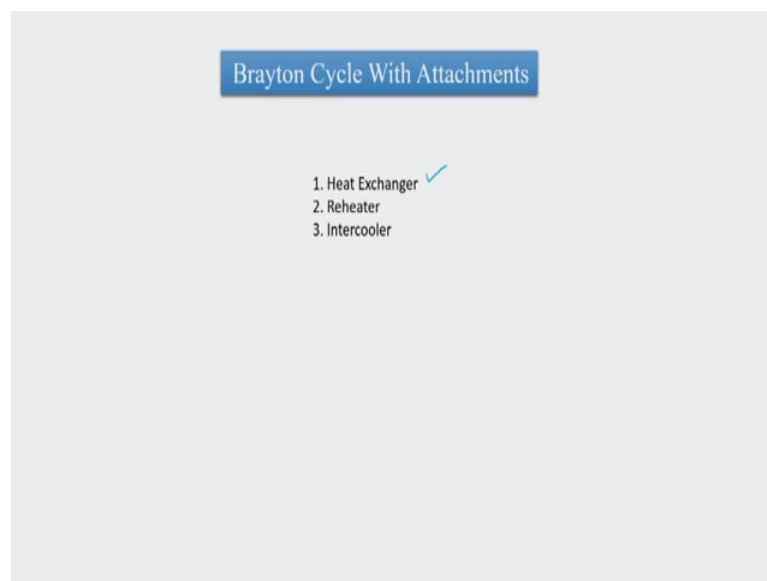


IC Engines and Gas Turbines
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Lecture - 37
Brayton Cycle with Intercooler

Welcome to the class. In last class we have seen that there would be Brayton cycle which will be with certain attachments.

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And in last class we have seen that Brayton cycle with Heat Exchanger attachment and then we found out that if heat exchanger effectiveness is 1, then how to find out the thermal efficiency. Then we saw that there is Brayton cycle with Reheater and if we have reheater based Brayton cycle then how to find out w_{net} , Q_{in} and then what is the optimum condition for the reheat pressure for which we can get w_{net} maximum. So, this we have seen. Basically, heat exchanger is used for getting maximum efficiency and reheater is used mainly for getting maximum work output.

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$$\begin{aligned}
 \eta_{\text{eff}} &= \frac{\text{Actual heat transfer}}{\text{Max. possible transfer}} = \frac{C_p (T_5 - T_2)}{C_p (T_4 - T_2)} \\
 \eta_{\text{eff}} &= \frac{T_5 - T_2}{T_4 - T_2} = 1 \rightarrow T_5 - T_2 = T_4 - T_2 \rightarrow \boxed{T_5 = T_4} \\
 \eta_{\text{eff}} &= \frac{T_4 - T_6}{T_4 - T_2} = 1 \rightarrow T_4 - T_6 = T_4 - T_2 \rightarrow \boxed{T_6 = T_2} \\
 w_{\text{net}} &= C_p (T_3 - T_4) - C_p (T_2 - T_1) \\
 Q_{\text{in}} &= C_p (T_3 - T_5) = C_p (T_3 - T_4) \\
 \eta_{\text{th}} &= \frac{w_{\text{net}}}{Q_{\text{in}}} = \frac{C_p (T_3 - T_4) - C_p (T_2 - T_1)}{C_p (T_3 - T_4)} = 1 - \frac{C_p (T_2 - T_1)}{C_p (T_3 - T_4)} \\
 \eta_{\text{th}} &= 1 - \frac{T_2 - T_1}{T_3 - T_4} = 1 - \frac{T_1 (r_p)^{\frac{\gamma-1}{\gamma}} - T_1}{T_3 - T_3 / (r_p)^{\frac{\gamma-1}{\gamma}}} = 1 - \frac{T_1}{T_3} \left[\frac{(r_p)^{\frac{\gamma-1}{\gamma}} - 1}{(r_p)^{\frac{\gamma-1}{\gamma}} - 1} \right] (r_p)^{\frac{\gamma-1}{\gamma}} \\
 \eta_{\text{eff}} = 1 &\rightarrow \boxed{\eta_{\text{th}} = 1 - \frac{(r_p)^{\frac{\gamma-1}{\gamma}}}{\beta}} \dots r_p = \frac{P_2}{P_1} \text{ or } r_p = \frac{\beta}{P_3} \text{ if } \beta = \frac{T_3}{T_1} = \frac{T_{\text{max}}}{T_{\text{min}}} \\
 &\quad r_p = \frac{P_{\text{max}}}{P_{\text{min}}}
 \end{aligned}$$

