IC Engines and Gas Turbines
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Lecture – 38
Real Brayton Cycle, Solved Example for Ideal Cycle

Welcome to the today's class. Today we are going to deal with Brayton cycle with non-ideal components. In the gist what we have covered till time, we have covered the ideal Brayton cycle, Brayton cycle with inter cooling, Brayton cycle with heat exchanger, Brayton cycle with reheating. We have seen there are advantages, why we should not have Brayton cycle with inter cooling, why we should have heat exchanger and what why we should have reheater. Every component has its own advantages and disadvantages.

Further we have also seen that if we integrate everything; like there is a single power plant with everything together; like heat exchanger, intercooler and reheater then what would happen, then how to do the cycle analysis that we have seen, but now in today's class we will be seeing that, if we are having the non-ideal components as part of a Brayton cycle. The fact to be understood over here is that, basically we have 4 components in Brayton cycle; one is compressor other is turbine, then we have a combustion chamber and then we have heat loss.

So, or in case of close cycle we have pre cooler, but within that we are having two components which are rotary machines and those two components are compressors and turbines. And now if these two components are non-ideal then how would be the cycle analysis? This is the topic of discussion for today's class and then we would see to solve some examples. So, let us see if we have Brayton cycle with non-ideal plants, non-ideal components.

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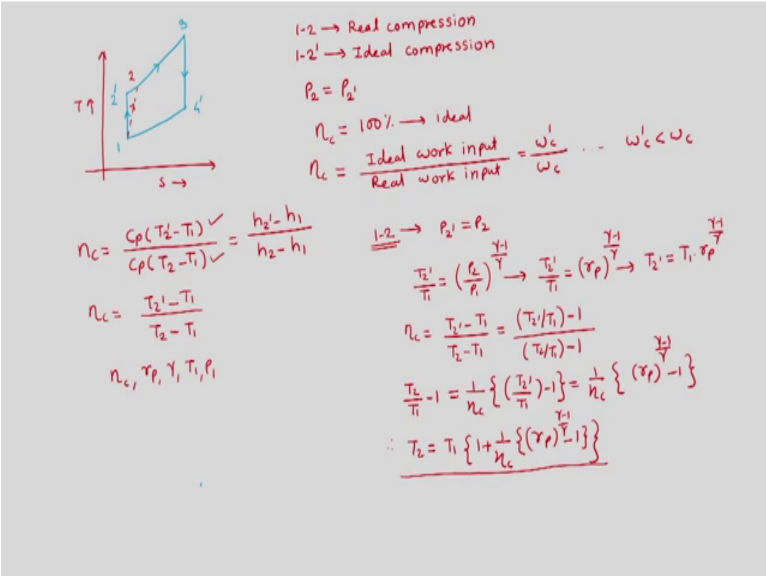

So, let us first consider the T S diagram as what we already always considered. So, we have T axis and we have S axis. So, the process initially what we would write, is all the processes what we already know, those processes we will plot for. So, we will initially plot for ideal cycle, so this is 1 to 2 is compression, so but will not say now two will say it has 2 dash is a compressor, then 2 to 3 is heat addition process then 3 to 4 is a expansion process in turbine. So, this is what we have already seen, but now we will not say it has 4, but we will say it as 4 dash. So, we have 1 2 3 4 as our cycle. Now having said this as our cycle we will see now if we are having non-ideal component. So, suppose compressor is non-ideal. So; that means, we are not going to reach the point 2 dash after the process of compression.

