IC Engines and Gas Turbines
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Lecture - 36
Brayton Cycle with Attachments: Heat Exchanger, Reheater, Intercooler

Welcome to the next class. In today's class, we will see about Brayton Cycle with some Attachments, and in previous class what we had seen was about Brayton cycle, its calculation for network output, its calculation for thermal efficiency and then we went on finding out what is the optimum condition when we can get maximum network of the Brayton cycle. And now we are supposed to find out what is the effect of different attachments on the Brayton cycle.

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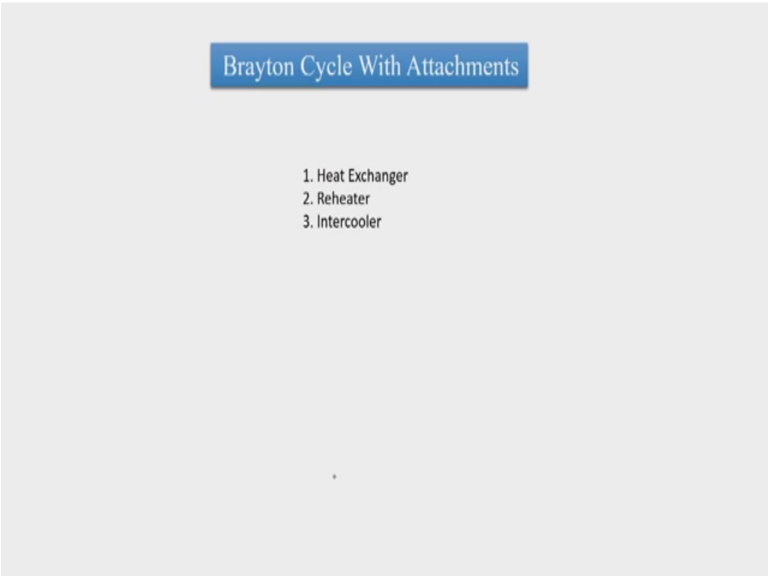

So, first attachment is Heat Exchanger, next attachment is Reheater and then Intercooler and then we will see what is the mixed or combined effect of all this thing. We should remember in our earlier lecture we had discussed about heat exchanger. We know that from last derivation efficiency of Brayton cycle depends upon pressure ratio. So, if we increase the pressure ratio or operating pressure ratio of the cycle efficiency of the cycle increases. Similarly, if we attach heat exchanger attachment of heat exchanger also increases the efficiency of the cycle. We had also seen that heat exchanger attachment is not that good idea if we go for high-pressure ratio since there are losses with it.

Attachment of reheater is more for getting more work output and attachment of intercooler is to reduce the work input. So, these 3 attachments have 3 different reasonings and then we will see one attachment each.

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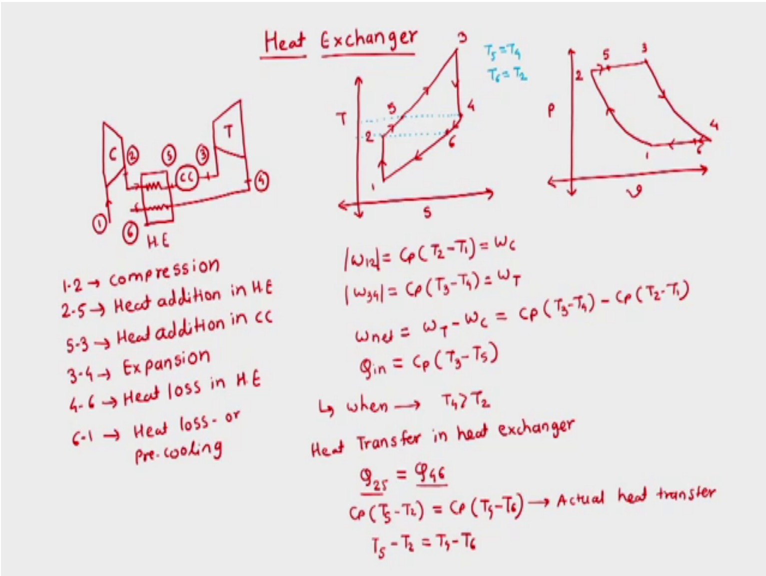

So, Brayton cycle with Heat Exchanger. So, let us see Brayton Cycle with Heat Exchanger, Cycle with Heat Exchanger. So, the attachment is this way if we see ,we first have compressor, we know that there is a compressor. So, we will denote it as C. So, air comes in the compressor and then it goes out from the compressor to the combustion chamber, but while going out it will pass through a heat exchanger and then this is combustion chamber. Then air goes to the turbine, so this is turbine. And then turbine exhaust goes into the heat exchanger where there is heat exchange between the air which is actually going to the combustion chamber. So, this is heat exchanger.

