IC Engines and Gas Turbines
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Lecture – 35
Brayton Cycle- Efficiency, Work Ratio and Optimum Work Output Condition

Welcome to the class. So, we are at the topic of basics of thermodynamics and Brayton cycle. Within that, we had completed already the basics of thermodynamics and we started with Brayton cycle. We saw, what are the processes which involved to compose the Brayton cycle. Then, we saw also the heat and work interaction in the Brayton cycle.

Obviously, we are considering at this moment idea l air standard cycle. So, we first studied in last class about the Brayton cycle, how to write it is P v diagram, T S diagram, h s diagram and then we saw it is heat and work interaction.

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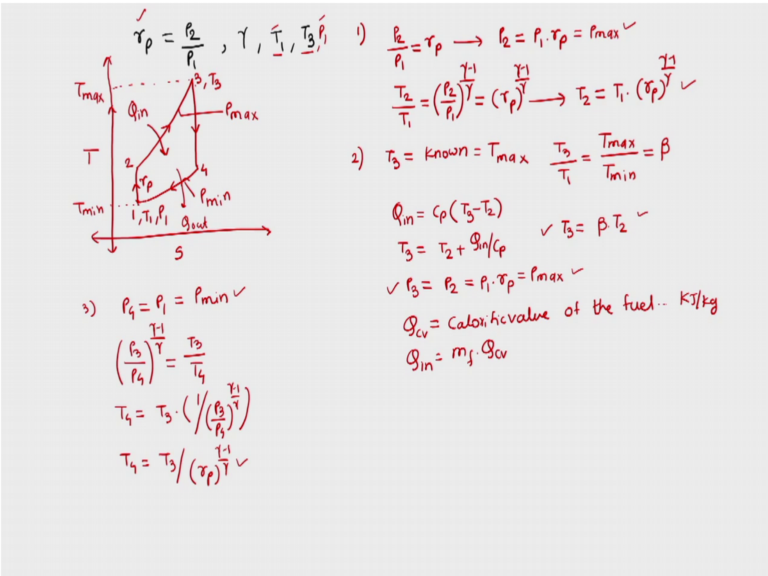

So, in today's class, we are going to discuss how to find out the different corner properties something like this; we are having Brayton cycle here.

So, in the Brayton cycle, we see that there are 1, 2, 3 and 4 as corners. So, we feel that we should know how are the properties at all the four corners; so for that, we will proceed for the calculation.

So, now let us see that what is known to us. From the design perspective, we would be known with r P which is a pressure ratio which is known as P 2 by P 1 and then we would know that we are working with gas which is suppose air or helium or any other gas CO 2.

Then, we know it gamma, then we know r p, we know gamma and then from the application point of view, we would know the inlet to the compressor which is T 1 and we would also know which is the maximum bearable temperature to the system what we are considering. So, this 1, 2, 3 and 4 these are Brayton cycle. In that, we know T 1 here and we know T 3 here and we also know what is r P here. So, knowing this quantities, we should find out what are the other quantity that other corners.

So, in the process 1 to 2, so what is happening? We should note a 2 and P 2 so, but we know P 2 by P 1 is equal to r p, but right hand side r P is known to us. That is why P 2 is equal to P 1 into r P. So, now, we know r P then we should know T 2; for that, we know the formula from isentropic relations T 2 by T 1 is equal to P 2 by P 1 bracket raised to gamma minus 1 upon gamma. So, this is isentropic relation. We can derive it from the point of view that in the isentropic process, we have P v raised to gamma is equal to constant. So, a P v raised to gamma is equal to constant, we have P upon rho raised to gamma is equal to constant.

Then, we can replace rho by T using the relation P is equal to rho r T. Then, we can get the formula T 2 by T 1 is equal to P 2 by P 1 bracket raised to gamma minus 1 upon gamma. But what is P 2 by P 1, it is r P. So, r P raised to gamma minus 1 upon gamma. So, this gives us T 2 which is equal to T 1 into r P raised to gamma minus 1 upon gamma. We should keep one point to be noted that r P is a pressure ratio of the cycle and it is P 2 by P 1; P 2 is the pressure after compression, P 1 is the pressure at the inlet of the compressor. So, P 2 is always greater than P 1. So, r P is greater than 1. Since r P is greater than 1, P 2 is higher. Further r P is greater than 1, so T 2 will be higher than T 1.

