IC Engines and Gas Turbines

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Lecture – 44
Thermodynamics Analysis of Centrifugal Compressors

Welcome to the class in the last lecture we had seen that the velocity triangles how do they exist for the Centrifugal Compressor. So, we are in the phase of understanding centrifugal compressor; in that case we have seen that centrifugal compressor has possibility of 2 types of inlet velocity trackers.

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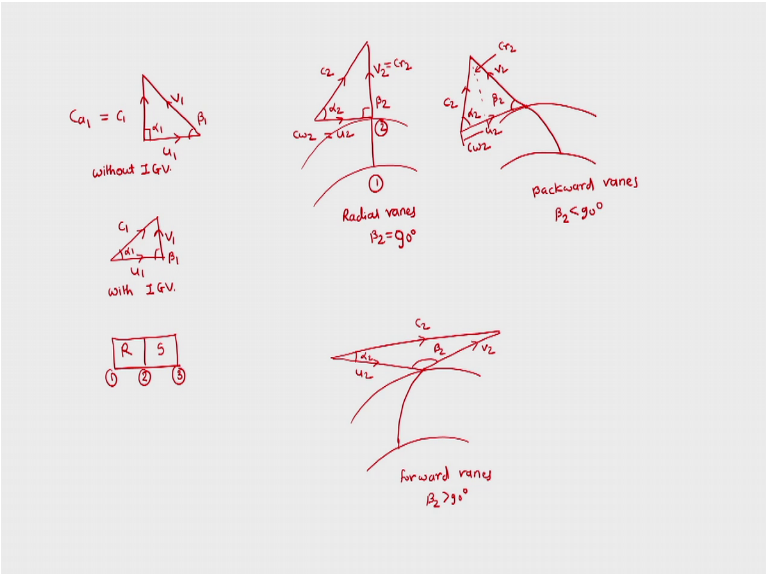

In one case we can have inlet velocity which is absolute velocity which is c 1 which can be axial velocity and then that velocity; that velocity is the inlet velocity at the inducer. And then, but the flow actually enters tangentially to the blades and then this is our u 1 rotational speed, so in this case we had seen that absolute velocity is perpendicular to that u, this is without inlet guide vanes this is without inlet guide vanes which we have seen without inlet guide vanes. But with inlet guide vanes we have instead of this we have absolute velocity which is coming like this due to the inlet guide vanes we initially had this u 1. So, we have this as v 1 and then this is c 1.

So, this is alpha 1 this is beta 1 here this is beta 1 and this is alpha 1. So, this is with inlet guide vanes this is what we have seen in last class. Then what we have seen that apart from these two possibilities of inlet velocity triangles we had 3 possibilities of outlet velocity triangles and then very first simple was the case of radial vanes. If we have radial vanes then what we would have is the velocity which is going out is v 2, but then we had this as u 2.

So, this will turn out to be c 2 and then if this is alpha 2 and then this is beta 2. Just for reminder what we had we are considering is the centrifugal compressor; so for centrifugal compressor we know that for any compressor we have first rotor and then we have stator. So, 1 to 2 to 3 this is what our arrangement is; so 1 to 2 what we are talking about this is 1 and this is 2. We are talking at the rotor inlet as 1 and we are talking about rotor outlet as 2. So, this is for radial range or we also know it as beta 2 is equal to 90 degree.

But in other case if we have the vanes to be non radial; then we had seen that in a case where they are backward faced; then in that case first we had this as u 2. We have to remember that u 2 is parallel to the arc of the circle whatever we are drawing. And then this is v 2 and then this turns out to be the c 2; v 2 velocity is tangential to the blades, u 2 velocity is tangential to the arc; since u 2 is the tangential velocity. And then this turns out to be alpha 2, this turns out to be the beta 2 or rather this turns out to be the beta 2 sorry ok.

So, after having a radial blades we had also seen that there is a chance that there will be blades which are backward facing blades. So, in case of backward facing blades we have a different velocity triangle. So, first we have u 2 and then we have v 2 and then we have c 2. So, this is the possible velocity triangle for backward facing vanes. So, this is beta 2 and this is alpha 2; here we had seen that this is backward, backward facing vanes backward vanes.