So, now, as our usual denotations this is 1, this point is 2, then this point we will say it as, we will denote this point as, this point as 2 as it was denoted earlier, then this point is 5 for us, since combustion exhaust is 3, turbine exhaust is 4, heat exchanger exhaust is 6. So, process 1 to 2, compressor Compression, process 2 to 5 is Heat addition, in heat exchanger process, 5 to 3, is Heat addition in combustion chamber process, 3 to 4 is Expansion, and then we have 4 to 6 this is in Heat loss in heat exchanger. Then, obviously, we will have 6 to 1 which is Heat loss, but we are considering in general open cycle gas turbine power plant. But if it would have been closed cycle then obviously, we would have Heat loss or pre-cooler, pre- Pre-cooling.

Now, our job is to denote this cycle on the T S diagram. Suppose, this is T axis, this is S axis, now we have 1 to 2, 1 to 2 is compression, then we have process 2 to 5 is heat addition in heat exchanger, but that is at constant pressure. Again, 2 to 3 is sorry 2 to 5 heat addition in heat exchanger. Then, we have 5 to 6 same pressure or 5 to 3. It will continue with the same pressure. Then, we have 3 to 4, we have 3 to 4 and then we have 4 to 6. This is heat loss in heat exchanger and 6 to 1 is pre-cooler. This is however TS diagram would look like.

Now, if I draw P V diagram for the same cycle then P V diagram will look like, we have P axis, we have specific volume as V axis and 1 to 2 isentropic compression. So, 1 to 2 is isentropic compression. 2 to 3 is isobaric heat addition, so 2 to 3 in the heat exchanger, then 3 to 5 sorry 2 to 5 is heat exchanger heat addition, 5 to 3 is heat addition in combustion chamber, then 3 to we have 4 expansion in the turbine and then we have 4 to 6 heat loss in heat exchanger and 6 to 1 we have heat loss in, heat loss in pre-cooler. So, these are the P V diagrams.

Now, we have to find out the network and efficiency formulation for the attachment with heat exchanger, Brayton cycle with heat exchanger. So, first the process 1 to 2 we will have same, we will have 1 to 2, w 1 to 2 is unaltered. So, w 1 to 2 is absolute value is C P into T 2 minus T 1. Again, this is for compressor, compressor work input. Similarly, w 3 4 which is C P into T 3 minus T 4 which is turbine work this is also unaltered. So, if we see, w net which is w T minus w C is unaltered and that is C P into T 3 minus T 4 minus C P into T 2 minus T 1. So, this formula is unaltered with the attachment of heat exchanger.

But what is change then? The change thing is Q in, which is amount of heat added into the system. In olden case without heat exchanger heat addition was C P into T 3 minus T 2 , that is from external source, but now we have heat addition from external source by combustion is only C P into T 3 minus T 5. So, this is the only amount of heat added into the process of heat transfer in combustion chamber. So, this is what it is going to be (Refer Time: 09:47) by an amount which is C P into T 5 minus T 2.

So, we will have efficiency will be higher since for efficiency formula we have w net upon Q in, but formula for w net is not changed, but formula for Q in shows decrement in Q in. So, efficiency of the cycle would increase. But then there are some issues which we should remember. Issue 1: when can we add heat exchanger? When? We had seen that attachment of heat exchanger is considered only for low-pressure ratio gas turbines, but when? For this when they seek requirement is you should have T 4 should be greater than T 2, then only we can add the attachment for heat exchanger if T 4 is greater than T 2 then heat exchange is not possible to the air which is going to the combustion chamber, it is also clear from this T S diagram. So, for that we need T 2 to be, T 2 to be lesser than T 4 or T 4 to be greater than T 2.

Then, what we next have to see is what is the heat transfer in heat exchanger, heat transfer in heat exchanger. There is, there are two elements in the heat exchanger one is air in the process 2 to 5. So, there is Q 2 5 and that has to be equal to Q 46 . Since this is heat loss other this is heat gain by the air which is going to the combustion chamber and this is heat loss by the gas which is going out from the turbine exhaust.

