Principles of Mechanical Measurement

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Module – 03
Digital Techniques in Measurement
Lecture – 08
Basics of digitization & number systems

Hello friends, welcome back to the 3rd week of our course on the topic of Principles of Mechanical Measurements. And this week we are going to talk about a very interesting topic that is Digital Techniques in Measurement. You all know that nowadays everything is moving towards digitalization or digitisation both transfer used interchangeably; so, we shall also be using both of them.

Now while earlier approach for most of the experimental works was to use some kind of analogue instruments; analogue means where directly we are getting the value in terms of either movement of an indicator over a dial or by movement of a fluid through a capillary tube or something similar; thereby directly giving us a value or rather we have to read the value from some scale which is inscribed on the device itself.

Nowadays all kind of measurement device and everything else is moving towards digital media where we are getting the numbers directly from the screen. And we do not have to deal with some continuous movement of certain indicator or something similar; rather we are just interested with the final number that we are getting. You can compare this to our modern life like something earlier days those movies used to be movie or music etcetera used to be coming in the record players which were something like an analogue media.

And later on or the cassettes may some of you may have seen the cassettes tape recorders, but it is unlikely that or it is very likely that many of you have not seen at all. Now everyone moved to CD players and later on the USB sticks where we are actually storing the entire content of a movie or a large quantity of music or maybe several movies into a say very small USB stick which may be of the shape of just a finger or even single nail, I should say or even smaller than that sometimes.

Like the memory cards that we use in cameras or in media in mobile phones they are extremely small in size and that power of miniaturization has been brought in by this era of digitisation. And therefore, it is very important that before you start talking about different measuring tools itself, we have to talk a bit about the digital techniques that we commonly use in the field of measurements. The way how we convert common analogue signals to digitals and how it recovered the digital data back to the analogue form and also several relevant topics associated with that will be our focus of discussion in this particular week.

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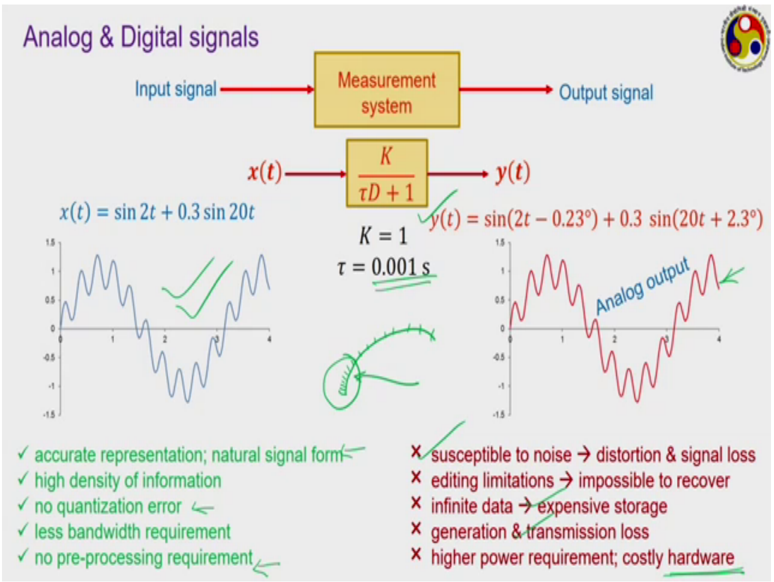

We now know from our discussion on our previous week is that any measurement system can be viewed to a some kind of mathematical operator; something which is known as the transfer function which takes one input, sometimes when multiple inputs by making an interface between the device and the physical media or physical system.

And that single input or multiple inputs goes through some kind of mathematical operation leading to the production of one or more number of outputs. In the previous chapter we restricted ourselves to singular input singular output devices because it is not possible to deal with more inputs or outputs simultaneously in a course rather that will more like that will become like a research topic.

So, and here also we are going restricting ourselves to single input single output kind of situation. Now we know that whatever input we provide depending upon the transfer function associate the measurement system, we get a corresponding output. Let us take an example; let us just think about a first order first order system and we know that the transfer function for a first order is given like this form K upon tau D plus 1 you already have seen the development of this equation and I am sure you have already used this one for solving some problems.

Now, let us see the x t which is a time dependent input signal. Let us assume that our input is a sinusoidal function of this form and our system the first order system is having its static gain to be equal to 1 and also a time constant of 0.001 second. Now we are supplying a periodic input to this and that a corresponding periodic signal can be viewed to be a combination of two harmonics. The first one having omega equal to 2 second one is having omega equal to 20 and also the corresponding amplitudes first one is having a amplitude of 1 and second one is of 0.3.

I hope you remember we solved an example problem there mostly to show the effect of tau on the form of the periodic outputs; it is precisely that problem. So, I am directly taking the solution from that you know how to develop the solution, where in the solution side also we are having extremely small time delay. So, 0.23 degree of time delay as we know I should say phase delay has been introduced in the first harmonics and 2.3 degree has been introduced in the second one and their amplitude remains the same.

So, this static gain and time constant; particularly the time constant corresponds to excellent frequency response, both phase and amplitude point of view. We are also familiar with their phase form from the example that we solved last week. This is the input side and this is the corresponding output side output is following the input quite in a fine manner. Now our objective here is not to focus on the shape of the output because that is what we have discussed. We now know that depending upon the magnitude of this stuff we can have different shapes.

But once we have this particular shape this continuous representation of the output is what we refer as the analogue output. Corresponding to this particular analogue signal that we are getting from the physical system, our system is receiving the analogue signal and also whatever mathematical operation it is doing following that it is giving us giving the output also as a analogue signal like this. Where, we have the value of y available at every possible time instant over the entire span of the experiment and also theoretically it is possible to have therefore, infinite number of data points that for any value of t, we have a corresponding value of y.

And hence any other once we know this particular mathematical form, then any other operation that we want to do on this at least theoretically we can definitely do that. So, analogue signal provides several advantages; the first one is it is the most obvious one; it is the natural signal itself and so it is a highly accurate representation of what we are trying to get. Now we have a very high density of information because theoretically we have infinite number of data points and we can extract the value of the output signal y at any possible times when starting from t equal to 0 to t equal to 4 in this example for any value of t; we have some y value available.

Then there is no quantization error; what is quantization? We shall be coming very shortly less bandwidth requirement and also there is no pre processing requirement for such kind of instruments or such kind of the signals rather. We are giving the input signal in this raw form like this one and whatever output we are getting; we take that in its raw form itself we do not have to go for any kind of pre processing of the input. And also in most cases we do not need any kind of post processing for the output as well. However, despite having such advantages the analogue signal has some disadvantages of its own as well.

The first one and the very very big issue; analogue signals are generally susceptible to noise leading to the distortion and loss of signal. Noise means say whenever you are doing some experiment, the noise refers to what you want to measure apart from that certain other parameters are also influencing your output.

Say for example, you are using a manometer to measure the pressure; you are using a mercury manometer and because of the pressure difference between the atmosphere and the physical system that you are handling because of the pressure difference there is a deflection in the mercury column. And by measuring the height difference between the mercury column in the 2 arms of the u-tube manometer; we can get the idea about the pressure in the physical system.

However, the height of the mercury column in a manometer can also be influenced by; if there is a large change in temperature across the 2 arms or maybe in the local location like whatever value of deflection mercury column, you are getting for a certain temperature; if the temperature of the surrounding increases by significant amount mercury may suffer some amount of thermal expansion leading to a different value of the deflection.

Also if there is some kind of vibration available in the if your if there is a vibration of the floor where you are doing the experiment that may get reflected; another big issue maybe there. Suppose this is you mercury column; this is a u and your rather this is a u tube manometer and this is your mercury column this is the height difference that we are talking about. Now suppose this particular manometer is mounted on some vehicle; as long as the vehicle is moving with a constant velocity or it is stag it is stationary, then there is no issue.

But, if this mercury sorry if this vehicle starts to accelerate then what will happen? Your mercury manometer we tilted like this leading to a different amount of reflection maybe of the mercury column in this or if the vehicle is tilted or suppose our vehicle is not tilted or but or the mercury I should say the manometer is not tilted, but the vehicle is accelerating. Now because of the acceleration it may happen that the mercury column instead of being horizontal that appears in some sort of angle; now there is a big issue how to get the measurement; like whether should we take this particular value or should we take this particular value or should we take mean then exactly which one refers to a central line and there is lots of confusion to deal with.

So, the analogue signal is susceptible to noise; noise coming from generally from the surroundings and also sometimes maybe from the physical system itself. There is editing limitations once some portion of the signal is lost; it is not possible to recover that. Infinite data which is an advantage, but that leads to very expensive storage because when we are dealing with this kind of output data; it always looks good to see this kind of signal on a cathode ray oscilloscope kind of device.

But when we are talking about saving this then we theoretically have to save infinite number of data leading to extreme difficulty in storing and also extreme difficulty in post processing the same amount of data. Then generation and transmission loss particularly transmitting this kind of medium over a long distance will always lead to some loss in this; just think about the normal power supply. Generally, we get power supply to our houses in the form through the high tension lines in the form of 440 volt, 50 hertz signal.

But, if that same wire is supplying power over say 100 kilometre, then because of the joule heating and other kind of induction losses; a significant amount of voltage drop you may experience at the consumers side; that is what we refer as this transmission loss. And the power requirement can also be quite high particularly the hardware quite costly.

