IC Engines and Gas Turbines
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Lecture – 48
Solved Examples for Axial Compressors, Centrifugal Compressors and Turbine

So, in our previous class, we are done with the last part of our discussion regarding the component which we had considered and that component was rotary machine and that was turbine. So, we completed the two turbines. Turbine which is type of axial turbine and before that we completed axial and centrifugal compressors. So, here our objective for today's discussion is to look into the examples which will generally be solved in this course of related to gas turbines.

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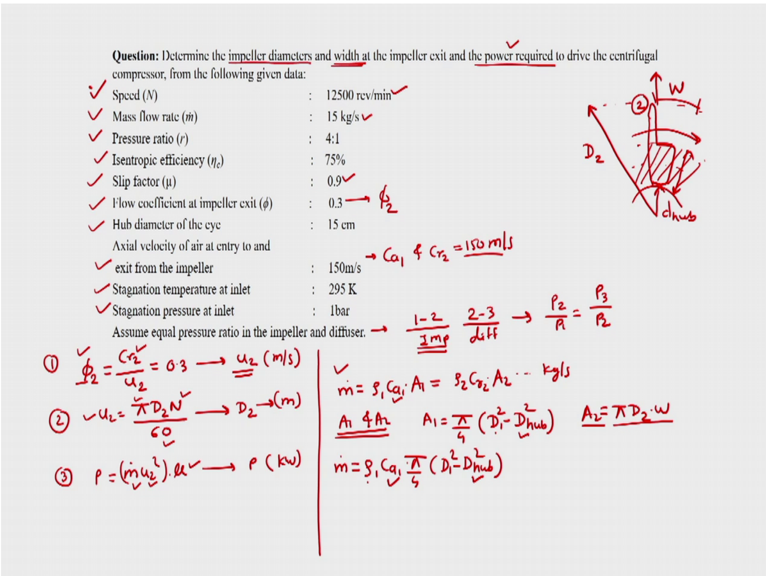

So, the first example is related to centrifugal compressor which says that determine the impeller diameter and width at the impeller exit and power required to drive the centrifugal compressor for given data. So, there is some data given. As per that data we are known with and which is speed which is given to be 12500 revolutions per minute; mass flow rate is given as 15 kg per second; pressure ratio is given to given as 4 as to 1. And then we are told that isentropic efficiency is at 75 percent, then slip factor is 0.9, flow coefficient which is 5 at the impeller exit. So, it is basically given as phi 2. Hub diameter at the eye is 15 centimeter; axial velocity at the entry to and exit of the impeller ok.

So, we are given with axial velocity 1 and we are given with C r 2 at the impeller exit. So, this is given 150 meter per second ok. And then we are given with stagnation temperature at the inlet 295 Kelvin; we are given with stagnation pressure at the inlet which is 1 bar. We are told that there is equal pressure rise obviously, the static pressure rise in impeller and diffuser. So, we know that 1 to 2 is impeller, and 2 to 3 is diffuser. So, we would say that p 2 by p 1 is equal to p 3 by p 2. So, this is what we would say. So, so this is the thing which are given for solving this example.

So, we will solve, we are told here that we are given with phi 2, and phi 2 we know phi 2 is equal to C a 2 divided by u or rather C r 2 C r 2 divided by u 2, this is phi 2. So, this is given to us which is 0.3. We are not given with u, but we are given with basically this velocity which is 150 meter per second. So, we know 150 meter per second; we know phi is 0.3, then this expression helps us to give you 2 ok. So, C r 2 is given which is 150 meter per second; phi is given which is 0.3. So, we can find out u 2.

So, if we remember then this is have and then this is and then this is the centrifugal compressor. This is rotating in this direction. This is hub and this is 2 ok. So, we are told that this is for (Refer Time: 04:26), this is diameter of hub, d hub. And this is diameter basically this is 3. So, this is radius corresponding to that diameter D 3 ok. So, this is 2. So, this is D 2 since 1 to 2 is impeller.

