**IC Engines and Gas Turbines**

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**Lecture – 42**
**Eular Turbomachinary Equation**

Welcome to the class. We have till time considered the thermodynamic aspects of the Brayton cycle and in that we have considered how the T-s diagram, PV diagram would look like for the Brayton cycle. How to find out the corner properties? And then we have evaluated different attachments and their performance.

So, having said all those things then we move toward the application of Brayton cycle which was in the examples we have seen that how does it give us w net which would be given to the generator for electricity generation. But, then we have seen in last class for the case where Brayton cycle is used for aircraft propulsion where w net would be the 0.

So, the enthalpy which was remained was used for the creating jet through the nozzle. So, after these things we will move next part which is the parts of Brayton cycle, which are compressor mainly and the turbines. So, in that when we are having compressors or turbines we have to again do their microscopic analysis or system analysis for the compressor and turbine.

So, in view of that first objective of today's class is to derive for the expression which is the Euler's turbine pump, on Euler turbine pump equation. So, for that Euler turbine pump equation we will go ahead and derive what does that equation wants to say to us.

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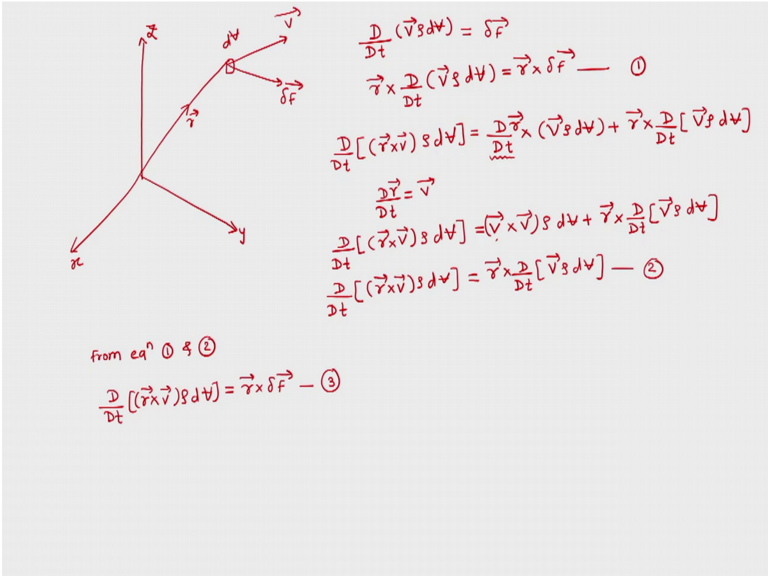


So, for that let us consider first a particle in a frame of reference, such that it is moving with velocity V these are the 3 directions this is the elemental particle which is moving with velocity V in this direction it is acted by force delta F in this direction. And, then this is the position vector of the particle and then this particle is in this frame of reference.

Now, we are going to use this as our reference case and write down the Newton's equation. Newton's second law when we want to write for this particle we would get D by Dt; D by Dt of V rho dV dV is the; dV is the volume of the particle elemental volume of the particle and rho is the density of the particle is equal to delta F. So, this is the Newton's second law as what we know.

Now, let us take the moment of this equation this is basically a moment in the equation. So, for that if we take momen, then moment is with respect origin of the coordinate system then r cross D by Dt of V rho dV is equal to r cross delta f, but from the identity we can write this equation we will give number 1 this is our equation number 1.

But from identity we can write D by Dt of r cross V rho dV is equal to D by Dt of r cross V rho dV plus we have r cross D by Dt of V rho dV this is the identity for the cross bar. Knowing this identity we got a term which is Dr by Dt, but we know that Dr by Dt is V which is rate of change of that position vector is a V velocity vector.

So, from that we can put here as capital D by Dt which is a material derivative for the particle r cross V rho dV is equal to V cross V rho dV plus r cross D by Dt of V rho dV, but we know that this cross product of the vector with itself is 0.

So, this term goes out, so we get D by Dt of r cross V vector rho dV is equal to r cross D by Dt of velocity vector rho dV. Now, this if we see equation number 1, then the left hand side of equation is here in equation 2 as right hand side. So, we can write from equation 1 and 2; from equation 1 and 2, we can write down that the D by Dt of r cross V rho dV is equal to r cross F delta.