So, analogue signals have certain issues on its own; there is another problem sometimes if we are dealing with an instrument which is having a dial gauge. Like this is a dial and we have an indicator moving over the dial gauge; like in several common electrical devices like voltmeter and ammeter, like in pressure dial gauges sometimes in certain temperature measuring instruments, tachometer which is generally used for measuring rpm all have this kind of device.

And most often you will find that the scale that we have on the device or not linear; rather the values are quite cramped up towards the initial part and then gradually getting relaxed as we are moving towards much higher values. So, linearity is not maintained and reading values with high precision within this zone can be very very difficult. So, analogue instruments have problems of their own while there best advantage is the absence of quantization error.

I should say absence of the quantization error and very accurate representation; the and also the absence or non requirement of any kind of pre processing the biggest problem is the susceptibility to noise to other external forcing systems very large storage requirement. And also the transmission losses plus the hardware are always very very costly; just compare just think about say you have one book; a book of 500 pages and the material of that book is also available in the form of an e-book which is generally come meaning in a PDF; as a PDF file.

Now, compare their sizes; where you digital item is can be stored anywhere you can store it in your computer, you can store it in some external hard disk, you can also store it in some in a very small say memory card kind of thing, but that book requires a specific storage, it also needs to be handled very very precisely. Because any mishandling may lead to damage of the book maybe some page may get damaged, may have permanent damage making it impossible to read. But there is no such problem with that e-book and also you can make any number of copies of that e book is allowed.

And so you can carry it with it anywhere you go which is not possible with the book itself that is a biggest advantage of having a digital media over an analogue one. So, because of all this problems particularly with storing and handling and transmitting the analogue media and also the susceptibility to the surrounding conditions the whole world is moving to the digital platform.

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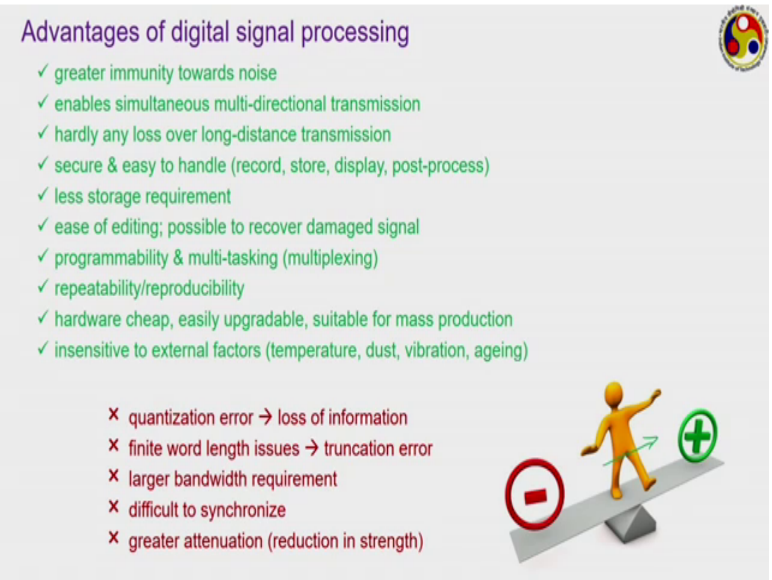

And let us just summarise the advantages that we may get with digital signal and corresponding processing. Firstly, greater immunity towards noise we shall be shortly seeing one example.

Digital signals are you can say they are totally immune to the surrounding effects; there by whatever input you are giving that comes out exactly in the same form or whatever form we want that. That enables simultaneous multidirectional transmission without any kind of transmission losses. Hardly any loss of a long distance transmission as I just mentioned; secure and easy to handle you can easily record store or store the data like think about recording a song in a CD or in a USB stick that is much easier than recording in a tapped medium or storing in a tapped medium.

The display and post processing power also quite easier; you can watch such data is available with you; you can save it in any place and then you can use any suitable software to do the editing. Like suppose, if you are want to edit say for a certain situations you have the marks of students in a particle class are available and you have stored that marks in a file in your computer. Then anytime you can go back to the file and whatever kind of post processing you want to do on that; you can do it using a suitable software and that same applies to any other kind of digital player media.

Less storage requirement; in fact almost negligible amount of storage requirement compared to the analogue one; we have already given example of that book. Ease of editing possible to recover damaged signal as well we shall be seeing one example also. Programmability and multitasking and multiplexing means the same digital media can be used for different kinds of purposes and they can also be used simultaneously. Then repeatability and reproducibility very easily we can make copies of that or we can reproduce the same thing into another device; hardware are also commonly cheap.

We can easily upgrade that and also the hardwares are generally suitable for mass production. Like think about a small chip, now it is much much easier to go for mass production of a small chip compared to a large tapped kind of storage media. And finally, the insensitivity to the external factors like temperature, dust, vibration hardly any ageing effective you will see, but these factors are always very much influential on the analogue signals. So, the digital signal comes with so many of advantages, but there are certain issues as well. One is the quantization error which was not present or which is not present with the analogue signal.

Quantization leads to the loss of information what we mean by quantization? I shall be coming to the next slide. Finite word length issues, leading to truncation error again I shall be discussing about this later on. Band width requirement general is a bit larger for the digital media; they are difficult to synchronise.

Synchronisation refers to suppose you want to record one live incident going on. Now there are two things that you have to record; one is of course, the movement or whatever is going on in form of light and that is the scene itself and other is a corresponding sound. And now, you are recording them generally in two using two different kinds of instruments and converting both of them to corresponding digital form, then you need to synchronise them properly other is what will happen?

You must have experienced of seeing a very badly synchronised movie where the sound comes maybe after 10 seconds of 10 seconds after the actual speaker has already spoken. So, it is very very irritating to face this such kind of situation, but that synchronisation is a bit difficult and requires proper expertise which generally is not at all required in an analogue media.

And greater attenuation leading to reduction in the signal strength, but despite such disadvantages there are ways we can overcome disadvantages and the weight of the advantages are much much more compared to disadvantages forcing the world or inventors to move towards the digital side. Effort is on or there are standard practices of tackling these disadvantages means how to reduce the quantization error, how to have better synchronisation or less attenuation of the stored media transmitted media. There are quite standard procedure and newer methods are coming up every day; so hopefully this errors also will not be that much significant times to come.

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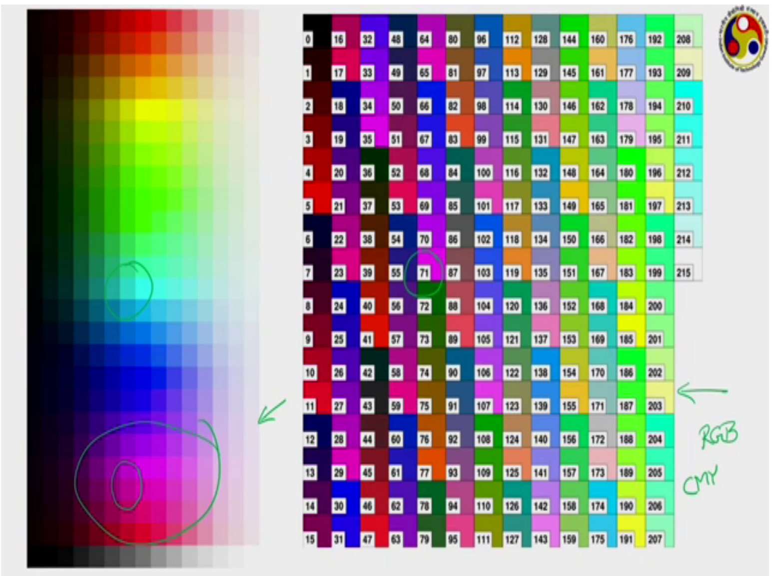

So, let us now the some examples that we are talking about. The on the left hand side suppose you want to paint your house or one particular wall of your bedroom and the corresponding supplier supplies you or provides you with a coloured bar chart. Now two kinds of bar chart he has given to you in one the one on the left that is my left that is what I am talking about this one. Here you are having so many shades of colour. But suppose if you want to pick up any one of them there it is very very it will very difficult for you to explain that.

Like suppose you have decided on this colour; now without the help of the card itself how can you make the painter understand about your choice? It will be very very difficult; it will be very very painstaking. Look at how many shades of pink you have like suppose you have decided to take this particular one, but how you will make it understand?

Because all these arounds are having more or less similar shade they all can be categorised as pink. And so you have to take this card and physically you have to show, but then after you have selected this and you want to tell your friend about your choice of colour then again you are in a soup. Because, you do not know the name of this one or rather only idea that you can give is a its a pink light or dark or something, but there are so many surrounding shades which are also satisfying the same criterion.

Now look at what we have, here each colour has been given certain number. And therefore, if you want to select any one of them then you can easily refer by number. Like suppose you have selected this particular one then you do not need to at all talk about that this is pink or green light or dark; rather you can always say that I have selected colour number 71.

And this way we if this particular chart is a standard one then without taking the physical chart physically with you can tell the painter that you have you have chosen 71 you can go to the shop and purchase paint number 71; you can also inform anyone else anyone interested about what is your choice.

So, that is what is digitisation are the advantage of going for digitisation where everything is represented in terms of numbers. We are not depending upon our senses to get a proper idea or overall idea about our selection, rather we are giving a number assigning a number to everything else and in terms of number itself we are giving all possible characteristics. Like you must have heard about this kind of format RGB; RGB refers to Red, Green, Blue.