So, what is Q 2 to 5? It is C P into T 2, C P into T 5 minus T 2 and this is equal to C P into T 4 minus T 6. So, this is equal. But C P we are considering calorically perfect gas, so C P is same. So, we practically get T 5 minus T 2 is equal to T 4 minus T 6, this is what we are getting.

Next thing what we have to see is, there is a concept called as effectiveness of heat exchanger.

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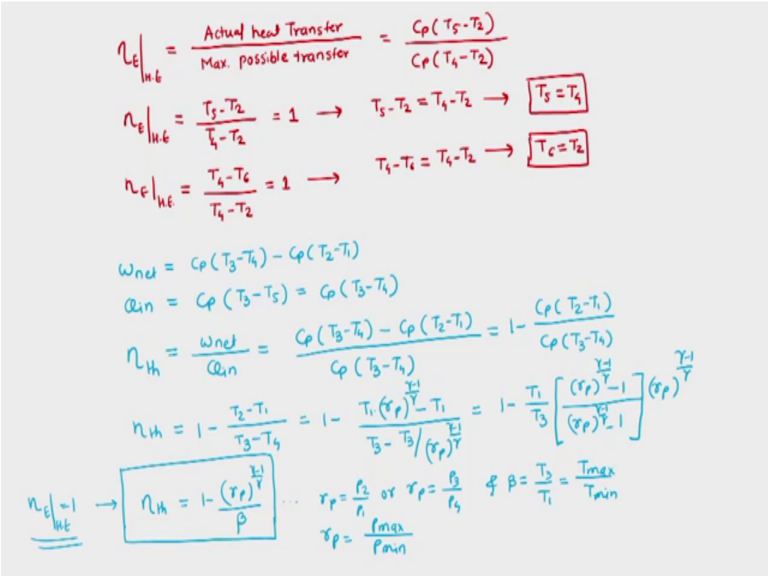

Effectiveness of heat exchanger, and effectiveness of heat exchanger is a Actual heat transfer by Maximum possible heat transfer. And what is actual heat transfer? Actual heat transfer is what we know it is from last slide. What is actual heat transfer? C P into T 5 minus T 2 is actual heat transfer or C P into T 4 minus T 6 is also actual heat transfer. So, actual heat transfer is C P into T 5 minus T 2. But what is maximum possible heat transfer? Maximum possible heat transfer is T 4 minus T 2. So, basically this is the maximum temperature in this at the in the heat exchanger divided by minimum temp minus minimum temperature in the heat exchanger. So, this is the maximum or big temperature difference in the heat exchanger. So, C P into T 4 minus T 2. So, this gives us the formula for effectiveness of heat exchanger. So, effectiveness of heat exchanger practically for us is by cancel C P we get T 5 minus T 2 divided by T 4 minus T 2.

Now, we are going to consider the cases where we will have ideal assumption for the heat exchanger. So, let us consider the heat exchanger to be ideal. So, heat exchanger will have efficiency which is 100 percent for effectiveness to be 1. So, let us consider that this to be 1. Once this is 1, it leads to T 5 minus T 2 is equal to T 4 minus T 2 and then this leads to T 5 is equal to T 4 this is one outcome. But effectiveness, can also be written as for the heat exchanger instead of T 5 minus T 2 it is also is equal to T 4 minus T 6 divided by T 4 minus T 2. This is also heat transfer in the heat exchanger we have seen it in the last slide. Heat transfer is either C P into T 5 minus T 2 or C P into T 4 minus T 6.

This is heat gain by the air from the exhaust and this is heat loss by the exhaust to the air. So, this is actual heat transfer both are we can say here as actual heat transfer. So, what we will have here is also for ideal case it is 1, and then this would lead to T 4 minus T 6 is equal to T 4 minus T 2. So, we will have T 6 is equal to T 2 for ideal case.

So, what is going to happen in ideal case? So, we will go back and we will see what is going to happen in ideal case. In the ideal case we will we can clearly see that there are few things which will be like this, in case we have ideal case leads to T 5 is equal to T 4 and T 6 is equal to T 2. So, what we are having is T 5 is equal to T 4 and T 6 is equal to T 2. So, temperature is same. So, this is how ideal heat exchanger would operate. So, we have to keep this point in mind.