So, we mean that, what are the reasons of being non-ideal? The major reason what we will consider over here is a friction. So, frictional losses are there in the case of compressor due to which compressor would not let us go to the point 2 dash and then we would go to some other point and let it be some other point it is 2. So, instead of growing from 1 to 2 dash we went to wrong 1 to 2. We know that process 1 to 2 is a real process it is an irreversible process. So, 1 to 2 is showed by a dotted line

So, as what we know 1 to 2 is a real compression process and 1 to 2 dash is ideal compression process, but one point we have to keep it in mind that pressure 2 is equal to pressure 2 dash, we did not get landed in a different pressure condition. So, pressure at 2 which is a static pressure at 2 is equal to static pressure at 2 dash. So, this is what we have achieved by the still having non-ideal component, which is a compressor, but then when we say that we have compressor which is an ideal we mean that compressor thermal efficiency is 100 percent for ideal, but then what is the definition of compressor efficiency.

So, compressor efficiency can be defined as ideal work input to real work input. We have to keep one point in mind and rather remember that compressor is a work input machine, so here we are giving some work input. Ideally we always would need less work for input than the real condition. So, ideal work being less what we call it as w dash c divided by w c, but we know that w dash c is always less than w c, so efficiency is always less than 100 percent. So, in real case we need to supply more pressure to get same pressure at the outlet of the compressor.

So, practically we are supplying more worker input to the compressor to get same pressure, but ideally we would need less work input to get the same pressure. In that aspect we know that compressor efficiency, we know work input to the compressor C p into T 2 minus T 1, this is real work, ideal work then C p into T 2 dash minus, sorry. So, we have T 2 dash minus T 1 and T 2 minus T 1. So, this is ideal work and this is real work

So, basically we would also have written it as h 2 dash minus h 1 divided by h 2 minus h 1 this is the enthalpy based formulation for the work, we directly wrote and considering the gas to be a perfect gas with we are having and we are also assuming that C p is and C v is not function of temperature. So, we are having calorically perfect gas as an assumption if we go from here to here. So, compressor efficiency would ultimately become for calorically perfect gas as h 2 dash minus h 1 divided by h 2 minus h 1.

Then we will deal with the examples, then we would understand that compressor efficiency would be a known parameter for us. And parallely we also know that pressure ratio is also a known parameter for compressor. So, pressure ratio is known, compressor efficiency is known and then we would also know what are the inlet conditions of the compressor. So, practically what we would be knowing is, compressor efficiency, pressure ratio, gamma and T 1. These things are P 1, these thing would be known to us. Having these things known our objective over here is to find out what is T 2 dash. Since we are going to do some more derivations based upon the non-ideal components. So, we know that for process 1 to 2. We know that P 2 dash is equal to P 2, this is known to us. And then now we our objective is to find out what is T 2 so, but we know that T 2 dash upon T 1 is equal to P 2 upon P 1 bracket raised to gamma minus 1 upon gamma. So, this is for the point 2 dash which is in a ideal point. So, we know that T 2 dash upon T 1 is r p raised to gamma minus 1 upon gamma

So, we know that T 2 dash is equal to T 1 into r p bracket raised to the gamma minus 1 upon gamma. This we have already seen, but now we know efficiency, so we are supposed to find out T 2 dash or T 2. So, we know that efficiency is equal to T 2 dash minus T 1 divided by T 2 minus T 1. So, here we can take, we can take T 1 as common then what we can get is T 2 dash upon T 1 minus 1 divided by T 2 upon T 1 minus 1 ok. So, what we can write as T 2 upon T 1 minus 1 is equal to 1 upon compressor efficiency into T 2 dash upon T 1 minus 1, but we know T 2 dash upon T 1 is equal to r p raised to gamma minus 1 upon gamma. So, this is equal to 1 upon compressor efficiency into r p raised to gamma minus 1 upon gamma minus 1.

So, we have T 2 is equal to T 1 into 1 plus 1 upon compressor efficiency into r p bracket raised to gamma minus 1 upon gamma minus 1 and then all brackets complete. So, this is the expression for calculating temperature at the exit to the non-ideal compressor. So, here all the quantities are known to us on the right hand side, so we can find out T 2 which is a actual temperature at the outlet of the compressor

So, now we will go with the next point which is the non-ideal turbine. So, 3 is known to us by the way which is a T max point or which is the point after the combustion chamber, we assume combustion chamber or heat addition and heat rejection processes are ideal, so we have to just work out with turbine.

