

IC Engines and Gas Turbines
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Lecture – 39
Solved Examples for Real Brayton Cycle

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Example

A gas turbine has minimum and maximum temperature as 50 °C and 950 °C. Compressor and turbine efficiencies are 82% and 87% respectively.

Calculate: Condition for maximum Net work, Net work and Thermal Efficiency

Given →

$$T_1 = T_{min} = 50^\circ\text{C} = (50 + 273)\text{K}$$

$$T_3 = T_{max} = 950^\circ\text{C} = (950 + 273)\text{K}$$

$$\eta_c = 0.82$$

$$\eta_t = 0.87$$

$$\beta = \frac{T_{max}}{T_{min}}$$

$$(\gamma_r)^{\frac{\gamma}{\gamma-1}} = \sqrt{\eta_t \eta_c \beta}$$

$$\beta = \frac{T_{max}}{T_{min}}$$

$$\gamma_r \rightarrow \gamma_{r, optimum}$$

$$\gamma_r = \frac{P_2}{P_1} \rightarrow \frac{T_2}{T_1} = (\gamma_r)^{\frac{\gamma}{\gamma-1}} \rightarrow T_2 \rightarrow \text{ideal}$$

$$\eta_c = \frac{T_2' - T_1}{T_2 - T_1} \rightarrow T_2 \rightarrow \text{Actual}$$

$$W_c = C_p (T_2 - T_1)$$

$$\frac{T_3}{T_4} = (\gamma_r)^{\frac{\gamma}{\gamma-1}} \rightarrow T_4 \rightarrow \text{ideal}$$

$$\eta_t = \frac{T_3 - T_4}{T_3 - T_4'} \rightarrow T_4 \rightarrow \text{Actual}$$

Welcome to the class. In today's class we will see some more examples, but the examples what we have seen in last class were related with the ideal Brayton cycle, where we had most of the times considered that the compressor and turbine have highest efficiency which is 100 percent efficiency and then we solved examples accordingly.

Now, we are going to see the new example, that in the first example, it is said that there is a gas turbine which has minimum and maximum temperature as 50 degree Celsius and 950 degree Celsius. Compressor and turbine efficiencies are 82 percent and 87 percent respectively and we are supposed to find out maximum network, condition, then network and thermal efficiency.

So, the things which are given to us are T_{min} , which we are actually having, given conditions and in that we are having T_{min} which is 50 degree Celsius, so we have it as 50 plus 273 as Kelvin. Then T_{max} we are told as 950 degree Celsius, so it is 950 plus 273 Kelvin, so we are given with this. Then along with that we are given with isentropic efficiency of compressor is 0.82 and isentropic efficiency of turbine is 0.87.

So, this example is based on the derivation which we did for the network to be maximum condition. So, in that we had derived that $r_p^{\gamma-1} \gamma$ is equal to $\eta_t \eta_c$ into beta. Here beta is equal to $T_{\max} / T_{\min}$. So, we know that beta is the ratio of maximum and minimum temperature, but in this example maximum and minimum temperatures are given to us. So, practically in this example beta is also given to us; $T_{\max} / T_{\min}$. So, this is known to us. So, beta is known, η_t is known, η_c is known, so we can find out r_p . Basically this r_p is r_p optimum, so we know that what is the condition for maximum net work output. So, this r_p gives us the condition for maximum net work output. So, condition is known to us.

Now, what we can do is we can draw the T-s diagram. In this T-s diagram this is isentropic 1-2-3, this is ideal cycle and then this is 2-dash and 2 and this is 4-dash and 4. So, now, we know everything at T₁, we know only pressure at T₂, we know temperature as T₃ and we do know temperature at 4. So, we can use the formulas what we used and what we derived for getting the properties at all the corners.

So, in this case we know now pressure ratio. So, pressure ratio means, we know P₂ by P₁ and that is our r_p , and in this case it is optimum. So, r_p optimum is known to us. So, we can take initial condition as 1 bar, we will assume that then we know P₂, in that case once we know P₁, we know P₂, since we know r_p knowing this we can find out T₂ by T₁ and that is equal to $r_p^{\gamma-1} \gamma$. So, we know r_p which is r_p optimum, we know T₁ which is minimum temperature. Basically from the cycle this is T₁ and from the cycle this is T₃.

