IC Engines and Gas Turbines
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Lecture - 46
Axial Compressor: Different factors, Degree of Reaction and Free Vortex Condition

So, welcome to the class. In the last class, we have seen that we started with Axial Compressor. And for axial compressor, we have seen how the velocity triangle is, how to find out the work interaction in case of axial compressor, and then how to find out stage temperature rise and pressure ratio for given conditions ok.

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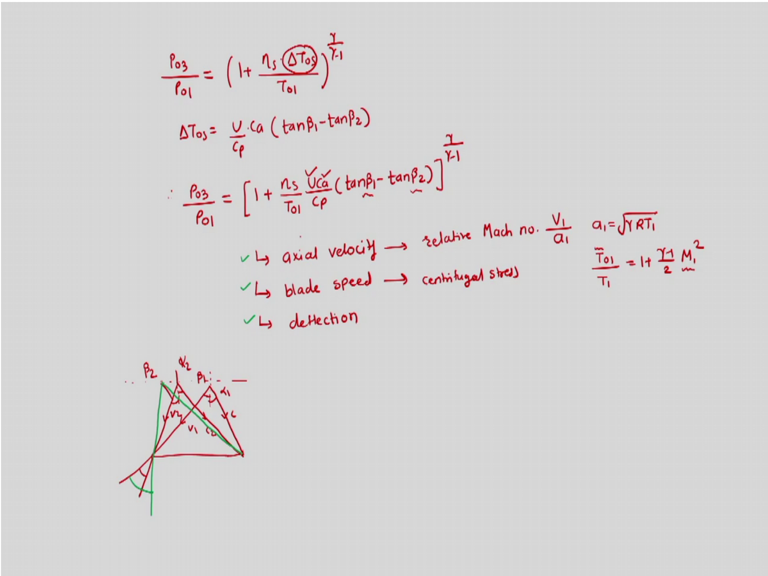

So, then we had obtained a formula in last class about the pressure rise as P naught 3 upon P naught 1 is equal to 1 plus stage efficiency into delta T naught s stage divided by T naught 1 bracket raised to gamma upon gamma minus 1. So, this is stage temperature rise. Rather we had found out that this stage temperature rise is U upon C a upon C p, where stage temperature rise is U upon U upon C p into C a U upon C p into C a into tan of beta 1 minus tan of beta 2. So, this was our formula for stage temperature rise.

So, we got a formula for pressure ratio as P naught 3 upon P naught 1, which is 1 plus efficiency stage efficiency divided by T naught 1 into U C a divided by C p into tan of beta 1 minus tan of beta 2 bracket raised to gamma upon gamma minus 1. So, here we can see that for a pressure rise to happen with required magnitude, there are different factors which affect.

First factor which affect is the axial velocity, which is C a axial velocity. Axial velocity if it is more, then we can see that we can get more pressure rise. But, axial velocity is restricted or there is a limitation on axial velocity due to relative Mach number. And relative Mach number is defined as relative Mach number is defined as v 1 divided by a 1. So, this is the relative Mach number at the blade, so entry. So, this number should be subsonic such that we should have minimum number of losses minimum amount of losses, so for that C a will be restricted.

So, we basically need to find out, what is a 1. So, basically we know that a 1 for gas is equal to gamma R T 1, and we should know what is T 1. So, T 1 we know formula at T naught 1 upon T 1 is equal to 1 plus gamma minus 1 by 2 M 1 square. So, we would know the Mach number at the entry, we would know the total temperature, then we can find out T 1. Putting that T 1, we will get a 1. And then we will get the value of relative Mach number, which would have restriction to have any amount of axial velocity. So, only limited amount of axial velocity will be allowed.

Then the second factor which can help to raise the pressure more is blade speed which is U, but blade speed is also restricted by stresses centrifugal stresses in the blade. And this stress is directly proportional with U square. So, there should not be again very high amount of blade speed. Third factor which is affecting is this which is called as deflection. We had seen in last class that beta 1 minus beta 2 is called as a deflection. And if we can have larger deflection, then we can have larger pressure rise, but larger deflection would create the problem of flow separation, so there would be problem.