Then, at point 3 basically, now we know that P 1 and T 1 are known to us. But we found out P 2 and we found out T 2. Now, we have to find out P 3 and T 3, but T 3 is known. There would be one more thing which sometimes, it would be given how would be T 3 is known to us? T 3 is the temperature which is a maximum temperature of the cycle and that temperature is experienced in the combustion chamber and hence it would be experienced by the high pressure side of the turbine or high pressure turbine but turbine is a rotary element we have seen. It is a turbo machine.

So, in the rotary application of it is, operation of it is if we expose it to high temperature it will get damaged quickly. So, for that, there is an upper limit for temperature which it can get exposed to. So, for that all fact the materials which are used to design the turbine blades will define the maximum temperature at the entry to the turbine. So, T max is known to us from that material perspective. There is one more way of saying that there is a quantity which is designed quantity which is T 3 by T 1 which is rather T max by T min. Temperature 1 is at the inlet to the compressor and at the inlet to the compressor, we know that T 1 is the temperature. So, that is the minimum temperature in the cycle.

So, this is T min and then this if we extend, then this is T max. So, this is T min and T max they are known to us. And that we will call it as beta in the present course of power. So, we would either know beta or directly know T 3.

There is one more perspective that we would be knowing is amount of heat added which is Q in that is also known to you. So, if Q in is known, we know the formula Q in is equal to C P into T 3 minus T 2. Then, in that case, T 3 is equal to T 2 plus Q upon C P or Q in upon C C P. This Q in is kilo Joule per k g. That is how we can know T 3 or if other way, we know beta then T 3 is equal to beta into T 2. So, this is how we can find out T 3. But for P 3, we know P 3 is equal to P 2 is equal to P 1 into r P; process 2 to 3 is a isobaric process or pressure is constant.

So, P 2 is equal to P 3. One more point to be thought over here, how to find out Q in if it is not given to us? Q in is the amount of heat added in the process of combustion is done between the fuel which we are going to add and with the air which we are supplying at the inlet to the combustion chamber. So, there is one thing which is called as Q C v which is calorific value of the fuel, calorific value of the fuel. It is amount of energy liberated if we burn 1 k g of fuel. So, it is given in kilo Joule per k g.

So, we can find out Q C v is equal to mass of fuel into sorry we can say that Q in is equal to mass of fuel into Q c. But we have to keep a point in mind that in our present cycle we call it as air standard Brayton cycle, we are considering everywhere air as the working medium. So, mass of fuel will not come anywhere in calculation except if required for calculation of Q in, that is how we know now T 3 and we know P 3 and how to find out our next element which is 4.

So, for 4, again we know that P 4 is equal to P 1. So, this is known to us. There is one more thing to be reminded remembered that P 2 is equal to P max and also P 3 is also equal to P max. So, P 1 and P 4 is equal to P min. That is the minimum pressure is here and this is P min and then this is P max, maximum cycle pressure. Then, we have to find out what is T T 4. So, how to find out T 4? So, to find out T 4, we will use again the formula which is P 3 by P 4 bracket raised to gamma minus 1 upon gamma is equal to T 3 by T 4. So, T 4 is equal to T 3 into 1 upon P 3 by P 4 bracket raised to gamma minus 1 upon gamma bracket come to it. But what is P 3 by P 4? P 3 by P 4 is r P.

So, so T 4 is equal to T 3 upon r P bracket raised to gamma minus 1 upon gamma. So, again r P is more than 1. So, this number is more than 1 in that denominator that will get divided by T 3. So, you get T 4 which is lesser than T 3. So, we will keep this formula as in mind for our further calculation. Since, now we know everything, we are given with as I said we are given with P 1 and T 1. So, P 1 and T 1 are already known to was in this corner we are also given with r P. So, using that r p, we will found out what is the value of P 2 and what is the value of T 2. So, we know everything at corner 2 also. Knowing Q in or knowing T 3 or knowing beta, we found out how to find out T 3. And then, P 2 is equal to P 3. So, we will find out P 3 also, then we find out what is P 4 and what is T 4.

So, this is the procedure for calculation in case of the Brayton cycle. But once this is done, we will do the next calculation which is for the efficiency calculation of the cycle.