So, this keeping this point in mind we have to modify our formula for w net and we have to modify our formula for efficiency of heat exchanger. So, let us write down the formula for network w net. What we wrote is C P into T 3 minus T 4 minus C P into T 2 minus T 1 there is no alteration in it, but Q in is equal to C P into T 3 minus T 5, but what is T 5 we can see this C P T 5 T 3 minus T 5. So, what is T 5? We have seen T 5 is equal to T 4. So, C P into T 3 minus T 4. So, now, let us define the efficiency. Thermal efficiency with heat exchanger and then it is w net upon Q in. So, it is C P into T 3 minus T 4 minus C P into T 2 minus T 1 divided by C P into T 3 minus T 4. So, this would lead to 1 minus C P into T 2 minus T 1 in divided by C P into T 3 minus T 4, ok.

So, thermal efficiency of heat exchanger will equal to 1 minus C P C P cancels out we have T 2 minus T 1 divided by T 3 minus T 4. So, we know formulas for T 2 and T 4. We can use those formulas. And as per those formulas 1 minus T 1 into r p, bracket raise to gamma minus 1 upon gamma minus T 1 divided by T 3 minus T 3 divided by r p bracket raise to gamma minus 1 upon gamma. So, this is equal to 1 minus we can take T 1 common. So, T 1 we can take T 3 common, so T 3. So, this in the bracket r p bracket raise to gamma minus 1 upon gamma minus 1 divided by r p bracket raise to gamma minus 1 upon gamma minus 1, but then this r p will go in that and then we will get r p into bracket raise to gamma minus 1 upon gamma.

So, thermal efficiency is equal to 1 minus r p bracket raise to gamma minus 1 upon gamma divided by beta, where we know r p is equal to P 2 by P 1 or r p is equal to P 3 by P 4 and beta is equal to T 3 by T 1 or T max by T min. Similarly, r p is equal to P max by P min. So, this formula is for heat exchanger attachment for the Brayton cycle based power plant, but constraint it heat exchanger has effectiveness 100 percent or is in case heat exchanger effectiveness this formula is for heat exchanger effectiveness is equal to 1. So, this is the constraint.

Having said this, now if we are giving with an example or if we have to design a thermal power plant where we are supposed to use the heat exchanger then we can do the all the calculations and find out w net and efficiency effect thermal efficiency of the power plant. Having done with the heat exchanger we will see the next attachment as Reheater.

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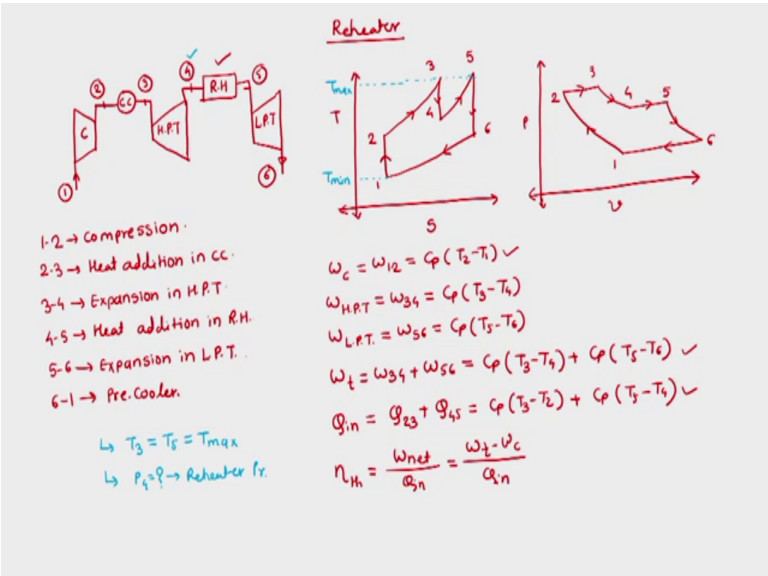

Our next attachment is reheater. Brayton cycle based power plant with reheater. As I said earlier use of reheater is more for increasing the power of it of the Brayton cycle based power plant. So, here as well we will have compressor, here we will come into the compressor, so this is C from the compressor it will go to the combustion chamber, so it is C C then it goes to a turbine and we call it as highpressure turbine. So, initially air would go to high-pressure turbine. High pressure turbine exhaust is committed to a reheater. So, here is the concept of reheater. Then from reheater it goes to the low-pressure turbine. So, this is low-pressure turbine and then air is exhausted. So, this is how we will have Brayton cycle based power plant for the attachment with reheater.