So, we know now T₂. From this formula we know T₂, so now, we know T₂ we know P₂. Once T₂ and P₂ are known anyway we know T₃, so then we can find out. But then there is a problem, this is T₂-dash is what we know, but we do not know T₂. So, for using we have to use formula for the compressor efficiency. We know compressor efficiency formula is ideal work $T_{2\text{-dash}} - T_1$ divided by $T_2 - T_1$

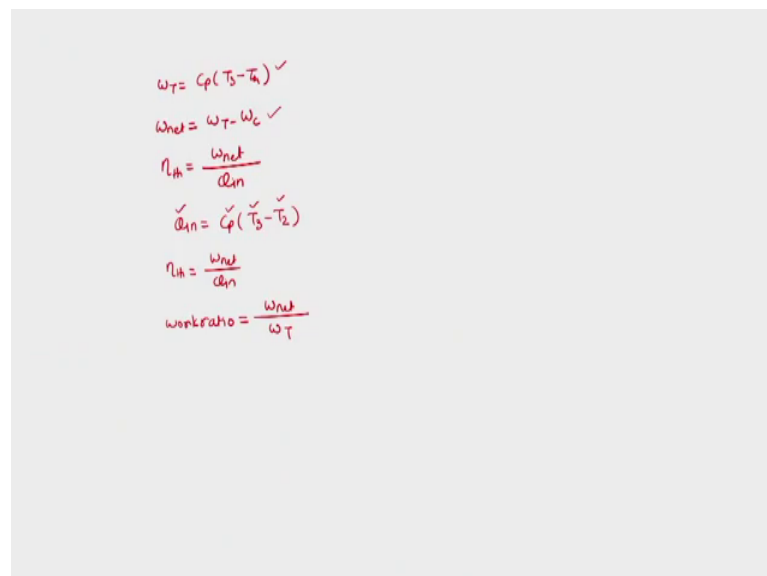
So, in this formula from the isentropic relation; T₂-dash is known to us, T₁ is known to us already, T₁ is also known to us here, compressor efficiency is given. Then this using the isentropic efficiency formula we can find out T₂. So, one point to be remembered here that isentropic formula are only usable for the ideal temperature at the exit of the

condition, but then we will use compressor efficiency to formula to get actual temperature at the exit of the compressor. So, then we know T 2.

Previously from isentropic we know T 2 dash, we know T 3. So, then basically we can know what is the work of the compressor w_c is C_p into T 2 minus T 1. We know air specific heat of air is 1.005 kilo joule per k Kelvin. So, knowing this we know the net compressor work. Having the compressor work known then we can find out turbine work, but for turbine work we should know T 4. So, to know T 4 basically first what we have to do, is we know r p.

Once we know r p we know T 3 by T 4 dash is equal to r p raised to gamma minus 1 upon gamma. So, since T 3 is known then we can find out from this formula T 4 dash. Once T 4 dash is known then we can use the formula for turbine efficiency. And turbine efficiency is actual work T 3 minus T 4 divided by ideal work which is T 3 minus T 4 dash. So, in this formula T 4 dash is known, T 3 is known, turbine efficiency is given, then we can find out T 4. Thus T 4 which is actual temperature is found out from the turbine efficiency formula and in case of isentropic we found out ideal temperature at the exit of the turbine. So, thus we can now find out the turbine work. So, w_T is equal to C_p into T 3 minus T 4.

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$$\begin{aligned}
 w_T &= C_p(T_3 - T_4) \checkmark \\
 w_{net} &= w_T - w_c \checkmark \\
 \eta_{th} &= \frac{w_{net}}{q_{in}} \\
 q_{in} &= C_p(T_3 - T_2) \checkmark \\
 \eta_{th} &= \frac{w_{net}}{q_{in}} \\
 \text{workratio} &= \frac{w_{net}}{w_T}
 \end{aligned}$$

So, here again we are using or assuming that air is the gas which is present in the turbine, so 1.005 into T 3 minus T 4; so, we got turbine work, then we know w_{net} which is

turbine work minus compressor work. So, this is how we can find out turbine work minus compressor work which is network, then it is told us to find out the thermal efficiency.

So, thermal efficiency of the plant is also simple to evaluate, since we know thermal efficiency is w_{net} upon Q_{in} , but the problem is we do not know Q_{in} . So, Q_{in} is equal to C_p into T_3 minus T_2 . So, it is not T_2 dash, you can see this is the process of heat addition. Heat is getting added from 2 to 3. So, heat is getting added from 2 to 3. So, T_3 is known, T_2 is known, C_p is known, so we know Q_{in} . So, knowing the Q_{in} we can find out thermal efficiency.

So, this is how we would have solved the problem. And this problem is a special problem, since this problem deals with optimum network condition, and this formula we have derived. There would be one more thing which some people would ask in the example and then that is work ratio.

So, work ratio, we have seen that work ratio is w_{net} upon w_T which is turbine work. So, turbine work is found out, w_{net} is found out, so we can find out work ratio. And we had seen long back in one of the classes what is the usefulness of work ratio. We know that work ratio has to be closer to 1. Higher the work ratio it is good, the plant is insensitive to or less sensitive to the component efficiencies we have seen that. So, this is done for this example.