So, how would be the larger pressure direct larger pressure gradient created? Larger pressure ratio created, we can see here by creating deflection. This was the velocity triangle, which we had seen in earlier case in our earlier graph, we had seen that the velocity triangle is of this kind. So, same velocity triangle, we are trying to draw here. So, we have C 1, and we have V 1, we have C 2, and we have V 2. And this angle, we had called as beta 1 minus beta 2, since this is alpha 1, this is beta 1, then this is alpha 2, and then this is beta 2.

Now, if we want to have higher deflection, then we should have our V velocity V 2 should be in the different place. So, we can have V 2 to be like this. If V 2 is like this, then we can see that there will be more deflection. If there is more deflection, then there will be more pressure rise. So, obviously we can also know that we would have more delta C w in this case. So, there will be more work absorb, and then there will be more pressure rise.

So, there are three factors which would affect the pressure ratio. One is axial velocity, but that is restricted by relative Mach number. One is blade speed, but that is restricted by stress in the blade. And another is deflection, and large deflection would lead to flow separation. So, these are the restrictions which will be we should be knowing which would affect the pressure rise in a stage.

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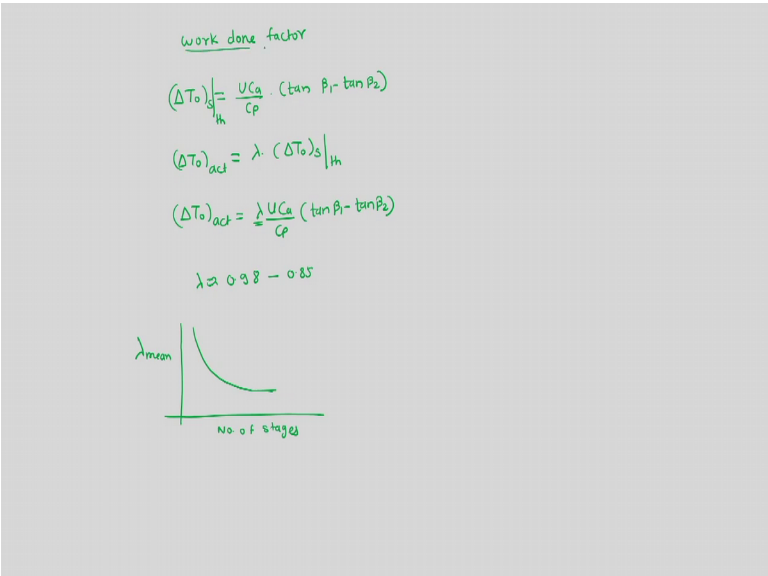

Now, we would see that there is a term which is called as work than factor. So, work done factor for the compressor, work done factor is related with the work supplied and work received. So, work received is always lesser than the work supplied. So, what we would have is delta T naught, delta T naught stage which we have we had found out as U C a upon C p into tan of beta 1 minus tan of beta 2. This is stage temperature rise, but that is ideal stage temperature rise, so we will get actual stage temperature rise is equal to lambda into delta T naught stage or which is rather theoretical. So, this is theoretical.

And then this leads to delta T naught actual is equal to lambda into U upon C a divided by C p into tan of beta 1 minus tan of beta 2 ok. And this lambda actually has a range generally in between 0.98 to 0.85; this number is less than 1. So, actually we will get lesser temperature rise than the theoretically which we would have got, otherwise if there would be that much amount of work which would be have got stored in the compressor.

But, this lambda as variation over number of stages, and this is mean value of lambda, and this lambda mean decreases over number of stages. So, as the number of stages increase, lambda mean decreases. So, lesser and lesser temperature rise in actual case would be observed than the comparator compare theoretical value, if we have higher value of number of stages. So, this is one more term which we should be knowing in case of axial compressor.

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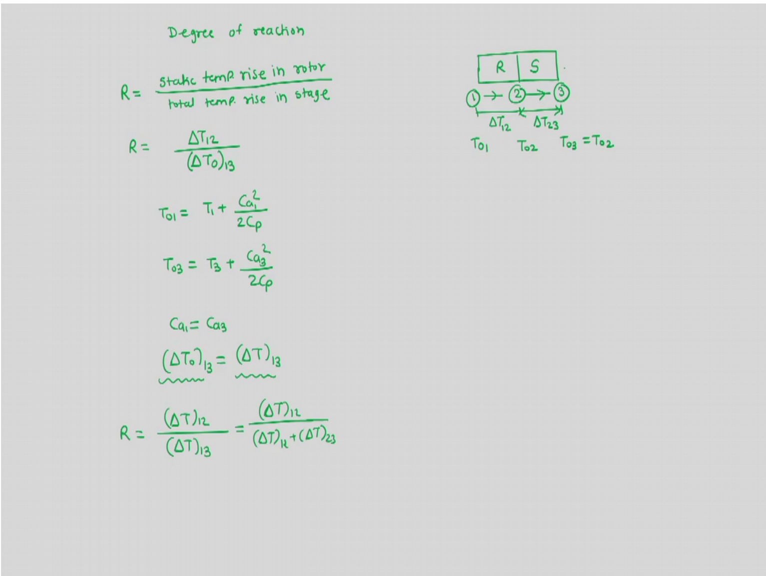

