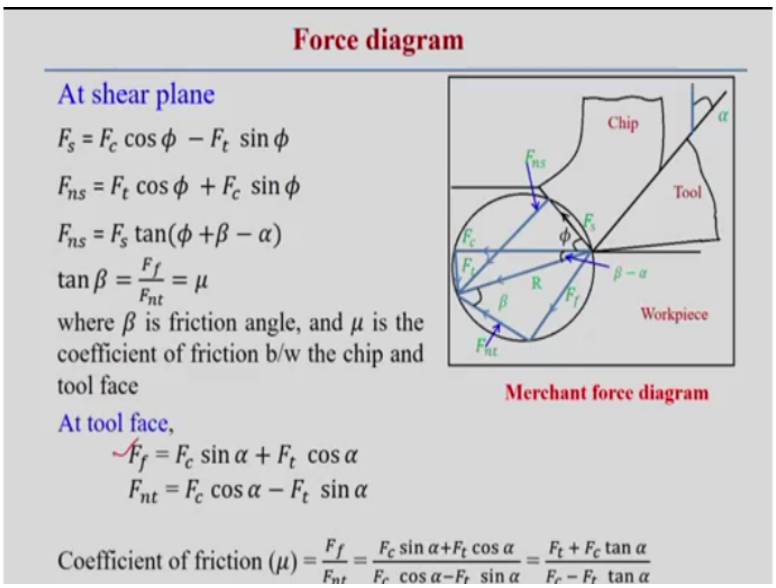

Here we can see that frictional force, which can be estimated in terms of these two components, cutting force components, one is that Fc which is parallel to that and normal to that force is the Ft. So Fc sin alpha + Ft cos alpha, in that form we can find out from this diagram. The frictional force and the normal force can also be estimated in terms of this Fc, Ft and along with the rake angle alpha.

Here we can see that frictional force is represented basically in terms of mainly the cutting force components as well as a function of the tool rake angle. So from here we can estimate the coefficient to friction, which can be estimated by Ff/Fnt that ratio of these two forces, this force and this force ratio, which can be estimated if you put this value, both of those values, we can estimate the Ft + Fc tan alpha Fc - Ft tan alpha.

So these are the different correlation and we also already explained that this coefficient friction is also equal to tan beta. If you know the friction angle beta, there also you can find out what is the coefficient of friction or the coefficient of friction can be estimated with all the force components as well as the rake angle. Now if we see overall all these expression that in this diagram, that it depends on the 3 different angles, we have considered.

One is the shear angle, another is the friction angle, another is the rake angle. All expression are in that form and it is very clear that in this expression, the force diagram or components of the force, we correlate which is independent of the clearance angle or other angles, which we have defined when we tried to define a tool signature. So all these 3 angles, only this rake angle is actually used to estimate this or to find the relation of this force components.

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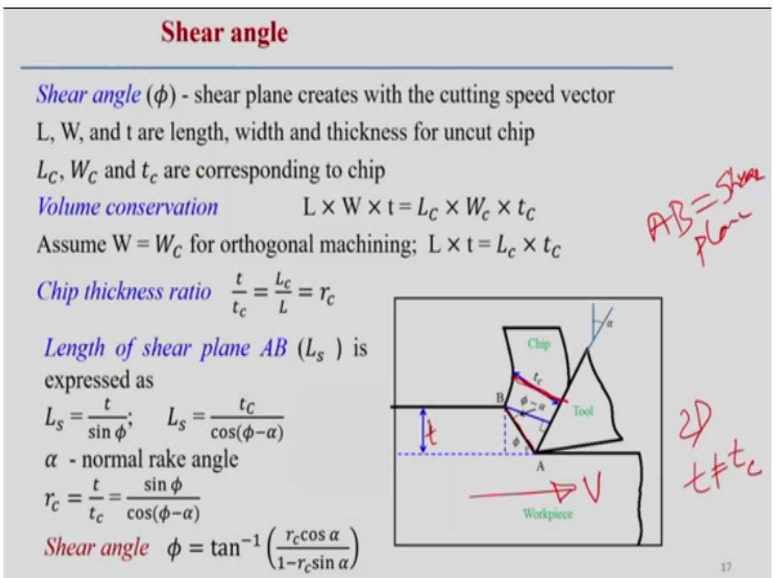