So, now, we know D 2 we are supposed to find out two things which is impeller diameters. So, impeller diameter means, we have to find out what is this diameter at the I, we are supposed to find out diameter at D 2 or diameter here. Then what would happen is we can find out u 2 is equal to pi D 2 N by 60. So, N is given to us which is 12500 rpm. So, knowing the N, knowing this value 60, and u 2, we can find out this step 1. In step 2, we can find out D 2 in meters, u 2 in meter per second.

So, we get D 2 out of this exercise. Then we are supposed to find out power required. Then we can find out power required as formula m dot into u u 2 square into u or our power coefficient. So, this is given to us. And this rather slip factor. So, slip factor is given to us which is 0.9, m dot is given to us which is 15 kg per second, u 2 is known to us from step 1, and then this will give us power in kilowatt or watt depends. Now, this is found out, this is this one diameter is found out and then we have to again find out width.

So, for that to find out 1 diameter and to find out the width, we need to use the formula for mass flow rate. So, formula for mass flow rate is basically rho 2 rho 1 C a 1 into A 1 which is equal to rho 2 C r 2 into u 2 sorry A 2, so these are the formulas for mass flow rate. This is in kg per second, where A 1 and A 2 are the different areas at the inlet and outlet.

For area at the inlet, we would know one thing which is hub diameter which is this is know. So, area at the inlet is this. So, for that A 1 is equal to pi by 4 D 1 square minus D hub square. And area at the outlet, so this is the approach for the fluid which is coming axially into the inducer, so for that this area which is this area in which fluid is at getting admitted, so for that we are considering the annulus area which is pi by 4 D 1 square minus D hub square. And for that we have area pi D 2 pi D into w which is width between this and next blade. So, we have this as area at the outlet. So, we can use these formulas mass flow rate is known.

So, here we know axial velocity, we know D hub, but we do not know D 1. So, let us consider m dot is equal to rho 1 C a one into pi by 4 D 1 square minus D hub square D hub is known, D 1 is not known, c 1 is known, rho 1 is not known, but we can find out rho 1. So, how to find out rho 1? So, for that we need to work out for the fact that we should know what is the pressure and temperature at the inlet. Once we know pressure and temperature at the inlet, we can use the formula p is equal to rho R T to find out density at the inlet. So, we know the we are given with stagnation temperature at the inlet.

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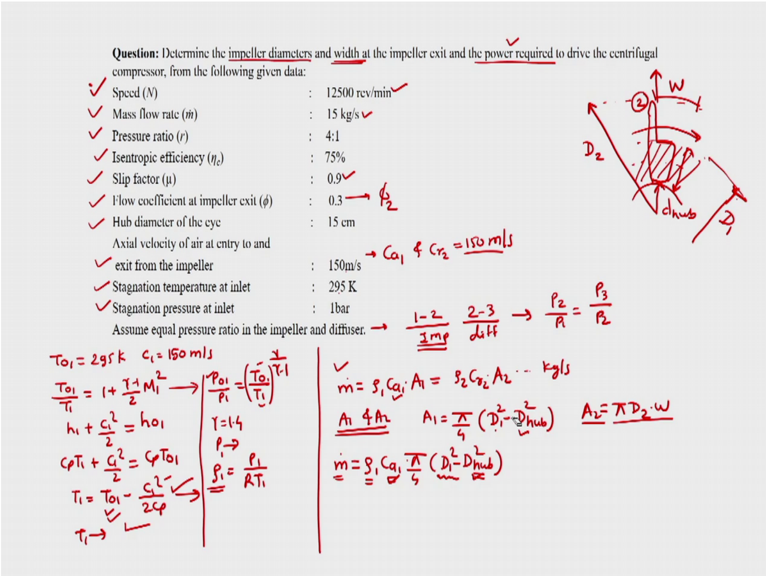