So, here we will again number 1, we will number it as 2, we will number it as 3, we will number it as 4, 5, and 6. So, the process 1 to 2 is Compression, process 2 to 3 is Heat addition, in combustion chamber then process 3 to 4 is Expansion, in HighPressure Turbine then process 4 to 5 is Heat addition in reheater and then process 5 to 6 (Refer Time: 24:39) power plant is Expansion in Low Pressure Turbine, but thermodynamically we will have process 6 to 1 in the Pre-Cooler.

So, now we have to draw thermodynamic cycle for this reheater attachment. Again we will draw T S diagram. So, this is T axis, this is S axis. So, process 1 to 2 process, 1 to 2 is isentropic compression, ok. So, then process 2 to 3 is isobaric heat addition, so this is 3. 3 to 4 isentropic expansion, so this is 4. Then 4 to 5 is isobaric heat addition, so this is 5; 5 to 6 is isentropic expansion, so this is 6. 6 to 1 is pre-cooler. So, here 1 to 2 is isobaric, 2 to 3 is 1 to 2 is isentropic, 2 to 3 is isobaric, 3 to 4 is isentropic, 4 to 5 is isobaric, 5 to 6 is isentropic, 6 to 1 is again isobaric. So, this is T S diagram for Brayton cycle with reheater attachment and this is the reheater.

Now, we can draw P V diagram for the same attachment. So, this is P, this is specific volume V in the P V diagram first we have 1 to 2 as isentropic compression, 2 to 3 is constant pressure heat addition, 3 to 4 is isentropic expansion, 4 to 5 is constant pressure heat addition, 5 to 6 is isentropic expansion and 6 to 1 we have isobaric heat rejection. So, this is Brayton cycle based power plant thermodynamic cycle for reheater arrangement.

Now, we have to again do the same calculation for w net and Q in and so we have to find out what is the thermal efficiency. We have to keep one point in mind that we have to use same formulas what we use to use for calculation of all corner quantities. So, if we know 3, T 3 and P 3 then we should be able to find out P 4 and T 4 from isentropic relations. Similarly, if we know everything at 4 then we can find out everything other thing at 5. However, there are certain constraints which would be known to us later on.

In this phase wire w, now we will have w c which is equal to w 12 and that is equal to C P into T 2 minus T 1 this is an altered. But w T is changed now we have not just w T we have w High Pressure Turbine and that is w 3 4 that is C P into T 3 minus T 4 and then we have w Low Pressure Turbine that is w 5 6 that is C P into T 5 minus T 6. So, we have total turbine work w T is equal to w 3 4 plus w 5 6. So, we have C P into T 3 minus T 4 plus C P into T 5 minus T 6. Parallely we have Q in and Q in is equal to there are two heat addition processes, one is Q 2 3 plus Q 4 5. So, we have C P into T 3 minus T 2 plus C P into T 4 sorry C P into T 5 minus T 4. So, these two formulas are changed. So, efficiency with thermal efficiency of Brayton cycle based power plant with reheater rearrangement it change.

And now we have w net upon Q in and w net is equal to w t minus w c divided by Q in. So, w t can be found out from here, w c can be found out from here, Q in found can be found out from here, and then we can find out thermal efficiency of the Brayton cycle based power plant.

But for the reheater arrangement we have to know one thing rather there are two things. First thing, generally whenever we are using Brayton cycle based power plant with reheater arrangement then we should know one thing that we generally get T 3 is equal to T 5 is equal to T max. These two temperatures are same. These two temperatures are same. So, having same temperatures we can have, so this is T max, this is T max and this is T min.

There is one more point to be noted that what is P 4? What is the value of P 4? Means we should ask our self a question that when we should stop expanding the gas into the low pressure or as a high pressure turbine. At what condition it should enter into the reheater. So, what is the pressure of reheater? So, P 4 is reheater pressure. What is that? So, what is reheater pressure? So, this is a question and we will do a derivation for this question that how to find out what is the optimum condition for reheater, so that we can get a given objective to be achieved.

So, what is our objective in case of reheater based attachment? Our objective is that there is certain amount of oxygen remained into the air which is uncombusted. So, for it has not taken part in the combustions we have to burn that and by burning that we are going to get more work output. So, getting more work output is our basic objective function over here. So, for that we have to find out what is the P 4, what is the condition in, what is the pressure of reheater such that it would lead us to maximum network. So, let us define this, so what our objective over here is to find out what is the reheater pressure which would lead to maximum network.