Now look into that what is shear angle here? Shear angle is basically the shear plane, if you schematically see these 2 dimensional figure, the shear angle can be defined that, so suppose there is a velocity of work piece relative in this direction V and then this is the shear angle, that basically this is the shear plane. Here you represent AB is the shear plane. So that plane making angle with respect to that velocity vector, that actually indicates the shear angle that is phi, well defined.

The uncut chip thickness, uncut chip thickness means just what is before cutting, before making the chip formation what was the thickness we define and basically what thickness we are supposed to cut. That is defined is the T here and chip thickness also you can measure, which is this chip thickness after, basically the shear plane is the tangential from the undeformed work piece to the deformed chip.

And that actually all deformation happens within a specified plane in specific to orthogonal cutting situation, that is assumptions, that is on the AB plane. Here if we assume the chip thickness is not necessary that T may or may not be equal to the chip thickness. Normally chip thickness, if we consider the expansion of the material deformation and maybe heat generation and that actually responsible to make the uncut chip thickness that means before cutting.

And after cutting what is the chip forms, the thickness of the chip may be different. So T not equal to Tc in this case. Now here you can see the angle, this angle is phi and here we define the other parameters also that L, W, and T are the length, width and thickness of the uncut chip, that means we assume that before cutting, the W equal to the normal to the diagram that is the width of the chip and length, particular time if we define the length of the chip.

And T is the thickness of the chip and that is uncut chip and after formation of the chip, then we can define that Lc is the length of the chip, Wc is basically the width of the chip and Tc is the thickness of the chip. So here we define the length maybe you can assume per unit time what is the length or per unit rotation what is the length of the chip that anything we can assume that, based on that we can make that volume conservation.

That means before cutting and after cutting total volume must be consumed. So in that sense this must be equal LWT should be equal to LcWcTc. Now assume that W = Wc, this is assumption that we assume that before cutting and after cutting the width of the chip remains same. So if this is the case, though L into T is Lc into Tc from here you can reach and we can find out this parameter, which is called the chip thickness ratio, T/Tc which is equal to Lc/L.

Or we can say this defined by Rc. So Rc is basically chip thickness ratio. Now length of the shear plane AB can also be calculated by other way also. Here you can see the length of the shear plane AB, so I think AB is the transformation zone, maybe from uncut chip to the formation of the chip. So then L, this AB if you know there is a known value T and phi, then from there we can find out Ls = that being basically length of the shear plane LsT/sin phi.

Other way also we can estimate Ls that means we look into the parameters after formation of the chip. So but if you look that here, this is the normal distance, that actually represents the chip thickness, which is normal to this rake face and of course this is the angle we have already defined. Here is alpha and here also this is the rake angle. So if you look into that, then it becomes, this is phi and this total angle becomes phi and this angle basically phi-alpha.

So from here if you measure this phi-alpha and this line is perpendicular to this rake face. So then that is equal to the chip thickness, so that Ls equal to that Tc/cos phi - alpha or we can say simply cos phi - alpha=Tc/Ls. So from here you can find out Ls = Tc/cos phi - alpha. So both make it equal, alpha is the normal rake angle and we can find out the chip thickness ratio is the ratio of T/Tc, which can be estimated from here, that sin phi/cos phi - alpha.

So here you can see the chip thickness ratio also depends on that shear angle and the rake angle. From here we can estimate the shear angle, if you know the chip thickness ratio. So actually experimentally it is easy to measure the chip thickness ratio because before the cutting we can measure the uncut thickness and after formation of the chip from there we can measure the thickness of the chip. So if it is known to us, what is the chip thickness ratio.