So, T naught 1 is given to us as 295 Kelvin, but we know that this is the stagnation temperature. We know also velocity which is 15 meter 150 meter per second ok. Velocity c 1 is 150 meter per second. So, this is also known to us. And we know the formula that T naught 1 upon T 1 which is equal to 1 plus gamma minus 1 by 2 M square. So, this is the formula which we can use, for that we need to know Mach number. But we can differentiate this formula or we can disintegrate this formula in other factor or in terms of we can use this formula where M will be taken as u C a upon square root of gamma r T, where T 1 would be found out. But instead of that we can also use this at this inlet which is h 1 plus c 1 square by 2 is equal to h naught 1, where h 1 is C p T 1 plus c 1 square by 2 is equal to C p T naught 1.

So, T 1 is equal to c 1 T naught 1 minus c 1 square upon twice C p. Here T naught 1 is known to us which is 295 Kelvin, c 1 is known to us which is 150 meter per second, C p is known to us for air which is 1.005 kg per kilo joule per kg Kelvin. So, this helps us to give T 1 directly without any solving non-linear equation as what we would have done here. So, these are the two methods by which we can find out this T.

But then we need to find out what is the density. So, for that we have to take help of pressure. So, we are also known that stagnation pressure is 1 bar. So, p naught one upon p 1 is equal to T naught 1 upon T 1 bracket raised to gamma minus 1 upon gamma, gamma for air is 1.4, T naught 1 is known 295 Kelvin, T 1 is found out from here. So, T

1 is known, T naught 1 is known, gamma is known, then p naught 1 is known which is 1 bar. So, we can get p 1 from here; from here we got T 1. So, we got p 1, we got T 1. So, we get rho 1 is equal to p 1 upon R T 1 p 1 upon R T 1. R is known for air which is 0.287 or 287 joule per kg Kelvin; T 1 is known to us from this step. So, this gives us density.

So, now knowing the density, knowing the velocity 150 meter per second, knowing the mass flow rate, knowing the diameter of the hub, we can find out D 1 which is the diameter at this state which is the diameter this is D 1. So, we know this is D 1 diameter.

Now, our objective is to find out D 2 or width w 2. So, now we know the formula that we have basically mass flow rate is equal to rho 2 A 2 into the root into u 2 or c 2 or C r 2 this is the mass flow rate formula. Basically u means velocity we have to keep some velocity which is normal to the area at the outlet. And for normal to the area at the outlet we have C r 2 velocity.

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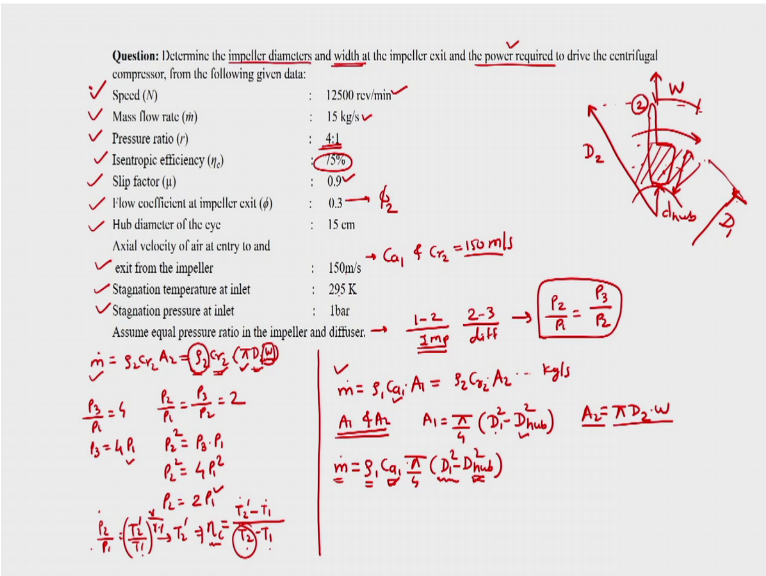

So, m dot is equal to rho 2 C r 2 and A 2, where rho C r 2 into area is pi D 2 into w. Mass flow rate is known; C r 2 is known 150 meter per second. And then this value is known, D 2 is known to us we have found out D 2 rather, and then we just have to find out w 2. So, we have to find out w 2. D 2 was found out in earlier step where we are used u 2 is equal to pi D 2 n by 60, but before that we have to find out rho 2. So, to find out rho 2, we have to need pressure and temperature at T pressure and temperature at 2.

