IC Engines And Gas Turbines
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Lecture - 47
Complete Analysis of Axial Flow Gas Turbine

Welcome for the class. So, till time, in last class, we have completed the two major types of compressors. Initially, we had considered centrifugal compressor, and then we had seen axial compressor which are the parts of gas turbine power plant. Now, we will be seeing the next important component which is turbine. So, for that all sake, today's class is basically upon axial gas turbines. So, axial gas turbines will be connected to the compressor, as the compressors will be connected to axial gas turbines such that turbine will deliver the necessary power required to run the compressor.

And we had seen in our earlier discussion that turbines will be having the favourable pressure gradient in the direction of the flow. So, what would happen is we do not need to worry about the losses which would occur due to modular separation since there is favourable pressure gradients. So, number of stages in the turbine would always be lower than the number of stages in the compressor.

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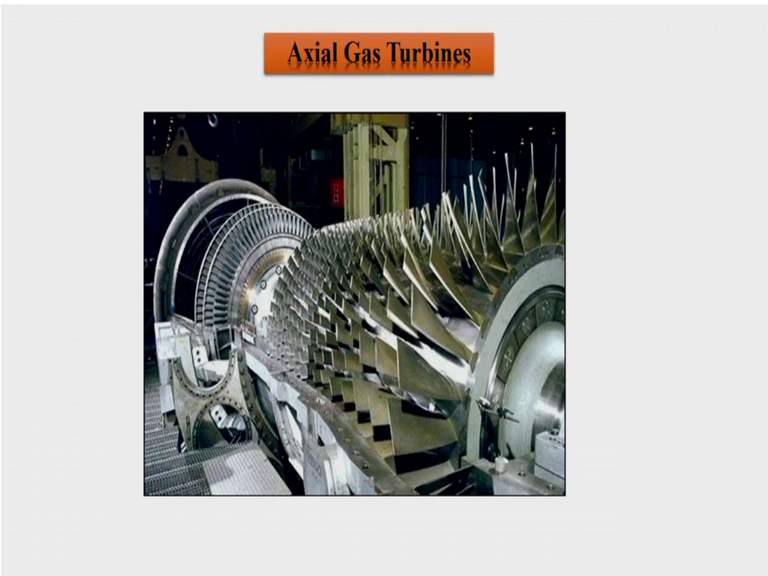

However, we are interested to find out certain performance aspects of the axial turbine from the basic elementary theory of axial turbine.

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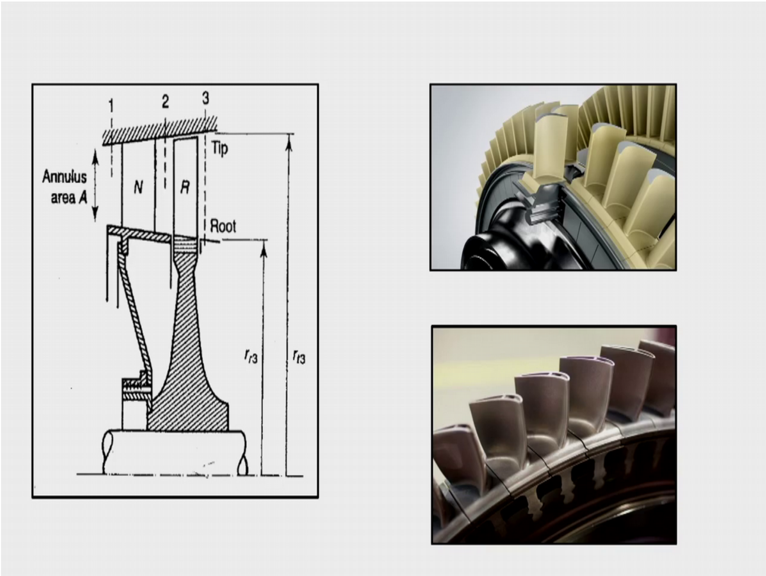

So, as we can see that axial turbine would be composed of again two parts; one is called as nozzle and other is called as rotor. So, in between in the axial turbine itself, we are having two components. Every stage of axial turbine is comprised of two major components; one component is called as nozzle other component is called as rotor. So, when the flow enters into the nozzle, as the name suggest nozzles job is to increase the kinetic energy of the flow. Here we expect the flow should have decrement in pressure and increment in velocity.

So, this high velocity flow would flow over the rotor, it is exactly opposite as in case of compressor. In case of compressor, we had again one stage of compressor was comprised of two components; one was rotor and one was stator. But, rotor was upstream and stator was downstream. But, in case of turbine nozzle is upstream and rotor is downstream.