This is one of the standard colour format used by computers where every colour can be categorised as in either RGB or depending upon how much percentage of red how much percentage of green and how much percentage of blue has been selected or has been combined to get that final colour; using that we can give numbers to any colour or any arbitrary colour. There is a quite common another standard CMYK or sometimes just CMY; which refers to Cyan Magenta and Yellow. Here gain the colours are cyan magenta and yellow and generally black associated with that.

So, again we can combining this 3 and sometimes black; we can develop any shades of colour and taking into account the fraction of C, fraction on M and fraction of Y; we can give some name to that. So, this way something like colour which generally is not considered as a proper technical quantity; we can also give them a technical look which is not possible in the analogue framework, but it is very much possible in the digital framework; so the method of digitisation.

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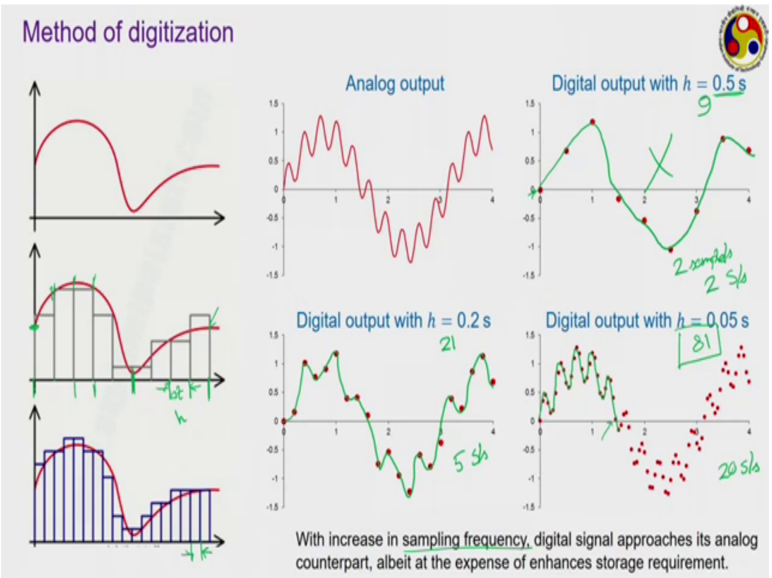

We know that I am sure all of you have some idea about what do you refer by digitalization. We also briefly refer to this in the first week itself; this one is our analogue signal. For the purpose of digitisation instead of plotting or measuring or rather getting such continuous representation of the out signal; we get the value of the signal only at certain time interval.

Say at this particular time instant we have taken the measurement and this value is coming up to be this, then at this particular instant we have measured the value to be like this, then at this instant we have measured the value as this instant we have measured the value with this and this continuing this way, this is the final instant where we are getting the value to be this. So, once you have done that you are available or you are ending up with a table where time; your is one side and the value of y at that particular time interval is on the other side.

And you can store this table in whatever format you want and then you can use any kind of software to produce a graph corresponding to this. Of course, the value of t or the choice of t definitely matters. Like in this case you can see that we are taking any successive measurement after this particular time instant upward; this is delta t or sometimes commonly referred as h.

So, after every delta t apart or this is h apart we are getting the measurement; delta t means constant, but the value of delta t itself is very important the value of this h if the value of delta t is too large; then we may end up with very few number of data. Like suppose if your choice of delta t is large you may have one phase somewhere here, then one here, then another one here another one here and finally, one here.

Now, it will be very difficult to plot the curve using this because you do not know the do not have the all the interior details. So, your curve may look something like something like this ah; which completely ignoring this particular curve part of the curve and also primarily giving a quite wrong representation.

Look at this particular case here if we use the selected values of delta t; we are getting a quite nice representation of the curve because it is able to capture this peaks also, it is also able to capture this trough and we are going to get a decent representation. But you can make it even better if we chosen even smaller delta t; just check out the delta t in this particular case. Here our delta t is much smaller actually half of the previous case leading to the development of a double number of data points.

So, we are having a much rich data set in terms of y versus t here and it is expected that we are going to get a much better representation, if we use this data to reproduce the original curve. Let us see another example another work out example; so this is a data set that we had originally which is the analogue output.

Now, with an h value of 0.5 second here the total span is of 4 second. So, with a time span of point time gap of 0.57; we have measured the values. And you can see where you are getting the first one at t equal to 0 and then after every 0.5 second apart we are getting the measurement leading to a total of 9 number of data points; we are having 9 datasets one corresponding at t equal to 0 and then after every 0.5 second apart subsequently.

And if we now plot this 9 data points and join them by the suitable line, then we are most likely to end up with a curve something like this. Is it looking similar to the original analogue input or out output? Not at all; it is looking completely different. So, we let us make h smaller; so that we can have more number of data points we are taking h equal to 0.2 second. So, how many total number of data points that we are going to have with?

In this case for every time second every one second apart, we are getting for every gap of 1 second you are getting 2 data points; so you are going to have 5. So, total number of data points you are going to have is equal to 21; one corresponding to t equal to 0 and then 5 more corresponding to each of 1 second added.

And you can see this does not look quite, does not look bad rather it is a much more improved one and if we plot it may be looking somewhat like this, which is definitely much more closer compared to the earlier one. So, this is definitely wrong, but this is not that bad, but still it is missing certain useful information from the original output. We can easily compare this one with the original analogue signal and we can see despite having 2 in; despite having almost 2.5 times more number or sorry or double number of data points.

More than double number of data points; we are yet to reach the final precise step. So, we increase it further we are choosing the h equal to 0.05 second; that means, for every second we are having 20 number of data points and leading to a total of sorry we are having a total span of 40 seconds 20 into total span of 4 seconds; so 20 into 4 is 80 plus 1; we are having total 81 data points. And look at this it is definitely a very good representation of the original signal; like if we plot it will if you join this data points, it will be something like this definitely we are getting a quite good representation.

Here I have joined only the points appearing in the initial part, but the first point where it is crossing this or adopting negative value; that is also being quite accurately predicted which probably was not there in the previous two cases and it is definitely looking a much better representation. So, as the h becomes smaller more and more number of data points comes into play we bridge a much better position, rather our I should say that the digital output or digital signal keeps on approaching the analogue signal.

This h are corresponding the inverse of that is sometimes is referred as a sampling frequency. So, in the first case your sampling frequency is just 2 because you are having only 2 or a common way of representing this one is 2 samples per second or 2 capital S per s.

In the second case we are having a sampling frequency of 5 sample per second; in this case our sampling frequency is 20 samples per second. So, as the sampling frequency keeps on increasing; we are approaching a much better scenario, but of course, how many number of data points you are going to deal with depends on how much accuracy we want. Like we may not be needing so many data points; like you can clearly see in this case definitely you are getting much more accurate results, but we have we have to handle 4 times more number of data points compared to the previous case and that is why we need much larger storage for this.

So, such accuracy may not at all be required for the application which is of our concern and hence our choice of the sampling speed or choice of the number of data points that we can deal with that depends on the application also. But the principle of digitisation remains the same here instead of having a continuous measurement we are measuring the data only at certain time intervals.

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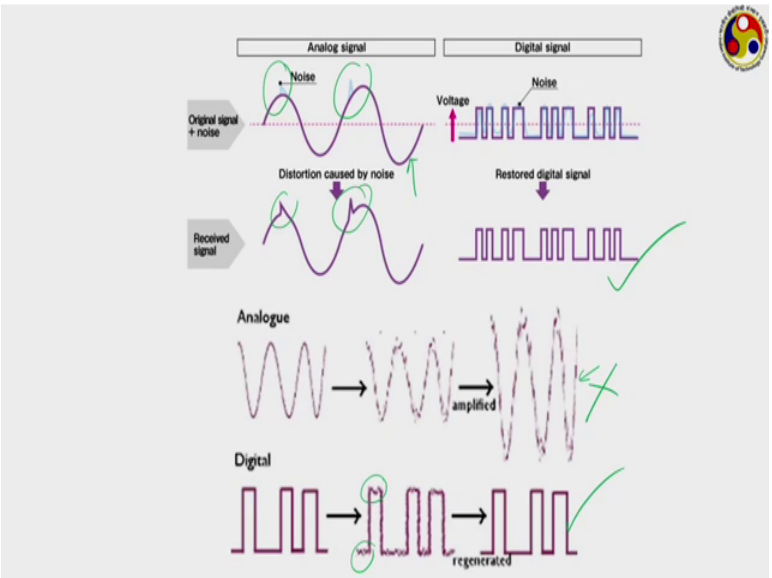

This is one an advantage for digital signal over analogue. In the analogue signal you can see there are certain noise appearing one here another one here and accordingly our signal you can see that is a distortion in this point and in this point.

But compared that with the digital signal; it is also been experiencing significant amount of noise, but as a data has been digitised or instead of dealing with the continuous representation like this, we have converted this to a set of numbers. So, that hardly gets influenced by their surroundings and we get exactly the same representation or the correct representation on the output side.

Another advantage; this is an analogue signal which we are amplifying and because of amplification; it is losing its power or the energy level. And once some part is some part of the signal has been damaged it will be very very difficult to recover that; practically impossible to recover the damage part. But the for the digital part you can see there is some distortion, but very quickly we can regenerate using the original data set and we get a correct representation; which is not possible with the analogue one; so, these are just 2 advantage of the digital signal which we have already discussed.