So from there, we can find out in this machining process what is the shear angle. Because rake angle is defined property and in a particular tool, rake angle is always defined. Then if you know rake angle, chip thickness ratio from here we can find out what is the shear angle for a particular machining process. To some extent if rake angle is fixed, if you assume particular tool and if rake angle is the same rake angle we are using, so therefore the shear angle depends on mainly the chip thickness ratio.

So that means before cut and after formation of the chip, what is the thickness of the chip, it depends on that ratio.

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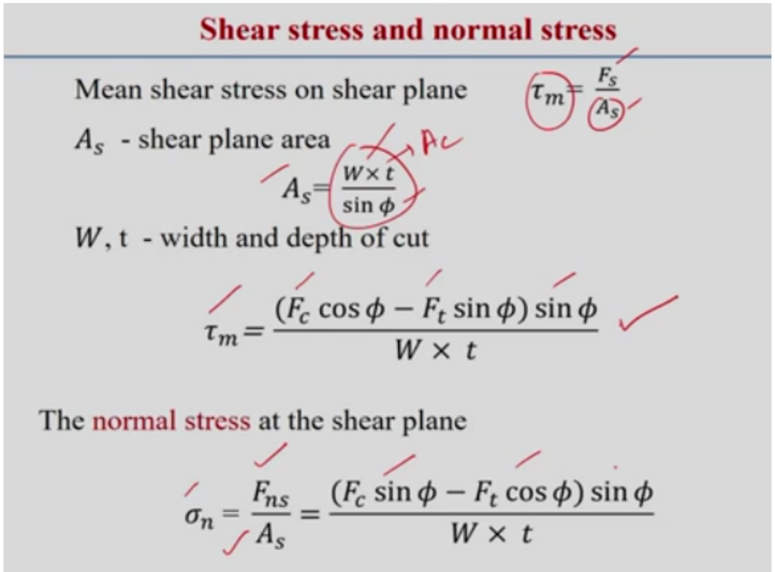

Now shear stress and normal stress during the machining process also we can estimate to prove you already estimated the shear force and then from here if you know from the Merchant's circle diagram, we know the shear force and from the shear force, we can estimate the mean shear stress on the shear plane. Here the mean shear stress on the shear plane, Fs/As. So Fs is the shear force acting on the shear plane.

And that basically the shear force is acting parallel to the shear plane and As is the Ad of the shear plane. If we come back to that this is the shear plane. So Ad of the shear plane we need to consider and then shear force is acting along this plane. So from there we can estimate what is the shear stress value or normal stress value. Now As can be related with this thing Wt/sin phi. So Wt into Ad of cutting that means before.

So W into T, basically this is the uncut chip thickness and normal to that, that is the W and that is the cross section area and that cross section area if you know that cross section area divided by sin phi that indicates the area of this shear plane, basically the area of the shear plane. Here we can see if the shear plane area, this area that means T into W normal to that divided by sin phi that is the area of the shear plane.

WT the depth and width of the cut and from here we can estimate the shear stress equal to, you know the expression of the shear plane from a Merchant's circle diagram, it is a function of Fc cos phi and Ft sin phi and if you put this WT/sin phi, this is the expression for normal shear stress, mean shear stress, sorry this is called the mean shear stress on the shear plane. Now normal stress is at normal stress acting on the shear plane that we know.

That from the Merchant's circle diagram that what is the normal force acting on that. So this is the shear force. Shear force is acting. So normal to the shear force, this is Fns. So that is the normal force acting here and once we know the normal shear force and cross section of the area, normal that means that we need to consider here the cross section area, which is normal to the force. So that indicates the normal stress. So normal stress at the shear plane.

So here the expression from the Merchant's circle diagram, we can easily put the expression in terms of Fc sin phi/Ft cos phi sin phi divided by As = W/Tintosin phi. So from here we can estimate the normal shear. So basically if you know the force components, all the 2 components of the force in the 2 dimensional machining process or orthogonal machining process, and of course it is known, to shear angle is known to us, and all these information given, then we can estimate what is the normal stress acting on the shear plane or shear stress acting on the shear plane.