Now, moving on to the next topic of discussion, which is called as degree of reaction degree of reaction. We would define degree of reaction as a R, suppose an R is degree of reaction or in some places degree of reaction is also defined as lambda, so that we would term as capital lambda maybe as in other case. Here it is also small lambda, and then we will define this as R or maybe capital lambda.

So, degree of reaction is defined as temperature rise in the rotor which is static temperature rise in the rotor, static temperature rise in rotor divided by total temperature rise in stage. So, this is the definition of degree of reaction or static enthalpy rise in the rotor divided by total enthalpy rise in the stage. So, this is what, so it is so it is comparing the pressure rise in the rotor divided by total complete pressure rise in the stage. So, this is how degree of reaction is defined.

Now, what we should be remembering is what we have in case of axial compressor is like this; we first have a rotor, then have stator. So, then this is 1, this is 2, and this is 3 for us ok. So, 1 to 2 is rotor, 2 to 3 is stage. So, we would say that this is delta T A, so temperature rise static temperature rise is delta T A, and this static temperature rise or rather we can call it as delta T 1 2, and this is delta T 2 3. So, this is what the temperature rise which would be taken place in the rotor, and the stator ok.

And then we know that at one, we have T naught one total temperature, at 2 we have T naught two total temperature, at 3 we should have T naught 3, but T naught 3 is equal to T naught. Since in the stator, there is no work interaction only, there is diffusion of the energy, and it would rise the pressure. So, we have this as a variation of total temperature in a stage, this is the change in static temperature in the stage. So, we have basically static temperature rise in the rotor divided by total temperature rise in the stator as the formula for degree of reaction.

So, we have delta T 12 divided by delta T naught 13 delta T naught 13. We should keep it in mind that there is no total temperature in the numerator, total temperature used only in the denominator for the fact that total temperature rise take place only in the rotor, otherwise degree of reaction would have been always one ok.

So, now we will see that we know T naught 1 is equal to T 1 plus C a square divided by twice C p, axial velocity square divided by twice C p, this is the temperature, total temperature in stage 1 ok. We are assuming that the flow is axial completely at the inlet. So, we are having similarly T naught 2 is equal to T 2 plus C a square upon twice C, which is a rather if we go to 3. If we go to 3, then we can write T naught 3 is equal to T 3 plus C a 1 C a 3 square divided by twice C p. C a 3 is here it is entry to the next stage, but we know that C a 1 is equal to C a 3.

So, in that particular case, what we would have is delta T naught 1 3 is equal to delta T 1 3. So, total temperature rise in the stage is equal to static temperature rise in the stage, so degree of reaction R can again be written delta T 1 2 divided by delta T 1 3. So, we will have delta T 1 2 divided by delta T 1 2 plus delta T 2 3, so this would be the formula for the degree of reaction.

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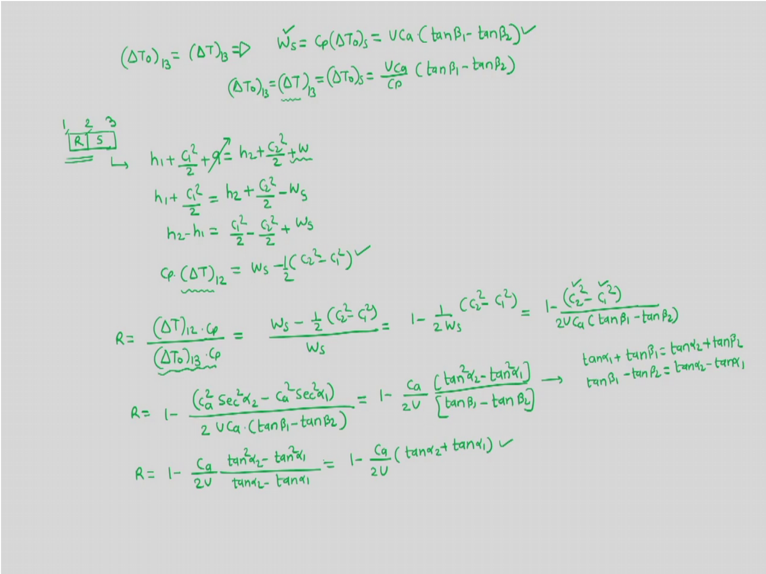

