IC Engines and Gas Turbines
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Lecture - 49
Radial Flow Turbine, Solved Example of Free vortex Condition

Welcome to the class. We have till time covered the portion of a different turbines where different aspects of axial turbine rather where we had seen mainly de-axial turbine can be of impulse type or reaction time depending upon the degree of reaction. Then we covered some design aspects of the axial flow turbines.

Now, after covering the part of the axial turbine we will just have brief very brief look at the other type of turbine which is radial flow turbine and then we will solve a typical example for the case where we are going to see about the axial flow turbine in through the example. The concept of axial flow turbine, through the example. About the radial flow machine in one of the earlier lectures rather very first lectures we had seen that the turbo machines can be of two types one is axial and one is a radial.

So, we had seen in similar line so, axial flow compressors and centrifugal compressors. So, there was an entity called a centrifugal compressor which was the outward flow machine where the fluid was coming through the eye of the impeller and then it was going outward and then while going outward it was gaining the work which was through the impeller and then it was going to the volute casing. So, that was the idea of centrifugal compressor. Then, we moved toward the axial compressor where the flow is going to along the axis of the compressor.

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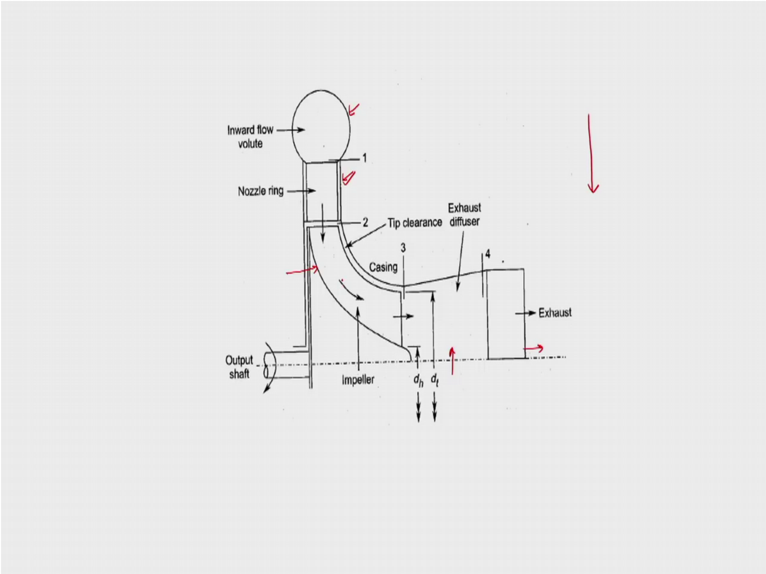

Now, same way as we had axial flow compressor and radial flow compressor which is a centrifugal compressor we have axial flow turbines and radial flow turbines. So, in the radial flow turbines unlike axial compressor where we had a flow which is going outward and we are having which is flow coming inward. So, inward radial flow turbines. So, are generally worked out in the incompressible flow regimes. So, the gases generally have inward radial flow turbines.

So, in the inward radial flow turbines as we can know there will be again inward flow coming through the volute and then we have the here is the volute and then we have the flow after volute will come through the nozzle; once it passes through the nozzle it will come into the casing and then after passing to the casing it will go to the diffuser and then it will go to the exhaust. So, these are the different components of radial flow turbines.

So, you can see that flow is inward. It is coming along the radius toward the center of the circle. So, it is like inward toward the center of the output shaft. So, the parts which were doing which were having functions in the centrifugal compressor exactly would have inverse function in case of the radial flow turbines. So, here initially we have nozzle which were earlier in case of centrifugal compressor very diffuser vanes. So, nozzle would initially convert the pressure high pressure to the necessary kinetic energy here in the part which is called as nozzle.