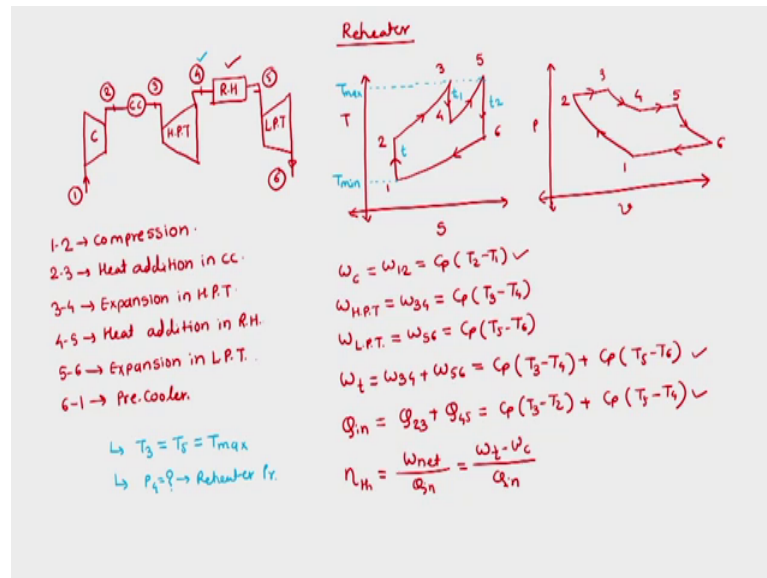
So, in last class there was one point which we saw was to find out efficiency with heat exchanger. This is the formula for it is efficiency of heat exchanger, then we saw for reheater and then we found out that this is the w_{net} formula for reheater.

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$$\begin{aligned}
 w_{\text{net}} &= C_p T_1 \left[(2\beta + 1) - \frac{\beta}{t_1} - \frac{\beta t_1}{t} - t \right] \\
 \frac{d}{dt_1} [w_{\text{net}}] &= 0 \rightarrow w_{\text{net}}|_{\text{max}} \\
 + \frac{\beta}{t_1^2} - \frac{\beta}{t} &= 0 \\
 t_1 &= \sqrt{\frac{\beta}{t}} \quad t_1 \cdot t_2 = t \\
 t_2 &= \sqrt{t} \\
 \boxed{t_1 = t_2 = \sqrt{t}} &\rightarrow w_{\text{net}}|_{\text{max}} \rightarrow \boxed{T_1 = T_6} \\
 w_{\text{net}}|_{\text{max}} &= C_p T_1 \left[(2\beta + 1) - \frac{\beta}{\sqrt{t}} - \frac{\beta}{\sqrt{t}} - t \right] \checkmark
 \end{aligned}$$

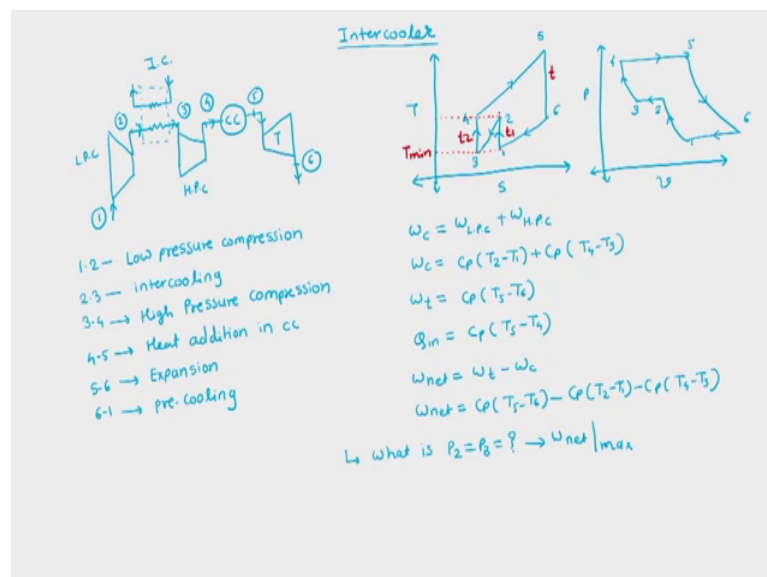
We also saw that t_1 is equal to t_2 is equal to root t . We have to just remember that what is t_1 . t_1 is the pressure ratio for the compressor. So, this was t_1 , t_2 sorry this is t , t_1 is the pressure ratio for the high-pressure turbine, T_2 is the pressure ratio for low-pressure turbine.