So, this is our equation number 3, now this is our equation number 3 which state that a rate of change of momentum of the particle and its moment rate of change of movement of the momentum of a particle is equal to rate is equal to the moment of the force applied on it. So, having known this equation we can further reduce this equation, we can further change this equation for the system, but for that we have to apply the Reynolds transport theorem. So, we will say that let us apply Reynolds transport theorem.

Basically what is our target over here? We have derived this expression for one particle which is of volume delta V and density rho, but in the case of turbine and compressor we will have in finite number of particles which would be entering in which would be leaving this system. So, we have to write down the expression for a system and not for a particle. So, Reynolds transport theorem helps us, to write down the system equation with the help of the equation what we would have written for the particle.

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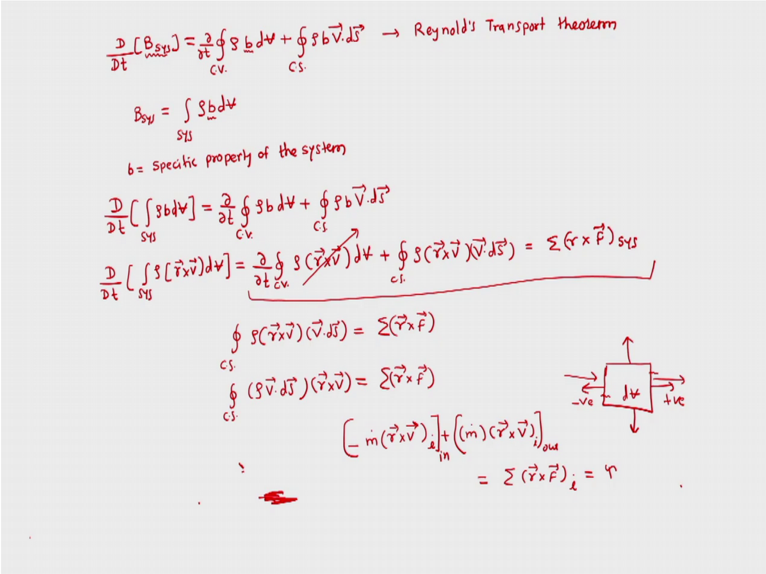


But for that, let us first write down what is the Reynolds transport equation? Reynolds transport theorem says that material derivative D by Dt of some property B of the system is equal to rate of change of some property of the particle maybe is equal to integral dou by dou t of CV Control Volume rho b dV.

B property is here which is a specific property B, B is a total property of the particle and this b is the specific property of the particle and then it is equal to control surface rho b V dot s. This is if a particle is having property B and then it is changing with respect to time, but when this system matches with the control volume.

Basically here we are considering that the system is composed of infinite number of particles and the laws what we can write for the system can be applied for the right hand side which is for the control volume and for that movement system and control volume both are matching with each other. So, the B system is basically the property which is summed or all the particles of the property all the particles of the system.

So, B of a system basically B of system is equal to integral over the system rho b dV. So, if we integrate it over the all particles for a system let us say b b is some property of the system, which is a specific, so b is a specific property of the system. So, if we consider momentum, then we have to consider that this is the momentum of the system then we have to consider b should be specific momentum, so it will be V and rho dV is the mass of a particle.

So, mass of a particle into velocity of a, so if we integrate over it for the all particles then it becomes momentum of the system and this momentum of a system is changing with respect to time as it progresses, but for that moment system is mapped with surrounding or control volume. So, we can write down this using Reynolds transport theorem.

So, this is Reynolds transport theorem. So, according to this now we can write down this as D by Dt of integration over system rho b dV is equal to dou by dou t of control volume rho b dV. This equation whatever we are writing is a general form of Reynolds transport theorem, we will evaluate it with respect to our equation number 3.

So, now, compare this equation with our equation number 3, this is our equation number three. So, we are having D by Dt of some terms which are inside the differential and then we can see here that rho dV is the same thing which will appear in the Reynolds transport theorem, but b is sitting over here, so, this is our b. So, our b is r cross V our b is r cross v, so we can write down D by Dt of system rho r cross V dV is equal to dou by dou t of control volume rho r cross V dV plus control surface rho r cross V V dot dS this is our equation.

And then we will further know that from equation number 3, this is just the left hand side, but right hand side is r cross dF, but that also needs to be integrated over the system. So, for that will be like this is equal to r cross F for the system; here r cross F is the movement of the force which is acted upon the system, earlier F was the force acting on the particle and r cross f was again the movement of the force acting on the particle, but now when we integrate all such movements for the particles.