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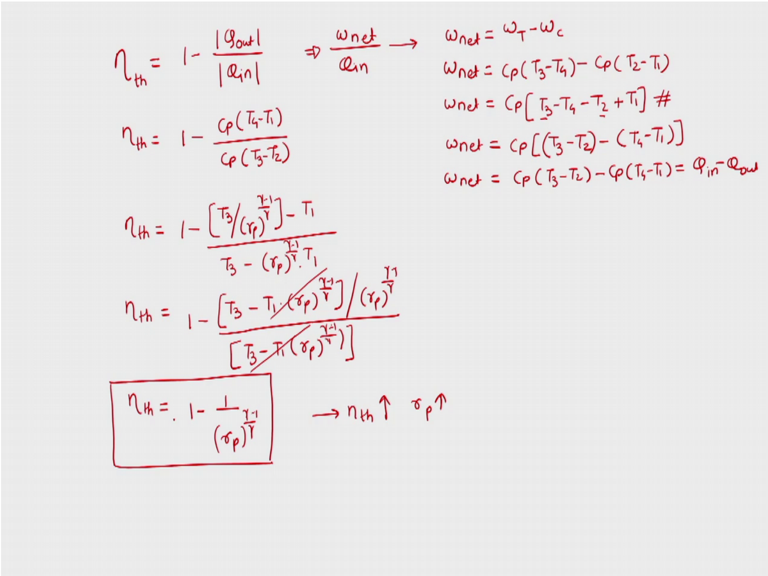

So, efficiency which is thermal efficiency of the cycle, what is thermal efficiency of the cycle, we know that thermal efficiency is 1 minus Q out upon Q in, that is heat lost divided by heat input which is this ratio is subtracted from one. So, thermal efficiency we know the formulas for heat interaction.

So, we can find out. So, from the Brayton cycle if you see the Brayton cycle over here, then we know that this is Q in and this is Q out. So, Q in is equal to C P into T 3 minus T 2, Q out is equal to C P into T 4 minus T 1. So, knowing this we can calculate C P into T 4 minus T 1 divided by C P into T 3 minus T 2. This is one of the formulas for finding out heat efficiency of the cycle.

But there is one more way we can write efficiency is equal to W net upon Q in. This is other way to write the formula but what it this formula is going to lead to us. So, this formula is going to lead to us what is W net; W net for us is equal to W turbine minus W compressor. Again if we go back and see Brayton cycle, then turbine were C P into T 3 minus T 4 and compressor work is C P into T 2 minus T 1. So, W net is equal to this but we can rearrange this terms and then we can say C P to be common, then we have T 3 minus T 4 minus T 2 plus T 1.

So, having said this, we get W net is equal to C P into we will rearrange the terms we will say T 3 minus T 2 we are taking this and this together. Then, we can take other terms together which is minus we will say T 4 minus T 1 but this W net can again be said to be

T 3 minus T 2 into C P minus C P into T 4 minus T 1 which was practically Q in minus Q out and this is what the left hand side formula has already given us. We will remember this formula which is C P into T 3 minus T 4 minus T 2 plus T 1 as W net formula this is where we would need to calculate one more expression in the down line. Then, what would happen? We know, now there are few thing which are known to us.

So, among those 1 minus C p, C p, C P get cancelled. So, we do not have to mention C P now. So, we have T 4 minus T 1 but what is T 4 from this we know the formula for T 4 here and then we should know also the formula for T 2 which would be required. So, so we will write down that formula for T 4 which was T 3 divided by r P raised to gamma minus 1 upon gamma minus T 1. And similarly, we have T 3 minus r P bracket raised to gamma minus 1 into T 1.

So, this is the formula for T 2, this is the formula for T 4. So, thermal efficiency is equal to 1 minus T 3 minus T 1 into r P bracket raised to gamma, gamma minus 1 upon gamma and this would get divided by r P bracket raised to gamma minus one upon gamma divided by T 3 minus T 1 into r P bracket raised to gamma minus 1 upon gamma. So, we can see that this bracket and this bracket would get cancelled. So, thermal efficiency of Brayton cycle is 1 minus 1 upon r P raised to gamma minus 1 upon gamma. So, this is the final expression for finding out thermal efficiency of the Brayton cycle.

Now, we can understand some aspects or here. What is that thermal efficiency here, increases with increase in r P. So, if we increase the pressure ratio of the cycle, then denominator increases. So, this one upon this number will decrease. So, you will get more thermal efficiency. So, it is expected to our at higher pressure ratio to get higher efficiency for Brayton cycle. So, this is the formula for Brayton cycle.