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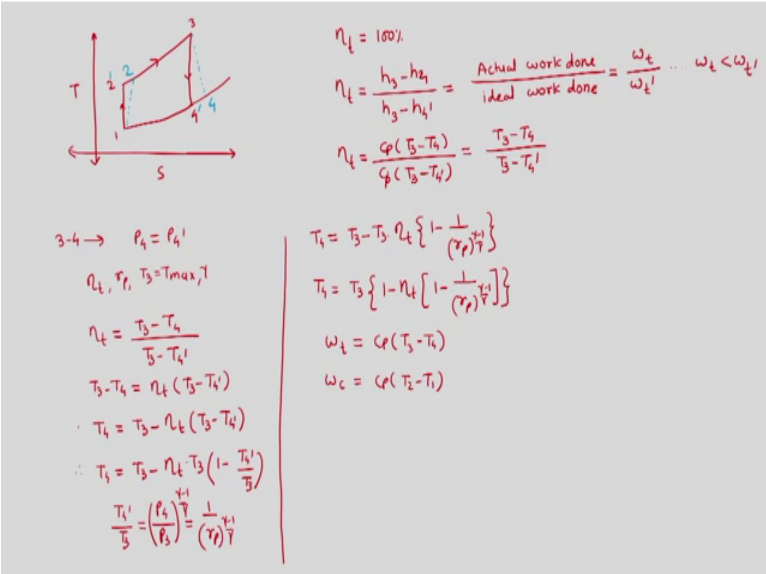

So, in the case of turbine what we will have, again we will try to draw the T S diagram for the next slide. So, we have T axis, we have s axis, then we have 1 to 2 compressor, then we have 2 to 3 and then 3 to 4, but what we are seeing now is non-ideal components. So, for non-ideal components we have 1 2, basically this is 2 dash, so 2, this is 4 dash, so this is 4. So, this is the non-ideal turbine case.

So, in case of non-ideal turbine now again we have know that we have known that, if the turbine efficiency for non-ideal ideal case will be again 100 percent. So, again we have to define what do we mean by efficiency of turbine. Again here we are considering the losses due to friction in the case of turbine. So, it is an isentropic efficiency of turbine. So, turbine efficiency, if we define in terms of enthalpy then it will be h 4 minus, it will be sorry h 3 minus h 4 divided by h 3 minus h 4 dash. So, practically it means actual work done divided by ideal work done.

So, it means w t divided by w t dash and then we know that w t is always less than w t dash. We again would remember that turbine is our producing machine. So, in case of work producing machine we always would need the, we always would get lesser work output than the ideal case. So, turbines would have non hundred percent efficiencies.

Again if we try to again assume the gas to be calorically perfect then we will have turbine efficiency as C p into T 3 minus T 4 divided by C p into T 3 minus T 4 dash. One more point to be remembered here that as in case of compressor we did not land in different pressure. The pressure at 4 dash and pressure at 4 is same, turbine exit which having same pressure whether it is ideal expansion or non-ideal expansion. So, we have the expression for it efficiency of turbine is T 3 minus T 4 divided by T 3 minus T 4 dash. Now, for the process 3 4 the same thing, what we did for earlier case for the process 3 4. We know that P 4 is equal to P 4 dash, but for again finding out the temperature T 4 we know that we are knowing turbine efficiency, we are knowing pressure ratio, we would know T 3 which is a T max by some means, then we know gamma. So, these are things which are known to us

So, now what we can find out is turbine efficiency is equal to T 3 minus T 4 divided by T 3 minus T 4 dash. Our objective is to find out T 4, so T 4 minus T 3 is equal to turbine efficiency into T 3 minus T 4 dash. So, T 4 is equal to T 3 minus 1 second. So, this is equal to T 3 minus T 4 dash. So, we have T 3 minus T 4 is equal. So, efficiency is equal to T 3 minus T 4 divided by T 3 minus T 4 dash.