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Example

A heat exchanger based gas turbine has minimum and maximum temperature as 300K and 1300K. Pressure ratio is 10 and mass flow rate is 10 kg/sec. Compressor and turbine efficiencies are 82% and 85% respectively. If the thermal ratio of the heat exchanger is 0.8

Calculate: Net work and Thermal Efficiency

$\hookrightarrow$ Heat exchanger
 $\hookrightarrow T_{min} = T_1 = 300K$
 $\hookrightarrow T_{max} = T_3 = 1300K$
 $\hookrightarrow r_p = 10.0$
 $\hookrightarrow \dot{m} = 10 \text{ kg/sec}$
 $\eta_c = 0.82, \eta_t = 0.85$
 $\eta_{HE} = 0.8$

$r_p, T_1 \rightarrow T_2' \rightarrow \eta_c \rightarrow T_2$
 $r_p, T_3 \rightarrow T_4' \rightarrow \eta_t \rightarrow T_4$
 $\eta_{HE} = 0.8 = \frac{C_p(T_2 - T_4)}{C_p(T_3 - T_2)} \rightarrow \text{Actual}$
 $\eta_{HE} = 0.8 = \frac{C_p(T_5 - T_4)}{C_p(T_3 - T_2)} \rightarrow \text{maximum}$

$\eta_{HE} = 0.8 = \frac{T_5 - T_4}{T_3 - T_2}$
 $\hookrightarrow T_5$
 $w_c = \dot{m} C_p (T_2 - T_1) \rightarrow \checkmark$
 $w_t = \dot{m} C_p (T_3 - T_4) \rightarrow \checkmark$
 $w_{net} = w_t - w_c \checkmark$
 $\eta = \frac{w_{net}}{Q_{in}}$
 $Q_{in} = \dot{m} C_p (T_3 - T_2) \rightarrow \checkmark$
 $\eta \checkmark$

So, for next example we will see that, in this example it is told that there is a heat exchanger based gas turbine power plant, and it has minimum and maximum temperature as 300 Kelvin and 1300 Kelvin. So, what is given to us in this example is, we are told that there is heat exchanger, it is told to us. And then it is told to us that $T_{\min}$, so for us it is equal to T_1 is equal to 300 Kelvin. We know that $T_{\max}$ is equal to T_3 is equal to 1300 Kelvin, then pressure ratio is ten. Then it is also told that r_p is equal to 10, mass flow rate is also 10 kg per second. We are told that mass flow rate which is $\dot{m}$ is 10 kg per second.

Now, we are given that thermal efficiency of compressor is 0.82 and thermal isentropic efficiency of compressor is 0.82 and isentropic efficiency of turbine is 0.85, but there is one more thing extra given that thermal ratio is 0.8 thermal ratio. I will denote it as capital T for heat exchanger and that is told as 0.8.

So, this is new example for us, where now we are having three components which is a compressor, which is a turbine and which is a heat exchanger and both are, or other all are non ideal. So, let us draw first T s diagram. So, in this T s diagram what we have is, first one, 1 to 2 dash, 2 dash to 2, 2 to 3, 3 to 4 and 3 to 3 to 4 is real process, 3 to 4 dash is ideal process. Then we have 5 here and then we have 6 here. So, these points correspond to the heat exchanger process. We know that 2 to 5 is the heat addition in the heat exchanger before going to the combustion chamber. So, this is heat exchanger, this is combustion chamber. So, 5 to 3 is only heat added in the combustion chamber

Now, we are supposed to derive or find out the network and thermal efficiency. So, for that all sake, let us try solving this example. What is known to us is T_1 , what is given to us is r_p . Again knowing the r_p , knowing the T_1 , we know that we can find out T_2 dash, this is what we have seen from the isentropic relation r_p raised to $\gamma - 1$ upon γ is equal to T_2 dash upon T_1 . So, T_1 is known r_p is known γ ; we are going to take it for air which is 1.4 for the sake of simplicity since it is again not told to us.

So, simplest assumption is for air, so we know T_2 dash. But now using the T_2 dash and T_1 we can use further compressor efficiency formula and find out T_2 . We know that compressor efficiency is ideal network input net, ideal work requirement divided by

actual work requirement. So, ideal work requirement is $T_2 - T_1$ and actual work requirement is proportional with $T_2 - T_1$, so this will give us T_2 .