Again nozzle whatever we are talking about is the component of the stage of a turbine which is a fixed component, so it does not add any work, does not add any energy to the flow rotor is moving or rotating component of this stage. And it takes away the energy, it takes the work of the flow that is where we expect the work to be done by the turbine. So, this is what one stage of the turbine would be composed. So, 1 to 2 is nozzle, 2 to 3 thermodynamically is the rotor. We had seen that this turbine; which is axial turbine will be fixed inside the rotor using such an arrangement. We had discussed this point, when we were talking about the difference between the radial and axial machines in one of the classes ok.

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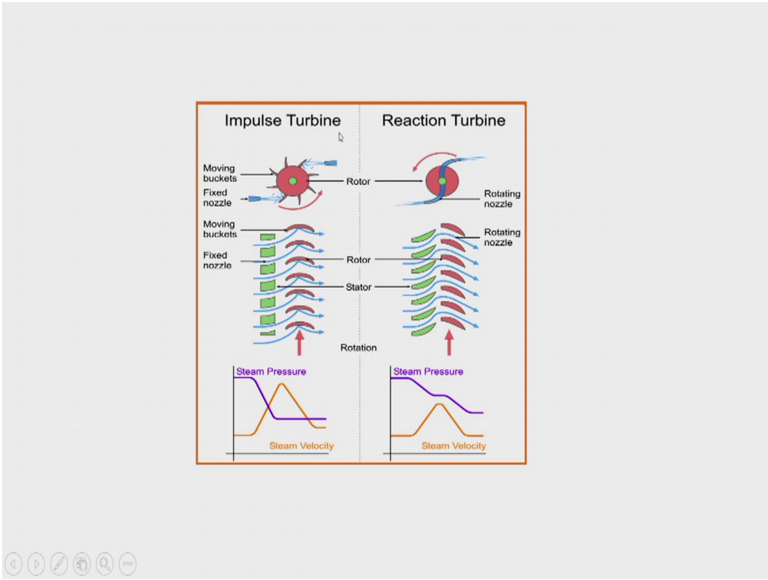

Now, we will be knowing that there are two basically component; two basically types of the turbine; one is called as impulse turbine another is called as reaction turbine. In the gas turbine, we know that we are having nozzles which are green in colour over here, then we have an rotor which is red in colour over here. Similarly, in reaction turbine also, we will have nozzle in green colour and we will have rotor in red colour. So, this is moving in the upward direction that is what it is rotating in this direction.

So, in case of impulse turbine flow, while of coming out rather after coming out of the nozzle, it will directly get imparted on the turbine blade as what we would see in case of hydro turbines. So, in case of hydro turbines, water jet would impart its energy, we will impact on the blade and then water turbine will rotate same thing.

In gas turbine, here we expect that the flow should be imparting its force on the blade and blade would automatically as a outcome of that force, this blade would rotate. But, in case of reaction turbine, motion is smooth in the rotor blades. There is abrupt change in the motion in case of impulse turbine. But, in case of reaction turbine, motion is smooth. We can see the reaction nozzle like this, where jet is coming out in one direction and then we would have rotation accordingly in opposite direction. Same thing here flow is going in downward direction and then blades will rotate in upward direction. So, this is a general diagram for the impulse and reaction turbines ok.

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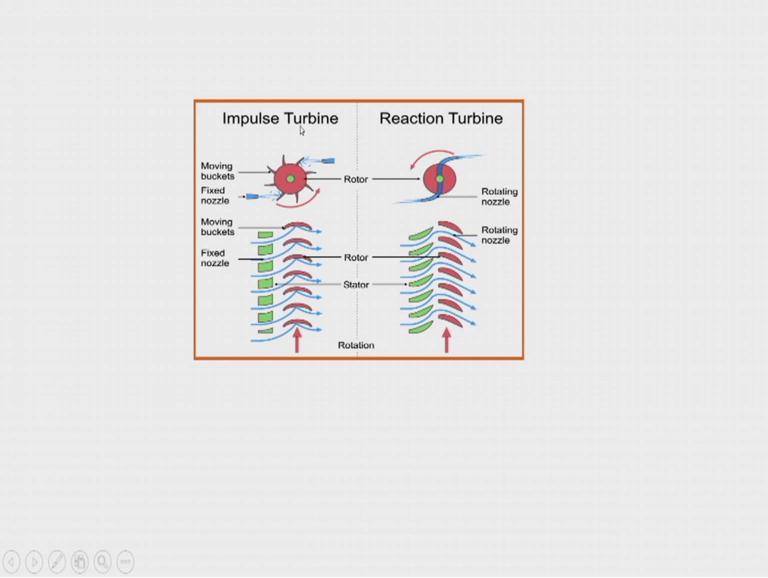

So, as what we would see that there are two components two types of turbines, which are gas turbines; one type of gas turbine is called an impulse turbine and other type of gas turbine is called as a reaction type of gas turbine. So, in these gas turbines as what we had seen, there is nozzle which is green in colour, then there is rotor which is red in colour. In the same case, here as well in reaction turbine green is the stator or the nozzle, and red is the rotor.