So, we want to find out T 4. So, for that T 3 minus T 4 is equal to efficiency into T 3 minus T 4 dash. So, T 4 is equal to T 3 minus efficiency into T 3 minus T 4 dash. So, what we would have is, T 4 is equal to T 3 minus turbine efficiency into T 3 into 1 minus T 4 dash upon T 3, but we know that T 4 dash upon T 3 is equal to P 4 upon P 3 bracket raised to gamma minus 1 upon gamma and that is 1 upon r p bracket raised to gamma minus 1 gamma. So, what we ultimately would have is T 4 dash, T 4 is equal to T 3 minus T 3 into turbine efficiency into 1 minus 1 upon r p raised to gamma minus 1 upon gamma.

So, T 4 is equal to T 3 into 1 minus turbine efficiency into 1 minus 1 upon r p raised to gamma minus 1 upon gamma. So, this is the formula for finding out temperature at the exit of the turbine. Here we know all the quantities; like again we know what is the pressure ratio, what is the gamma what is the turbine efficiency and what is a T 3 which is T max. So, having known this we can find out the actual corner properties which are take temperature and pressure

So, now we know what is turbine work output? Turbine work output is T C p into T 3 T 3 minus T 4. So, T 4 is known to us, similarly compressor work C p into T 2 minus T 1 again T 2 new formula for T 2 in terms of compressor efficiency is known to us. Having known these things we should now find out what is the optimum condition for the maximum work output in case of the compressor and turbine based plant where both are non-ideal. So, having said this we should find out w net. So, let us work for this.

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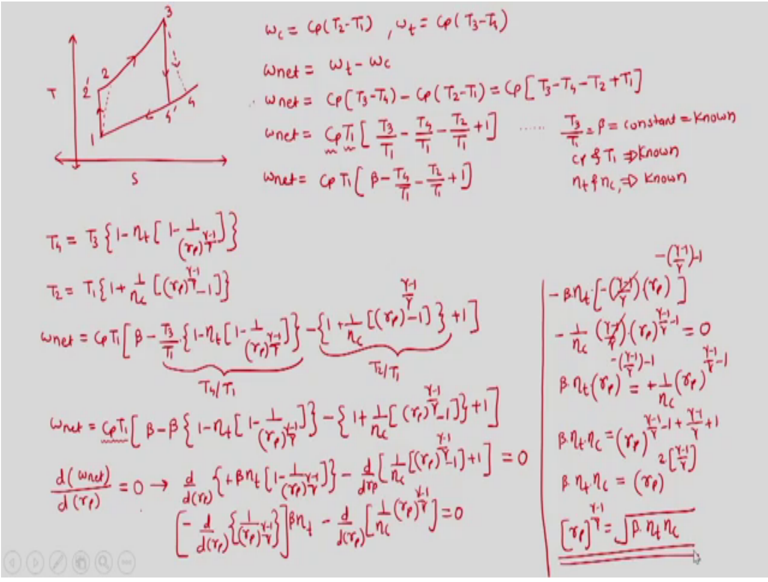

So, we are now having non-ideal plant where we have turbine and compressors with non-ideal efficiencies. So, what we have, this is a, this is a turbine and then what we have, this is a compressor; 1 to 2 dash 1 to 2 1 to 2 to 3 3 to 4 and 3 to 4 dash. So, these are the processes in case of non-ideal. And we saw that compressor work is C p into T 2 minus T 1 and turbine work is C p into T 3 minus T 4. Now, we are supposed to find out w net. We know that w net is w t minus w c, so w net is C p into T 3 minus T 4 minus C p into T 2 minus T 1.