So, now we will go for the next derivation which is for the work ratio. So, this formula once it is derived, we have to find out next performance parameter for the Brayton cycle and that next performance parameter is work ratio.

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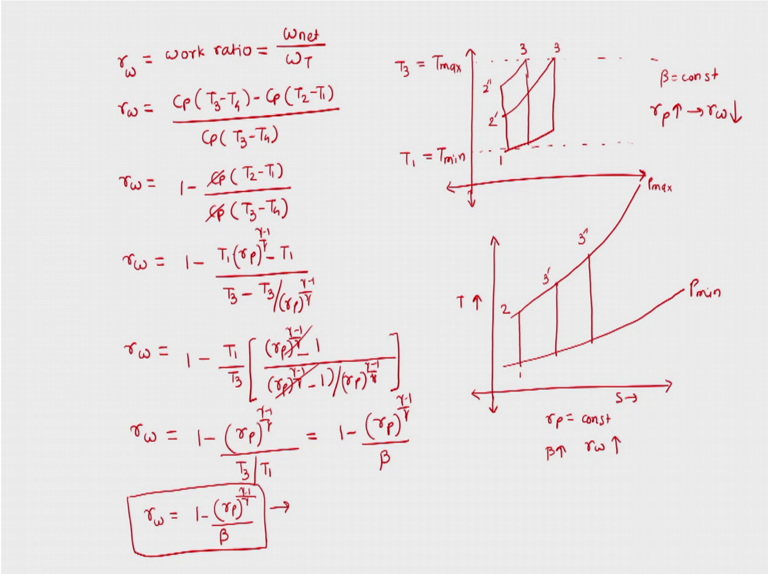

I will denote r W which is work ratio and work ratio is we have said that W net upon W T which is network upon turbine work. But we have found out, what is network. So, r W what is network C P into T 3 minus T 2 which is turbine T 3 minus T 4 which is turbine work minus C P into T 2 minus T 1 is compressor work divided by C P into T 3 minus T 4 which is turbine work. So, r W can be written, we can split this denominator for two numbers.

And then, we can get 1 minus C P into T 2 minus T 1 divided by C P into T 3 minus T 4, C P C P would get cancelled. And then, we get r W is equal to 1 minus and T 2, T 2 can be replaced as T 1 into r P bracket raised to gamma minus 1 upon gamma minus T 1 divided by T 3. T 4 again can be replaced as T 3 divided by r P raised to gamma minus 1 upon gamma.

Then, r W which is a work ratio is 1 minus. We can take T 1 common from the numerator. We can take T 3 common from the denominator. So, it is r P bracket raised to gamma minus 1 upon gamma minus 1 divided by again we can write r P bracket raised to gamma minus 1 upon gamma minus 1 but we will have upon r P raised to gamma minus 1 upon gamma. But then, this bracket and this bracket would get cancelled. So, r W is equal to 1 minus we have r p, this will go in the top r P bracket raised to gamma minus 1 upon gamma divided by we can write this T 1 by T 3 as 1 upon T 3 by T 1.

So, 1 minus r P bracket raised to gamma minus 1 upon gamma divided by beta. So, this is the derivation for work ratio. So, if we know what is the pressure ratio or operating pressure ratio for these cycle which is Brayton cycle and then if we know what is T max upon T min for the Brayton cycle which is beta temperature ratio for the maximum and minimum, we can find out work ratio for the Brayton cycle. Now, what is the impact?

Now, we will consider two cases for drawing the T h diagram. In one case, we will fill that beta is constant and if we vary r p, what will happen to r w? And in other case, we will fill that if r P is constant and if we vary beta, what will happen to the work ratio. Then, we can draw T h diagram for this two facts. So, let us consider a fact that we are having constant beta. So, if beta is constant, then we will mark we will say that this is T min and this is T max. So, this is maximum temperature which is T 3 and this is minimum temperature which is T 1. So, these are known to us.

Now, we initially compressor from here to here, then we get T 2, this is 1, this is I will say 2 dash and I can go from here to here. And then, I can come back and my cycle is done. So, this is for one beta. So, now, I will draw next cycle for the same beta, but with increased r P. So, I can go from here to 2 double dash, but now my beta is same. So, I cannot go more than this. So, I will have to come back from here. So, it is evident from this diagram that here in the first case, in the first case, we had lower turbine outlet temperature and much more turbine we have lower compressor outlet temperature and higher turbine inlet temperature. So, for that, what we can get is higher work ratio ok.