Both we can estimate from the force components as well as from the cross section of area. So definitely all these phi shear plane angle, we need other information, such that we can estimate the phi.

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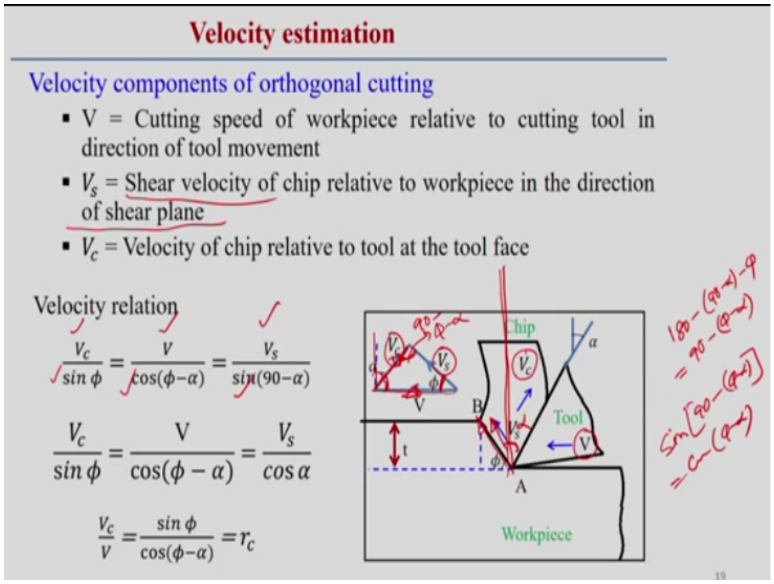

Of course if you look into the velocity, that what are the velocity components in the ideal orthogonal cutting situation, if you look into that process, two dimensional process the tool is moving with respect to velocity V. Of course, the chip also is flowing at a particular velocity, if we assume the velocity vector, which is parallel to the erect phase and that is called the chip velocity Vc and tool velocity with respect to opposite is V.

Of course the shear velocity also we can assume that V as the shear velocity of the chip related to the work piece. So shear velocity of the basically as the deformation happen at the particular plane, so then we assume some velocity components and that velocity components say the shear velocity, but shear velocity of the chip, which is with respect to the work piece in the direction of the shear plane. So that is the definition of the Vs.

So Vs is basically the shear velocity. We can say the shear velocity with respect to the reference to the work piece in the direction of the shear plane. So if you assume all these components acting during this process, Vs, V and Vc. We can draw a velocity triangle also from here and then as a steady state situation and this makes a close triangle and then this is the Vs corresponds to that shear velocity, V cutting velocity and Vc is the chip velocity.

So then there must be some angle between V and Vs, the angle is phi. That is the shear angle and of course normal to this and this we can say alpha the rake angle, the tool rake angle is, we define the tool rake angle normal to that velocity vector and the face making angle alpha. So this angle is equal to alpha with respect to the chip velocity, we define this alpha and now, we can easily find out the velocity relation from this close triangle.

So then we can say the sin law we can put it here. So Vc, the velocity components and opposite to this angle is phi. So Vc/sin phi we can put it here, which is equal to cutting velocity V and opposite angle is this one. So this angle if we estimate, it is basically phi-alpha, because if this is phi, this is alpha, so this angle is 90 - alpha. So this angle is 90 - alpha, so this angle definitely 180 – 90 – alpha - phi, so basically it is coming like that 90 – phi-alpha.

So this angle 90 - phi - alpha, so if you put sin this V/this angle, so sin 90 – phi - alpha, so this basically cos phi - alpha. So here we can V cos phi - alpha and Vs, this opposite angle is basically 90 - alpha. So Vs sin 90 - alpha, so from here we can find out Vc/sin phi V/cos phi – alpha = Vs/cos alpha. So we can relate if we know the cutting velocity is known to us, we can estimate the chip velocity, if we know the V, phi and alpha.