Then we can parallelly do it for the turbine. So, we know r_p for the turbine, we know T_3 which is maximum temperature of the cycle and then from that using the isentropic relation we can find out T_4 . So, T_4 is known to us now from isentropic relation. Then we can use turbine work formula to find out T_4 . Again turbine work, turbine efficiency which is isentropic efficiency is equal to actual turbine work divided by maximum turbine work. Actual turbine work is $T_3 - T_4$, maximum turbine work is $T_3 - T_4$. Only T_4 is unknown so we can find out.

Now, in this example there is a new thing which is told to us which is turbine thermal ratio of the heat exchanger; of the heat exchanger is 0.8. So, this is what it is told to us thermal ratio of the heat exchanger. So, we know that when we derive the formula we had taken it as 1, and based upon that we derived some formulas. What is this? This is actually transfer divided by maximum possible transfer. What is actually transfer? Actually transfer is, basically it is a constant pressure heat addition in the heat exchanger, so C_p into $T_5 - T_2$ divided by. What is maximum possible heat transfer? It is C_p into $T_4 - T_2$, C_p into $T_4 - T_2$. So, this is actual heat transfer. This is maximum heat transfer.

We have seen if we would have ideal turbine and compressor then we should have 5, T_5 is equal to T_4 and then we should have T_6 is equal to T_2 . This is what in ideal case we had seen. So, thermal coefficient of heat exchanger being 0.8, so we have $T_5 - T_2$ divided by $T_4 - T_2$. So, in this formula 4 is known to us, 2 is known to us, so this formula would give us T_5 .

So, we know now everything at 1, everything at 2, everything at 5, everything at 3 and everything at 4. So, we now know all the properties at all the corners which are for the cycle. So, this example is basically telling us why it is important to write or draw the power plant cycle thermodynamic cycle before solving the example. So, by first drawing the cycle and then proceeding with the example would help us in knowing what properties are known to us at all the corners, so this should be explicit. Then we can start finding out work. We know that compressor work is C_p into $T_5 - T_2$ minus T_1 , this is known to us or some people can use the formula which is C_p into $T_2 - T_1$,

but in that case they have to divide this by compressor efficiency. Then we know T_2 , we know T_1 we know C_p , we found out C_p we found out w_c which is compressor work, then we can find out turbine work which is C_p into T_3 minus T_4 . Here as well T_3 is known, T_4 is known, C_p is known we found out turbine work.

So, we know that w_{net} is equal to turbine work minus compressor work. So, we eventually found out network then now we have to find out the efficiency of the power plant. So, for that we know efficiency of the power plant is w_{net} divided by Q_{in} . So, since we do not know Q_{in} we can find out Q_{in} as C_p into T_3 minus T_5 . So, we should know T_5 . So, T_5 is known to us from the heat exchanger formula, so we know T_5 , we know T_3 , we know C_p , so we know Q_{in} . So, putting the Q_{in} we know efficiency of the thermal power plant.

Basically without or with heat exchanger we know that network would not have what altered, what has got altered is Q_{in} . So, since Q_{in} has decreased efficiency would increase. So, this is how we have to solve the examples which are there with the components being non ideal. So, every component would have certain efficiency associated with. So, in case of the heat exchanger it will be called as thermal ratio or some people would call it as effectiveness of the heat exchanger, then these numbers will be associated with actual divided by maximum possible heat transfer data.

So, then in case of the thermal power plant or gas turbine based power plant with exchanger plus intercooler plus reheater. We; obviously, would have efficiencies of multiple compressors which might be different, multiple turbines which might be again different.

So, such example would lead to additional complexities since we have two turbines with two efficiencies, and we have two compressors with two efficiencies, but problems are simple from the perspective that we now have established the procedure how to solve the example.

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- ↳ draw the cycle
- ↳ Mark known
- ↳ Isentropic relations
- ↳ component efficiencies
- ↳ η_t , η_{net} , η_{ratio}

So, we have to first what we have to do, is how to solve the example. In this case first draw the cycle and then mark what are the known things. Then in this case we have to first use isentropic relations to find out the properties of the ideal cycle corners. So, ideal cycle has corners 2 dash, 4 dash 3 4. So, those corner properties we can find out from isentropic relation. And then we have to use component efficiencies to find out the temperatures at the actual corners. Actual corners are 2 4 5 6, so these are actual corners. So, we have to first find out isentropic relation then go for actual efficiencies which are component efficiencies and then find out corner properties, and then find out performance parameters. So, this performance parameters are for the cycle.

So, first performance parameter will be thermal efficiency of the cycle, and then there will be w_{net} of the cycle, then there will be work ratio of the cycle. So, these properties we can find out. So, having said this, this ends today's class where we have understood how to solve the examples for the gas turbine based power plant if there are non ideal components.

Thank you.