So, what we have is C p into T 3 minus T 4 minus T 2 plus T 1. So, w net is equal to. Known quantities are taken out, so we have T 3 upon T 1 minus T 4 upon T 1 minus T 2 upon T 1 plus 1. Here C p is known to us T 1 is known to us. Practically T 3 is also known to us. So, we know that we have used T 3 upon T 1 as beta and then that is constant or that is known

Similarly, C p and T 1 are also known. So, we are working out the w net formulation or constraint for maximum w net with these fix things. So, w net is equal to C p T 1 into beta minus T 4. We can write T 4 by, we can write T 4 by T 1 minus T 2 by T 1 plus 1, but what we know? What we know is T 4 is equal to, from last slide we can take it T 4 is equal to T 3 into 1 minus eta T r p raised to gamma minus 1 upon gamma. So, this is T 4.

Similarly T 2 is equal to is known to us, T 1 into 1 plus, T 1 into 1 plus compressor efficiency r p raised to gamma minus 1 upon gamma minus 1, 1 upon compressor efficiency.

So, these two things are known to us, so we can use them to find out the w net. So, w net is equal to C p T 1 into beta minus T 4 by T 1. So, that will be T 3 by T 1 into 1 minus eta T into 1 minus 1 upon r p raised to gamma minus 1 upon gamma. Then we have minus T 2 by T 1, so we have 1 plus 1 upon compressor efficiency into r p raised to gamma minus 1 upon gamma minus 1 plus 1 then bracket complete. So, this is basically T 4 by T 1, and this is practically T 2 by T 1

Then now we need to differentiate this formula with respect to r p, but before that we know that w net is equal to the quantities with constants can be separated or known quantities can be separated. So, we have C p T 1 into beta minus beta into 1 minus eta T 1 minus 1 upon r p raised to gamma minus 1 upon gamma bracket complete minus 1 minus 1 upon compressor efficiency r p raised to gamma minus 1 upon gamma minus 1 plus 1. So, having said this we know that beta is a constant. So, since beta is a constant its differentiation with respect to r p will be 0.

So, here we have to keep in mind that apart from our things eta c and eta T are also known. Having these things known we can now differentiate d w net by d r p, and then we can equate it to 0. So, having said this what we will get. So, this quantity is a constant, so C p T 1 is a constant, its differentiation would be 0, so we can neglect that and then we will have this also to be 0. So, we directly would end up in this way C p d r p of minus minus minus plus beta into it eta T into 1 minus 1 upon r p bracket raised to gamma minus 1 upon gamma minus. So, this differentiation will be 0, so minus minus plus. So, d by d r p of 1 upon compressor efficiency and 2 r p raised to gamma minus 1 upon gamma minus 1 plus 1 and then this, this equal to 0

So, having said this we can differentiate it and then we can find out that within that this first two quantities when would multiplied with 1, they will have their differentiation to be 0. So, what we ultimately would have is minus d upon d r p of 1 minus r p raised to gamma minus 1 upon gamma, and then this quantity would be multiplied by eta T plus and beta plus. Then we would again have, differentiation will be 0 for this one. So, we will have d upon d r p of only this quantity which is 1 upon eta c into r p raised to gamma minus 1 upon gamma, and this term would be equal to 0. Having said this we can differentiate it and then find out what is happening.

So, its differentiation will be beta minus beta eta T into minus gamma minus 1 upon gamma into r p bracket raised to minus gamma minus 1 upon gamma minus 1. So, this is the differentiation of this term and then differentiation of this term would lead to plus 1 upon eta c into r p into gamma minus 1 upon gamma into r p bracket raised to gamma minus 1 upon gamma minus 1 equal to 0. So, the term in the bracket which is gamma minus 1 upon gamma would get cancelled

So, what we would have is beta into eta T into r p bracket raised to minus gamma minus 1 upon gamma minus 1 is equal to minus 1 upon eta c into r p bracket raised to gamma minus 1 upon gamma minus 1. So, we have beta, eta t, eta c in equal to r p raised to here we will have gamma minus 1 upon gamma minus 1 plus we will have gamma minus 1 upon gamma plus 1 ok. So, there is a slight mistake, here it is 1 plus 1 upon eta c, so here it should be 1 plus 1 upon at eta c. So, here it will be minus, here it will be minus, so here it will be minus accordingly and then this will lead to minus over here, and then it would lead to plus over here then we will have beta eta T eta c is equal to r p raised to 2 into gamma minus 1 upon gamma.