So from here we can find out Vc/V, any relations in terms of V cutting velocity, that is the sin Vc/V is basically sin phi/cos phi - alpha, which is equivalent to the chip thickness ratio. That we have already shown this chip thickness ratio, because here you can see that chip thickness is equal to sin phi/cos phi-alpha. So we can put it here. So Vc/V is basically Rc, so cutting velocity is basically V into Rc. If you assume chip thickness ratio Rc is less than 1, so Vc/V is basically less than 1.

So Vc<V, so the chip velocity is always less than the cutting velocity if Rc is less than 1. That means after the formation of the chip, which is expanded, thickness is more than with respect to what was uncut chip thickness. So in that situation, then Vc is the chip velocity is always less than the cutting velocity. So here similarly, Vs can be represented in terms of the V, so Vs it can be estimated that V cos alpha/cos phi - alpha.

If phi and alpha is known to us, then you can easily estimate what is the shear velocity also. From here velocity triangle, we can easily find out the different velocity components in the orthogonal cutting situation.

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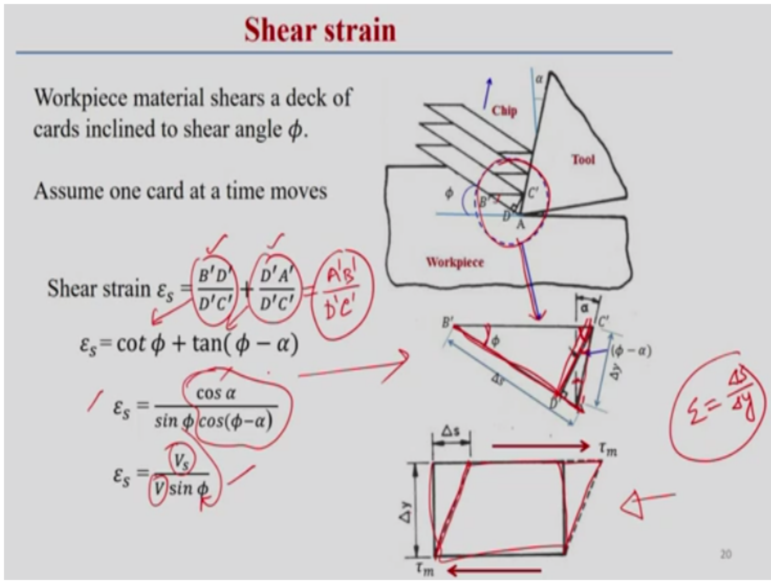

Now come to that point, the shear strength, but when we estimate the shear strength, here you can see the work piece material shears a deck of cards inclined to the shear angle phi. So we assume that a deck of cards which is equivalent to the formation of a chip and of course it is always questionable that up to what thickness you can know that means what should be the thickness of this deck. We assume some finite thickness deck.

But if you look into this movement of a deck of cards inclined at a shear angle, then we can make the schematic like that. When it is interacting the tool with the work piece, then it creates this situation such that if you zoom it, this part during the shearing part, so we assume that one card is, not the multiple cards is moving at a time, rather we assume that single card is moving at a time. Then we can estimate what is the strain and strain rate generation during this cutting process.

So if you zoom it and we can find out that at the point A is exactly contact with the tool tip and then A to B dot, it moves from one point A to B dot. So one says that and then we can find out that different angles here, this is the rake angle, so we will define the rake angle and this angle with respect to this is a tool face and this is perpendicular, so this angle is rake angle, this is perpendicular, so this is rake angle and this angle we can easily find out.

It is phi-alpha and this is making L is phi. So here the shear angle phi and this angle, it is the shear angle phi, so this is perpendicular and this angle is 90-phi, so this angle must be phi-alpha because this angle is alpha and from here the shear strength is basically that which can be estimated that it should be. If we assume this exactly A dot, C dot, if we say that A dot to B dot/D dot C dot.

So here you can estimate the shear strength, the total move during this process from point A dot to B dot during these movement of the one card at a time and then that normal distance is, when this is movement normal to that distance is basically normal to A, A dot B dot that is equal to C dot D dot. So that actually indicates the total shear strength during this process. So that shear strength can be A dot B dot can be estimated in these 2 components.