And this is beta 2 which is less than 90 degree, in this case basically v 2 is equal to cr 2 here this is cr 2 and then this is cw 2. But in now we will draw for the as what we have drawn earlier for forward facing vanes; we have this, we have this as u 2 and we have this as v 2 and so it turns out to be this as c 2. So, where we get beta 2 is more than 90; so this is forward vanes where beta 2 is greater than 90 degree.

So, we had seen that there are 3 options for the outlet velocity triangle and there are 2 options for the inlet velocity triangle. Having said this we should now find out what are the outcomes of this velocity triangles.

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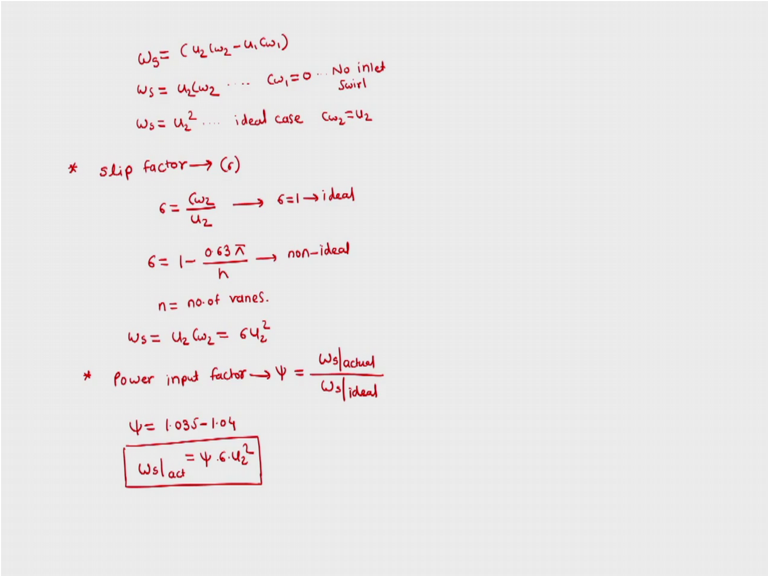

First is we should know what is w for a stage that is work input for a stage and that work input for a stage we knew that m dot into u 2 c w 2 minus u 1; c w 1, but we will consider it to be specific work input. So, for specific work input we will not have m dot. So, we will have u 2; c w 2 minus u 1 c w 1. Now when we consider the inlet velocity to be coming axially then we have a chance that we might not have in general we might not have inlet one velocity c w 1 might be 0 you know the simplified case.

So, we can neglect this inlet cw 1 repeated 0. So, if there is no inlets 1 then c w 1 is equal to 0 and then in that case w s becomes u 2; cw assumption is we have c w 1 is equal to 0 which is no inlets swirl; no inlets swirl. So, in the case of no inlets swirl we will have swirl velocity or whirling velocity at the inlet is 0, but here we would have some outlets swirl velocity u 2 cw 2. Practically speaking if we go back and if we see the radial case then in this case u 2 is actually equal to c w 2 in radial case ok.

So, having known as u 2 is equal to c w 2 we can write w stage for the centrifugal compressor is equal to u 2 square; this is for ideal case ideal case where c w 2 is equal to u 2. But it is not valid in general since c w 2 is not equal to u 2, there is a reason for that and the reason is called a slip factor. There is a term called a slip factor and this has to do with inertia of the flow. The flow has certain inertia and due to that inertia it is not going to follow faithfully with the blades.

So, the blade has velocity u 2 at the outlet of the vane, but the flow will have velocity cw 2; which is not equal to u 2 and then associated difference or associated ratio between the 2 velocities is called as slip factor; which is accounting the inertia. So, inertia slip factor which is sigma and this sigma is equal to c w 2 upon u 2 and in ideal case sigma is equal to 1. However, this there is a correlation which said that sigma is equal to 0.63 pi divided by n for non ideal case; a real case where n is equal to number of vanes. We can see from here that as number of vanes are increased; we will get higher value of sigma ok.

So, inertia effect will be reduced and this should be pointed out here that there is this effect of inertia will leads to sigma which is slip factor. But there is no role of the frictional loss here sigma is due to inertia. So, we would have w s which is stage work is c u 2; c w 2 is equal to sigma u 2 square; this is the stage work. But there is one more problem which is due to friction and then that is called as psi which is called as power input factor.

So, this is one and then there is other which is called as power input factor and that is psi. And psi is defined as actual work upon ideal work; so psi is equal to actual w stage actual w stage actual divided by w stage ideal. And then this psi is mainly due to the frictional loss or friction between the vanes and the casing. And this psi has a range which says that psi is going to vary between 1.035 to 1.04; the psi accounts working frictional measure.