So, after passing through the nozzle the flow which was at high pressure at the entry to the nozzle would turn out to the high velocity at the exit to the nozzle and then the flow passes through the turbine blades. So, here we can see that the turbine will rotate and then the enthalpy drop necessary required for the work output would be taking place into the turbine which is impellers again. So, turbine blades here would again will be impellers.

Then, there will be certain amount of kinetic energy at the exhaust to the turbine or at the exit to the impeller if that is very hard and there will be diffusion blades and there will be diffuser which will reduce that kinetic energy to the required energy at the exhaust of the complete radial flow machine. So, this is general configuration of the radial flow turbine and then there are T-s diagram would be as similar as that earlier case where we will have isentropic expansion in the nozzle and then we will have gain as an expected isentropic work into the work interaction into the impeller blades.

So, there will be since nozzle is non-interact non work interacting part there will not be any work interaction over here, but in case of impeller blades we will have work extractor. So, total pressure decreases when we come from 2 to 3, but total pressure would partially decrease from 1 to 2 since we are having loss due to friction. But, total pressure decrease or total temperature decrease between 2 to 3 is mainly due to work interaction and partially or very small amount due to losses. So, this is how we would have the T-s diagram for 1 to 3, but 3 to 4 is again a diffuser.

So, in case of diffuser we have again no work interaction. So, we will have partially the loss in total pressure down to friction again. So, this is the gross perspective of the radial flow machines radial flow turbine. As we have seen that axial flow turbine is very compatible and generally preferred for aircraft since it has lesser drag for a given diameter. But, here as we go for higher pressure ratios the diameter needs to be increased for the axial flow compressor or axial flow turbine. So, there drags are higher. So, these machines are not generally preferred. But, the fact is the per stage work interaction is higher in this case. So, this can be preferred if we are working with lower work output of the turbine. So, this is how we have the typical radial flow turbine composition and aspects of the typical radial flow turbine.

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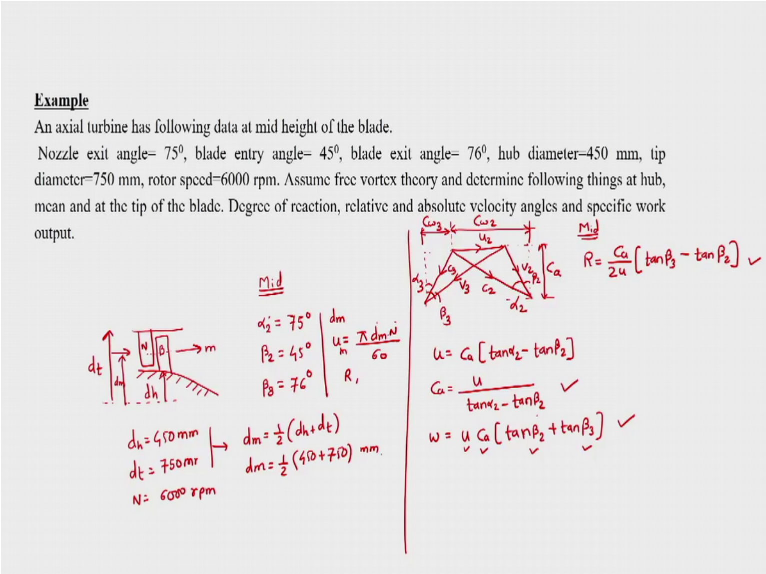

So, after this we will see an example for the typical turbine case and then example reads like, an axial turbine as following data at the mid height of the blade. We should remember this example is purposely chosen for the fact that we had seen something which is called as the radial equilibrium and free vortex theory. So, this example is based on that free vortex theory and radial equilibrium. It helps us to understand different concepts.

So, at the mid plane of the height; so, blade is we know that typically this is our turbines. So, we have blade over here this is the stage. So, we would have first nozzle we would have first nozzle and then we have blade. So, this is blade. So, this is the typical configuration of a stage in case of the axial flow turbine. So, this is how the flow moves along the axis.