So, for that, we will take help of this expression which is given as constrained to us which say that there is equal pressure rise in impeller and diffuser. And the pressure rise for the compressor is 4 as to 1. So, this p 3 upon p 1 is four, but what is that equal pressure rise. So, we would expect both two be 2. So, p 2 by p 1 is equal to p 3 by p 2 is equal to 2. So, this is what it is known to us. So, we know that p 3 is equal to four p 1 p 3 is equal to 4 p 1 from here ok. So, p 1 is known to us. So, we can find out p 3 ok.

So, knowing this we can find out p 2 square is equal to p 3 into p 1, but p 3 is equal to 4 p 1. So, p 2 square is equal to 4 p 1 square. So, p 2 is equal to twice p 1, anyway this was known to us. We know p 1 which is static pressure. Knowing the static pressure, we can find out p 2 which is the static pressure at the exit of the diffuser.

Now, our interest is to find out static temperature at the p 2 is the static pressure at exit of the impeller. So, we should find out static temperature at the exit of the impeller. So, we can use isentropic formula which is p 2 by p 1 is equal to T 2 by T 1 bracket raised to gamma upon gamma minus 1; p 1 is known; T 1 is known; then p 2, then p 2 is also known from here we can find out T 2, but this is ideal temperature which is isentropic. So, knowing this, this step gives us T 2 dash which is isentropic temperature, but we are told at isentropic efficiency of compressor is 75 percent.

So, we know that compressor efficiency is ideal work divided by actual work. So, T 2 dash is known; T 1 is known; T 1 is known; T 2 is unknown. So, for the unknown T 2, for the unknown T 2, knowingly knowing the compressor efficiency we can find out T 2. So, thus we have found out T 2 what is our requirement. So, T 2 is known, p 2 is known, then rho 2 can be found out using the formula p 2 is equal to rho 2 R T 2. So, p 2 is known, T 2 is known, we would find out rho 2. So, rho 2 is known here, C r 2 is known, pi value is known, D 2 is known for pi D n by 60, so we can find out w ok. So, here this example is this way solved.

So, the main thing what we should remember in this example is how to take area for mass flow rate calculation which area is taken at the inlet in which area is taken at the outlet. We have axial inlet and we have radial outlet. So, this is axial inlet which is normal to the plane of the board; we have radial outlet which is in the radius of the centrifugal compressor ok.

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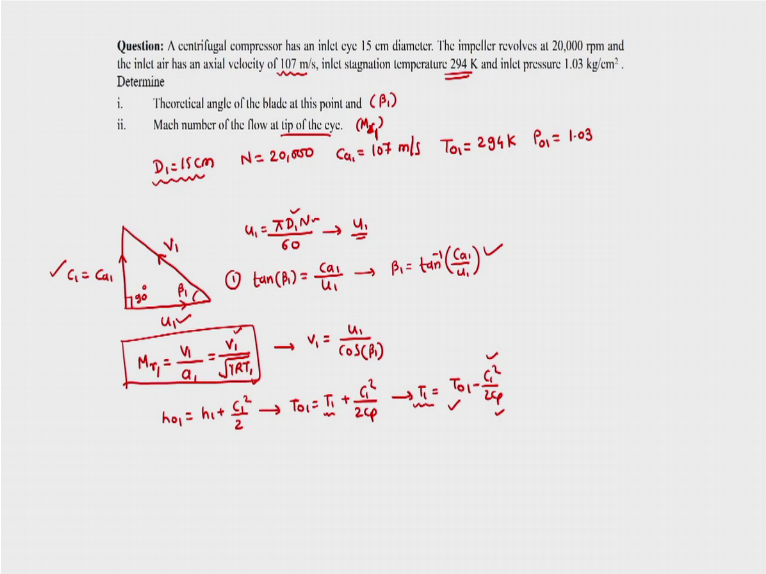