And which is B dot D dot and D dot A dot. We can find out that B dot D dot/D dot C dot, so this component can be estimated in terms of phi, cot phi and A dot D dot/D dot C dot, which can be estimated by tan phi-alpha. So here you can see that shear strength is basically cos alpha sin phi cos phi-alpha or we know that cos alpha cos phi-alpha, which is equivalent to the shear velocity also, velocity ratio, shear velocity/V that ratio.

So Vs/V and then with that we can look from here also that V cos alpha/cos phi - alpha is the shear velocity. So from if you put cos alpha cos phi-alpha is basically Vs/V and from there we can find out sin phi is there and you can find out the shear strength expression of this thing, which depends on the shear plane velocity and cutting velocity as well as the angle this thing. So what do we define this normal because if elemental form, how you can define the shear strength?

Basically if we check, this is the element and there is the application of the shear stress. So the elemental movement move and this is the distance, now this is the final step after movement. So distance movement Ds and delta Y. So shear strength is basically this elemental delta S by delta

Y. So delta is the movement which direction the shear stress is acting. The displacement along this direction divided by normal distance, this is delta Y, which indicates the shear strength.

Similar analogy we put here also that this is the movement and a normal distance is this one. So from that point of view, we can estimate the shear strength during this process.

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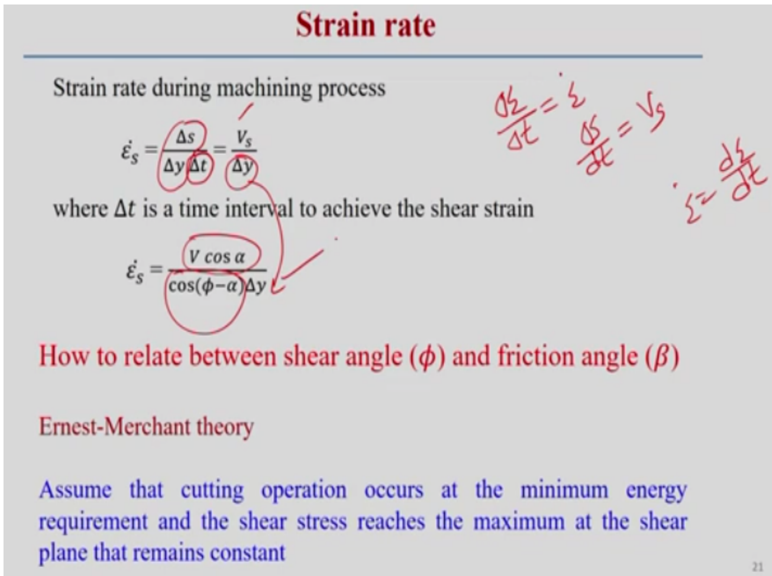

Now strain rate. The strain rate during the machining process can also be estimated. So it is simply strain rate is basically we can say that change of strain with respect to time d epsilon/dt that indicates the strain rate. So strain rate can be estimated by delta s/delta y with respect to divided by elemental time delta t. So because delta s/delta y is basically the delta epsilon and then delta epsilon equal to delta t. So this indicates the strain rate.

So this is the expression of the strain rate. Now delta s/delta t, so delta s the movement by delta t it basically indicates the change of plane with respect to time that is equivalent to the shear plane velocity at the shear that is equal to the Vs. So Vs/delta y that is elemental length, what we can estimate the strain rate here also. So delta t is the time interval to achieve the shear strength. So within that time interval delta t, what is the shear strength achieved.

From that point of view, you can estimate the strain rate. So now Vs can be represented by cutting velocity V cos alpha/cos phi - alpha and into the delta Y is here. So that it depends the strain rate, we can estimate what the value of delta we are taking. So that is why when are assuming that, assume that one card moves at a time, but what is the thickness of this card, so what is the thickness delta Y here.