Now, we should be knowing rather the delta T naught 1 3 in case of axial compressor. So, delta T naught 1 3 which is equal to delta T 1 3 is equal to or rather can be found out from stage work, which is C p into delta T naught stage. And then that is for our case, this is U C a into tan of beta 1 minus tan of beta 2. So, we knew that delta T naught stage is equal to U C a divided by C p into tan of beta 1 minus tan of beta 2. So, we know how to find out delta T naught stage, which is delta T naught 13, and which is equal to delta T 13.

Now, we got the formula of R like this. So, we found out the denominator, we need to find out the numerator. So, numerator can be found out by considering the energy equation between the rotor, and the stator such that we know that this is rotor, this is stator this is 1, 2, and 3. Let us apply energy equation between steady flow energy equation between point station 1 and station 2. We can write down that h 1 plus C 1 square C 1 square by 2 plus q is equal to h 2 plus C 2 square by 2 plus w, but this is an adiabatic process, so q is not there. Further this w is received by the system, so we have h 1 plus C 1 square by 2 is equal to h 2 plus C 2 square by 2 minus w stage.

So, we would have h 2 minus h 1 is equal to rather, we would have w stage h 2 minus h 1 is equal to C 1 square by 2 minus C 2 square by 2 plus w s ok. So, we have C p into delta

T 1 2 is equal to w stage plus or we can also write it as minus C 2 square half here, and then we can write it as C 2 square minus C 1 square. So, this is the formula, till now what we have got. So, here we got the numerator, here we got the denominator.

So, now R is equal to delta T 1 2 which is equal to delta T naught 1 3 is equal to w s minus half. So, we can multiply both the sides by C p, so we will get C p into delta T naught 1 2 divided by C p into delta T naught 1 3, so this is here. So, we can write it as w s minus half C 2 square minus C 1 square, but this factor is w s.

So, we will have w s. So, what we get is one minus half 1 minus 2 w s into C 2 square minus C 1 square, but we can know that w s is equal to this. So, we can use that and say that 1 minus C 2 square minus C 1 square divided by U C a into 2 into tan of beta 1 minus tan of beta 2 ok. So, this is the formula till time for R.

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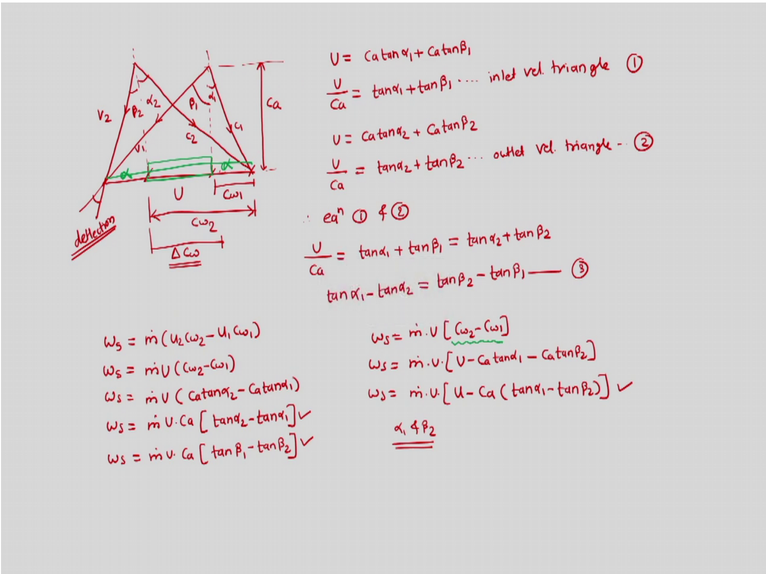

Now, we have to find out what is C a, what is C 1. We will go back to the velocity triangle, what we had drawn. So, in this velocity triangle if we see what is C 1, and what is C 2. So, C 1 is C a sec alpha 1, and C 2 is equal to C a sec alpha 2. So, from the velocity triangle, we know that C 1 is equal to C 1 is equal to R is equal to 1 minus C a square sec square alpha 2 minus C a square sec square alpha 1 divided by 2 into U into C a divided by tan beta 1 minus tan beta 2.