So, the idea of digitisation is instead of dealing with a continuous representation of the signal; we convert the that to a set of data, a t versus y kind of representation where we shall be having the magnitude of the measurement under consideration available with us after a particular time instant apart over the span of our interest. And then once we have the data set with us we can deal with it in whatever way we want.

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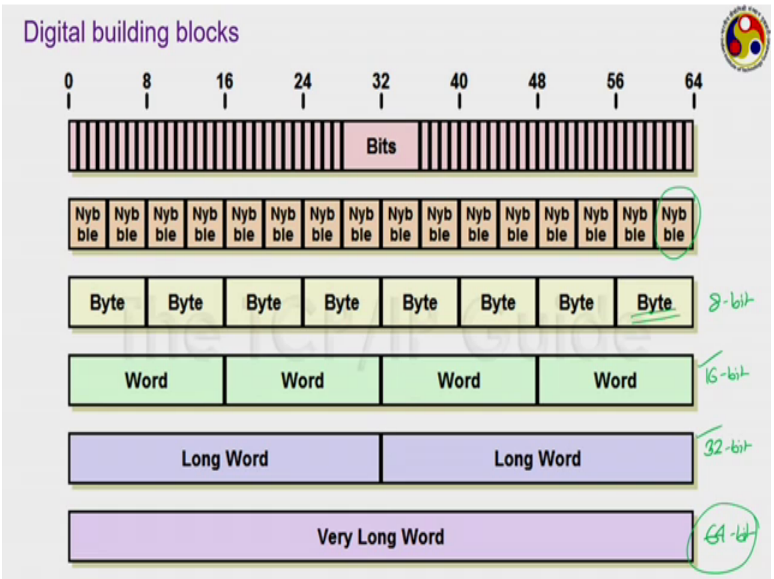

So, just some very basic about the digital building block of our computers or any such digital instruments; probably you are familiar with this names the smallest building block is called a bit; 1 bit can store only 1 digit, 1 bit can store a single digit. When we combine 4 bits together we call them nybble and 8 bits together it is called a byte.

So, a byte refers to 8 bits together commonly one number is used or sorry or 1 byte is used to store one full number whereas, a 1 bit can store a single digit of a number; 1 byte which comprises of 8 bit it can store one full number of course, depending upon the value of the number; of course will depend upon what kind of number system we are following.

Because, the number must has to be represented by 8 digits; if the number of digits are more then 1 byte alone is not sufficient; we have to go for double precision or 2 bytes or even more kind of representation. Word refers to any small functional element of a digital circuitry or digital components; like here shown a word can be combination of 2 bytes or it can be also combination of 4 bytes commonly referred as a long words or you can have all this 8 bytes coupled together to form a very long work.

If you think about this is more an 8 bit kind of representation a byte, so, here we have 16 bit, this is 32 bit and this is 64 bit. Like you probably have heard earlier that digital instruments computers etcetera used to be in 16 bits, then converted into 32 bit and now most of them are coming in 64 bit. The 64 bit refers to precisely this where we have a very long word which comprises of 64 bits.

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$2^{10} \approx 1024$ 1 k = 1000 1 kg = 1000 g, 1 km = 1000 m

Byte (B)	Kilobyte (KB)	Megabyte (MB)	Gigabyte (GB)
1 character of data 8 Bits	1024 bytes 1 page of plain text	1,048,576 bytes or 1024 KB (1 MB ≈ 1024 KB ≈ 2^{10} KB) About 4 books or 200 pages of text	1,073,741,824 bytes or 1024 MB (1 GB = 2^{10} MB) About 4,500 books, or over twice the size of Sir Isaac Newton's library (considered very large at the time)
Terabyte (TB)	**Petabyte (PB)**	**Exabyte (EB)**	**Zettabyte (ZB)**
1,099,511,627,776 bytes or 1024 GB about 4.6 million books	1,125,899,906,842,624 bytes or 1024 TB About 4.7 billion books, which would fill the Library of Congress 140 times	1,152,921,504,606,846,976 bytes or 1024 PB About 4.8 trillion books, which would occupy the size of the states of Maryland and Delaware	1,180,591,620,717,411,303,424 bytes or 1024 EB 4.9 quadrillion books: a library big enough to house this would be 1 million square miles LARGER than all of North America

Even larger units can also be formed by using by adding all suitable prefixes like 1 bite refers to a single character of data or just one number as I have mentioned which has 8 bits. And now when we have 1024 bytes combined, we call it a kilobyte.

Now one question may be coming to your mind commonly 1 k refers to 1000; like 1 kg refers to 1000 gram, 1 kilometre refers to 1000 metre. Then instead of 1000 why you are having 1024? This 1024 comes from 2 to the power 10; this 2 to the power 10 is 1024; why 2 to the power 10? I shall be coming shortly. So, instead of using 1000; this 1024 which is miserably close to 1000 or 2 to the power 10 is used as a conversion unit for a while giving this prefix k m g etcetera.

So, 1 megabyte will be equal to 1024 kilobyte or 2 to the power 10 number of kilobyte. This way 1 gigabyte is equal to 2 to the power 10 number of megabyte and it continues this way; instead of using 1000 megabyte 1 GB refers to 2 to the power 10 megabyte. So, 1 kilobyte typically corresponds to a page of plain text, 1 megabyte it can be 4 books or

200 pages of texts or can refer to some other media also, 1 gigabyte is quite large it is about a whole library or even bigger, it can store that much amount of books.

1 terabyte refers to several millions 4.5 to 6 millions of books can easily be corresponding to. So, that can be all the libraries of the world combined with each other together; then we can move to petabyte, exabyte even zettabyte amount of units. So, this way we can form different units and all of them there for refers to how much storage you are providing to store all this digital elements or the digital numbers.

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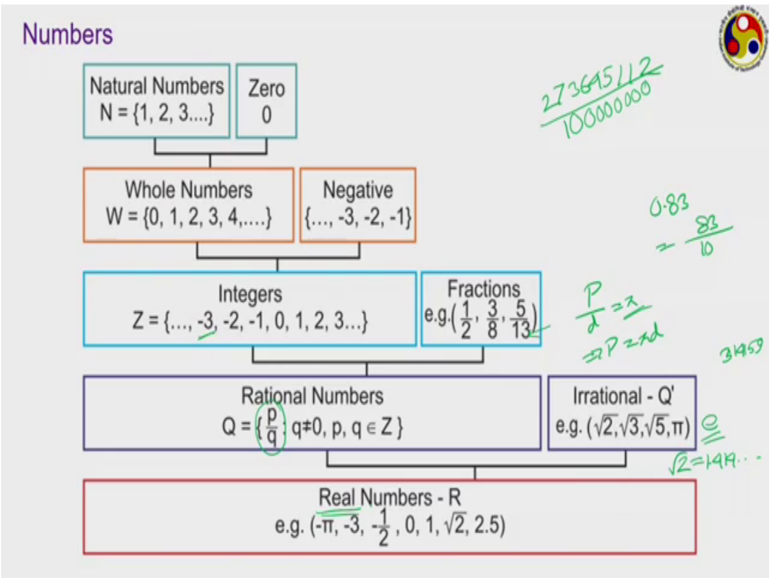

Now, we are talking about numbers because in digitalization we are converting a continuous signals into numbers, so, let us quickly go through numbers and their properties. We know about natural numbers and 0; natural number refers to 1, 2, 1, 2 up to 9 and there is 0. There can be whole numbers or negative numbers; we can have integers. Integers refers to there is no fractional part; it is just the number alone may be with a symbol; like here this minus 3 has a symbol of its own the other part can be fraction.

Most of the common number can be represented as a fraction as well and therefore, it is one important term here this is a denominator has to be a non 0 quantity for it to be a proper fraction. Both integers and fractions are called rational numbers; the term rational numbers commonly refer to any number which can be represented by ratio like this. Like suppose, if you are talking about 0.83 we can always write this as 83 upon 100 or suppose you are talking about 0.4; we can always write this as 2 by 5.

So, the fractional loss any decimal point based numbers can always be converted to a suitable fraction. And therefore, they are all rational numbers and integers are also definitely rational numbers, but it is possible that there can be certain cases where the number is number cannot be represented by a ratio and they are called this irrational numbers. Like this root 2, root 3, pi e the base of natural logarithm, they are all irrational numbers.

What is pi? Can you say; I hope you remember pi is definitely not 22 upon 7, this is not correct, pi equal to 3.14 etcetera that is also not correct. What is the definition of pi? Pi is the ratio of perimeter of a circle to its diameter. Perimeter of a circle P by diameter is equal to pi leading to perimeter is equal to pi into d. So, pi is always the ratio of the perimeter to diameter of a circle; this quantity was measured to be constant long back hundreds of years back people working on geometry measured this.

And they are surprised to see that this particular ratio always remains constant for any circle of any diameter size and that was given the name pi. So, pi is one of the irrational numbers; while rational numbers can have integers and fraction kind of representation, irrational numbers definitely cannot have any such representation rather they are commonly represented by their own. This 22 upon 7 or 3.14 whatever may be the number of digits you want to keep on adding that will only give you an approximate value of the pi; it is a never ending series.

And 22.7 is never correct and 3.14159 dot dot dot dot that will keep on continue for infinite length. The same for this e or any other like what is root 2? 1.414 dot dot dot; so it is an infinite series. So, these are all irrational numbers; if we find an end of the series we can always represent them to rational form. Like suppose you are dealing with the number of 2.7364512112 something like this.