Then this is the hub where it is mounted. So, what we are given is at the mid height. So, this is mid and then we are told that the nozzle exit angle which is alpha 2 is 75 degree, we are again told that blade entry angle blade entry angle is beta 2 which is 45 degree and then we are given that blade exit angle which is beta 3 which is 76 degrees. So, these things are given at the mid height ok.

But, apart from that we are given that the diameter hub diameter d h that this d h is 450 mm, we are told that then tip diameter d t is equal to 750 mm and n is equal to 6000 rpm.

So, this is something which is given to us here we are not told about the height at the mid plane. So, we can find out the height at the mid plane from this data where we can say that d m diameter at the mean is equal to half d h plus d t. So, d m is equal to half, d h is 450 plus 750 in mm.

So, now, at the mid we are also knowing that what is d m in mm, but then we know that u which is the rpm which is the velocity tangential velocity at the mid plane u m is pi d m N by 60. So, we are knowing N, we know d m, so, we can find out u m. So, these things are given. So, further it is told that assume free vortex theory and determine following things that mean and tip of the blade. So, we are asked about degree of reaction R, we are asked about angles relative and absolute velocity angles and specific work output.

So, we are asked about these things. So, having known this we will proceed with the example. So, before that we should first draw the velocity triangle and then I will describe how we are going to solve this example. So, for the gas turbine axial gas turbine we have seen that this will be our typical velocity triangle where we have here C 2, this is V 2, this is u 2 and we say that we are drawing the velocity triangle at mid plane height we are drawing the velocity triangle.

So, this is C a and this is we are drawing at mid plane, mid height and then this is for all sake this is C w 2. V 2, u 2 this is C w 2 we know that this angle is beta 2 and this angle is alpha 2. So, similarly then if we go to the outlet velocity triangle, then for outlet velocity triangle this is V 3, this is C 3 and then we have this as alpha 3 and then this has beta 3. So, this will be C w 3. So, this is what the velocity triangle as what we have seen.

So, now, we have to find out everything at mid blade height. So, at mid blade height we have found out we are rather given with alpha 2, beta 2 and beta 3 we just have found out what is u. So, we just have to find out what is R. So, for R we know formula for R which is C a upon twice u into tan of alpha 2 minus tan of beta 2. So, this is the formula for the R which is degree of reaction, but we do not know C a. So, we can find out C a here by using the formula u is equal to C a into tan of alpha 2 minus tan of, 1 second, formula for R is u C a upon twice u into tan of beta 3 minus tan of beta 2. This is the formula for R.

But, we need to find out C or here and C a can be panned out from formula which is tan of the alpha 2 minus beta 2. We have derived this formula from the velocity triangle itself, where we had seen that degree of reaction is enthalpy drop in the rotor divided by total enthalpy drop in a stage. So, from that we have derived this formula. Further this u by u is equal to C a into tan of alpha 2 minus beta 2 can be found out from this velocity triangle again.

So, we get C a is equal to u divided by tan alpha 2 minus tan beta 2. So, we know u which is found out from pi d m by 60 for mid. So, you have to take mid height over here and find out this. We know alpha 2, we know beta 2, so, C a can be found out. Once C a is found out we will put this C a over here and then we will find out what is R. So, R can also be found out. Since, C a is known, u is known, beta 3 is known, beta 2 is known we can find out. So, we need to find out just work interaction.

So, for work interaction the known equation is u into C a into tan of beta 2 plus tan of beta 3. So, specific work interaction at the mid plane height is u is known, C a is known, beta 2 is known, beta 3 is known then we can know W. So, by this formula we can find out everything at mid plane height, but our objective again here is to find out all these things at the tip and the hub also. So, let us find out this thing that hub.

So, for that we have to use the free vortex theory. As per the free vortex theory we know that C w into R is equal to constant this is what free vortex theory derivation we had seen. Further we are saying that there is radial equilibrium once we have radial equilibrium in exist and free vortex theory we can also take that C a is equal to constant in the complete stage.