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And when T_1 is equal to T_2 ; that means, we have a constraint which we can fill here T_1 is equal to T_2 means T_3 by T_4 is equal to T_5 by T_6 . But we already know that T_3 is equal to T_5 . So, we all we will have T_4 is equal to T_6 . So, this constraint practically would lead to T_4 is equal to T_6 . So, this is an important point which we have to remember.

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Now, we will go ahead and see the Brayton cycle with intercooler. For intercooler, basic objective is that we have to reduce the work input. We know that w_{net} which is the

network output of the Brayton cycle is equal to w_t which is turbine work minus w_c , which is compressor work.

Reheater increases w_t and intercooler job is to reduce w_c . So, reduce the compressor work, this is the job of intercooler. Again, we will draw schematic of the arrangement for intercooler with Brayton cycle with intercooler. So, this is now low-pressure compressor, air comes in and then it goes into a intercooler, then it goes to the high-pressure compressor. From there it goes to the combustion chamber, from the combustion chamber air goes to the turbine and then from turbine it goes to the exhaust. So, this is the path of the air, we have low-pressure compressor, high-pressure compressor and then we have combustion chamber and turbine, but there is a intercooler and then this intercooler is going to reduce the temperature of the air which is going from low-pressure compressor to the high-pressure compressor intercooler. And again, process of inter cooling is isobaric. So, 1 to 2 we will target to (Refer Time: 04:54), then 2 to 3 inter cooling, 3 to 4 high-pressure compression, 4 to 5 combustion chamber, 5 to 6 turbine.

So, process 1 to 2 we have high we have low-pressure compression, low-pressure compression. And obviously, this is an isentropic compression. 2 to 3 is inter cooling and obviously, this is isobaric process, 3 to 4 is high-pressure compression. Then we have 4 to 5 this is heat addition in combustion chamber. Then we have 5 to 6 which is expansion in turbine which is; obviously, isentropic expansion. 4 to 5 is again adiabatic heat addition. 6 to 1, in case of flow cycle power plant it is a pre-cooling.

Having done this, we will again like last time try to draw the T S and P V diagram for the inter cooling based Brayton cycle. So, we have 1 to 2 compression process. And 2 to 3 is isobaric heat rejection in 2 to 3, then 3 to 4 is again isentropic compression, 4 to 5 is heat addition at isobaric condition, 5 to 6 is expansion process in the turbine, and 6 to 1 is the process which is pre cooling process for close cycle gas turbine based power plant. Having said this we can as well draw the P V diagram where we will have 1 to 2 as isentropic compression, 2 to 3 is isobaric heat rejection, 3 to 4 is isentropic compression, 4 to 5 is isobaric heat addition, 5 to 6 is expansion in the isentropic expansion in the turbine, 6 to 1 is pre cooler process for close cycle gas turbine power plant. So, this is T V, T S and P V diagram.

Now, again we can find out what is the work interaction. What is the w_c ? Here see here there are 2 compressors. So, w_c is w low-pressure compressor plus w high-pressure compressor. So, w_c is equal to $C_p (T_2 - T_1) + C_p (T_4 - T_3)$. Then we have w_t is equal to $C_p (T_5 - T_6)$ and then we have Q in which is $C_p (T_5 - T_4)$. So, if we are said nothing about intercooler on constraints and optimization then we can go ahead and do the calculation as what we can do here. But the job of intercooler as I told is to reduce the compression work. So, we have to maximize the reduction.