The delta Y is basically what delta we should consider, the accuracy calculation depends on that, what is the value of delta we are actually considering based on that. So how to relate, this way we can estimate the strain rate also during the machining process. When I come to the other point, so we have discussed that there are several angles involved when you try to express the different components of the forces, that shear angle, then friction angle and rake angle also there.

Rake angle is basically explicitly defined by the geometry of the tool, but shear angle and the friction angle, it depends, mainly the shear angle, it depends on the process also and it depends on other parameters. Now how to relate between the shear angle and the friction angle, so that relation can be found out which is called Ernst Merchant theory that actually depends on the minimum energy consumption.

So assume the cutting operation occurs, then minimum energy requirement in that situation, the shear stress reaches the maximum at the shear plane. So shear stress reaches the maximum with the shear plane, but after reaching that it remains constant value. So by these assumptions and when I try to minimize the energy requirement during the machining process, if you follow that, then we can find out some correlation between the shear angle and friction angle.

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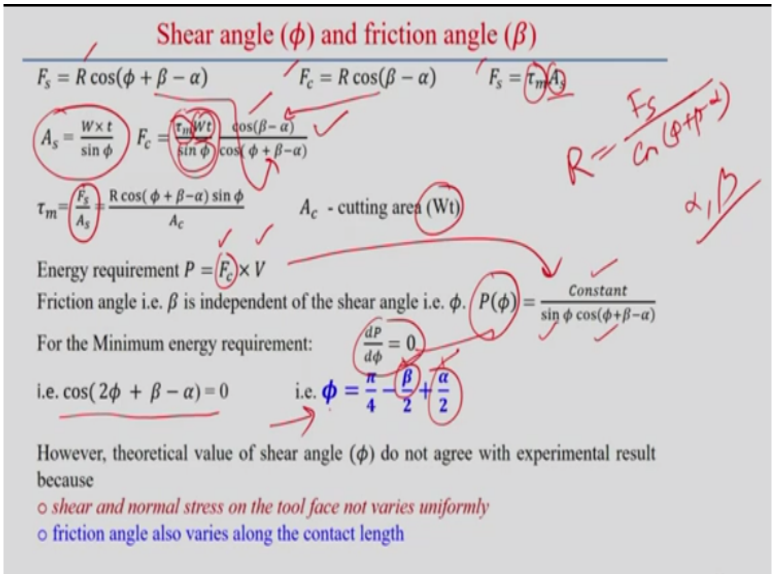

Let us see how we can estimate all these things. So shear force we can estimate that shear force equal R cos phi + beta - alpha that we have already estimated from the Merchant’s circle diagram and the cutting force Fc is basically R is the resultant force, cos beta - alpha from there and we need to use the relation of the shear force is equal to shear stress assuming it is a constant and the cross section area, shear area basically. So that indicates this expression.

Now As that we already defined the shear plane area is the Wt/sin phi already defined. Now cutting force estimate the R equal to, R you can estimate that R equal to Fs/cos phi + beta - alpha. So once we put the R value and then put the Fs value, then we can find out, Fs is shear stress as well as the As value. As value = Wt/sin phi. So this is basically the shear stress value and this is the other component cos beta-alpha, these components and these components comes here.

Cos phi + beta - alpha, so that is the expression of the cutting forces in terms of all these parameters. Now shear stress can also be estimated from this other way also, Fs/As. If you assume this is a constant, but this can be estimated from here s is the cutting area, which is equivalent to the W into T. Now we are estimating the other way also. We are assuming the shear stress as a constant.

And we assume the shear stress of a particular material maybe, which is known to us, and if knowing this value, how we can estimate and optimize the power consumption during this machining process. So then, energy requirement or power requirement during this process is equal to cutting force into velocity force into velocity that is the equivalent to the energy requirement here.

Now friction angle, we assume that beta is independent of the shear angle and of course in a particular material, particular tool and the work piece material, then power consumption it can be a function of only the shear angle phi. Shear angle is already very well known here. So we can put the expression, if you put the expression of Fc already given here and velocity V, we can find out, it is a constant sin phi cos phi + beta - alpha.