And then now instead of considering the forces acting on the particle we are considering the F as the force which is acting upon the system at a location where r is the position vector such that r cross F is the movement of all those forces acting on the system which would ultimately lead the same movement, which otherwise would have considered if the forces were considered for the particle. So, having said this we got this as our expression which states that, now this becomes our expression. Earlier we were considering it for the particle and then we wrote down this for the particle.

But now we are trying to write it for the control volume since we consider the turbine hand pump as our open systems where control volume analysis would be done. So, for that all sake using the Reynolds transport theorem, we reduced the equation of the particle equation for the particle to the equation for the control volume. So, now, this is our equation moment of momentum equation which say that rate of change of moment of momentum and the summation with the fluxes is equal to the moment of the forces in submission over the system.

So, then but now, we will consider a steady state system. So, if the system is in steady state this part of the equation would vanish, then we will get rho r cross V into V dot ds is equal to summation r cross f for the control volume. Then this is for the control volume, so summed all the forces for the control volume which are acting upon ok. So, now, this is a vector within that we have basically system control surface, we have rho V dot ds and then r cross V is equal to r cross F. Then this is mass flux rho V dot ds is the mass flux elemental mass flux going with small elemental surface area ds.

Now, we are going to integrate for all such surfaces, so practically what we would have is summation of the fluxes of the moment of momentum going through small surfaces and then that would be equal to the moment of forces acting on the control volume. In view of this we having this vector equation can decompose it into number of components and we will consider for the turbo machine we are interested only in the tangential part.

So, if we consider this to be in tangential for the force which is leading to the torque which is in the axial direction; so, the movement which is created in the tangential velocity, which would create it in the axial direction. So, all the component in this direction if we make then we can write down this to be summation of all the masses elemental masses through such surfaces.

And then their cross product of the position vector with velocity so, but for that now we will consider for the turbine or for the compressor there is only 1 inlet, there is only 1 outlet. So, we will have only 1 inlet and we have only 1 outlet. So, we have only 1 inlet, so for the inlet V cross ds will be if this is our control volume for that all sake and then this control volume has this as control surfaces where these are the normal to the surfaces which are outward and now we will consider the velocity vector.

So, for this inlet this velocity vector is positioned such that velocity flux is coming into the control volume of volume dV and then this velocity vector is going out of the control volume. So, this surface turns out to be the inlet this surface turns out to be the outlet, but here velocity vector and normal vector both are in opposite direction. So, V dot ds will have negative sign here and for outlet V dot ds will have positive sign since both the vectors are aligned with each other.

So, what we would have is minus m dot and then we will have r cross V in a particular direction of our interest plus for the outlet; for the outlet we have m dot into r this is for inlet and then r cross V for outlet in a particular direction of our interest is equal to summation r cross F between inlet and outlet in a particular direction of our interest and that is what we will call it as torque and this is torque of our interest.

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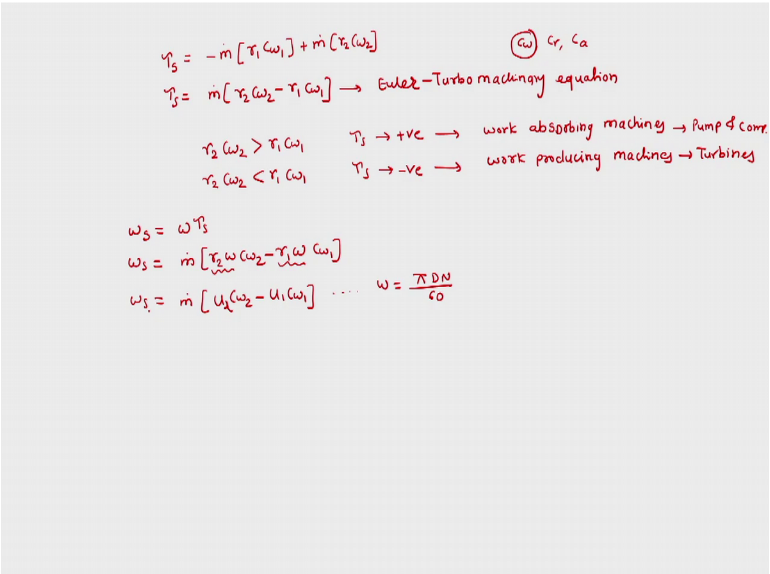


So, what we got is torque or shaft torque acting on the shaft or rotor of our interest is minus m dot, but now we are considering the steady state of the system. So, since we are having steady state we consider same mass flow rate at the inlet and outlet. So, what we would have is m dot, now we are considering the velocity and the position vector which would create the tangential velocity, so we will have r 1 C w 1 plus m dot r 2 C w 2, for the V velocity of the part of the system we have decomposed it into 3 components basically velocity v, whatever we have considered till time for the derivation was basically having 3 component one is whirling the component, one is axial component another is the radial component.