So, in case of impulse turbine, the fundamentally the motion would be governed as what we can see in case of water turbine, where jet of water will impart the force on the water on the blades of the turbine and then the blade will rotate. So, here as well the nozzle will generate high kinetic energy flow, and then the flow would get heat to the turbine blades, and due to which the turbine moves in upward direction in the figure means, it would move in this direction to us.

So, similarly in case of reaction turbine, we have green as nozzle and red as the turbine blades. And the dynamics is same thing as the jet over here, where jet is coming out from the sprinkler in one direction and as a reaction the rotor is rotating in opposite direction. So, the same thing nozzle would pass the high pressure; pass the low pressure high velocity flow toward the turbine blade.

But, here motion or movement of the flow is smooth in the turbine blades. Unlike in case of impulse turbine, where we will see that there is abrupt change in the streamline pattern, but here stream line is moving smoothly in the turbine blades, but here flow is moving in downward direction. And other reaction there is rotation of the turbine blade in opposite direction. So, this is how the impulse and reaction turbines are operating for the gas turbine.

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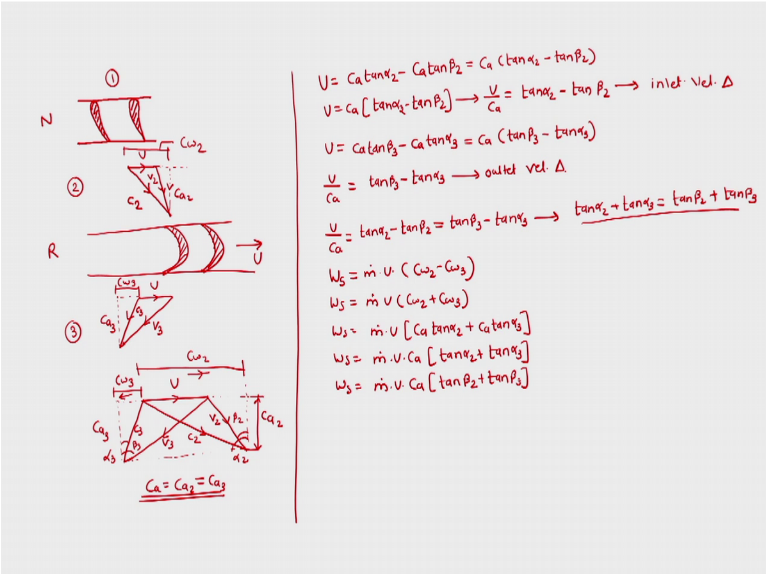

So, now having said this, we will see now how we can draw the velocity triangles for the general gas turbine, which would be either impulse or reaction. So, let us draw a schematic of the first nozzle. So, let us say we are working with the nozzle, so let us draw a schematic of the nozzle.

So, let us say that this is our nozzle and in case of nozzle, these are our blades; these are our nozzle blades which are actually having conversion portion, the flow is taking place in the nozzle which are having convergent portion. [noise] So, this is nozzle, so what we would have is the velocity of the flow which is coming out like this. So, this is absolute velocity c 2, but we are having rotor which is moving in the direction like this. So, we are having basically rotor blades, we are having so the flow is approaching the rotor with absolute velocity c 2.

So, let us draw, the rotor blades, so rotor blades are like this. In case of compressor, we had seen that there is small deflection, when the flow passes the rotor. But, in case of turbine, there will be large deflection. So, what would be happening here is basically, we are having u velocity of the flow in this direction, so that is giving us relative velocity which is so from the nozzle flow is approaching the rotor which absolute velocity c 2.

So, let us draw the rotor blades. So, rotor blades will be like this, we had seen that the rotor blades would lead to very small deviation in case of the axial compressors. But, in case of axial turbines, there would be large deviation to the flow. Since, we are having favourable pressure gradient, so this is the v 2. So, we are having state 1 here, we are having state 2 here, we are having state 3 here and this is rotor. So, u is the velocity by which rotor is moving in this direction, so this is inlet velocity transducer. So, practically what we would have is this as this is v 2, so what we would have is this as c a 2, and what we would have is this as c w 2.

So, now we are at the outlet. In the outlet also, we are having same u in the same direction. And then we have the relative velocity first which will be coming out for the flow and we will have absolute velocity. So, this is v 3, then this is c 3. So, this is how our velocity triangle would look like. So, then in the presence of this velocity triangle at the outlet, we again would have c a 3, and we would have this as c w 3. So, this is the velocity triangle at the inlet and outlet for the axial compressor turbine.