So, in general w stage actual what we are having is equal to psi into sigma into u 2 square. So, this is the formula for work input for a stage for a centrifugal compressor where u 2 will be the velocity at the tip of the impeller and sigma is the slip factor and psi is the power input factor. So, having said this now we have to draw the TS diagram for the centrifugal compressor.

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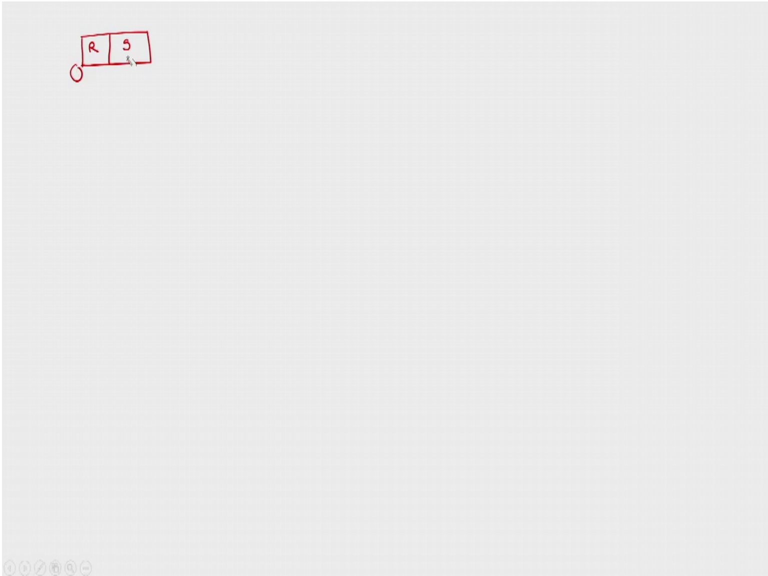

Again we will remember that we have in the centrifugal compressor, we first have rotor and then we have stator.

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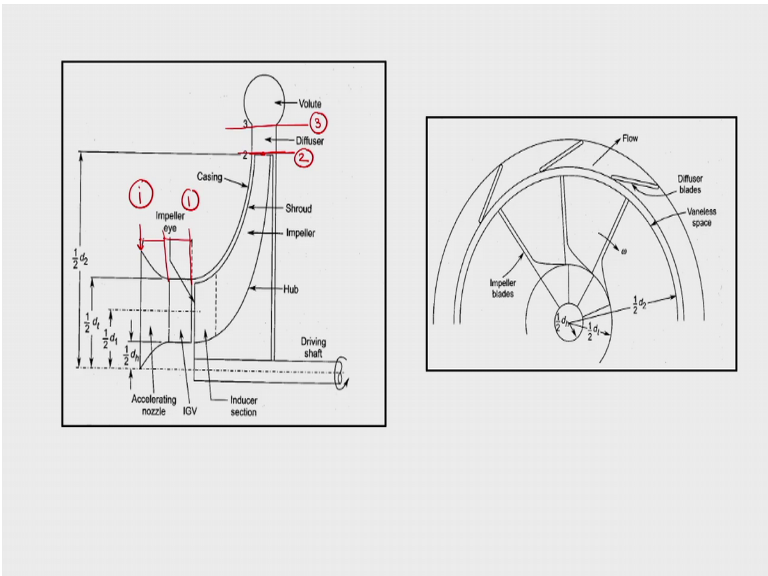

So, if we go back and then see if we go back and see the; see the diagram what we had seen earlier which is the configuration of the centrifugal compressor; where we can see that this is the I inlet of the centrifugal compressor. And then till this point from here to here we have the inlet accelerating nozzle from here to here we have inlet guide vane.

So, these parts where there; however, we did not consider them for our velocity triangle analysis and we consider them for the rotor outlet, we drew the velocity triangle at the section 2. We had drawn the inlet velocity triangle with and without inlet guide vanes. So, we see that this is i which is inlet; this is 1, which is at the outlet of the inlet guide vane and this is 2 which is at the outlet of the impeller and then this is 3 which is at the outlet of the diffuser. So, we will try to draw the velocity; the TS diagram for the centrifugal compressor with these points in mind where first flow is going from i to 1, 1 to 2 and 2 to 3.