So, r p raised to gamma minus 1 upon gamma is equal to square root of beta eta t eta c. So, this is the constraint on the optimum pressure ratio for the maximum network in case of non-ideal components of the Brayton cycle. So, if we have non-ideal components of the Brayton cycle, we would get ultimately the pressure ratio is equal to square root of pressure 2 raised to gamma minus 1 upon gamma is equal to square root of beta eta t into eta c as constraint on optimum pressure ratio. So, such pressure ratio which satisfies this is called as optimum pressure ratio. As what we would remember that if these efficiencies are 100 percent, then we had seen that for ideal cycle we had r p optimum bracket raised to gamma minus 1 upon gamma was square root of beta. So, we just got an additional components with beta as eta t and eta c

So, here we practically end the part what we were talking about, as if we are having non-ideal components. So, we have to keep again a point in mind that the things will not change, except what we will have is a turbine and compressor efficiencies. So, those turbine and compressor efficiencies will be used to find out the temperature at the end of the turbine or temperature at the end of the compressor, rest of the things would remain same. There is one more point to note that now heat addition would not start from 2 dash, but it will start from 3 to, so heat addition will be C p into T 3 minus T 2.

Similarly in case of heat rejection it will not start from T 4 dash, but it will start from T 4, then Q out is equal to T 4 C p into T 4 minus T 1. So, efficiency would accordingly change. So, efficiency would be 1 minus Q out upon Q in. So, 1 minus C p into T 4 minus T 1 divided by C p into T 3 minus T 2. So, we know now T 3, we know now T 2, we know now T 4 and we know T 1, so accordingly we can find out efficiency. So, this is how the chapter or the part for non-ideal components is. Now we will see some examples

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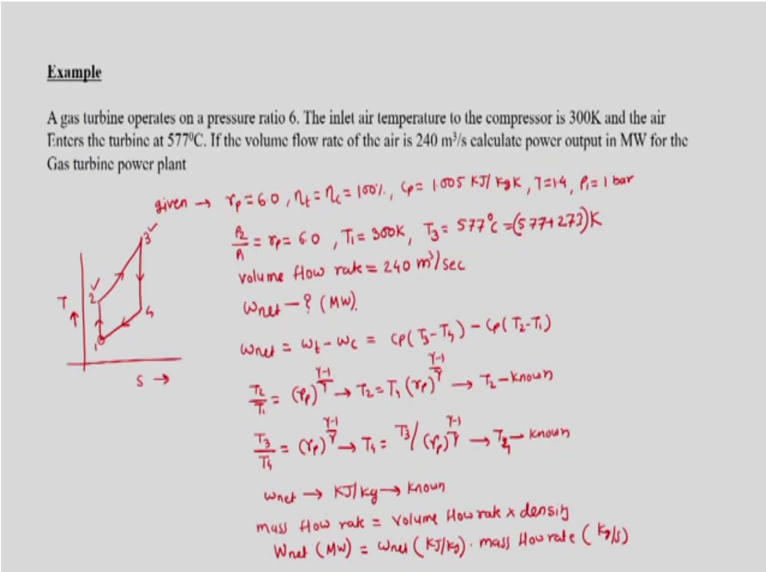

We will here see how to solve the example or what are the things which would be given in general in an example. So, an example says that there is a gas turbine which operates on a pressure ratio 6. The inlet air temperature to the compressor is 300 Kelvin and the air enters the turbine at 577 degree Celsius. If volume flow rate of air is 240 meter cube per second then calculate power output in megawatt for the gas turbine power plant. So, what is given to us, so what is given to us is given in the example. In the example we are given with r p and that r p is equal to 6.