So, now if we combine these two velocity triangles, then u can form common base, this is u and then in the presence of u as what we can see, this is c 2 and then this is v 2. And then we are having this as c a 2, and then we have this as c w 2, where we would have this as beta 2 and this as alpha 2. So, then we are having this velocity as v 3, this velocity as c 3 as a virtue of this, so we should have the height of the turbine, height of the blade, we should have the height of the triangle should be same at the inlet and at the outlet.

Since, we expect the flow to be going in the same axial direction with same velocity. So, this is v 3 and this is c 3, so for us this becomes c a 3 and then this becomes c w 3. And we expect c a is equal to c a 2 is equal to c a 3. So, we expect the velocity and the axial direction should be same or constant at the inlet and the outlet. So, here we can see that c w 2 is in the this direction and c w 3 is in this direction. So, this should be remembered, since we will need this.

So, having said this let us consider what is u from inlet velocity triangle. From inlet velocity triangle u is equal to c a into tan alpha 2 minus c a into tan of beta 2. So, from inlet velocity triangle u is equal to c a into tan of alpha 2 minus tan of beta 2. So, u is equal to c a into tan alpha 2 minus tan beta 2. So, u upon c a is equal to tan alpha 2 minus tan beta 2 and this is from inlet velocity triangle.

Now, let us consider outlet velocity triangle. There we have c a into then here we will have this as beta 3, this as alpha 3. So, we would have c a into tan beta 3 minus c a into tan alpha 3 ok. So, we would have this is equal to c a tan of beta 3 minus tan of alpha 3. So, we have u upon c a is equal to tan of beta 3 minus tan of alpha 3. So, this is from outlet or exit velocity triangle.

So, from inlet and outlet velocity triangle, we can get u upon c a is equal to tan alpha 2 minus tan beta 2 is equal to tan beta 3 minus tan alpha 3. So, this leads to the fact that we have tan alpha 2 plus tan alpha 3 is equal to tan beta 2 plus tan beta 3. So, this is a known identity or rather a famous identity, what we have obtained from the inlet and outlet velocity triangles.

So, now we will find out what is w s which is shaft work or stage work for the turbine, we know that the formula is m dot into c w u into c w 2 minus c w 1 that is what the formula we had derived for. But, in that case w was the work input to the system, but we had found out that for the turbine it is m dot into u into c w 1 minus c w 2 that is the formula, what we had obtained from the Euler turbine from expression. So, w is equal to m dot into u into c w 2 minus c w 3, this is the stage work for the turbine.

But, here we can see that c w 2 and c w 3 are in opposite direction, so we have m dot into u into c w 2 plus c w 3 so, this is in general. Then we can write c w 1 is equal to c w rather 2 is equal to m dot into u c a tan of alpha 2 plus c w 3 is equal to c a tan of alpha 3. So, we have this w s is equal to m dot into u into c a tan of alpha 2 plus tan of alpha 3. So, this is what we have worked, further we can know from this relation the tan alpha 2 plus tan alpha 3 is equal to tan beta 2 plus tan beta 3.

So, we can also write it as m dot u c a into tan of beta 2 plus tan of beta 3. So, this is what the stage work expression, what we would need for the calculations or we should know from how to obtain stage work turbine work from the velocity triangles having said this, we should now find out, what is the temperature drop in the turbine and what is the pressure rise in case of turbine.

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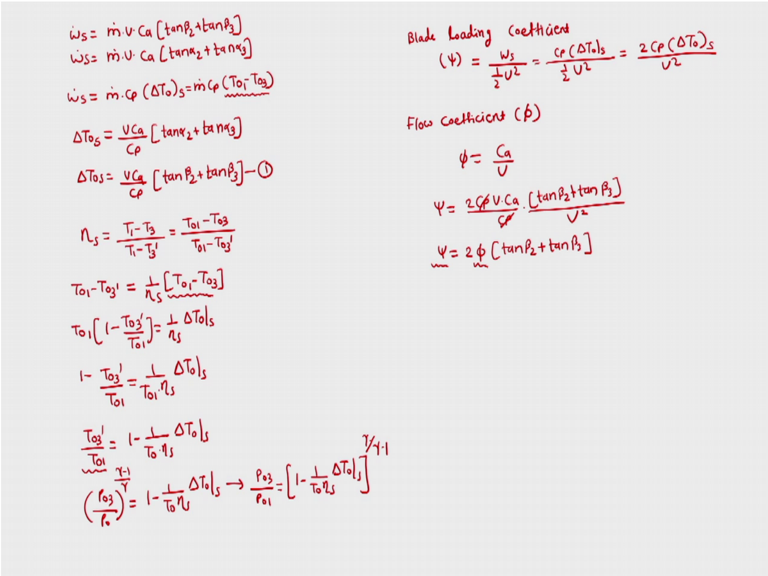

So, let us take this expression. And we will write down this expression again, where we have seen that is equal to m dot into u into c a into tan of beta 2 plus tan of beta 3 or w s is equal to m dot u c a tan of alpha 2 plus tan of alpha 3. So, this is what expression we had found out.