So, it is a very long number, then we have a fractional representation of this; it is very easy. We will just check how many number of digits we have after the decimal point; here we have 1, 2, 3, 4, 5, 6, 7, 8. Let me remove the decimal point itself you had 2 before that and now as there are 8 numbers 8 digits after this decimal points, so in the denominator put 8 0s; 1, 2, 3, 4, 5, 6, 7, 8 and it is a ratio now; where I will be giving the same value.

We can simplify it whatever you want and you are having a perfectly rational numbers. And so rational numbers and irrational number together give us the real numbers which can take any kind of value.

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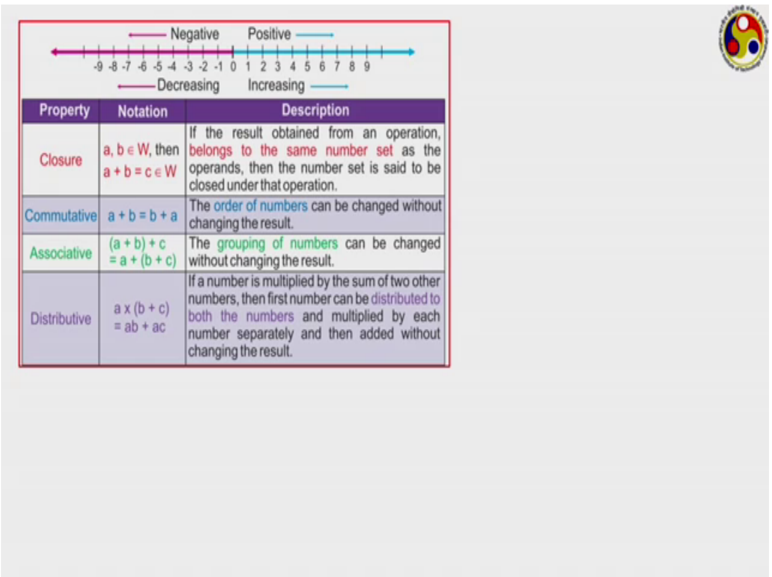

Property	Notation	Description
Closure	a, b ∈ W, then a + b = c ∈ W	If the result obtained from an operation, belongs to the same number set as the operands, then the number set is said to be closed under that operation.
Commutative	a + b = b + a	The order of numbers can be changed without changing the result.
Associative	(a + b) + c = a + (b + c)	The grouping of numbers can be changed without changing the result.
Distributive	a x (b + c) = ab + ac	If a number is multiplied by the sum of two other numbers, then first number can be distributed to both the numbers and multiplied by each number separately and then added without changing the result.

Now, the numbers can have 4 kinds of properties; one is closer. Closer refers to if the result of an operation belong to the same number set as the operand then we said then the numbers it is closed. Cumulative a plus b that is equal to b plus a; that is if we change the order of the numbers, but the final result does not change, then we call it cumulative.

Then associative like the example shown here; if we change the grouping of the numbers without change that does not change the result. And finally, distributive if the number is multiplied by the sum of 2 other numbers, then the first number can be distributed to both the numbers and multiplied by each number separately. And then added without changing the result like shown here a into b plus c is equal to a b plus a c. Now it may sees a very obvious operation, but may not be possible for all kinds of numbers.

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Like for integers; integers the closer is ok; there are 4 common operation that we do addition, subtraction, multiplication and division with addition subtraction and multiplication this is ok, but for division you may not always be in the same set.

Of course, b is a non zero quantity, but despite that you may not reach the same data set. Like it may become the result of this one can correspond to a different data set or different set I should say. Cumulative is not true for subtraction and division, but true for addition and multiplication because a minus b and b minus a have different meaning, similarly, a by b and b by a are not equal to each other.

Associativeness is also non valid for subtraction and division because of the same reason and distributive term is valid for addition and subtraction of course, they are not applicable for multiplication and division.

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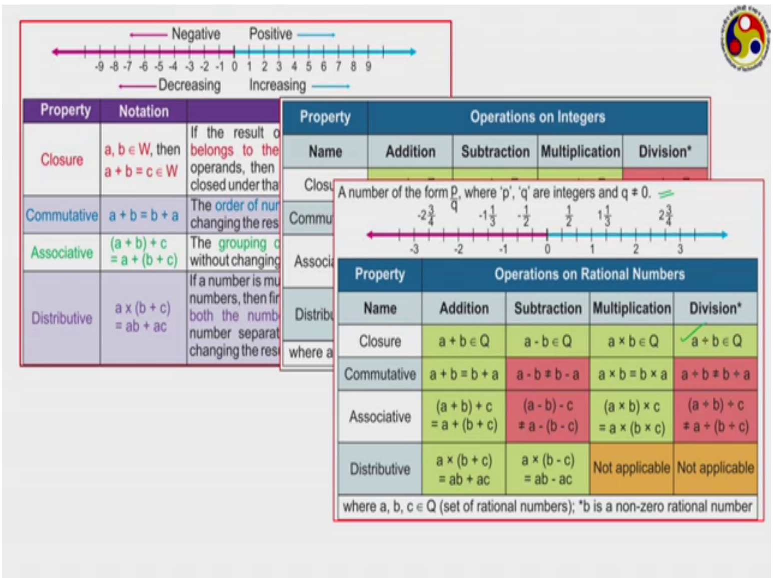

Look at what we have with rational numbers; for rational numbers it is just the extension of what we have for integers. If you are talking about rational numbers then we are going to get a rational data set also after rational data set also after division.

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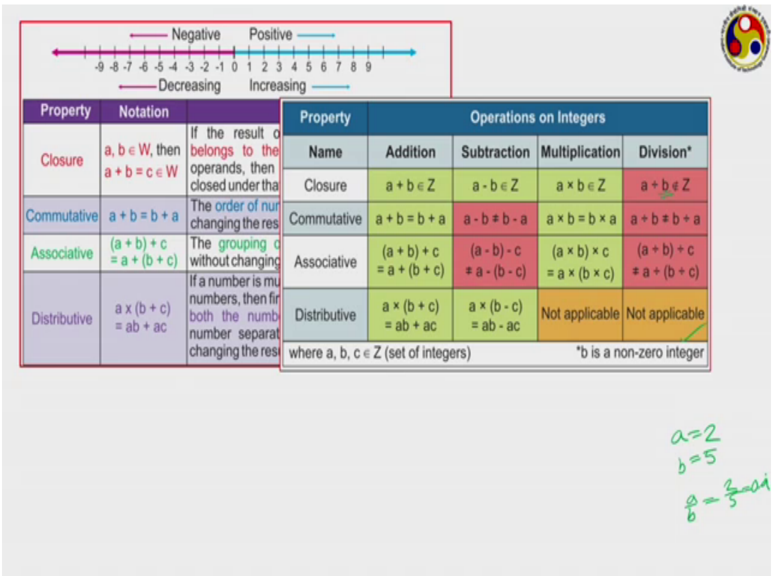

But in case of integer suppose your a equal to 2 and b equal to 5; then your a by b is not going to give you an integer. It is not an integer you are going to end up with 0.4 or maybe just this fraction this way therefore, the dataset does not hold. That means, this particular condition may be true sometimes, but generally not, but in rational case the fractions are also included; integers and fractions both are included in rationals case. So, it is the data set can be closed for any kind of operation; however, cumulative and associativeness does not hold for subtraction and division as well.

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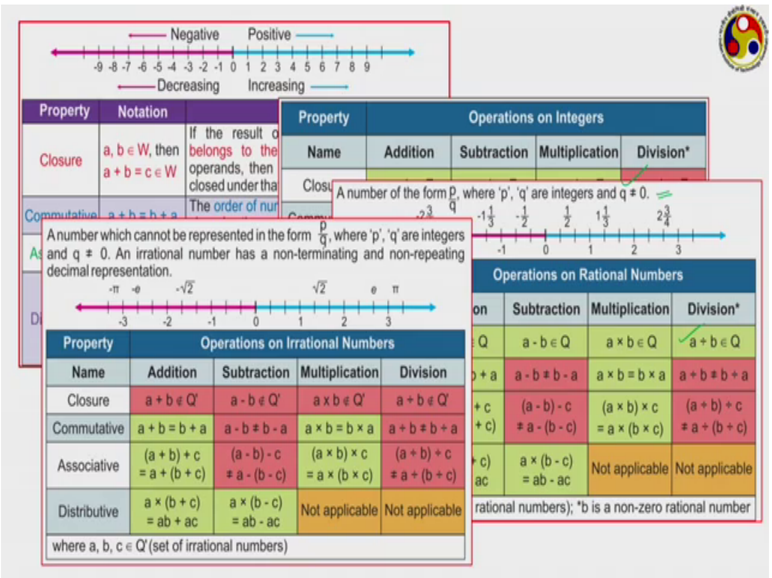

Now, we go to irrational numbers; irrational numbers none of the mathematical operation lead to the closer of the dataset because we; it the operation their operation will lead to a rational number itself or maybe something else. Cumulative and associate properties again does not hold for subtraction and division, so these are certain properties of numbers.