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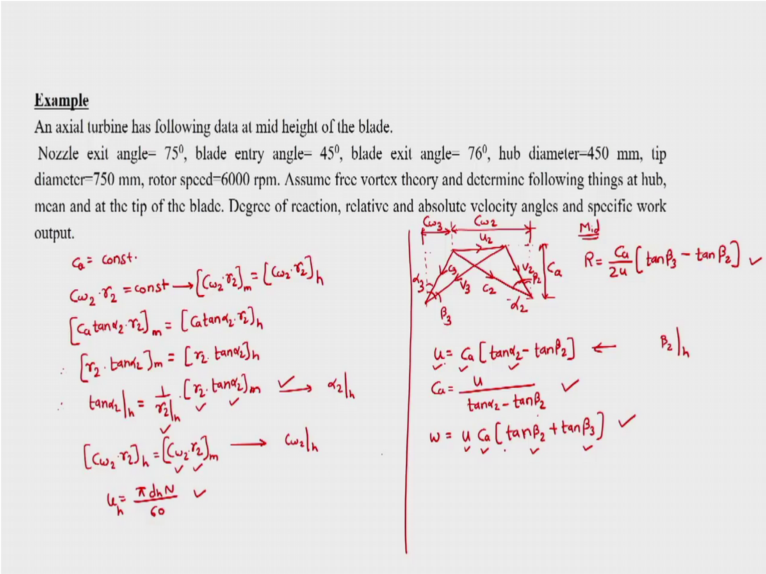

So, we will take C a is equal to constant. This is known to us we take it to have no axial thrust and then we are knowing that C w 2 into r 2 is equal to constant by all these means we mean C w 2 into r 2 for the mid is equal to C w 2 into r 2 at hub.

So, we know that how is what is C w C w 2? C w 2 is basically is equal to C a into tan alpha 2 C a into tan alpha 2 into r 2 at mid is equal to C a tan alpha 2 into r 2 at hub. But, C a is constant. So, we have r 2 into tan alpha 2 corresponding to mid is equal to r 2 into tan alpha 2 corresponding to hub.

So, we can find out tan alpha 2 at hub is equal to 1 upon r 2 at hub into r 2 into tan alpha 2 at mid. So, at the mid height we know r which is d m by 2 alpha 2 is known to us which is given to us and then height which is d d h by 2 at r which is also known to us. So, we know basically alpha 2 corresponding to the hub diameter at the hub diameter. So, we know that. Now, knowing this we can proceed with rest of the things.

Further, since we know this we also know C w 2 at mid C w 2 into r 2 this equation gives us alpha 2 at hub C w 2 into r 2 at hub is equal to C w 2 into r 2 at mid. So, at mid we know C w 2, at mid we know d 2. So, from this equation we know C w 2 at hub. So, this is also known to us. Now, we can proceed with these things further for that we should know u at hub is equal to pi d h into N by 60. So, by this we know what is u at hub. So, we can use this formula where we have to say that this is at hub, this is constant and for 2 at hub and beta 2 at hub. So, we should know what is beta 2 at hub.

So, for that we can use basically this formula to find out beta 2 at hub. u at hub is known, C a is known, alpha 2 is known. So, we can find out beta 2 at hub using this formula rather this formula can help us now in finding out beta 2 at hub. Now, our objective is to find out r at hub and w at hub. So, for r at hub we should know beta 3 at hub. So, for finding out beta 3 at hub we can carry forward the same method of calculation which is C w 3 into r 3 at hub is equal to C w 3 into r 3 at mid.

So, then we will have C C w 3 r 3 at mid C w 3 into r 3 at mid is equal to C w 3 into r 3 at hub.