So, we have R is equal to 1 minus, then this formula can be further replaced sec square alpha in tan square alpha, and then we can get it again replaced using the relation of sec and tan, we can get it as C a upon twice U into tan square alpha 2 minus tan square alpha 1 divided by tan beta 1 minus tan beta 2.

But, in earlier sections, what we had seen from the velocity triangle that there is a relation which says that tan alpha 1 plus tan beta 1 is equal to tan alpha 2 plus tan beta 2. So, we can write down that relation which says that tan alpha 1 plus tan beta 1 is equal to tan alpha 2 plus tan beta 2. So, we get tan of beta 1 minus tan of beta 2 is equal to tan of alpha 2 minus tan of alpha 1.

So, we can write down it over here, and we can say that R is equal to 1 minus C a upon twice U into tan square alpha 2 minus tan square alpha 1 divided by tan of alpha 2 minus tan of alpha 1. [noise So, we get 1 minus C a upon twice U into tan of alpha 2 plus tan of alpha 1. So, this is our formula for R, further this formula can also be reduced in terms of betas. So, this is how we can find out R for the compressor, which is the degree of reaction for a compressor.

This formula can further be utilized to understand, how the R is going to vary with respect to the blade height. So, here we know that our formula for degree of reaction is this. And here we have C a tan alpha 2 and C a tan alpha 1 ok, we will do it. After one more derivation which is called as the free vortex theory ok. So, till time what we were trying to do was for the two dimensional case, where we had seen that we were working with the velocity triangle on a plane which is parallel to the axis of the compressor. So, we were having this analysis as two dimensional.

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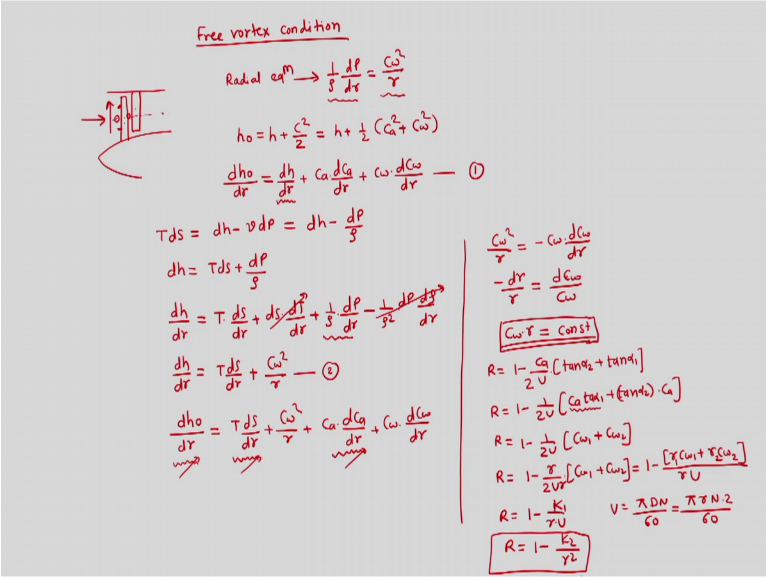

But, we can do further a glimpse of three dimensional analysis, where we will use a theory which is called as free vertex condition or free vortex theory ok. So, we will say that this is free vortex condition. And we will derive this condition, as per this condition. Now, this condition is applied for the compressor axial compressor as well as for the axial turbine. So, this part of discussion is common between axial compressor and axial turbine.

So, what do we mean by free vortex condition, we using this condition we are trying to find out the relation between the triangle velocity triangles, whatever we would have drawn in a plane at a condition. So, this is the blade as we know, this is a blade, and this is a stator, this is rotor, this is stator. We know that we are working at this height of the blade. So, if we are working at this height of the blade, then we are drawing the velocity triangle here and here, then knowing the velocity triangle at these locations how to find out the velocity triangle at other locations or the of the rotor. So, this is what our objective through this free vortex condition is.