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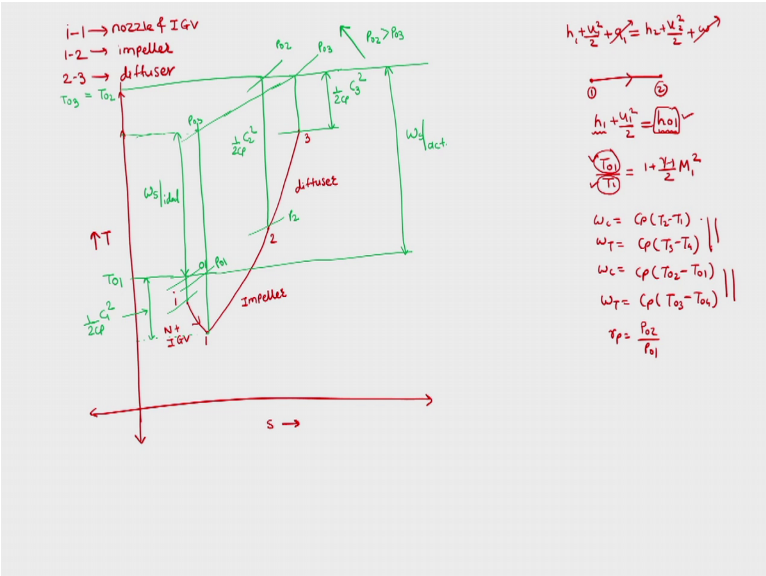

So, let us continue that and then see. So, we have flow which is i to 1 then 1 to 2 and then 2 to 3. Here the flow is between accelerate in nozzle flow is between nozzle and inlet guide vanes it is in the impeller and it is in the diffuser. So, i to 1 1 to 2 and 2 to 3; so this is our flow passage. So, for that all let us try to draw the TS diagram.

So, here we have y axis as T, which is temperature and x axis which is S as entropy; before drawing the TS diagram we will try to say understand one point here which is called as stagnation quantities and here that is why this temperature onwards would be represented as not as T but T naught; in some cases what is the stagnation condition.

Now we have written the steady flow energy equation as h plus v square by 2 or u square by 2 plus q is equal to 1 between two stations 1 and 2; we stated that it is h 2 plus u 2 square by 2 plus w. So, there is some work and heat interaction between two stations 1 and 2 if flow is going between two stations; this is general streamline.

And then here we see that if there is no heat interaction flow is adiabatic if there is no work interaction. And in this case if we feel that we can isentropically stop the flow at station 1, then what would happen? The velocity of the flow will become 0 and then that velocity which is decreased to 0 would help to rise the enthalpy. So, afforestation 1; if you isentropically stop the flow in an imaginary manner then we will have h 1 plus u 1 square by 2 is equal to h naught one. So, this is the stagnation enthalpy for the station 1; so h is called as static enthalpy. We have in the thermodynamics now there are two types of terms which is called as static and stagnation.

So, in the thermodynamics we just know what is the static temperature, static enthalpy, static pressure. But when we try to account it with gas dynamics then we have to account the kinetic energy associated with that static temperature and then if we add the enthalpy and the kinetic energy then the resulting term would be called as stagnation enthalpy. So, accordingly we get a relation which is therefore, the relation between T naught 1 and T.

And then the isentropic relation famous relation for T naught 1 upon T 1 is equal to 1 plus gamma minus 1 by 2 m square. So, this is the relation between the total temperature and static temperature; this relation can be proved from this expression where we are taking h is equal to C p into T where C p is assumed to be constant gas is assumed to be calorically perfect. And then accordingly if we derived n we will relation which is 1 plus gamma minus 1 by 2 m square; so here we are having static temperature here we are having total temperature. Whenever we were interacting for the Brighton cycle in earlier chapters; we were considering TS diagram and for the TS diagram we were considering only static temperature.

So, for us compressor work was C p into T 2 minus T 1 our turbine work was C p into T 3 minus T 4. This was a simplified analysis where we had also stated in that time that in both the cases we are considering gas to be calorically perfect plus we are considering flow to be adiabatic plus we are considering that the kinetic energy is negligible in terms of the enthalpy. Since kinetic energy is negligible in terms of enthalpy; we had neglected the temperature to be considered as total temperature.