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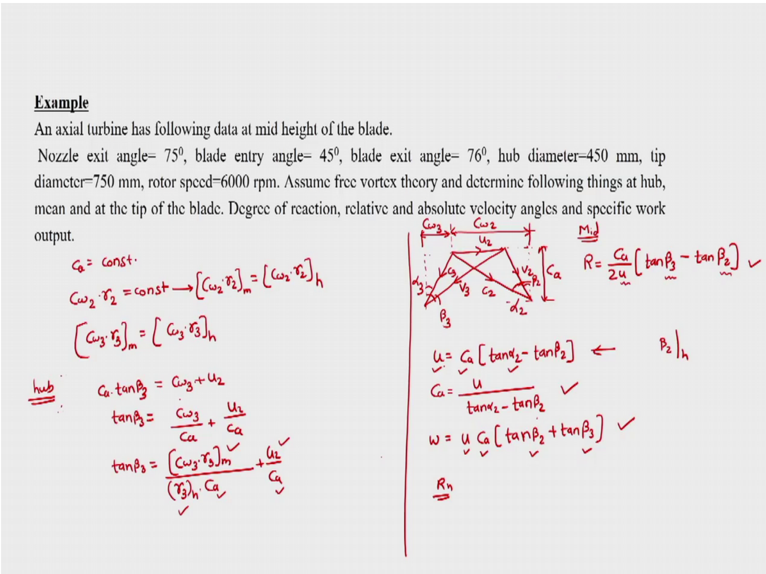

So, at hub we know that. So, this is one thing. So, we can we need to find out beta 3. So, we can say that u C a into tan beta 3 at hub we are doing it for hub, but velocity triangle can be similar. So, we will say C a into tan beta 3 at hub is equal to C w 3 plus C u 2. So, basically this is what C a tan beta 3. So, we have tan beta 3 is equal to C w 3 upon C a plus u 2 upon C a.

So, this u 2 is found out by us. So, we have tan beta 3 which is C w 3 into r 3 at mid divided by r 3 at hub into C a plus u 2 upon C a. So, you we know this, we have calculated everything at the mid. So, we know this, we know this, we know this, u is pi d

N by 60 for the hub and then we know C a. So, we know beta 3. So, knowing beta 3 we can use this formula r is equal to C a upon twice u into beta 3 tan beta 3 minus tan beta 2. Here beta 3 beta 2 corresponds to hub now and then we know this u is also corresponding to hub. So, we can find out r at hub.

Similarly, we can use formula which is w is equal to u into C a into tan beta 2 plus tan beta 3. So, here u at hub is known, C a is constant, tan beta 2 at hub is known to us and then we know tan beta 3 hub also. So, knowing tan beta 2 and tan beta 3 at hub we can find out the w which is work interaction at the hub. So, this whole procedure is based on the fact that we know everything a the mid. Once we know everything at the mid we can proceed and find out the things at the hub. Similar thing can be repeated for the tip also.

So, we should know first diameter at the tip which is known to us. Once we know diameter at the tip we can find out u pi d u is equal to pi d N by 60. So, u 2 will be known to us at the diameter of the tip and then we can use the free vortex theory where we will say that C w 2 into r 2 is equal to C w 2 into r 2 corresponding to mid and corresponding to tip. So, then we will find out alpha 2 using that. Since we know that as we have seen over here r 2 into tan alpha 2 is equal to r 2 into tan alpha 2 corresponding to mid and tip. So, we can find out tan alpha 2 at the tip.

So, once we found out tan alpha 2 at the tip we will find out that beta 2 at the tip using the formula u upon C a ok. So, that will give us tan beta 2. So, then we have to find out tan beta 3. So, tan beta 3 again we will use same formula C a, tan beta 3 is equal to C w 3 plus u 2. So, everything would be repeated for the tip. So, this is how we can solve an example in case of an axial turbine. So, this lecture which dealt with the calculation for the axial flow turbine or rather in case of turbine and specifically some discussion about the radial flow turbine ends here. And, then in the next class we will see about the other components of the gas turbine which would include as intake and the nozzle.

Thank you.