But here we assume the alpha, beta constant. Beta is independent of the shear and alpha is also defined over the geometry of the tool. So it cannot be a variable. Now if you express in terms of that, then for minimum energy requirement, if you do the first variable to be equal to 0, if you do the first derivative for this equation as a function of phi, then you can find out cos 2 phi + beta – alpha = 0, which finally reaches to this expression phi = phi/4 - beta/2 + alpha.

So from here this is the relation between the shear angle, rake angle and the friction angle. So if rake angle is defined and of course friction angle is defined from then we can estimate what is the rake angle, but this rake angle we can estimate the shear angle, but this shear angle, we can find out but assumption is that minimization of the energy requirement during the machining process. From that point, we can get this expression or this relation.

But theoretically, this is the estimation by energy minimization principle, the shear angle but theoretically the value of the shear angle may not actually do not agree with the practical value or the experimental values. Because we assume the shear stress remains constant during this process. So that is not a good assumption. Shear under normal stress on the tool face does not vary uniformly, so shear strain may not vary uniformly actually.

Of course other point is that friction angle also varies along the contact length. So because of that we assume that with this pre-defined assumptions, the relation does not vary, follow practically most of the cases. So then what happens, then Merchant's.

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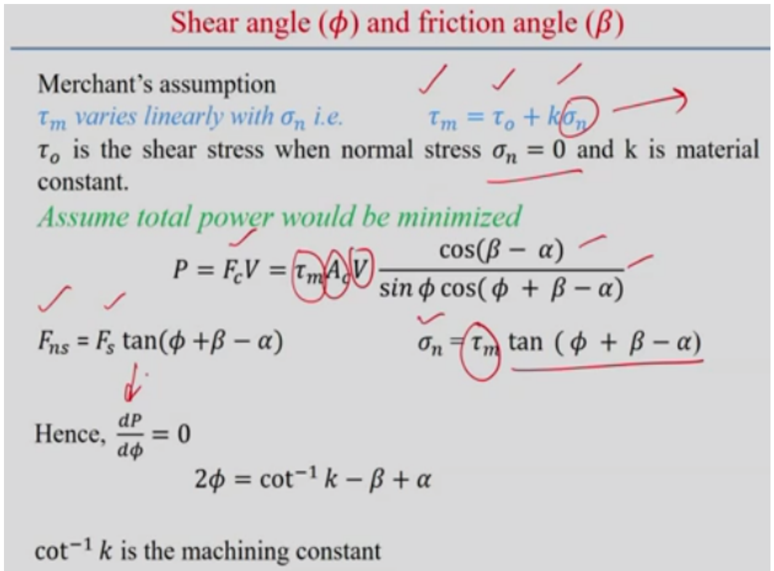

Then we propose other theory that Merchant's assumptions. We assume that shear stress is not constant, but the shear stress varies linearly with respect to the normal stress that means shear stress, we assume the shear stress equal to some constant value + K into sigma N. That means it varies linearly with respect to the normal stress value, normal stress on the shear plane. So tau 0 is the shear stress and normal stress equal to 0 and K is the material constant.

So this is the expression linear variable. See if we do the, now if you try to minimize the power consumption during the machining process, if you follow the similar exercise, then if you put power P equal to cutting force into velocity that is from here you can find out tau m into S into V and if you put all these expressions, here and of course, here you can find out that Fns equal to Fs also, this relation also you know and Fn equal to the relation between the normal stress and the shear stress value in terms of other angle.

And similarly if you put all these value here and minimize the power consumption 0, we can find out the 2phi + cot inverse - K beta - alpha. So these are the expressions which is different from the earlier expression. So here cot inverse K is sometimes called the machining constant. So in that way, there are several theory that exist actually during this machining process and we try to minimize the energy during this consumption.

But with assumptions, we can reach the different expression between the shear angle, friction angle and rake angle. Of course all these cases, rake angle we assume as a constant, because it is properties of the tool, we can assume it is the properties of the tool, but shear angle and friction angle may vary depending upon the process. So thank you very much for your kind attention. Now we look into the next part of this conventional machining process.