So, for this optimization again our objective is to increase the w_{net} and w_{net} is equal to w_t minus w_c . So, w_{net} is equal to $C_p (T_5 - T_6) - C_p (T_2 - T_1) - C_p (T_4 - T_3)$. So, this we need to maximize. So, again the question over here as what we asked for reheater that, what is the intercooler pressure. What is the intercooler pressure? So, what is P_2 is equal to P_3 is equal to what and for the w_{net} max. So, this is the question we have to ask our self. And for this question we have to derive an expression and find out. So, for that we will go ahead. So, for this question of what would be the intercooler pressure to get the maximum w_{net} , we have to follow the same procedure what we followed for the reheater.

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$$\begin{aligned}
 w_{net} &= w_t - w_c \\
 w_{net} &= C_p (T_5 - T_6) - C_p (T_2 - T_1) - C_p (T_4 - T_3) \\
 w_{net} &= C_p [T_5 - T_6 - T_2 + T_1 + T_3] \\
 \text{Constraints for optimization} &\rightarrow * C_p, T_1 \rightarrow \text{known} \rightarrow \text{constant} \\
 &\quad T_p \rightarrow \text{known} \rightarrow \text{constant} \\
 &\quad \beta = \frac{T_{max}}{T_{min}} = \frac{T_2}{T_1} = \text{constant} \\
 w_{net} &= C_p T_1 \left[\frac{T_5}{T_1} - \frac{T_6}{T_1} - \frac{T_2}{T_1} + 1 - \frac{T_4}{T_1} + \frac{T_3}{T_1} \right] \\
 w_{net} &= C_p T_1 \left[\beta - \frac{T_6}{T_5} \cdot \frac{T_5}{T_1} - \frac{T_2}{T_1} + 1 - \frac{T_4}{T_5} \cdot \frac{T_3}{T_1} + \frac{T_3}{T_1} \right] \\
 T_1 = T_3 &\rightarrow \text{Inlet temperature to both compressors is same} \\
 w_{net} &= C_p T_1 \left[\beta - \frac{T_6}{T_5} \cdot \beta - \frac{T_2}{T_1} + 1 - \frac{T_4}{T_5} + 1 \right] \\
 w_{net} &= C_p T_1 \left[\beta - \frac{T_6}{T_5} \cdot \beta - \frac{T_2}{T_1} - \frac{T_4}{T_5} + 2 \right]
 \end{aligned}$$

So, w_{net} is equal to w_t minus w_c . So, w_{net} is equal to $C_p (T_5 - T_6) - C_p (T_2 - T_1) - C_p (T_4 - T_3)$, ok. So, w_{net} is equal to C_p we can

take common. So, T_5 minus T_6 minus T_2 plus T_1 minus T_4 plus T_3 . Again, we will look up on the constraints what we had to think, think off. So, constraints for optimization the constraints what we saw last time was, one is C_P and T_1 are known, so constants. So, next constraint is r_p is known, so it is constant and next constraint is β which is T_{\max} upon T_{\min} and in present case T_{\max} is T_5 upon T_1 is constant. So, for these 3 constraints we will do the optimization.

So, here we will get w_{net} , w_{net} is equal to C_P we can take T_1 common, then it becomes T_5 by T_1 minus T_6 by T_1 minus T_2 by T_1 plus 1 minus T_4 by T_1 plus T_3 by T_1 . So, T_5 by T_1 is β . So, w_{net} is equal to $C_P T_1$ and then this is β . So, minus T_6 by T_1 . So, we will do same thing T_6 by T_5 into T_5 by T_1 , T_5 by T_1 , T_6 by T_5 into T_5 by T_1 , plus minus T_2 by T_1 plus 1 minus T_4 by T_1 . So, we will write T_4 by T_3 into T_3 by T_1 ok, and plus T_3 upon T_1 .