So, we if have for the same beta. So, if beta is constant and if we increase r P beta is constant, if we increase r P then what would happen r W decreases which is evident from in this diagram. So, we are increasing r P to instead of 2 dash, we went to 2 double dash. And since we went to 2 double dash, we have we have problem in that we have increased compressor over output input, but in that context, not much turbine work has been increased. So, work ratio has decreased. So, next is if I have constant r P and if I vary beta, I am going to vary beta. So, I am, but my r P is constant.

So, now I will write these as 1 and this as other this is P min and this is P max. So, this is constant for means I went from 1 to 2 or here and then I can stop here. This is my T max and then I come back. This is my one cycle. But instead of this fact for the same pressure ratio, instead of stopping here at 3 dash, I can stop over here which is 3 double dash, then

I can get this as my turbine work out. So, we can see that if we are having beta increased for same r p, r P this is temperature and entropy. If r P is equal to constant and if we have beta increased, then we see that r W decreases sorry increases r W increases. So, these r 2 major point which we should remember,

So, now as we can see we have derived two relations; first relation what we derived was for thermal efficiency of Brayton cycle and next relation what we derived was for work ratio of Brayton cycle. We had to remember one point over here that the work ratio what we have made derivation for in this class is more important when we will get introduction to the non-ideal cycles.

So, in present case, we have turbine and compressors are ideal turbine and ideal compressor but what would happen is turbine and compressor their efficiency would not be 100 percent. And if their efficiencies are not hundred percent, then work ratio is an important parameter to compare. Problem is this if I want to compare two cycles which are operating at different pressure ratios, now how can I compare this. That is where if two cycles Brayton cycles have two sub Brayton cycle based plants have same pressure ratio then thermal efficiency can be helpful for us to compare the cycle which has higher thermal efficiency is a better plant.

But if we have two different pressure ratios then thermal efficiency would not be directly useful. In that case we have to compare with work ratio and then we can define the cycle to be good which is having higher work ratio since higher work ratio means the cycle is insensitive or less sensitive to the component efficiencies that are turbine efficiency and compressor efficiency; so this is what we have to discuss.

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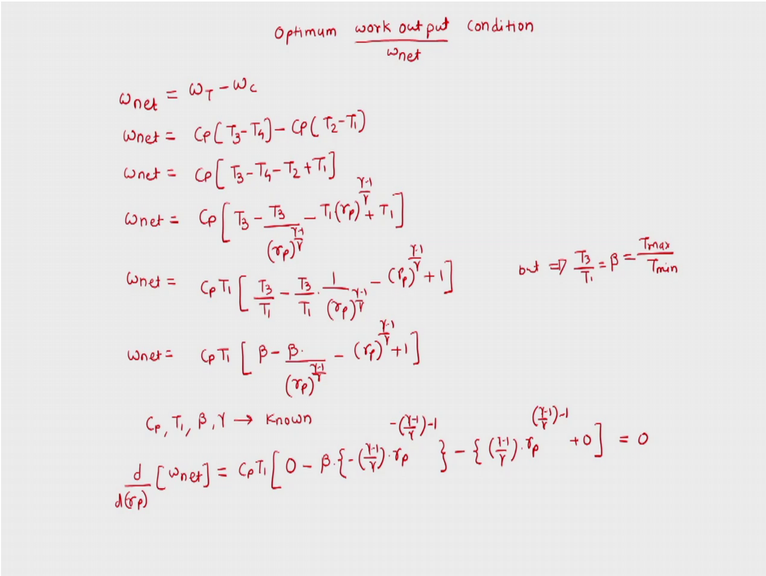

The next point to talk over here is optimum work output condition. What we have seen till time is that we can find out W net, so which is work output. So, we are we practically means optimum condition for W net, when W net will be maximum or T is the condition at which W net will be maximum. So, that condition we have to find out. So, let us start for that condition.

So, for that we first have to write down what is W net W net is equal to W T minus W c. So, W net is equal to C P into T 3 minus T 4 minus C P into T 2 minus T 1. So, then we can say W net is equal to C p, we can take it common, then T 3 minus T 4 minus T 2 minus T 1. We can do one thing over here that, we can use the formulas for T 4 and T 2 to replace their existing form.