First we should always draw the T S diagram and this example, since we are not given with turbine and compressor efficiencies we can assume turbine efficiency and compressor efficiencies as 100 percent, so they are one. So, for us we are working with ideal gas turbine power plant 1 2 3 4. So, what is given to us is r p. So, we are knowing P 2 by P 1 is equal to r p is equal to 6.

So, we parallely know the inlet air temperature to the compressor means, we know T 1 is equal to 300 Kelvin and then we know turbine entry temperature which is T 3, T 3 is equal to 577 degree Celsius, so it is 577 plus 273 Kelvin. So, I am just explaining the process what we are given is volume flow rate and volume flow rate is 240 meter cube per second, and we are supposed to find out what is w net, but the usual formula of ours would lead w net in kilo Joule per k g, but here we are supposed to find out w net in megawatt. So, this we have to remember. So, what we can do

We need, we know that w net is equal to w t minus w c. So, we know that w t is equal to C p into T 3 minus T 2 minus C p into T 2 minus T 1. We again should know that C p for air is 1.005 kilo Joule per k g Kelvin. This is information which either would be known to us or would be given to us. So, sorry, so turbine work is T 4. So, turbine work is T T 3 minus T 4 and compressor work is C p into T 2 minus T 1. Then for us we do not know what is T 4 and we do not know what is T 2, but we know T 2 by T 1 is r p raised to gamma minus 1 upon gamma.

So, we know r p, we know T 1, we can find out T 2. So, T 2 is equal to T 1 into r p bracket raised to gamma minus 1 upon gamma, since we are told with air. For air gamma is 1.4, so we know r p, we know gamma, we know T 1. So, now, T 2 is known to us, then T 2 is known to us, then T 3 is also given to us as it is told. So, now, we just have to find out T 4. So, for T 4 also we know formula T 3 by T 4 is r p raised to gamma minus 1 upon gamma, but this leads to T 4 is equal to T 3 upon r p bracket raised to gamma minus 1 upon gamma. So, here as well T 3 is known to us, gamma is known to us, r p is known to us

So, now T 3 is known to us. Knowing T 3, T 3 not T 4 is known to us. So, knowing T 4, knowing T 2, knowing T 3 and T 1 we can find out w net, but this w net in kilo Joule per k g is known. Our object over here is to find out what is the w net in megawatt. So, here we have to multiply it with mass flow rate or volume flow rate. So, what is the volume flow rate? Volume flow rate is given to us which is 240 meter cube per second, but this is volume flow rate. So, we have to find out mass flow rate. So, for mass flow rate we have to multiply by density.

So, mass flow rate is equal to volume flow rate into density. Then where we have to find out density? We have to find out density at 1. So, we know we will take pressure as 1 bar at the inlet and then we know temperature 300 Kelvin then we can find out density. So, for that we will assume P 1 is equal to 1 bar. So, if we take P 1 as 1 bar then we know P is equal to rho r t. Then knowing the pressure and knowing the temperature we can find out density at the inlet.

So, we know, once we know the density at the inlet we know volume flow rate, then we know mass flow rate, then w net in megawatt is equal to w net in kilo Joule per k g in mass into mass flow rate which is k g per second. So, this is how we can solve this example where we are supposed to find out what is the power output in megawatt. So, having said this, we will see how the next example is.