So, now we know that w dot s is equal to m dot into c p into delta T naught stage which is equal to m dot c p into T naught 1 minus T naught 3 so, this is the temperature drop in the stage. So, we know now temperature drop in the stage is equal to u c a upon c p into tan of alpha 2 plus tan of alpha 3. So, this is temperature drop in a stage or temperature drop in a stage, we can also write u c a upon c p into tan of beta 2 plus tan of beta 3. We can write down either expressions for the temperature drop in stage.

Now, we know stage efficiency or isentropic efficiency of turbine is equal to temperature drop with rather, we can write it in terms of work which is actual work divided by ideal works. Actual work is T 1 minus T 3 divided by T 1 minus T 3 dash. So, we have this as we can also we would also write it as T naught 1 minus T naught 3 divided by T naught 1 minus T naught 3 dash.

So, here we can take T naught 1 minus T naught 3 dash is equal to 1 upon isentropic efficiency T naught 1 minus T naught 3. So, expression for T naught 1 minus T naught 3 is here, we will name it as number 1. So, we can take common as T naught 1, so T naught

1 1 minus T naught 3 dash upon T naught 1 is equal to 1 upon stage efficiency into delta T naught stage.

So, we can write down it as 1 minus T naught 3 dash upon T naught 1 is equal to 1 upon T naught 1 into stage efficiency delta T naught stage. So, we have T naught 3 dash upon T naught 1 is equal to 1 minus T naught into stage efficiency into delta T naught stage. But this is P naught 3 upon P naught 1 bracket raised to gamma minus 1 upon gamma is equal to 1 minus T naught into stage efficiency into delta T naught stage.

So, we can write down this expression as P naught 3 upon P naught 1 is equal to 1 minus T naught into stage efficiency into delta T naught stage bracket raised to gamma upon gamma minus 1. So, this is how we can find out the pressure drop into the turbine by the virtue of work interaction into the turbine.

So, here we would need a help of equation one to put the delta T naught stage. So, there are two parameters, which one should know while working with the turbine. And first parameter is called as blade loading coefficient and it is denoted by psi its expression is stage work divided by half u square. So, this is equal to; this is specific work so, what we would have is no m dot here, so we would have it as c p into delta T naught stage divided by half u square delta T naught by stage, we can put it over here. And we can calculate, what is the psi or blade loading coefficient in case of a stage of the turbine.

Similarly, there is other constant which is are called as flow coefficient, and it is termed as phi. And phi is equal to c a by u, so c a by u is equal to phi. So, we can write down our expression of psi as 2 c p delta c T naught s is u into c a divided by again c p into tan of beta 2 plus tan of beta 3 and then this is joined by u square, this is divided by u square delta T naught u square. And then we would have psi is equal to c p would cancel, then we would have psi, we can define psi in terms of phi, here where we can get 2 into c a by phi into tan of beta 2 plus tan of beta 3.

So, this is what the expression between relation between psi, which is blade loading coefficient and relation with psi. So, this is how we can write down the expression ok. So, we will need this thing, when we are going to work with some examples in case of the axial turbine ok.

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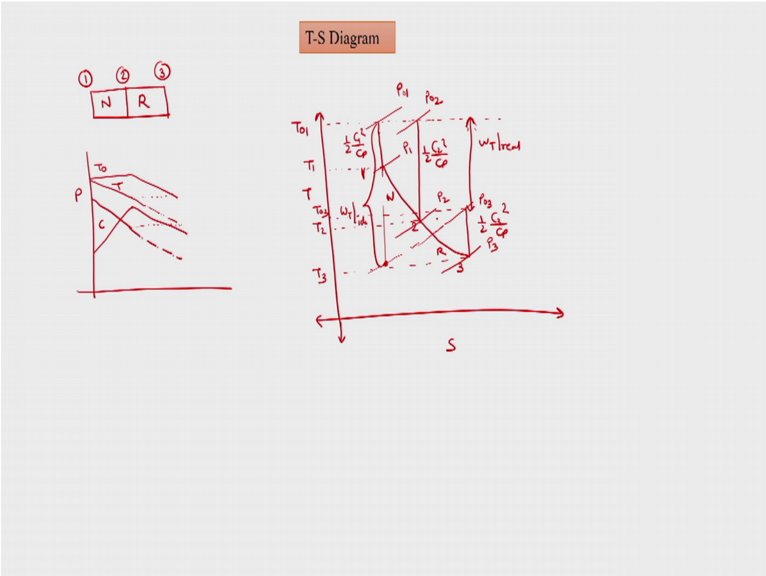

So, now next point to be discussed is the T-S diagram or the axial turbine. So, for T-S diagram, we know that we will write down our schematic as what we did for the compressor, we first have nozzle, then we have rotor, 1 to 2 is nozzle and then 2 to 3 is this rotor. So, we know that what is going to happen in turbine.