So, W net is equal to C P into T 3 minus T 3 upon r P bracket raised to gamma minus 1 upon gamma minus T 1 into r P bracket raised to gamma minus 1 upon gamma, we have plus sorry, here we have plus T 1. So, it is like this. So, having said this, we can write C P into this bracket. But in this bracket we will take T 1 as common. So, T 1 so, T 3 by T 1 minus T 3 by T 1 into 1 upon r P bracket raised to gamma minus 1 upon gamma minus r P bracket raised to gamma minus 1 upon gamma plus 1. So, this is what we have arrived at.

Now, we will use the term which we had already used saying that, but what we know is T 3 by T 1 as beta which is T max upon T min. This is the design constraint. So, once this constraint is known to us, we can write W net is equal to C P into T 1 then beta minus beta upon r P bracket raised to gamma minus 1 upon gamma minus r P bracket raised to gamma minus 1 upon gamma plus 1. Now, our objective is to find out when W net can be maximum. So, let us make an assumption for the finding out derivation and that assumption is C P is known, T 1 is known.

And beta is known and gamma is known. So, these are known quantities. So, since they are known quantities, this expression will have this all quantities to be constant. So, C P into T 1 is constant, beta is constant and then gamma is constant, then W net practically is function of only r P. So, let us find out and then we need to write. Since it is only function of r p, I can differentiate with respect to d by d r P of W net. And this can be then, we can use this term and then we have four terms here 1, 2, 3 and 4 everything will get multiplied by C P T 1, but C P T 1 is a constant. So, we need not worry about it. So, that constant will appear as C P T 1 over here, then beta is not function of r p, then that is 0 minus this beta is as it is.

And then, we will have this term can be differentiated and written as minus gamma minus 1 upon gamma into r P bracket raised to minus gamma minus 1 upon gamma minus 1. This is differentiation of second term minus differentiation of third term.

So, differentiation of third term will be gamma minus 1 upon gamma into r P bracket raised to gamma minus 1 upon gamma minus 1. Then, last term is one for which differential will become 0 but our job is to find out the condition for maximum W net. So, this r P condition will lead to maximum W net. So, this differential will be 0. Having said this differential to be 0, we can make use of I will rub this and I can use this part and then this differential can be made to 0 and then C P T 1 can be divided since it is a non-zero number basically.

Then, we have only two terms to get the W net condition.

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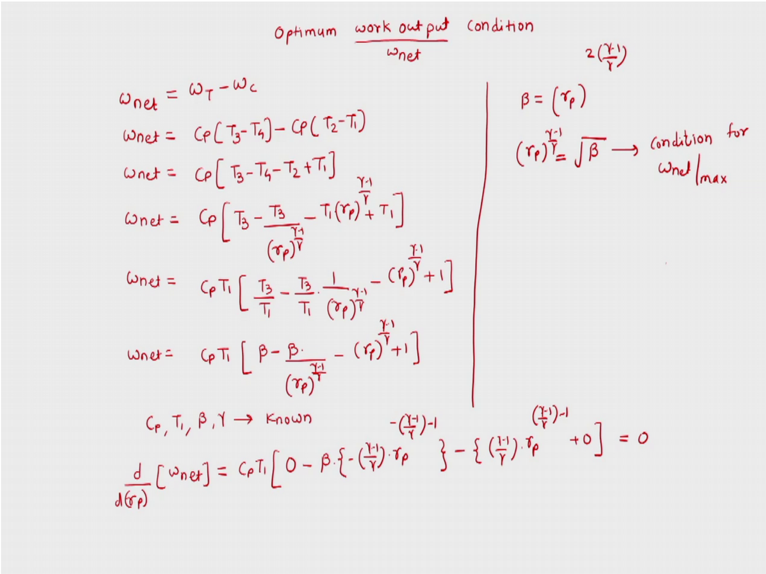

And those two terms would lead to if we rearrange these two terms, we will get that C P that T 1 would go and then we would have this gamma minus 1 upon gamma would also go and then, we can rearrange the ratios and then we get beta is equal to r P bracket raised to 2 upon gamma minus 1 upon gamma. So, r P raised to gamma minus 1 upon gamma is equal to square root of beta. This is the condition for W net to be maximum ok. So, we will go to the next slide and we will write down the condition for W net to be W max, if we have r P raised to gamma minus 1 upon gamma is square root of beta.