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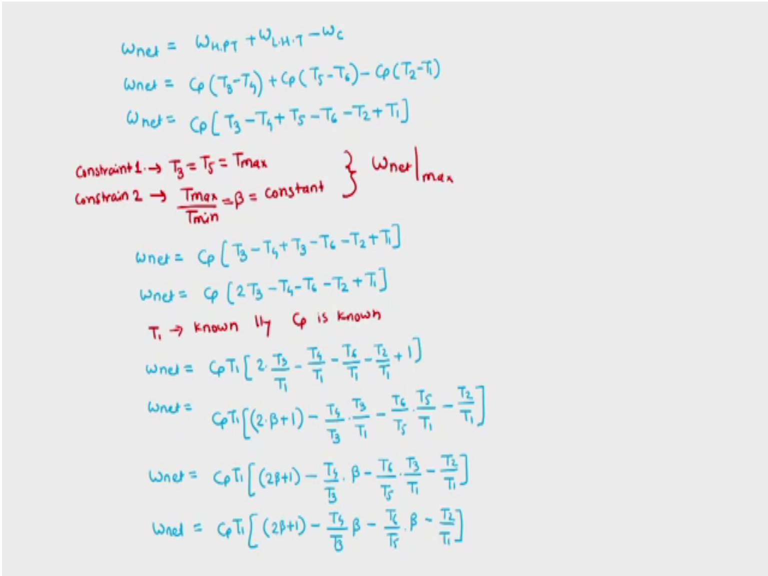

So, let us, work out what is w net, w net for us is w high pressure turbine plus w lowpressure turbine minus w compressor. So, w net it is equal to C P. What is highpressure turbine work? It is T 3 minus T 4 we should remember it here. T 3 minus T 4 T 5 minus T 6 and T 2 minus T 1. T 3 minus T 4 plus C P into T 5 minus T 6 is low pressure turbine work and then we have compressor work is C P into T 2 minus T 1.

Now, having said this we can take C P common and write down T 3 minus T 4 plus T 5 minus T 6 minus T 2 plus T 1. Now, there is one more thing we will put here a constraint which says that T 3 is equal to T 5 or what we will write our constraint as T 3 is equal to T 5 is equal to T max and we will also say the constraint 1, constraint 2. We have T max upon T min which is beta and then that is constant. So, our objective is to find out what is the w net max for these two constraints. If T 3 is equal to T 5 is equal to T max and T max upon T min which is beta is equal to constant. So, for these two constraint we will carry forward our derivation and as per that we have w net is equal to C P then T 3 minus T 4, then we have T 5, our T 5 is equal to T 3 minus T 6 minus T 2 plus T 1. So, w net is equal to C P into twice T 3 minus T 4 minus T 6 minus T 2 plus T 1, but we again know that conditions at the inlet are known,we know that conditions at the inlet are known, so for that all fact we have T 1 is known.

Similarly, C P is known. So, we will continue this derivation and in that now we can get w net is equal to C P into T 1 divided by 2 into T 3 by T 1 minus T 4 by T 1 minus T 6 by T 1 minus T 2 by T 1 plus 1. So, w net is equal to C P T 1. So, we can write it as 2 into beta. T 3 by T 1 is beta. And similarly, T 5 by T 1 is also beta minus, we can see here T 4 by T 3 into T 3 by T 1. So, we have divided and multiplied by T 3. Why did we do so? Since, we can see 4 and 3 are in isentropic line. So, there is a relation which is known between temperature at 4 and temperature at 3 and further by doing so, we got introduced with beta. So, beta is known which is T 3 by T 1 and then relation would have also be known. Similarly, T 6 is here, so we can replace it, we can replace it and say that this is basically T 6 divided by T 5 into T 5 divided by T 1.

But what is T 5? T 5 is basically T 3. What is T 5 is basically T 3so, my plus 1. So, w net is equal to C P T 1 twice beta minus T 4 by T 3. What is T 4 by T 3? 1 upon r P bracket raise to gamma minus 1 upon gamma into beta, ok. So, minus T 6 by T 3 which is T 6 by basically T 5. So, T 6 by T 5, T 6 by T 5 is also same way we can say it as T 6 by T 5 is also 1 upon r p bracket raise to gamma minus 1 upon gamma into beta plus 1. So, we get this formulation for the w net.