We will again consider one thing as an assumption that inlet both the compressor is same. We will put a constraint which says that T_1 is equal to T_3 , which says that inlet temperature to both compressors is same. So, having said this we can go ahead with the next constraint and we can write down w_{net} is equal to $C_P T_1$ β minus T_6 by T_5 into β minus T_2 by T_1 plus 1 minus T_4 by T_3 into T_3 by T_1 is 1, so plus 1. So, we have w_{net} is equal to $C_P T_1$ β minus T_6 by T_5 into β minus T_2 by T_1 minus T_4 by T_3 plus 2. Now, we have to carry forward this expression for the further differentiation. So, for the same thing again we need few things.

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$$\begin{aligned}
 \omega_{net} &= C_p T_1 \left[(\beta + 2) - \frac{T_2}{T_1} \beta - \frac{T_2}{T_1} - \frac{T_1}{T_2} \right] \\
 \frac{T_2}{T_1} &= t_1 \quad \frac{T_4}{T_3} = t_2 \quad \frac{T_1}{T_6} = t \\
 \frac{P_2}{P_1} &= r_{p1} = \left(\frac{T_2}{T_1} \right)^{\frac{\gamma}{\gamma-1}} \quad r_{p2} = \left(\frac{P_4}{P_3} \right)^{\frac{\gamma}{\gamma-1}} \quad r_p = \frac{P_5}{P_6} \\
 t_1 &= (r_{p1})^{\frac{\gamma-1}{\gamma}} \quad t_2 = (r_{p2})^{\frac{\gamma-1}{\gamma}} \quad t = (r_p)^{\frac{\gamma-1}{\gamma}} \\
 r_{p1} \cdot r_{p2} &= \left(\frac{P_2}{P_1} \right) \cdot \left(\frac{P_4}{P_3} \right) = \frac{P_4}{P_1} = \frac{P_5}{P_6} = r_p \rightarrow r_p = r_{p1} \cdot r_{p2} \\
 &\quad t = t_1 \cdot t_2 \\
 \omega_{net} &= C_p T_1 \left[(\beta + 2) - \frac{1}{t} \beta - t_1 - t_2 \right] \\
 u_{net} &= C_p T_1 \left[(\beta + 2) - \frac{1}{t} \beta - t_1 - \frac{t}{t_1} \right] \\
 \frac{d}{dt_1} [\omega_{net}] &= 0 \rightarrow \omega_{net}|_{max} \\
 -1 + \frac{t}{t_1^2} &= 0 \rightarrow \boxed{t_1 = \sqrt{t}} \quad \boxed{t_2 = \sqrt{t}} \rightarrow t_1 \cdot t_2 = t \quad T_4 = T_2 \\
 \omega_{LRC} &= C_p (T_2 - T_1) \\
 \omega_{HRC} &= C_p (T_4 - T_3) = C_p (T_2 - T_1) \\
 \omega_{LRC} &= \omega_{HRC}
 \end{aligned}$$

So, this expression we can take it as ω_{net} is equal to $C_p T_1 (\beta + 2)$, this is $\beta + 2$ minus T_6 by T_5 into β minus T_2 by T_1 this is T_6 by T_5 β minus T_2 by T_1 minus T_4 by T_3 minus T_4 by T_3 .

So, here again let us consider what we had considered earlier that T_2 by T_1 is equal to T_1 , T_4 by T_3 is equal to T_2 and T_5 by T_6 is equal to t . What we are actually saying is this, as per this diagram we are saying that here if we say we are saying that this is T_1 , this is T_2 and this is t . So, what we are saying is, what we are saying is P_2 by P_1 suppose it is r_{p1} is equal to T_2 by T_1 bracket raise to γ upon γ minus 1. So, it means that r_{p1} it means that t_1 is equal to r_{p1} bracket raise to γ minus 1 upon γ . Similarly, what we have is t_2 is equal to r_{p2} bracket raise to γ minus 1 upon γ where we are saying that r_{p2} is equal to r_{p2} is equal to if we go in this diagram r_{p2} is equal to P_4 by P_3 P_4 by P_3 . And then similarly we can say that t is equal to r_p bracket raise to γ minus 1 upon γ where we say r_p is equal to P_5 by P_6 , ok.