And instead of that we had considered the static temperature, but now we have to consider the energy which is complete energy and where we have to consider the kinetic energy as well. So, if we consider kinetic energy then compressor work will be treated as C p into T naught 2 minus T naught 1 and turbine work will be treated as T naught 3 minus T naught. The difference between those two formulas is just that we are considering the kinetic energy. So, when we replace the static temperature by total temperature; we have started accounting the kinetic energy.

Similarly, when we consider the pressure rise or pressure ratio, we had considered p 2 by p 1, which is the pressurize in a compressor, but here while dealing with the components since we have to find out their efficiencies; we will try to deal with the total condition instead of the static condition. So, our pressure ratio rp will also be P naught 2 upon P naught 1 ok. So, having said this as preliminary information for the stagnation conditions; we will use this for the analysis of the centrifugal compressor.

This part whatever we have dealt with is part of the course which is gas dynamics where it is elaborately treated that what do we mean by static condition and what do you mean by stagnation conditions. Here we would need these concepts for understandingly flow through the centrifugal compressor or axial compressor or this turbine. So, for that we just got the introduction of this point which is called a stagnation point or static temperature or stagnation conditions. Stagnation conditions are also called as total conditions, so we have two kinds of names to this we either will call h naught as the stagnation enthalpy or we will call it as h naught as total enthalpy ok.

So, for that all sake we were here at i from i to 1 we would go in the nozzle, but in that case flow is accelerating since flow is accelerating its temperature will decrease, but then there would be losses; so entropy would also increase. So, we will come from i to 1 in this phase and from 1 to 2 the flow is going through the impeller. So, while going through the impeller we are having the temperature rise; some part of the temperature rise some part of the enthalpy rise would be happening into the impeller and then we had 2 to 3 as the diffusion.

So, we are having nozzle plus inlet guide vane, we are having impeller, we are having diffuser ok. So, this is the velocity diagram this is the TS diagram for our centrifugal compressor, but this is not complete. Now if we consider the flow to be isentropically stopped flow to be isentropically stopped; then what we can get is this as T naught 1. So, we would have T naught 1 as the total temperature over here and then this can further we can further find out the total temperature, but this is this height is the kinetic energy corresponding to it. So, it is half C p into c 1 square. So, we would have stopped this kinetic energy that is why we have got naught. So, this is P naught 1 and then this is P 2.

Now we will stop the flow at 2 to go to the isentropic state and then we stagnation state. As we have seen that we are considering that by imaginary process which is an isentropic process, we are stopping the flow to go to the stagnation conditions. So, for that all sake we will reach here at P naught 2 and then corresponding temperature would be T naught 2. However, in this case we will reach here and this is T naught 2 and then this is half C p into c 2 square.

Similarly, if we stop isentropically at point 3; then also we will get same total temperature which is T naught 2 is equal to T naught 3. Since there is no work interaction between point 2 and 3 flow is going through the diffuser, but there is loss in total pressure due to friction in this direction total pressure increases. So, P naught 2 is greater than P naught 3; there is loss internal pressure. Since there is loss in total pressure we are having friction as a reason for that.

And then this is since flow is not completely stopped 1 upon 2 C p; c 3 square which is the velocity at the outlet of the diffuser. So, in all what we would have is we were here at naught and we reached here at naught. So, this is the actual shaft work what stage were what is supplied to the compressor actual. But ideally if we would have to go to this P naught 3; then we would have to supply this work.

So, this would have been the ideal; ideal stage work. So, this is the TS diagram for the centrifugal compressor. So, having said this now we will proceed we need these are total temperatures evaluations in the examples to be solved. So, only we have tried to represent them in terms of total temperatures; now our objective is to find out the actual temperature rise in the compressor.

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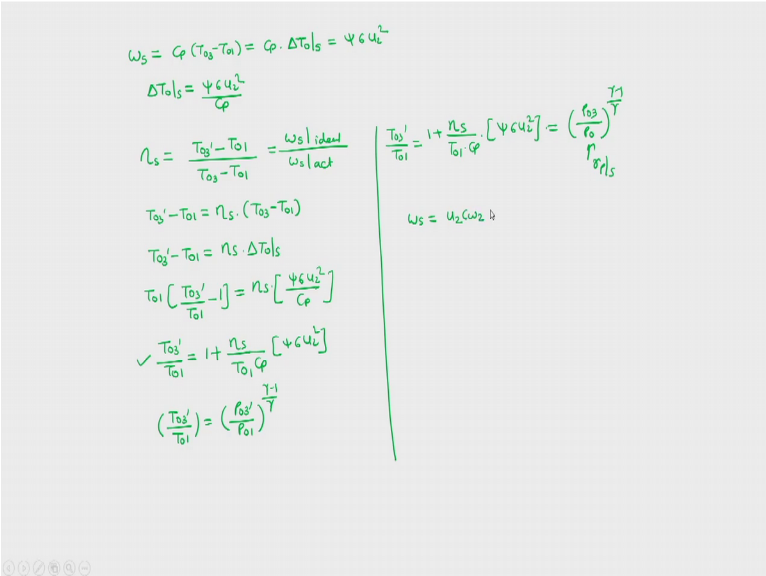