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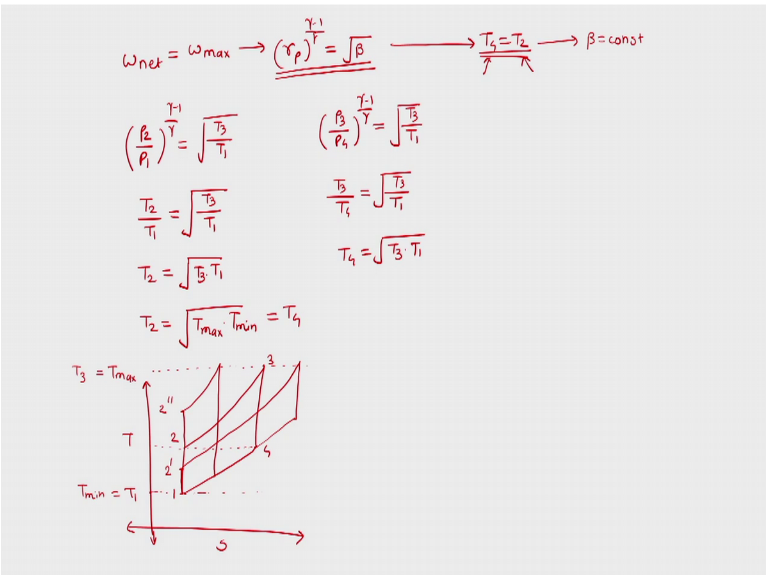

But what is this r P raised to gamma minus 1 upon gamma, we are trying to find out what does this condition thermodynamically means to us.

We know r P is equal to T 2 by T 1 bracket raised to gamma minus 1 upon gamma, but beta is equal to beta is 1 second r P is equal to P 2 by P 1 bracket raised to gamma minus 1 upon gamma and then we have, then we have square root of T 3 by T 1. And what is P 2 by P 1 bracket raised to gamma minus 1 upon gamma. So, this is equal to T 2 by T 1 and then it is equal to square root of T 3 by T 1. So, what we can get over here is equal to T 2 is equal to square root of T 3 into T 1. So, basically T 2 is equal to square root of T max into T min.

But there is one more thing. This r P can also be written as P 3 by P 4 bracket raised to gamma minus 1 upon gamma which is again square root of T 3 by T 1 or what is P 3 by P one bracket raised to gamma minus 1 upon gamma? That is T 3 by T 4 which is square root of T 3 by T 1. So, what we get out here is T 4 is equal to square root of T 3 into T 1. So, what we get is T 4 is equal to T 2 is equal to square root of T max upon T min. So, at the condition of this, we practically get T 4 is equal to T 2. What is T 4? T 4 is temperature at the outlet of the turbine. What is T 2? T 2 is the temperature at the inlet of the compressor, inlet of the outlet of the compressor. So, temperature at the outlet of the turbine temperature at the outlet of the compressor, they should be equal ok. So, this is the constraint.

Then, what we can see is this, we can draw the T S diagram for this constraint. And then, how would it look like basically what we can get out here is this. So, this is the constraint. This is T 1 which is T min and this is this is T max which is T 3, 1, 2, 3, 4 and then T 2 is equal to T 4. But we have to remember that we are having this constraint under the fact that beta is equal to constant. So, we have to remember that we are frozen where this T 1 and this T 3. Among this, we get if we vary pressure ratio, then what would happen? We can have lower pressure ratio.

So, in case of lower pressure ratio, I will have T 2 dash and then I can go from here and then, I will end up here. Then, what is happening in this case is that we are having more heat rejection, more heat rejection leads to lower efficient lower W net ok, there is loss. Further, we need to go to T 2 double dash for the same beta. And then, this would lead to very large compressor work output. So, one stage we have very large compressor work output and in other case, we have large heat rejection. So, these two constraints lead us to the fact that W net would not be maximum and for maximum W net, we need T 2 is equal to T 4

So, this is how we have derived two relations in today's class. First relation was about thermal efficiency of the cycle where we saw that thermal efficiency depends upon pressure ratio of the cycle, then second derivation was about work ratio of the cycle. We saw that work ratio depends upon beta and pressure ratio of the cycle and third derivation was on W net maximum condition and we saw that compressor outlet temperature and turbine outlet temperature, they should be same if we want to have the maximum work output condition.

Thank you.