In the nozzle, we would have pressure decreased and in the stator, if we are having impulse turbine, then we would have pressure to be constant. But, if we would have reaction turbine, then we would have it to be continued to decrease, so this is pressure. If we are having velocity, then velocity would be increased in the nozzle and velocity will be decreased in the rotor ok, this is c.

And then we are having total temperature, total temperature is constant in the nozzle and it would decrease in the rotor ok. So, static temperature would decrease in the rotor and then it would again depend upon the type of turbine. If it is impulse turbine, then it is constant, if it is reaction turbine, then it will decrease.

So, in the reaction turbine, basically we do not have any enthalpy drop in the case of impulse turbine, we do naught have any enthalpy drop or we do not have enthalpy to be converted into kinetic energy in the rotor having known this, we can draw the T-S diagram for the turbine. So, we would be here T S.

So, first we are in stage 1; stage 1 to 2 is nozzle so, in the stage and then 2 to 3 is the rotor. So, we are having nozzle here, we are having rotor here. So, in the state 1 we are having P 1 pressure at the state 2 we are having P 2 pressure, at the state 3 we are having P 3 pressure and this is T 1, this is T 2 and this is T 3. But we can again put associated velocity is here so, we can put half c 1 square upon c p such that we can go to T naught 1.

And then we would have P naught 1, but we have no energy interaction between nozzle; in the nozzle, so we would have same total temperature in the case of nozzle exit also except that due to friction, total pressure would decrease so, this is P naught 2. And then we would have some where T naught 3, P naught 3, so we would have P naught 3 here and then this would be our T naught 3. So, this would be our T naught 3.

So, see and this is actually half c 2 square upon c p; this is half c 3 square upon c p. So, if we would have gone isentropically, then we would have been reached here in the same p naught 3 from p naught 1. So, this would have been the turbine work in ideal condition, but this is our turbine work in real condition ok. So, this is turbine work in real condition.

So, this is the velocity a T-S diagram for the turbine which axial turbine, so we should know this point. But, however this T-S diagram has an assumption that we are working with the reaction turbine. If we would be working on the turbine which is impulse turbine, then 2 and 3 point would be same. Since, there is no enthalpy drop in the process 2 to 3, but we would go a little right since there will be friction loss.

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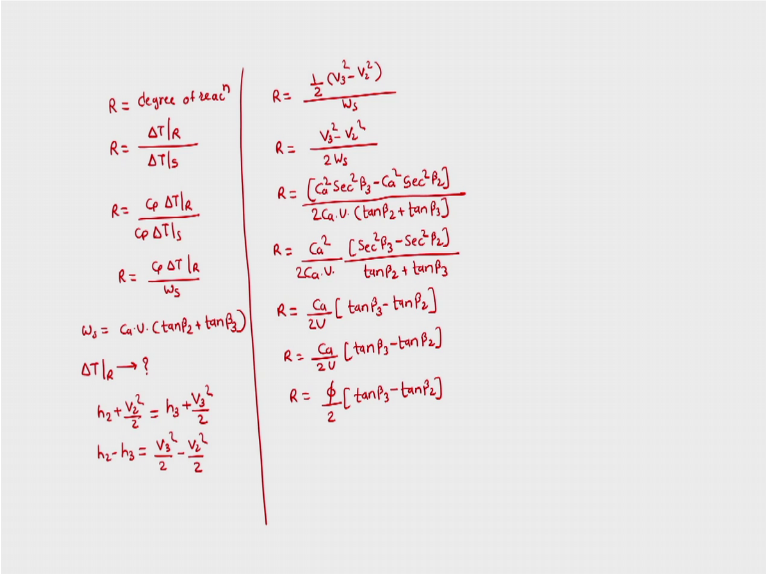

Now, we will go to the next part which is degree of reaction. We have defined R as degree of reaction. And then so R is our degree of reaction. So, we have defined R is equal to temperature drop in the rotor divided by temperature drop in the stage, this is what our definition of R is. So, we will multiply c p on both sides, so we get R is equal to c p into delta T into rotor divided by c p into delta T into stage. We have seen that static and total entropy temperature change or be identical as what we have seen in case of compressor. So, R here this is c p into delta T in rotor; delta T of rotor divided by stage work.

So, now we know the stage work formula w stage is equal to c a into u into tan of beta 2 plus tan of beta 3. Now, we need to find out, what is delta T across the rotor, so how to find out? So, we can go to the velocity triangle or from the T-S diagram, we know one point that as the absolute total temperature does not change in case of the nozzle, similarly relative total temperature does not change in case of the rotor.