So, we know that stage work this specific stage work; whatever we have supplied to the compressor is C p into T naught 3 minus T naught 1 which is C p into delta T naught stage in actual. And then that is equal to psi sigma u 2 square; we have to keep it in mind this psi sigma 2 square is a specific number corresponding to radial vanes. And we are considering psi which is a factor corresponding to friction then we are considering sigma, which is again a factor corresponding to inertia. So, considering these two factors we have found out psi sigma u 2 square for the stage work which is actual stage.

So, we got delta T naught in a stage is equal to psi sigma u 2 square upon C p. Then if we know velocity triangle and then we know psi sigma; then we can find out what is the temperature rise in the stage in actual case. But now we are interested in finding out what is the pressure ratio pressure rises in the case of centrifugal compressor from the velocity triangle.

So, for that we know stage efficiency of the compressor is T naught 3 dash minus T naught 1 divided by T naught 3 minus T naught 1. Actually what we mean for stage efficiency is w stage ideal divided by w stage actual. So, for w stage ideal we are having C p into delta T naught which is at ideal and divided by C p into delta T naught which is actual C p C p has got cancelled; so we have this as the formula for stage efficiency.

When we were considering earlier thermodynamic cycle, we were not considering total temperatures; we were considering static temperatures; however, this is an improvised formula where we are considering kinetic energies also. So, T naught 3 dash minus T naught 1 is equal to stage efficiency into T naught 3 minus T naught 1. So, T naught 3 dash minus T naught 1 is equal to stage efficiency into delta T naught stage in actual case. So, we will take T naught 1 as out common.

So, we will have T naught 1; we would have T naught 3 dash upon T naught 1 minus 1 is equal to stage efficiency. And we know that delta T naught stage is equal to psi sigma u 2 square upon C p and then we have T naught 3 dash upon T naught 1 is equal to 1 plus stage efficiency upon T naught 1; stage efficiency upon T naught 1 into C p divided by into sigma psi sigma u 2 square; this is isentropic temperature rise in a stage.

And for isentropic temperature rise we have a formula where T naught 3 dash upon T naught 1 is equal to P naught 3 dash upon P naught 1 bracket raised to gamma minus 1 gamma which is the pressure rise in a stage. So, we have 1 plus; we have T naught 3 dash upon T naught 1, 1 plus stage efficiency divided by T naught 1 into C p into psi sigma u 2 square. And then this is equal to P naught 3 upon P naught 1 bracket raised to gamma minus 1 upon gamma. And then this is the stage pressure ratio which is r p stage this is what we had to be found out; so this is what we were interested in.

So, having known the velocity triangle what else we can do? First we can find out the work required for the given compression from the velocity triangle; from known velocities we can find out what is the work input to the compressor. Then we can find out the temperature rise in the stage then we can find out pressure rise in the stage; all these things we can find out for the stage.

And then having said this we will introduce one more factor one more coefficients for the compressors. And then for that let us write down again ws formula what we had written down and then for that initially without ah; we had formula will go to the next slide.