So, for that we practically are considering this radial variation. And since this is an axial compressor or more intention was that it has only axial velocity. So, it is not having any radial velocity, but with this non-radial velocity being accounted, we will try to derive a constraint. So, for that we will first consider, there exist a radial equilibrium and radial equilibrium leads to 1 upon rho dp by dr dp by dr is equal to C w square by r. So, centrifugal force is balanced by the pressure gradient in the r direction for the compressor at a location ok. So, this is our assumption we could have derived that, but this can be taken to start with as a [vocalized-] assumption for radial equilibrium to derive the free vortex condition.

Now, let us consider h naught is the total enthalpy at a station which is equal to h plus C square by 2, but we know that C can be decomposed into two components which is C a square plus C w square, further we can differentiate this h naught, and then we get d h naught by d r is equal to d h by d r plus C a into d C a by d r plus C w into d C w by d r plus C w into d c w by d r, this is equation number 1.

Now, we know our thermodynamic relation which is combined first law and second law as per that Tds is equal to dh minus vdp, so that can be written as dh minus dp by rho, then we can write here as d h is equal to Tds plus dp by rho. Now, here we can see that there is a term dh by dr. So, we can differentiate this equation, then we can get dh by dr, so T into ds by dr plus ds into dT by dr plus 1 upon rho into dp by dr minus, we would have 1 upon rho square into dp into d rho by dr, this would be the term.

But, then this term is a second order, further this term is also second order will for the sake of convenience, we will neglect them being their magnitudes to be very small. So, we get dh by dr is equal to T into ds by dr plus 1 upon rho dp by dr, but we know that 1 upon rho dp by dr is equal to C w square by r.

So, what we get after this putting equation 2 in equation 1, we get d h naught by d r is equal to T into ds by dr plus C w square by r plus C a into d C a by d r plus C w into d C w by d r. Now, we know that when we are considering centrifugal compressor our C a is taken as constant with respect to r. So, this term is negligible in magnitude or 0 for us.

Further we are considering subsonic compressors. So, there is no entropy variation in r direction. So, these two terms would get cancelled. Further we are also considering that the dh naught by dr is negligible which is saying that we are having equal amount of work to be taken by the compressor at different heights. So, considering these terms to be negligible in the respect of other two terms, we get C w square upon r is equal to minus C w into d C w by d r, so which says that minus dr by r is equal to d C w by C w.

After integration, this expression leads to the fact that C w into r is equal to constant. So, if we know C w at a height, then we can find out C w at some other right. So, this is how this formula helps us to work out for finding out the velocity triangle known at one point to be finding out velocity triangle to be evaluated or estimated at the other height.

This formula can be having thought to having relation with the formula for what we have derived for the degree of reaction. So, degree of reaction formula, what recently we derived has 1 minus C a upon 2 U into tan alpha 2 plus tan alpha one. So, r is equal to 1 minus U into 2 C a into tan of alpha 2 plus tan of alpha 1, so it is R is equal to R is equal to C a by U C a by U. So, we have 1 minus twice U into C a tan alpha 1 plus tan alpha 2 into C a.

If we see velocity triangle, so C a tan alpha 1, C a tan alpha 1 would be C w 1. So, R is equal to 1 minus twice U, and this is C w 1 plus C w 2. And now we will multiply both the numerator and denominator by R. So, we will get 1 minus R upon twice U into C w 1 plus C w 2, which would lead to 1 minus r C w 1 plus r C w 2 divided by r into U. Then we know that r 1, and we can write it as r 2, since r 1 and r 2 are same, but this is a constant. Since this is a constant.

We get R is equal to 1 minus 1 upon r into U c that this constant is K 1, but we know that U is equal to pi D N by 60. So, U is equal to pi r N into 2 by 60. So, it turns out that R is 1 minus K 2 upon r square. So, this is the relation of degree of reaction with respect to R. So, if R is increased, degree of reaction will increase. If R is decreased, degree of reaction will decrease.

So, this is how we when do our 2D analysis for the velocity using the velocity triangle at mid bed height, free vortex theory helps us to translate that analysis with the constraint that we are having negligible change in axial velocity, we are having negligible change in the total enthalpy in the radial direction. So, we can take that C w into R is equal to constant. So, these are the topics which we will be needing for understanding the axial compression. And here we end the topic on axial compressor. We will meet for the next topic in the next class.

Thank you.