So, C w is whirl component, C r is a radial component and C a is axial component of the velocity till time what we were considering. So, for that we are having C w which is a whirling component or which is a rotating component of the velocity and it is the radius r of the corresponding inlet or outlet for the system for the control volume. So, this turns out to be the shaft which is torque acting on the shaft.

So, tau shaft is equal to m dot r 2 C w 2 minus r 1 C w 1. So, the velocity vector what we had considered as 3 components and this has led to this as our equation and this is called as Euler turbo machinery equation or Euler turbine pump equation. Here it is evident that if r 2 C w 2 is greater than r 1 C w 1, means outlet is having more; outlet is having more r 2 C w 2, then the inlet then what we would have is tau s as positive and if r 2 C w 2 is less than r 1 C w 1 we will have tau s as negative ok.

So, this is for the system, so this is what it would lead to the fact for us and then we can know that the machines which are work absorbing machines and then there are machines which are work producing machines. So, for the machines which are work absorbing machines what we will have is basically this. So, this r 2 C w 2 and tau shaft will be positive for the work absorbing machines and here this will be negative for work producing machines.

So, we will have this for pump and compressor and this is for turbine and accordingly we will change this equation by considering this to be positive or negative accordingly. Since we are interested in the magnate of the shaft power, having said this we can next move to find out the work. So, w which is a shaft work is equal to omega which is the angular velocity into shaft power.

So, shaft work is equal to m dot into r 2 omega C w 2 minus r 1 omega C w 1. So, r 2 omega and r 1 omega will lead to the tangential velocity at that location is equal to m dot u 1 rather u 2 C w 2 minus u 1 C w 1, where we know u is equal to pi DN by 60 omega is equal to pi DN by 60 ok.

So, this is RPM which would be given to us, then we can find out what is the velocity ok, in this case then we can find out we would have found out this power. Now, let us consider the fact that what is the idea of this expression, how can we interpret this expression of power, what we have derived? So, for that all sake let us consider a velocity triangle here onwards for our for that discussion.

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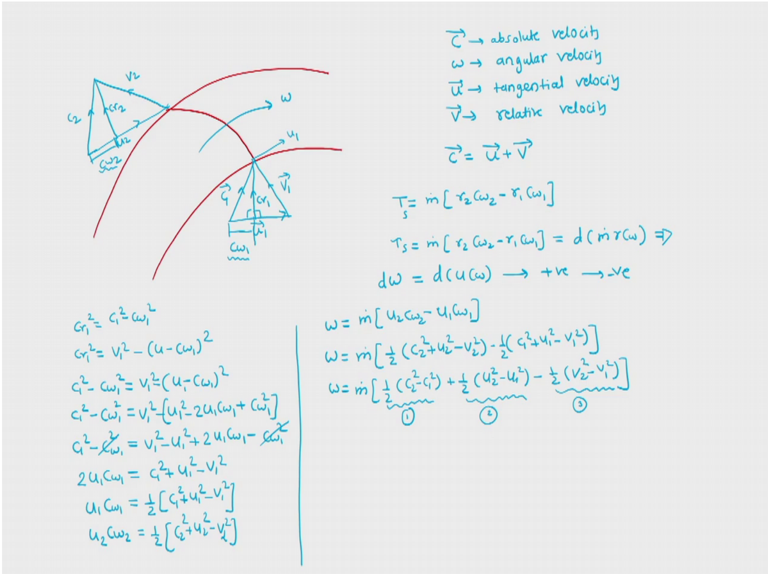


And as per that velocity triangle let us consider that, a general turbo machine, in that turbo machine this is the blade having this blade with us we will have this something as inlet. So, for that inlet we will have this as the velocity which is the relative velocity, this will be absolute velocity and then this will be the tangential velocity and this will be the radial velocity C r 1 ok.