So, relative total temperature means h 2 plus v 2 square by 2 is same as h 3 plus v 3 square by 2. So, we have h 2 minus h 3 is equal to v 2 is equal to v 3 square by 2 minus v 2 square by 2. So, this expression, we can use to find out R. So, R is equal to half v 3 square minus v 2 square by 2 divided by shaft work or stage work.

So, R is equal to half v R is equal to v 3 square minus v 2 square upon twice shaft work. So, we can go to the velocity triangle and then find out what is the v 3 and v 2. So, v 3 can be written as from the velocity triangle v 3 is this, v 3 can be written as c a tan beta 3, and v 2 can be written as c a tan beta 2. So, we can write down these expressions, it is c a sec beta 3. It is c a sec beta 3, it is c a sec beta 2. So, these are the expressions for v 3 and v 2. So, we would have c a square sec square beta 3 minus c a square sec square beta 2 divided by we have an expression c a into u into tan of beta 2 plus tan of beta 3.

So, we have R is equal to c a square upon c a into u into sec square beta 3 minus sec square beta 2 divided by tan of theta 2 plus tan of beta 3. So, R we can express this sec theta sec square theta in terms of tan square theta and then ultimately we would get c a upon u into tan of beta 2 tan of beta 3 minus tan of beta 2. So, this is we just have forgotten two here.

So, we have R is equal to c a upon twice u tan of beta 3 minus tan of beta 2, but we know that c upon u is equal to phi. So, phi upon 2 is equal to tan of beta 3 minus tan of beta 2. So, this is the expression for degree of reaction for the axial compressor the turbine. So, here we should keep one point in mind that as what we had seen, we are working on the mid blade height.

So, here all the velocity triangles are drawn at the centre of the blade. So, inlet velocity triangle is drawn at the centre, outlet or such a triangle is drawn at the centre. So, we do not consider any radial velocity to the flow, and we expect the c a to be same between inlet and the outlet of the turbine. So, we had seen one more thing in last class, when we were working with the compressor, we had seen that there is a concept called as free vertex condition.

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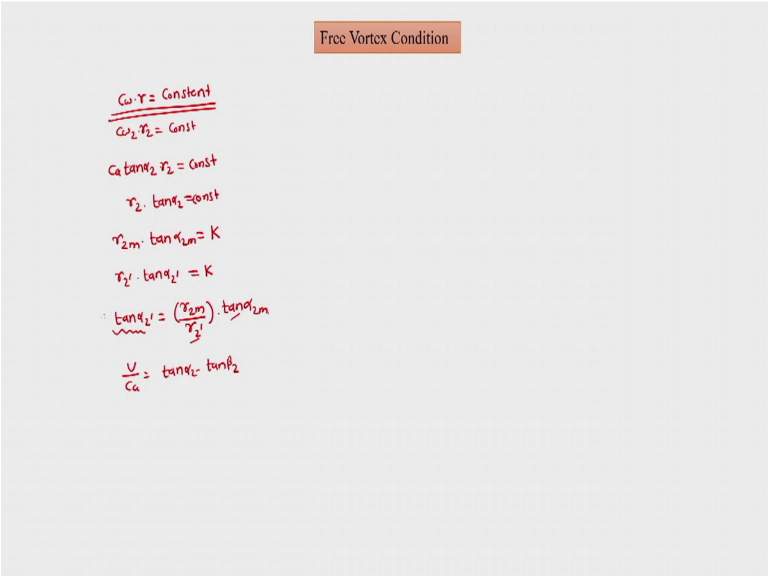

So, in the free vertex condition, we were having a constraint a condition achieved which was c w into r was constant using that condition, we can translate the velocity triangle drawn for one case, to the velocity triangle drawn for the other right, so that we can take and help.

So, what is c a, c w for that all sec. So, we can see c w 2 into r 2 is equal to constant. So, from the velocity triangle, we can find out what is c w 2? c w 2 is equal to c a tan alpha 2 . So, we can write down this expression for c w 2 which is equal to c a tan alpha 2 into r 2 is equal to constant. So, basically we can c is a constant, so we have r 2 into tan alpha 2 is a constant ok, this is from free vertex.

So, what we are having is if we are at the mid blade height, then r 2 m into tan of alpha 2 m is some constant k. And I want to work at some other height maybe r 2 dash, so into tan alpha 2 some dash is also k. So, we can find out tan alpha 2 dash is equal to r 2 m divided by r 2 dash into tan of alpha 2 m.

So, we can find out what is tan alpha 2, at some other height, when we know the radius of that height and if we know, what is tan alpha 2 m at the mid blade height. So, this is what we can utilize, the concept of free vertex theory, translate the velocity triangle from one stage to the other stage. But, there is one more thing which we have derived and that was tan of u upon c a is equal to tan of u upon c is equal to tan of alpha 2 minus beta 2.