So, we will solve the second example which is related to again centrifugal compressor. This example says that there is a centrifugal compressor which has an inlet eye which is 15 centimeters. So, we are given with D 1 which is 15 centimetre, then impeller revolves at 20,000 r p m. So, we are given N as 20000 rpm, and inlet air has axial velocity C a 1 is 107 meter per second, inlet stagnation temperature is given 294 Kelvin, inlet pressure is given, p naught 1 is given one point naught three kg per centimeter square.

So, we have to determine the angle of the blade at the theoretical angle of the blade at the at the inlet that means, we are supposed to find out blade angle which is beta 1 at the inlet ok. And then we have to find out Mach number at the outlet M 2. So, we are suppose to find out Mach number of the flow at the tip of the eye, but this Mach number is m 2 at the tip is basically related to relative velocity basis. So, this is basically M r 2 relative 2. So, this is what we have to find out.

So, now, for this all sake let us try what is given to us we can find we know that velocity triangle for the centrifugal compressor at the inlet is this is our C a 1 this is u 1. So, we have this as v 1. So, this is our velocity triangle. So, we have alpha as 90 degree, and we have beta 1. So, this is known to us. So, now, having these things known, we can find out basically this is C 1 is equal to C a 1, C w is 0 at the inlet. So, u 1 is equal to pi D 1 N by 60. We are known with D 1, we know N, and then we can find out u 1.

So, u 1 is known. And this velocity triangle u 1 is known. We are given with again axial velocity which is this. So, this is known. So, we can find out this, we can find out this; that means, we can find out beta, so which says that tan of beta 1 is equal to C a 1 divided by u 1. So, we know that beta 1 is equal to tan inverse C a 1 upon u 1. So, we know the first point which is theoretical angle of the blade at the inlet at the entry. So, this is known to us.

Now, we have found out the first answer which is this. Now, we are suppose to find out the second answer which is Mach number of the flow at the tip of the eye. So, for that we have to find out basically vel 2 which is velocity at the outlet. Once v 2 is found out we can find out the Mach number which is v 2. So, basically formula of Mach number M r 2 is v 2 upon a 2, so v 2 upon square root of gamma R T 2.

So, we should know basically v 2 and T 2 for this all fact. Once we know v 2 and T 2, we can find out M r 2. Here we are said that we have to find out Mach number at the tip of the eye. So, basically we have to find out not two, we have to find out one. So, since this is 1, this is again 1, and this is again 1, this is 1, and this is 1. So, we have to find out M r 1, which is v 1 upon a 1, so which is v 1 upon under root gamma R T 1, this Mach number we can find out here. Now, we have to find out v 1 first. And now v 1 can be found out since beta is known to us. So, v 1 is equal to u 1 upon cos of beta 1.

So, beta is known from here from step 1, u 1 is known before that then we know v 1. Now, our problem is to find out T 1. For that we can again take help and then we can find out h naught 1 is equal to h 1 plus c 1 square by 2. So, T naught 1 is equal to T 1 plus c 1 square by 2 c p. So, here we are supposed to know T 1. So, T 1 is equal to T naught 1 minus c 1 square upon pi C p, where we are told that stagnation temperature at the inlet is 294. So, this is known velocity, axial velocity is known 107; C p is known for the air. So, this is constant. So, this is known to us 1.005. So, know these things we know T 1. So, we know everything in this formula, and we can find out what is the Mach number relative Mach number at the tip of the eye of the compressor.