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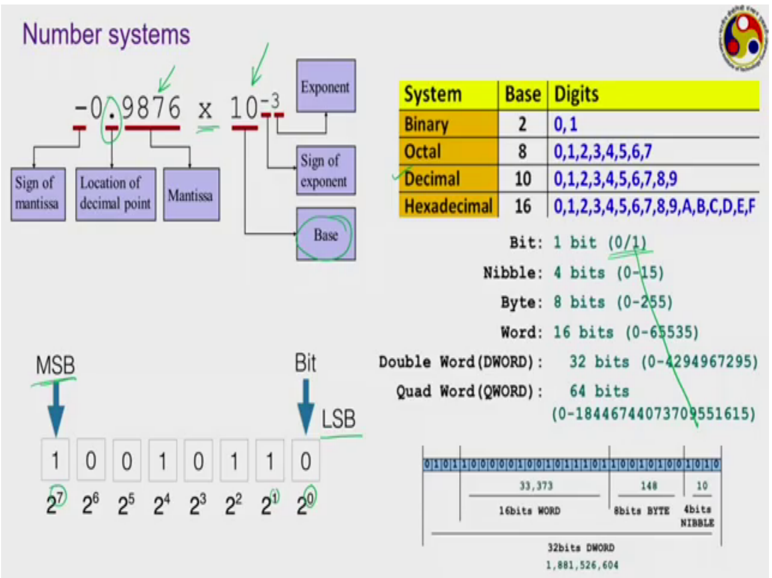

Now, how you represent the number commonly? This is the most scientific way of representing any general number, where we clearly have two parts; one this particular part is called the mantissa, this particular portion is called the exponent. In scientific notation we always have a 0 before the decimal point, then the decimal point itself and then whatever digits we have and this digits are referred to the mantissa.

This is a decimal point and we always have a leading 0 and there may be a sign of the mantissa; it can be negative or positive for positive numbers often the sign is ignored, but for negative numbers of course, you have to put. And then we have this multiplication sign and then 2 to the power minus 3; this minus 3, minus is a sign and 3 is a exponent and this 10 is called the base or quite often it is also called radix of the number. 10 base refers to the decimal number system, but note that is not the only possible number system that we have; we can have infinite number of data sets, but this 4 generally are the most common one.

Decimal refers to the base of 10; that means, there are 10 possible digits starting from 0 up to 9, but we can also have the binary data set where we have just 2 digits 0 and 1, we can have the octal dataset where we have 8 digit 0 to 7 and you can also and the hexadecimal data set starting from 0 to 9. And then A B C D E F; here this A corresponds to 10 in decimal, B corresponds to 11 decimal and this way we shall be shortly be seeing their correspondence.

So, decimal is only one of several possible number base that we can have, but still we generally talk in terms of decimal always because that is the most common kind of number base that we deal with in our day to day life. Just see how we can represents like if you are using 1 bit; if you stick to the binary notation, then we can in 1 bit; we can just write 0 or 1. Because these are the 2 digits like you for using decimal then any digit out of this 10, 0 to 9 can go to 1 bit.

Then let us just ignore this particular side just there are 4 bits; then how many you can have, how many numbers you can represent? You can represent there are 4 digits you can put. So, these two 9 9 9 9 these are all you can get or if you are talking about binary then the smallest you can have this, the largest you can have this.

If you are talking about octall this is the smallest you can get and this is the largest you can get. So, this way we can decide what is the maximum number that we can stored in 1 nybble or 1 byte or 1 word or 1 quad which is equal to 64 bit, but all the bits does not have the same weightage. Once you are storing the numbers in bit like here we are having 1 byte. So, there are 8 bits each of the bits are housing one number; it is shown in the binary system this is the 1 which is having the largest weightage if we generally start from the right hand side and their weightage is said to be 0.

Then 1 I am moving this way the leftmost one is having a weightage of 7. So, the one having an exponent of 0 like this is called the Least Significant Bit LSB or least significant bit. Whereas, the one on the leftmost left most side which is having the largest exponent or largest weightage that is called the Most Significant Bit, MSB, most significant bit.

LSB refers to Least Significant Bit, MSB refers to Most Significant Bit that is called the most significant because that influences the final value of this number the most whereas, LSB is the one which influences the least that is why these names are given. Let us try to see how you can convert across this different number systems.

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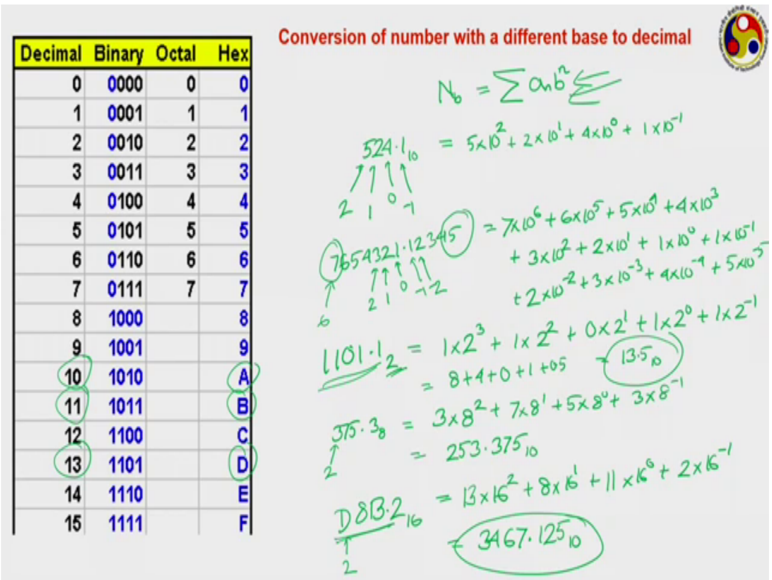

Decimal	Binary	Octal	Hex
0	0000	0	0
1	0001	1	1
2	0010	2	2
3	0011	3	3
4	0100	4	4
5	0101	5	5
6	0110	6	6
7	0111	7	7
8	1000		8
9	1001		9
10	1010		A
11	1011		B
12	1100		C
13	1101		D
14	1110		E
15	1111		F

Any common number can be represented or their decimal equivalent can be represented in a form like this. Say we have a number N b in any base system b, then we generally have summation of this particular form. Like let us say we start with a decimal case we have a number say arbitrary 524.1 and we use this subscript 10 to denote that this is something with decimal base.

If we in normal operation we do not have to use this because you are always talking about decimal base, but when you are talking about different kinds of bases; then we have to talk about this base. Then how to get the value of this? Now, the one which is immediately below the decimal point this one that should be given 0; this given 1, this given 2. Whereas, the one on the other side of the decimal point that should be minus 1; if anything appearing after that minus 2 minus 3.

Similarly on the other side it will continue as 3 4 this way. So, we have the to get the value we have 5 into base; that is 10 to the power 2 plus 2 into 10 to the power 1 plus 4 into 10 to the power 0 plus 1 into 10 to the power minus 1; we definitely will written the same values. It may seem very obvious to you, but whenever you are doing this that will be coming very shortly; let us take another example.

Let us say you have taken a very long number now how to write this in this particular form or in this particular form? So, this is the least significant or which is the least significant bit here? The one appearing the rightmost is the least significant the leftmost is the most significant. Why? Because of the weightage that we assigned; how to assign the weightage? The look at the one immediately on the left of the decimal point here we have this 1.

So, this is having a weightage of 0; next one having weightage of 1, next one having weightage of 2 and this continuous this way whereas, on the other side of the decimal point we pick a minus 1, then minus 2. So, if we write this what is the weightage for the most significant bit here? 1 2 3 4 5 6. So, it will be 7 into 10 to the power 6, plus 6 into 10 to the power 5, plus 5 into 10 to the power 4 plus 4 into 10 to the power 3, plus 3 into 10 to the power 2, plus 2 into 10 to the power 1, plus 1 into 10 to the power 0, plus 1 into 10 to the power minus 1, plus 2 into 10 to the power minus 2 plus 3 into 10 to the power minus 3 plus 4 into 10 to the power minus 4 plus 5 into 10 to the power minus 5.

So, this way we can expand any number in this summation form. Now look at the number of some other digits, other base I should say. Let us take one binary number we are talking 1101.1; we know we can have only 0 and 1, then how to get their decimal equivalent? This is a 1 with 0, the most significant 1 having weightage of 3. So, you are having 1 into; what is the base here? It is binary base is 2. So, 1 into 2 to the power 3, plus 1 into 2 to the power 2, plus 0 into 2 to the power 1, plus 1 into 2 to the power 0, plus 1 into 2 to the power minus 1.

So, get their values; this is 8 plus 4 plus 0 plus 1 plus 0.5 that is 13.5 in decimal base 10; that is how we can convert a binary number to its decimal equivalent. Let us try the same with an octal number let us say we have 375.3; octal base. So, what it will be? We now know that we have 0 1 and 2; the most significant bit is having weightage of 2. So, we have 3 into the base it is octal; so it is 8 to the power 2 plus 7 into 8 to the power 1 plus 5 into 8 to the power 0 plus 3 into 8 to the power minus 1.

And if we calculate this you are going to get something; actually I have noted down the result for this because I do not have a calculator the result for this calculation is going to be 253.375 in 10 decimal base. Let us see one example with the hexadecimal case; for hexadecimal case it is having a D8B.2; base 16. So, what is the weightage of the most significant bit? This is having a weightage of 2 here. So, we have D into what is the base? This hexadecimal, base is 16; 16 to the power 2 plus 8 into 16 to the power 1 plus B into 16 to the power 0 plus 2 into 16 to the power minus 1.