So, here what we are saying C is absolute velocity and this blade is rotating with omega; omega angular velocity, u tangential velocity and then we have V as relative velocity. For that all sake what we would have is C which is absolute velocity is basically tangential velocity and relative velocity summation. So, the flow is going from inward prouder we do not want to define this turbo machinery as compressor or turbine we are just trying to draw the velocity triangle for the inlet and outlet of this turbo machinery.

So, that we would apply the turbo Euler turbine pump expression for this turbo machine. So, we have fluid which is entering tangentially to this blade with relative velocity V and this fluid has some absolute velocity c, but this point has tangential velocity which is u, but all r 1 which are at the inlet, so we were are saying that it is basically the 1 it is basically the inlet.

So, for that all sake we would have tangential velocity at this location is u 2, then we have tangential component with the blade as V 2 and then we have absolute velocity which is coming out as C 2 and then we have radial velocity as C r 2. So, in this case this is the absolute velocity C 1 and this C r 1. So, this component is C w 1 and here we are having this component as C w 2. So, this is the whirling component at the inlet this is the whirling component at the outlet.

So, now, we can write down the torque expression as what we wrote torque on the shaft is equal to m dot r 2 C w 2 minus r 1 C w 1. So we know C w 1, we know C w 2, so this were C w 1 and C w 2 values and then we are having torque T shafts, but if we try to write down this expression as tau shaft is equal to m dot r 2 C w 2 minus r 1 C w 1, then this is basically d of m dot D of m dot r C w which is basically D of for the power for the power we have power d w is equal to d of u C w and then this would turn out to be positive for the compressor and this turns out to be negative for the turbine ok.

But now we will see that how to use this velocity triangle to further derive the expression for the work input. So, for that all sake we have from the inlet velocity triangle C r 1 square this is right angle over here. So, C r 1 square is equal to C 1 square minus C w 1 square or C r 1 square is equal to V 1 square, this is also at right angle. So, V 1 square minus u minus C w 1 bracket square ok. So, these are the 2 relations for C r 1 square. So, what we can write is basically C 1 square minus C w 1 square is equal to V 1 square u minus C w 1 bracket square.

So, u C 1 square minus C w 1 square is equal to V 1 square minus u u 1 square minus twice u 1 C w 1 plus C w 1 square. So, we have C 1 square minus C w 1 square is equal to V 1 square minus u one square plus twice u one C w 1 minus C w 1 square. So, C w n square C w n square would cancel out. So, we get twice u 1 C w 1 is equal to C 1 square minus u 1 square minus V 1 square. So, u 1 C w 1 is equal to half C 1 square plus u 1 square minus V 1 square. If we apply the same philosophy at the outlet we should be getting u 2 C w 2 is equal to half C 2 square plus u 2 square minus V 2 square, having said this we can put this into the expression for work.

So, we know work is equal to m dot u 2 C w 2 minus u 1 C w 1. So, we know that w is equal to m dot half C 2 square plus u 2 square minus V 2 square minus half C 1 square plus u 1 square minus V 1 square. So, w is equal to m dot into half C 2 square minus C 1 square minus plus half u 2 square minus u 1 square minus half V 2 square minus V 1 square.

So, work done or work input for the turbomachinery is related with the 3 changes in the kinetic energy. So, there are basically 3 effects sitting into the turbomachinery first effect is called as impulse effect, where absolute velocities based kinetic energy changes between the inlet and the outlet of the turbo machinery or the rotor part.

In this second effect we are having the centrifugal effect, first is impulse effect, the second is the centrifugal effect and this velocity change related to that there is some kinetic energy change this is mainly due to change in the radius of the fluid element which is between the inlet and the outlet due to centrifugal force and then third effect is called as reaction effect a relative velocity of the fluid changes while flowing over the blade from the inlet to the outlet.

So, work done which we have arrived at the formula only from the velocity triangle or from the perspective of Euler turbine pump equation has basically 3 effects sitting in it which have impulse effect, centrifugal effect and reaction effect ok. And then these effect become dominant in one case and then they may be summed or each other, one of the effects would be dominant in one case or then there is a combination of all the 3 sets in some of the cases we are going to see all these things in detail in the following classes.

So, here we end today's class with the derivation for the Eula turbine pump equation for any turbo machinery and then this expression we will use for the particular cases here onwards.

Thank you.