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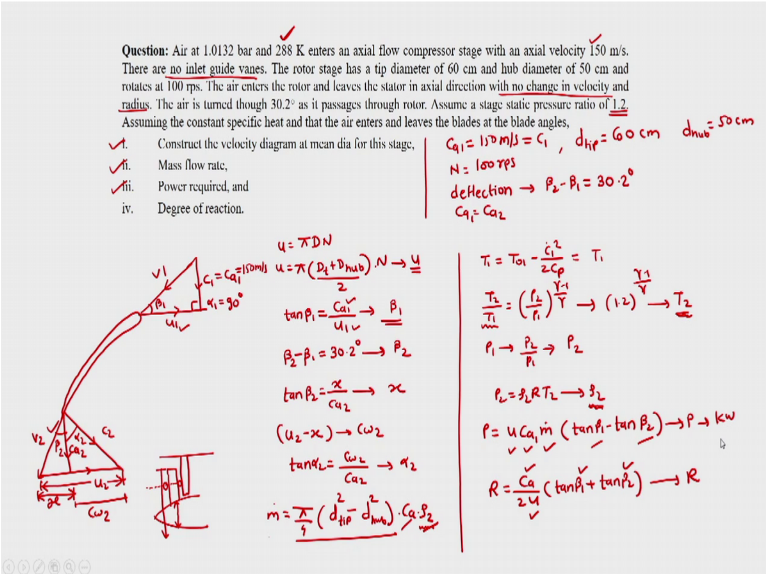

So, the example, next example says that air at a one point not one three two bar and 288 Kelvin enters an axial compressor stage with an so with an axial velocity of 150 meter per second. So, given thing to us is C a 1 is equal to 150 meter per second. There is no inlet guide. This C a 1 is equal to basically C 1. The rotor stage has a tip diameter as tip diameter D tip, D tip is equal to 60, centimetre and hub diameter D hub is equal to 50 centimeter. These things are given to us. And it rotates basically speed is given as 100 rps. Air enters the rotor and leaves the stator in axial direction with no change in velocity and radius. The air is turned through 30.2 degree as it passes through the rotor.

So, we are given with deflection. So, we are given with beta 2 minus beta 1 as 30.2 degree ok. Assume a static pressure rise of 1.2. So, we are assuming that there is a static pressure rise as 1.2, we are told to that. And assuming constant specific heat, and that the air enters and leaves the blade at leaves the blade at the blade angles ok, construct the velocity diagram at the mean diameter of the stage mass flow rate we have to find out pressure required and degree of reaction ok.

So, we have to first find out, but first we have to draw the velocity triangle. So, we know that this is the blade. So, for the blade v 1, and then we have u 1, and then we have c 1 ok, u 1, then we have c 1 is equal to C a 1. Then for that all sake we have this as v 2, and then we have this as c 2, and then we this has u 2, but here we have C a 2. So, this is the velocity triangle for us.

Now, here we are supposed to find out first we have done we have drawn the velocity triangle. Now, we are supposed to find out what is the mass flow rate, but before that we would try to make the angle this is beta 1, this is alpha 1 is equal to 90 degree. Why did we draw like this, since we are told that there is no guide vane. So, since there is no guide vane that is why it is coming directly in the axial direction. So, absolute velocities in the axial direction, since there is no guide vanes, it is mentioned over here in the example. Having said this we are now saying that this is beta 2, this is alpha 2 ok.

So, we would first say that u is equal to pi D N. Before that we should remember one thing that in case of compressor, we know this is hub, and then we have this as rotor and then we have this as stator ok. And then we are given with this as hub diameter, and we are given with this as tip diameter. But we should always work with the mid height of the with the mid height of the blade, so our all calculations belong to the mid height of the blade. So, we have to take mean of these two diameters to find out this mid height of the blade. Since we have drawn the inlet velocity triangle here and we have drawn outlet velocity triangle over here, where there is no change in radius as it is suggested. So, u is equal to pi D N.