So, we can calculate the value, but wait there is one problem here D and B are appearing which never appeared for binary or octal. So, we have to refer to the chart that is on the left; A refers to 10 in decimal, similarly B refers to 11 and D refers to 13. So, replace this D with 13 and replace the B with its decimal equivalent; which is 11. Now you do the calculation you are going to get 3467.125 in hexadecimal case. So you can clearly see as the base of the number using this particular summation notation, we can convert a number from any base to decimal, you are talking about decimal one and one side always.

Because that is the most common one that you use in daily life and other thing that you can see like in this example it is a very large decimal number, but quite smaller hexadecimal representation it is having a higher base. Whereas, this small decimal number its having a much larger or quite wider binary representation; because, we need more number of digits to represent this.

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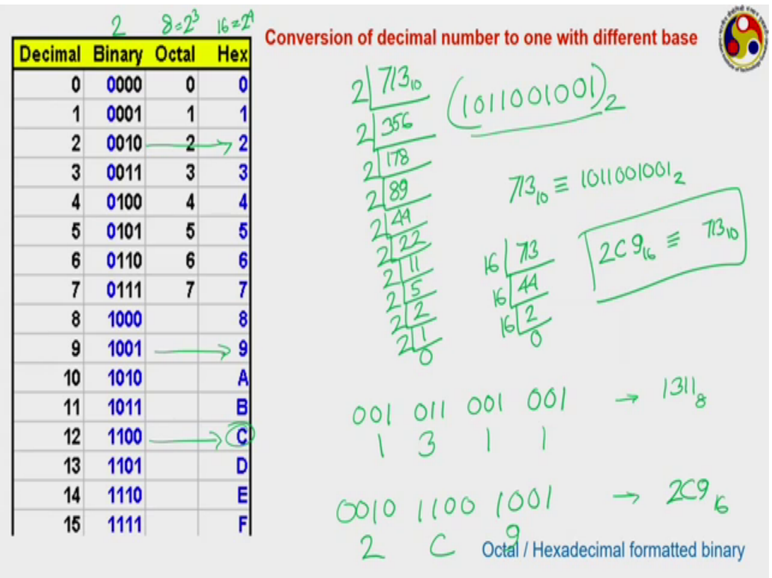

Let us quickly try the other one converting a decimal number to something with a different base let us take a random base. Let us take a we take a number randomly generate a number 713; it is having a base of 10 and we want to convert it to its binary equivalent. How can we do this? The idea is divide it by 2; divide it by the base of the system on to which you want to convert it. You are originally having a decimal one, you want to convert it to binary that is why we are dividing by 2. If you want to convert into octal we shall be dividing by 8.

So, if we divide it by 2; then what we are going to get? We are going to get 3 then 5, 6 but what is left out? That is 1; so let us note down the 1, divide it once more. So, we are going to get 1, 7, 8; what is left out? It is an even number nothing left out; so 0 right the 0 to the left of this one, then divide by 2; so we are going to get 8 and 9 nothing left out again 0.

So, write 2, divide by 2 again keep on dividing till you get a 0. So, if you divide 89; 1 2 3 step we have done, you divide by 89; you are going to get 44; 1 left out. Divide 44 by 2; you are going to get 22 and nothing left out as remainder; divide 22 by 2, you getting 11 again nothing left out your 11 by 2 you are going to get 5 now you are having 1 as remainder.

Divide 5 by 2 you are going to get 2 and 1 in the remainder, divide by 2 you are going to get 1 and nothing left. And now you divide this 1 by 2 you are going to get a 0 and a 1 as the remainder and what you have got? This is your corresponding binary equivalent. That is 713 in decimal is equivalent to 1011001001 in decimal, sorry in binary.

We can get anything else also in the same format; let us say if you want to get the hexadecimal equivalent for this. So 713 divide by 16; if we divide by 16 then what we are going to get? We are going to have a 4 and another 4 here left out is 9 divide 44 with 16. So, we are having a 2 and 12 will be left out here. Now 12 we cannot put because in hexadecimal there is system there is nothing 12, but this 12 actually corresponds to this C. So, let us write C and now divide this 16 again leaving a 0; leaving a 2 to. So, this 2 C 9 in 16 hexadecimal system is equivalent to that 713 in binary sorry decimal system.

This way we can convert a number in decimal system to a number of any other base. But one advantage we get is a once we have the binary equivalent, we actually do not have to calculate out octal or hexadecimal equivalent separately rather we can do it directly. Because look at this the base for binary system is 2, what is a base for octal system? That is 8; that means, 2 cube, what is the base for hexadecimal system? 16 which is 2 to the power 4.

We can take the advantage of this one; like the binary equivalent that you already had let us make divide this number or I should not say divide, let us write this number by separating the digits in groups of 3 starting from the leftmost. So, in the leftmost we have 1 0 0; then we have 1 0 0, then we have 1 1 0 and then there is only 1 left; so, but we need to have 3 digits in each group; so put leading 0s, we know that putting a leading 0s will not alter the values.

So, we have; it is a same binary number, but we have a retails by keeping a space after every third digit. And now look at them just try to find out the these are the binary numbers for each of this groups try to find out the octal equivalent using the table.

001 is this corresponding octal equivalent is 1; so write 1; 011 octal equivalent is 3, write 3. 001, octal equivalent is 1 and it is also 1. And now the octal equivalent for this entire system will be this 1311, base 8. If you want to do it in hexadecimal system; instead of doing 3 make groups I think you can guess. Write the binary number such that there are groups each group composing 4 digits; so, we can write 1001.

Then we have 0011, then we have 01 and no digits, so let us put 2 0s before this. So, the same binary number we have written such that there are 3 groups each comprising of 4 digits. And now try to find out the hexadecimal equivalent to each of them; for 0010; 0010, let me erase the earlier ones. So, 0010 is this is hexadecimal equivalent is 2 so it is 2.

1100, 1100; sorry 1100 hexadecimal equal is C; this is C and 1001, 1001 hexadecimal equivalent is 9; so write 9 and we get 2 C 9 as the hexadecimal equivalent. Just compare with the previous case this is what we have got already.

This things are called octal or hexadecimal formated binary; that is a binary number written in a format which allows straight forward conversion to octal or hexadecimal cases. So, this way we can convert any decimal digits to other number bases or any other number base to decimals.

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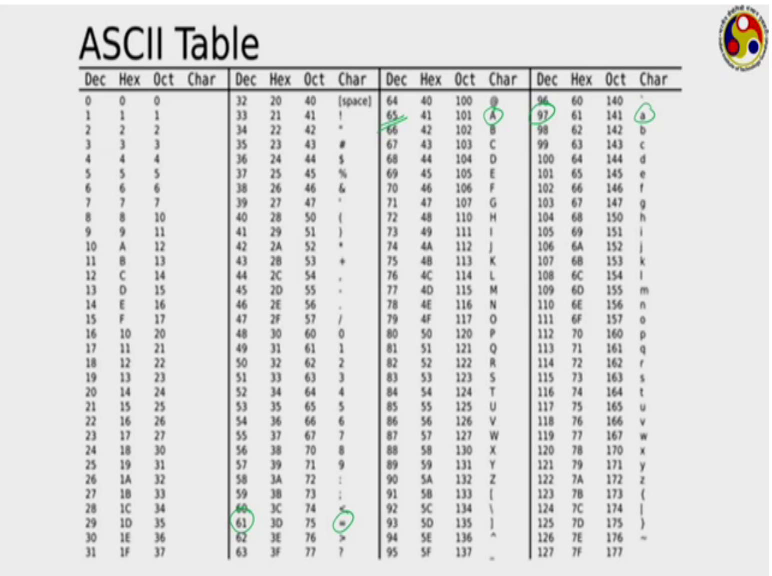

ASCII Table

Dec	Hex	Oct	Char	Dec	Hex	Oct	Char	Dec	Hex	Oct	Char	Dec	Hex	Oct	Char
0	0	0		32	20	40	[space]	64	40	100	@	96	60	140	`
1	1	1		33	21	41	!	65	41	101	A	97	61	141	a
2	2	2		34	22	42	"	66	42	102	B	98	62	142	b
3	3	3		35	23	43	#	67	43	103	C	99	63	143	c
4	4	4		36	24	44	$	68	44	104	D	100	64	144	d
5	5	5		37	25	45	%	69	45	105	E	101	65	145	e
6	6	6		38	26	46	&	70	46	106	F	102	66	146	f
7	7	7		39	27	47	'	71	47	107	G	103	67	147	g
8	8	10		40	28	50	(	72	48	110	H	104	68	150	h
9	9	11		41	29	51	)	73	49	111	I	105	69	151	i
10	A	12		42	2A	52	*	74	4A	112	J	106	6A	152	j
11	B	13		43	2B	53	+	75	4B	113	K	107	6B	153	k
12	C	14		44	2C	54	,	76	4C	114	L	108	6C	154	l
13	D	15		45	2D	55	-	77	4D	115	M	109	6D	155	m
14	E	16		46	2E	56	.	78	4E	116	N	110	6E	156	n
15	F	17		47	2F	57	/	79	4F	117	O	111	6F	157	o
16	10	20		48	30	60	0	80	50	120	P	112	70	160	p
17	11	21		49	31	61	1	81	51	121	Q	113	71	161	q
18	12	22		50	32	62	2	82	52	122	R	114	72	162	r
19	13	23		51	33	63	3	83	53	123	S	115	73	163	s
20	14	24		52	34	64	4	84	54	124	T	116	74	164	t
21	15	25		53	35	65	5	85	55	125	U	117	75	165	u
22	16	26		54	36	66	6	86	56	126	V	118	76	166	v
23	17	27		55	37	67	7	87	57	127	W	119	77	167	w
24	18	30		56	38	70	8	88	58	130	X	120	78	170	x
25	19	31		57	39	71	9	89	59	131	Y	121	79	171	y
26	1A	32		58	3A	72	:	90	5A	132	Z	122	7A	172	z
27	1B	33		59	3B	73	;	91	5B	133	[	123	7B	173	{
28	1C	34		60	3C	74	<	92	5C	134	\	124	7C	174	|
29	1D	35		61	3D	75	=	93	5D	135	]	125	7D	175	}
30	1E	36		62	3E	76	>	94	5E	136	^	126	7E	176	~
31	1F	37		63	3F	77	?	95	5F	137	_	127	7F	177	

And not only numbers rather any kind of symbols that are available to computers are generally given some kind of numbers following any number system this the ASCII character set; there are a few other character set available, but this the most common one.