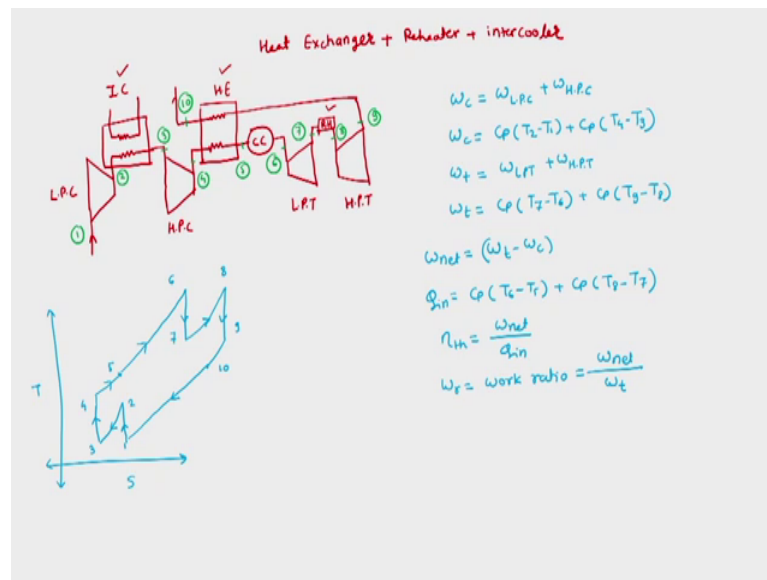
So, again we can see what is r_{p1} into r_{p2} ; r_{p1} is what we have said it is P_2 by P_1 . And what is r_{p2} ? It is P_4 by P_3 , but we can see here clearly that P_2 is equal to P_3 says process of inter cooling is isobaric. So, this is P_2 is equal to P_3 . So, this is P_4 by P_1 . But P_4 is equal to P_5 P_1 is equal to P_6 . So, this is r_p . So, what we have is r_p is equal to r_{p1} into r_{p2} . So, what we would have is t is equal to t_1 into t_2 . So, we can do

the same thing here w_{net} is equal to $C_p T_1 \left(\beta + 2 \frac{T_6 - T_5}{T_1} \right)$ what we said T_1 is upon t into β minus T_2 by T_1 , T_2 by T_1 is t_1 minus T_4 by T_3 is t_2 . So, this we have w_{net} is equal to $C_p T_1 \left(\beta + 2 \frac{t_1 - t_2}{t_1} \right)$. So, we can replace t_2 by t upon t_1 . And in we can differentiate this expression with respect to t_1 , w_{net} and we can get equated to 0 such that we expect w_{net} to be maximum. We will get a constraint of this. So, here again we have 4 terms, but in those 4 terms we have this term as constant and we have this term also as constant. So, we need to differentiate only for two terms. So, differentiation of this term will give us minus 1, minus this will give us plus t upon t_1 square is equal to 0, this would tell us again t is equal to square root of t_1 or rather what we get is t_1 is equal to square root of t . But what we have is t_1 into t_2 is equal to t . So, what we will have is t_2 is also equal to square root of t . So, this is how we have got a constraint or intercooler pressure.

So, if we go back and see what we have achieved. So, we got again that t_1 is equal to t_2 , but we said internally that these two temperatures are same. So, we said this is T_{min} , so this ratio is same; that means, this temperature is also same. So, what we got internally by this assumption is T_4 is equal to T_3 , sorry T_4 is equal to T_2 , T_4 is equal to T_2 . We should also know one thing that what is $w_{low-pressure compressor}$, is $C_p (T_2 - T_1)$. What is $w_{high-pressure compressor}$ is $C_p (T_4 - T_3)$, but T_4 is equal to T_2 , so $C_p (T_2 - T_1)$. So, $w_{low-pressure compressor}$ is equal to $w_{high-pressure compressor}$.