Now, we have to do few things about evaluation of this formula. So, let us work out, ok. So, what we can do here? That we can write w net is equal to C P T 1 is equal to into 2 into beta, we can take this one closure 2 into beta plus 1 minus T 4 divided by T 3. We are going to rearrange this term which is T 4 by T 1 as T 4 divided by T 3 into T 3 divided by T 1 minus. Similarly, we can do it for 6. T 4 by T 3 is for high pressure turbine and T 6 by T 5 is for low-pressure turbine. So, we can do it for here also which is T 6 by T 5 into T 5 by T 1 minus T 2 by T 1 plus, so this plus 1 is accounted. So, we will have bracket complete. So, w net practically would be C P T 1 into twice beta plus 1 minus T 4 by T 3 into beta T 3 by T 1 is beta minus T 6 by T 5. What is T 5? T 5 is T 3 by T 1 minus T 2 by T 1. So, w net is equal to C P T 1 in the bracket 2 beta plus 1 minus T 4 by T 3 into beta minus T 6 by T 5 into beta minus T 2 by T 1. This is the derivation for w net.

Now, we did not achieve our objective, yet we have to continue. So, let us continue we will take this w net here. So, w net is basically equal to C P T 1 into twice beta plus 1 minus T 4 by T 3 into beta minus T 6 by T 5 into beta minus T 2 by T 1 and then bracket complete. So, we have 1, 2, 3 and 4 terms and then we have 1, 2, 3 and 4 term correct.

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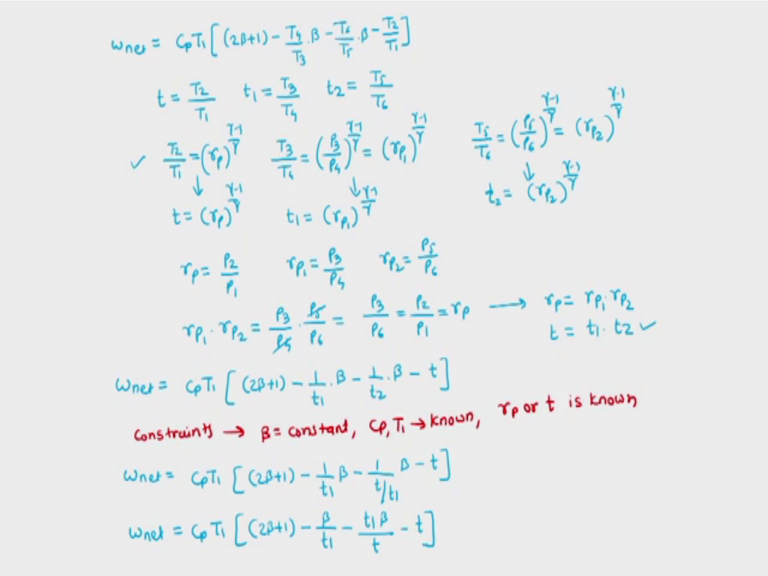

So, here let us start seeing let us redraw the diagram reconsider the diagram. And here let us see that this is for compressor this is T 2 by T 1, for high-pressure turbine this is T 3 by T 4 and for low pressure turbine this is T 5 by T 6. So, let us write this.

So, let t be the temperature ratio which is for compressor, let T 1 be the temperature ratio for high pressure turbine, let T 2 be the temperature ratio for low pressure turbine what we are going to achieve with this, we are going to achieve it this way. So, w net formula we can directly replace it by this. Here what we will have is this way. Basically, we know r p temperature T 2 by T 1 is equal to r p bracket raise to gamma minus 1 upon gamma. Similarly, we know T 3 by T 4 is P 3 by P 4 bracket raise to gamma minus 1 upon gamma. Let us say that P 3 by P 4 is r P 1 then that is equal to bracket raise to gamma minus 1 upon gamma. Then, we have T 5 by T 6 is equal to P 5 by P 6 bracket raise to gamma minus 1 upon gamma.

Let us say P 5 by P 6 as r P 2 bracket raise to gamma minus 1 upon gamma. So, t is equal to from this, t is equal to r p raise to gamma minus 1 upon gamma. So, from here t 1 is equal to r P 1 bracket raise to gamma minus 1 upon gamma. From here we have t 2 is equal to r P 2 bracket raise to gamma minus 1 upon gamma. But what is r P? r P is equal to P 2 by P 1. What is r P 1 we wrote? r P 1 is P 3 by P 4. What is r P 2 we wrote? We wrote it as P 5 by P 6. Then what is r P 1 into r P 2? It is P 3 by P 4 into P 5 by P 6, but if we go back and see the diagram then P 4 is equal to P 5 this we should remember. So, if

P 4 is equal to P 5 this will cancel out, we will get it as P 3 divided by P 6. But what is P 3? What is P 3? P 3 is equal to P 2. P 6 is equal to P 1. So, P 3 is equal to P 2 P 6 is equal to P 1. So, this is r P. So, we get r P is equal to r P 1 into r P 2.