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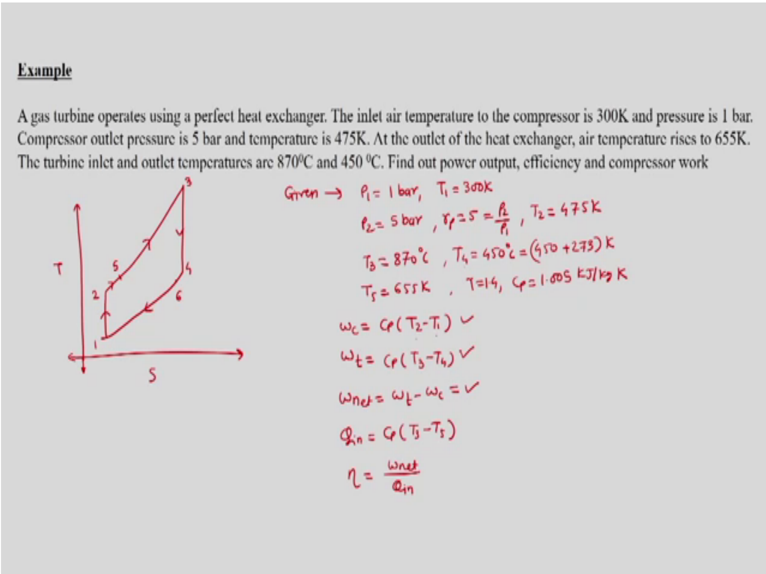

In the next example we are again told that there is a gas turbine which operates on a perfect heat exchanger. So, this example is with a gas turbine with heat exchanger. The inlet air temperature to the compressor is 300 Kelvin and pressure is 1 bar, compressor outlet pressure is 5 bar and temperature is 475 Kelvin. At the outlet of the heat exchanger air temperature rises to 655 Kelvin, the turbine inlet and outlet temperatures are 870 celsius and 450 celsius, find out power if output and efficiency of and compressor work. So, here as well first we will start with T S diagram. So, as per T S diagram we are at 1 2, but 2 to 3 is a heat exchange process then 3 to. So, we did not name it as 3 v 3 4, here we have 5 and then here we have 6. So, this is the ideal cycle with heat exchanger

So, now in the given things we are given with P 1 which is 1 bar T 1 which is 300 Kelvin, we are again told that P 2 is 5 bar. So, practically r p is 5 which is P 2 by P 1 it is given to us, but we are also given that T 2 is equal to 475 Kelvin. We are told that T 3 is equal to 870 degree Celsius and T 4 is 450 degree Celsius. So, keep this point in mind T 2 is 475 Kelvin, but T 4 is 450 plus 273 Kelvin for heat exchanger cooperate we always know that T 4 has to be greater than T 2, but we are told that T 5 is 655 Kelvin which is a temperature at the outlet of the heat exchanger.

Now, we are supposed to find out power output and efficiency and compressor work, which is compressor work is simple to find out w c is equal to C p into T 2 minus T 1. Again for all this case we have to take gamma is equal to 1.4 C p. For air 1.005 kilo Joule per kg Kelvin and then we know C p, we know T 2 and we know T 1, so compressor work can be easily found out. Similarly for turbine work is also C p into T 3 minus T 4 T 3 is also given, T 4 is also given, C p is also known. So, turbine work can also be found out. So, w net is equal to w t minus w c. So, turbine work is known, compressor work is known, then we can find out w net, but what is that we have to find out is efficiency. So, for efficiency there is only one glitch, only one problem that we have to find out Q in

So, Q in is C p into T 3 minus T 5 and its not T 2 that is what we have to remember, since combustion chamber entry temperature is T 5. So, T 3 is known to us, T 5 is known to us, C p is known to us. We know Q in, then efficiency is w net upon Q in. Thus we can find out how the different parameters of the cycle can be found out for every case. One thing we have to remember that first we have to draw the T S diagram or P V diagram of the cycle, then we have to find out what are the given things. From the given things we have to find out all the corner parameters like T 2 P 2 T 3 P 3 T 4 P 4.

Once those parameters are found out then we can find out heat and work interaction in each process; like compressor work, like turbine work, like Q in and Q out. And then we can find out what is the resultant performance parameters of the cycle which are efficiency w net like parameters. So, this is the fact which we should remember and we end here with the lectures for the fact where we were dealing with the compressor turbine, if they are non-ideal then how would be they affect the Brayton cycle.

Thank you.