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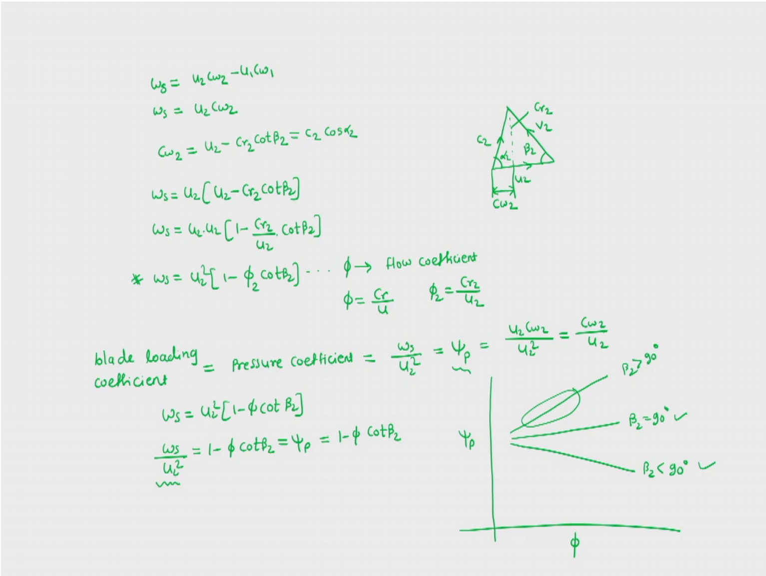

We had written down w s stage is equal to u 2 cw 2 minus u 1 cw 1 when we say that we will neglect the inlets well we will we say that ws is equal to u 2 cw 2. And if we go back and find out what is cw 2; if we try to find out what is cw 2 then we will go back and see what is cw 2. So, what is c w 2? c w 2 is this; so if you take a typical case of backward facing vane then c w 2 is u 2 minus this. So, c w 2 we will write down this velocity triangle here.

So, our velocity triangle was this was u 2, this was v 2 this was c 2 and this is c r 2 and this is beta 2 and this is alpha 2. So, this is further and then this is c w 2. So, for us c w 2 is equal to from this velocity triangle it is u 2 minus this distance which is c r 2 cot beta; cr 2 cot beta 2 which is otherwise c r 2 cos alpha 2; otherwise c R 2 which otherwise c 2 which is basically c 2 cos alpha 2 or otherwise u 2 minus cr 2 cot beta 2.

Then we will use this formula w s which is a stage work is u 2 into u 2 minus c r 2 cot beta 2 ok. Now we can take u 2 common w stage; this formula whatever we are writing here we are not considering any slip factor, here we are also not considering any power input factor. So, u 2 into u 2 into 1 minus c r 2 upon u 2 into cot beta 2; so stage work is equal to u 2 square 1 minus phi 2 cot beta 2. Here we have introduced a new term which is phi and it is called as flow coefficient.

So, phi 2 is the flow coefficient at the impeller tip or at the impeller exit and then phi has a definition which is cr divided by u; so phi 2 for us is cr 2 of 1 u 2; so this is flow coefficient. Now there is one more such coefficient and which is called as blade loading coefficient or it is also called as pressure coefficient for the compressor; centrifugal compressor.

And this pressure loading coefficient has a definition which say that it is a ratio of stage work divided by the square and this is named as psi p pressure coefficient which stage work divided by u 2 square. We know that stage work is cw 2 w to u 2 which is rather u 2; c w 2 divided by u 2 square; so it is c w 2 upon u 2. So, this is our formula for psi p which is pressure coefficient.

So, knowing this formula we can write down the expression for c w 2 upon u 2 rather cw 2 upon u 2 can also be written as 1 minus, we can see over here cw 2 u 2 is equal to 1 minus; in this case if we write down this expression then we have ws is equal to u 2 square 1 minus phi cot beta 2 then we have ws upon u 2 square is equal to 1 minus phi cot beta 2, but this is equal to psi p.

So, we have psi p is equal to 1 minus phi into cot beta 2. This pressure coefficient is helping us to compare the different centrifugal compressors and we can have an analysis using this pressure coefficient. But there is an interesting curve which is a characteristic curve for the pressure coefficient with respect to flow coefficient. And then this we can note we can see that this is a linear relation between them for a given beta 2; which is fixed and then beta 2 is equal to 90 degree for radial beta 2 is greater than 90 for forward and beta 2 less than 90 for backward case; backward range.

And we can see that phi p is decreasing with respect to phi in case of backward vanes phi p is constant in with in case of radial vanes and phi p is increasing in case of forward vanes. And then this gives us hint that this is an unstable design and that is why this would not be preferred. But the designs corresponding to radial and for backward facing means are stable since phi p is decreasing with respect to phi; so these designs will be preferred.

So, this is how we can analyze the centrifugal compressor and here we end the part of the centrifugal compressor and we will see other compressors other compressor which is axial compressor in the down the line.

So, thank you very much for the today’s class.