We would have used by 60 in using this formula if we would argue an N in rpm, but we are given N is rps. This is pi D N. So, but here we can u is here D 1 D tip plus D hub by 2 into N. So, we know N, we know these two diameters. So, we can find out u from here ok. So, we know now u. So, knowing u we can say that we can find out beta 1. We know u, we know c 1, which is C a 1 which is 150 meter per second. So, in this formula, we can say that tan of beta 1 is equal to C a 1 upon u 1. So, here we know C a 1, we know u 1, we can find out beta 1. But then we told that beta 2 minus beta 1, it is a deflection of the flow is 32.2 degree, so we get beta 2 also from this formula. So, beta 2 is known to us now.

Now, this is complete u 2, this is u 2 here, and we know now beta 2 ok. We are also in told that the there is no change in direction and velocity, there is no there with no change in velocity and radius. So, we basically say that C a 1 is equal to C a 2; there is no change in axial velocity. So, C a 2 is known to us. So, C a 2 is known to us. So, we can find out this distance, this, let us say this distance to be x. So, x is equal to we know that this distance is C w 2. So, x is equal to basically tan beta 2 is equal to x divided by C a 2, and that way we can get x from this formula since we know beta 2.

Now, since we know x now we can know u 2 minus x, which is rather C w 2. Now, knowing this C w 2 and this, we can know C w 2 we can know tan alpha 2 is equal to C w 2 upon C a 2. So, we can know from here what is alpha 2. We know what is alpha 2 from this formula. So, knowing this, we can take use of other parameters. And we can find out which is mass basically these things will be useful for us to find out power required. So, before that we can find out mass flow rate. And mass flow rate is pi by 4 D tip square minus D hub square into C a into rho 2. So, this is the formula for our mass flow rate which is supposed to be found out.

So, for that we should know rho 2. So, for that let us find out we are given with basically pressure ratio, so that we can make use of we are told that there is some pressure and temperature and velocity. So, we know that D 1 is equal to we were using this formula minus c 1 square upon twice c p. So, this formula is known to us. Here we are taking this as static temperature, which is we are taking it as total temperature, and we are taking it as velocity which is 150 meter per second. So, knowing these two things, we can find out T 1 ok, but we know that T 2 upon T 1 is equal to p 2 upon p 1 bracket raise to gamma minus 1 upon gamma, but pressure ratio is given to us as 1.2.

So, this is 1.2 bracket raise to gamma minus 1 upon gamma within that T 1 is known to us. So, since T 1 is known to us we know from this formula what is T 2. So, we know now basically T 2. So, T 2 is known. Knowing the T 2, we further knowing the p 1 and p 2 by p 1, we can find out p ok, p 2 is also known to us.

Now, having said this, we can further make use of rho 2 which is p 2 is equal to rho 2 R T 2, we can find out rho 2 from this one. So, once we know rho 2, once we know T 2, we can once we know rho 2, this is known, this is known everything is known, we can find out m dot. So, this is how we can find out m dot. So, once m dot is known, we can use this to find out power required. Since for power required, we have power required which is equal to u into C a into m dot into tan of beta 1 minus tan of beta 2. This formula is known to us; u is known; c is known; C a 1 or C a 2 is same; m dot is known; beta 1 is known; beta 2 is known. So, we can find out which is power required.

So, then we can find out degree of reaction which is C a upon twice u into tan of beta 1 plus tan of beta 2. So, here again C a is known; u is known; tan beta 1 is known; tan beta

2 is known. We can find out degree of reaction power will be kilowatt degree of reaction is a non-dimensional number.

So, we have just solve this example for axial compressor ok. So, this is how we have solved the examples for compressors. So, we have to work first with drawing the velocity triangle, we have to mention the given quantities along with the velocity triangle. We should know the parameters of the compressor, and then we can solve the example for the compressor.

Thank you.