So, if we want to maximize the w_{net} then we get a constraint that both the compressors should have same work output, work input. Similarly, when we were working for turbines we also should have got the same thing for reheater constraint that if we want to use the reheater for w_{net} maximum, then we had got that for the constant maximum condition in the turbine inlet we have to have same outlet temperature for both the turbines. So, both the turbine should give same work output. So, $w_{low-pressure turbine}$ is $w_{high-pressure turbine}$ for reheater arrangement and similarly we have $w_{low-pressure compressor}$ is equal to $w_{high-pressure compressor}$ for intercooler arrangement. Having said this we have next thing to discuss is what if we connect everything. So, we should do same calculation and similar way we should perform if we mix everything.

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So, we have now Heat Exchanger plus we have Reheater plus intercooler. In these we will just draw schematic which we had all already discussed, but we will write down some formulas where here. So, we have first a low-pressure compressor. So, it will have here inlet. So, it is low-pressure compressor. Before the outlet of low-pressure compressor, it will go to the intercooler, so this is intercooler, intercooler then full intercooler it will go to the high-pressure compressor. So, this is high-pressure compressor. From the high-pressure compressor air will go to the heat exchanger. So, this is heat exchanger.

Then it will go to the combustion chamber and from the combustion chamber it will go to the low-pressure turbine, this is low-pressure turbine. From low-pressure turbine gas will go to the high-pressure turbine, so this is high-pressure turbine. So, from that we will have an exhaust initially going into the heat exchanger and from there it will go out. So, this is heat exchanger. So, before that from low-pressure turbine to high-pressure turbine we will have a reheater. So, now, we have intercooler, we have heat exchanger, we have reheater. So, all the attachments are connected. So, let one be the condition at the inlet to the low-pressure compressor 1, then we will have outlet as 2, then we will have inlet as 3, then we will have outlet as 4, then 5 at the entry at the combustion chamber, exit of combustion chamber is 6, entry to reheater is 7, exit of high-pressure turbine sorry exit of the reheater is 8, then exit of high-pressure turbine is 9 and then we have 10 at the exit of the heat exchanger.

So, now we should write all the formulas what we were writing earlier. Our formulas are this way which states that we have w_c is equal to $w_{\text{low-pressure compressor}} + w_{\text{high-pressure compressor}}$. So, w_c is equal to $C_p (T_2 - T_1) + C_p (T_4 - T_3)$, condition of optimization if it is said then we could have put it. But in general, what would be the case if we have all the attachments that we are mentioning.

Then we have w_t , w_t is again $w_{\text{low-pressure turbine}} + w_{\text{high-pressure turbine}}$. So, w_t is equal to $C_p (T_7 - T_6) + C_p (T_9 - T_8)$. So, w_{net} is as usual which is $w_t - w_c$. But what is Q_{in} ? Here Q_{in} is again in two places, one in combustion chamber and one in reheater. So, combustion chamber is $C_p (T_6 - T_5)$ and reheater is $C_p (T_8 - T_7)$. So, this is the arrangement for all the attachments and this is the expression for Q_{in} . So, thermal efficiency is $w_{\text{net}} / Q_{\text{in}}$.

If ask to us or if required we can also calculate w_r which is work ratio and work ratio is equal to w_{net} / w_t , ok, but before that let us plot T S and P V diagram for this attachment. So, T S process 1 to 2 is isentropic compression, then process 2 to 3 is isobaric heat rejection in intercooler, then process 3 to 4 is isentropic compression, then we have heat addition in intercooler 4 to 5, so this is point 5. And 5 to 6 is constant pressure further heat addition in combustion chamber, so it is 6. Then from 6 to 7 we have high-pressure turbine then from 7 to 8 we have a reheater, then 8 to 9 we have low-pressure turbine and then we have initially 9 to 10 heat loss and then 10 to 1. So, here this is how our cycle will proceed.

So, you can practice for this parallelly you could also draw the P V diagram for this arrangement. So, this is how we have seen that if we consider first calorically perfect gas, second here is only working medium as constraint and then if we consider different attachments like heat exchanger, reheater and intercooler, what would be the work and heat interactions, how to find out all the corner conditions like conditions at all points and then what are the optimization constraints and conditions, and what are the performance parameters like w_{net} and efficiency and work ratio.

So, in next class we will see about what would happen if the cycle is not ideal that there are components like turbine and compressor who have efficiencies which are not 100 percent. So, let us meet in next class.

Thank you.