Here you can see some very common things like this say capital A has been given a decimal number of 65, small a has been given something of 97. Hexadecimal and octal are corresponding representations only or some abrupt symbols. So, this equal to that is similarly given a symbol of a number of 61; so if we want to give this equal to symbol to computer it will read it as 61; ASCII character 61 and then using this number we can do whatever we want this is just an example of doing this.

But final question that we have here we know all computers or any digital device used binary despite having other number base available. Why it cannot use decimal? Because that is a most common one that we use in our day to day life; why it cannot use octal and hexadecimal which or any other number system; that is because binary provides a distinct advantage.

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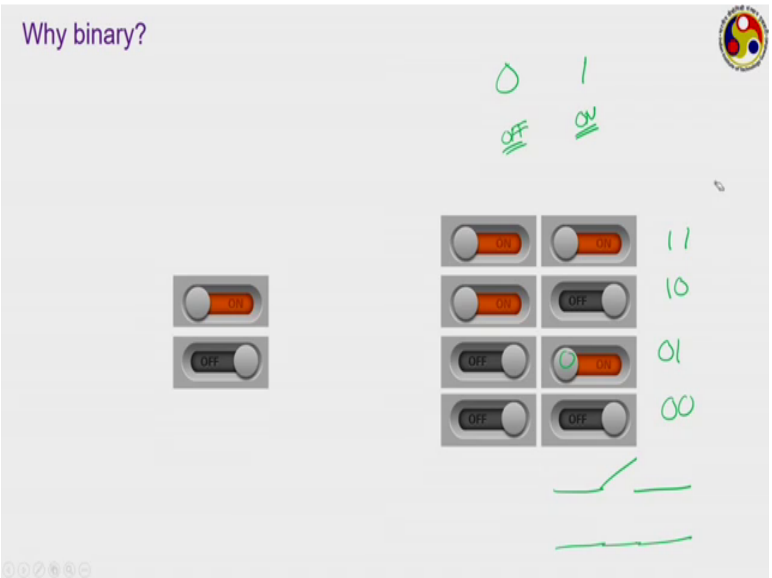

How many digit it has? Only 2; one is 0 another is 1. Now just relate that to our real lives 0 means what? Absence of something, 1 means the presence of something; that means, 0 can be something like an on; no I should say 0 can be absence of something means something like an off.

1 means the presence of the same quantity that is on and that is a precise idea that is why we use binary. Because our digital devices are operated by some current some electricity and now how we can make the computer understand some decimal digit? It will be much easier for a it to make us understand the octal digit, sorry the binary digit you can think about say we have a switch kind of representation.

So, if we want to make it understand 0, we shall be opening the switch there by no current is flowing. If we want to make it understand 1, we shall be closing the switch thereby allowing the current to flow. So, just by on or off position of the switch, you can make it understand this 0 or 1 and that is why interfacing becomes very easy we can easily make the computer understand what we want to give as input.

Just compare it with decimal here we have in binary we have only 2 logics; one is off other is on, but in decimal how many we have? Let us say our computer is having a total 10 volt system. So, our on will refer to 10 volt, off will refer to 0 or when the circuit is open; this 0 volt it will read as 0, when it is closed whatever voltage it is giving it is reading it will read as 1.

But if we want to make it understand decimal, then what it will do? Then this 10 volt this entire 10 volt span is to be separated into small components like when it is 0; it will read as 0. Now when the value is something like between 0 to 1 volt, then it may read it as 1, when it is between 1 to 2 volt; it will read it as 2.

This way there are 10 different logics we have to get which are very very combustion very very difficult to deal with. So, that is the main reason we go for binary system in any digital instruments and this is the you can visualise, one is the on situation other is the off situation. So, if you want to input give input for single bit; we shall be providing on one to represent 1 or an off to represent 0; which can be very easily done; we shall be seeing that in the next lecture how that is done.

If we want to give input for 2 bits; then we can have 2 simultaneous switch when both the switches are on your input will be 1 1. When first one is on second one is off, it is input is 1 0, first one is off second one is on, it is 0 1 and last case both are off so 0 0. So, very easily you can give such inputs ah; you just need 2 logics for each of these switches and then you can make any combination of that.

But, if we want to have the same using decimal or any other operation system other number system, we need to have lots and lots more logics. Like for decimal unit 10 logics for a single switch and now 10 square number of combinations for 2 such switch and that will exponential keep on increasing; which is not required in binary system.

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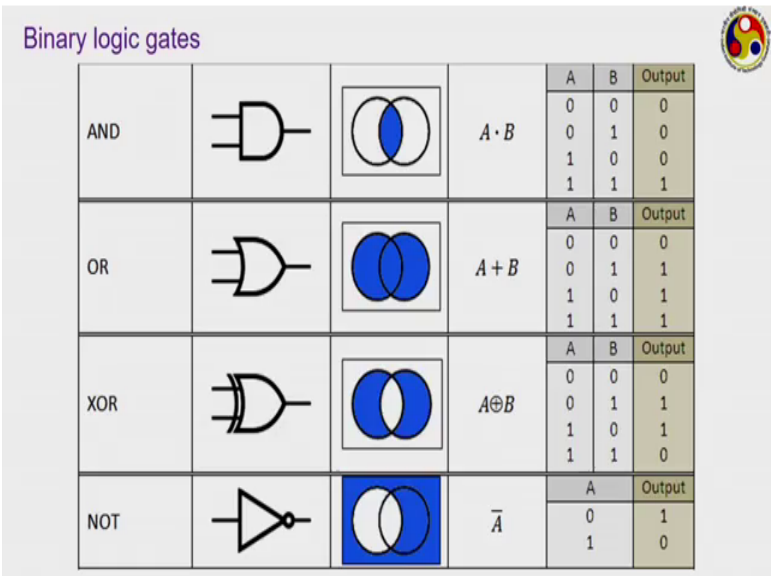

Some binary logic gates I thought about discussing it today, but I am skipping it today in the next lecturers the time is over. So, in the next lecture I shall be starting from this point onwards.

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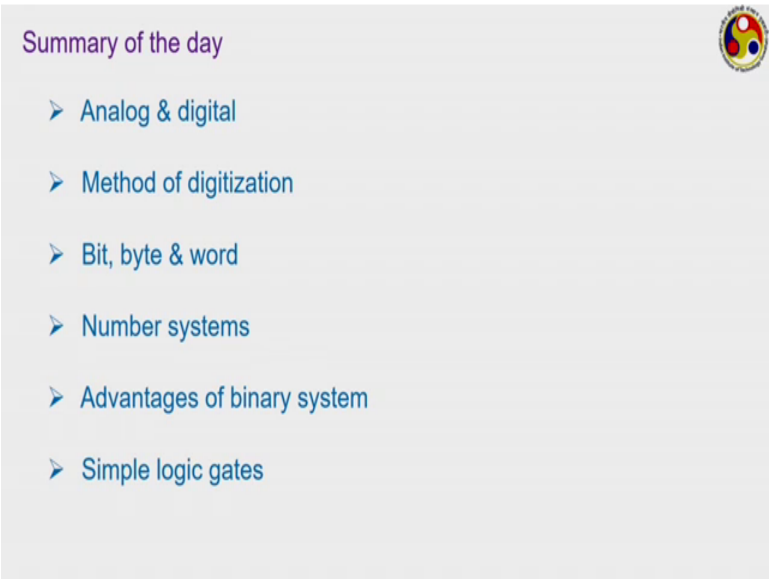

So, what we have learnt today? We have learned about the analogue and digital signals we have discussed in detail the advantages of digitalization or digitisation. We have discussed briefly about the method of digitisation then we talked about bit, byte and word which are the fundamental units.

Then the number systems and conversion from one system to another system that we have discussed about and we have discussed about the advantage of the binary system. My plan was to discuss about logic gates, but I shall be discussing in the next class because the I do not want to extend it anymore.

So, the lecture will be continued; tomorrow we shall be starting with the logic gates and then we shall be seeing how we can combine different logic gates to give much longer input. And then some fundamental components of electronic circuitry about how to achieve the steps of digitalization; thereby a proceeding further to analogue to digital or digital to analogue conversions. So, that is it for the day, thanks for your attention, hopefully you have enjoyed it; if have any query please write to me immediately and wait for the next lecture.

Thank you.