So, if we put this in the formulations for temperature over here in this row we will get t is equal to t 1 into t 2. We can prove this there is no problem, r P we will we can do this bracket raise to gamma minus 1 upon gamma, then bracket raise to gamma minus 1 upon gamma, bracket raise to gamma minus 1 upon gamma then we can get t is equal to t 1 into t 2. So, this is how we can use this. That is why w net is equal to C P T 1 twice beta plus 1 minus T 3 by T 4 by T 3, and T 4 by T 3 is 1 upon T 1 for us into beta minus T 6 by T 5. This is 1 upon T 2 for us into beta minus T.

But we can consider that we were working with, we will assume one more thing over here that we are working along with other constraints as for constraints, total number of constraints as what we said beta constant, C P T 1 known, similarly r P or t is known. So, if this constraint is there then t is a constant for this derivation, and then for this case we can get w net is equal to C P T 1 twice beta plus 1 minus 1 upon t 1 into beta. We can replace this T 2 by this expression which is T 2 is equal to 1 upon t by t 1 into beta minus t. So, w net is equal to C P T 1 twice beta plus 1 minus beta upon t 1 minus t 1 beta upon t minus t. So, this expression we have to remember. We have to keep it forward and then we can differentiate these w net with respect to T 1.

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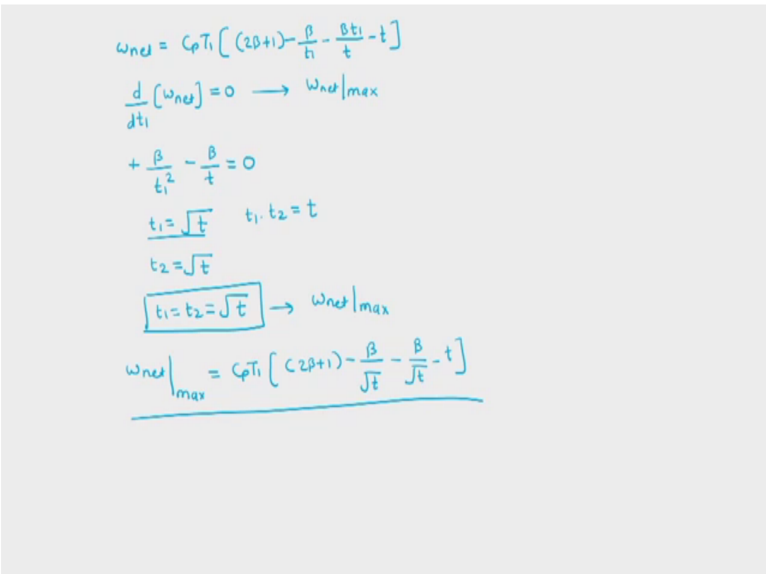

So, from this expression w net is equal to C P T 1 into twice beta plus 1 minus beta t 1, minus we have beta t 1 upon t minus t. Then for getting maximum w net let us differentiate this with respect to t 1 and then we create it to 0, so that we can get w net maximum w net maximum. So, there are few term, if where we have to equate it to 0 C P T 1 being constant it will go beta is a constant, t is a constant.

So, this term in the bracket would have zero derivative, this term will also have zero derivative. So, practically we have only two terms to get differentiated. So, we will have minus beta into plus beta upon t 1 square, plus beta upon t 1 square. This is differentiation of first term, minus beta t is equal to 0. So, beta beta would cancel out. So, we get t 1 is equal to square root of t, t 1 is equal to square root of t. But what we know? We know t 1 into t 2 is equal to t. So, it leads to t 2 is also equal to square root of. So, it leads to t 1 equal to t 2 equal to square root of t. So, this is the constraint for which we can get w net maximum.

So, w net max is equal to C P T 1 twice beta plus 1 minus beta divided by square root of t minus beta divided by again square root of t minus t. So, this is the expression for maximum w net. So, this is how we can arrive and the constraint for the (Refer Time: 51:56) based Brayton cycle power plant where we can get maximum w net. So, the arrangement for intercooler and rest of the thing we will see in the next class.