So, we can use this concept u upon c a is equal to tan of alpha 2 minus tan of beta 2. So, this concept we can use and then we can find out beta 2 at some other radius as well. Since c a is constant, alpha 2 is evaluated and u can be found out based on the radius and omega. So, this is how we can work out to find out different angles, if they are known at one height, we can find out them at some other height ok.

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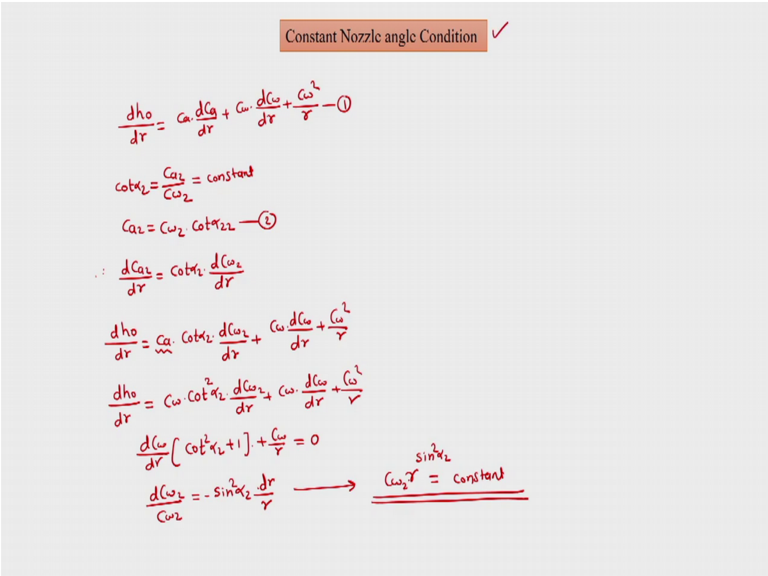

So, next thing, what we need to find out is apart from the condition of free vertex, there is one more condition for which nozzles are generally for which exile turbines are design is called as constant nozzle angle. We had seen in last class that there is an expression which says that dh naught by dr is equal to c a d c a by dr plus c w d c w by dr plus c w s upon r square plus c w square upon r. So, this is the expression we had seen.

But, now in case of the constant nozzle angle, we expect alpha 2 to be constant. So, for that all sake, we would consider that cot alpha 2 is equal to c a 2 upon c w 2 is equal to constant. And since it is a constant, we can have it same at different altitudes. So, we have c a 2 is equal to c w 2 cot alpha 2. So, we have d c a 2 upon dr, it is a constant into d c w 2 upon dr ok.

Then we can use this term in equation number 1, so we would have dh naught by dr, this expression we had earlier proved in case of the axial compressor. So, cot alpha 2 into d sub w 2 by dr plus c w into d c w by dr plus c w square upon r, but c a is equal to this from equation 2. So, dh naught by dr is equal to c w into cot square alpha 2 into d c w 2 by dr plus c w into d c w by dr plus c w square upon r. We can consider left hand side to be 0 by fact that h naught is not changing with respect to r. So, we can divide the expression by c w, then we can get d c w by dr into cot square alpha 2 plus 1 into c is equal to; plus c w upon r is equal to 0.

So, we can have an expression which says that d c w 2 upon c w 2 is equal to minus signs minus 1 plus cot square alpha will get adjusted and then one sine minus sine square alpha 2 into dr by r upon integration. This expression can be written as c w 2 into r raise to sine square alpha 2 is equal to constant. So, this expression which is c w 2 into r raised to sine square alpha 2 is equal to constant in the condition where constant nozzle angle will be maintained at the entry to the rotor ok.

So, these are the different aspects for the axial turbine. First we had considered, how the axial turbine has different components, how the flow take place depending upon the degree of reaction of the turbine. If it is impel, then there would be certain type of the flow. If there is reaction turbine and certain other type of the flow, then we drew velocity triangle for the axial turbine. Knowing the velocity triangle, we found out the work interaction, temperature drop, pressure rise then we defined two parameters. One is blade loading coefficient and other is phi which is the flow coefficient using this concept.

Then we drew the T-S diagram for the turbine, after T-S diagram, we try to derive an expression for degree of reaction in case of the axial turbine in terms of the parameters of the axial turbine. Then we found out; how to find out, if we know the velocity triangle at one state, then how to find out it at other state using the free vertex condition.

And now if we know constant nozzle angle condition, then what is that this is similar to the condition what c w into r is equal to constant which was free vertex condition instead of that we have c w 2 into r raised to sine square alpha 2 is equal to constant. So, here we end the part, what we were discussing about which is axial flow turbines.

Thank you.