Principles of Mechanical Measurement

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Module – 02
Lecture – 03
Response of Measurement Systems

Hello, friends. Welcome, to the third and last lecture of our second week where we are talking about the Responses of Measurement Systems. In the previous two lectures, you have been introduced to several dynamic characteristics of measurement system particularly for something of our importance of the amplitude and frequency responses and also we have developed a general mathematical structure relating the output and input of a single input single output device and from there by changing the order of the equation we have already discussed about the zeroth and first order system.

In the zeroth order system, we have seen that that is the most ideal scenario that we can have here output and input are having a straightforward linear relationship related by just a single parameter which is the static sensitivity. However, for first order system we have seen that there is a bit of storage characteristic for the system where along with the static sensitivity we also have the time constant to deal with. We have also discussed about two standard responses of first order system that is a step in ramp inputs.

So, today we shall be continuing with that first order system also very briefly we shall be discussing about the second order system, but before that a very quick recap.

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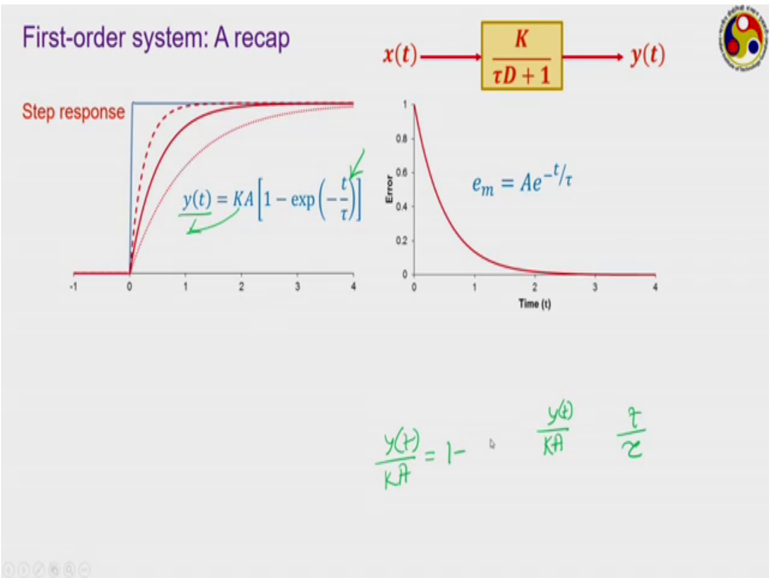

We have seen that for first order system the transfer function can be given in the form of K upon tau D plus 1, where K refers to the static sensitivity and tau is the time constant. So, ideally we want the static sensitive to be higher for any system and tau to be lower; the smaller the time constant beta is the response of the system or quicker is the response of the system.

And, when we judged that with respect to a step input something which is traditionally called the step response we have seen that the if the blue line represents the same curve which we have already discussed in the previous lecture. Here the blue line represents the step input and each of the red line represents the corresponding output when the time constants are varying.

This is the corresponding a mathematical form of the equation that we have already derived K being the static sensitivity, A being the height of the step or the size of the step and as the time constant keeps on increasing the response keeps on shifting in this side that is keeps on going further away from this. We have also derived an expression for the error which we have seen to be following a negative exponential function that as the time keeps on increasing the error also keeps on decreasing that is the difference between output and input keeps on reducing. So, the smaller the value of tau it is the quicker the system reaches the actual output.

Like when the tau is a small one if we look at the dotted line in the previous graph somewhere here the system output represents very close to the input whereas, when the tau becomes higher like in this case the three values of tau that we have selected at 0.25, 0.5 and 1. So, the first line, this one corresponds to toggle to 0.25, but if we talk about this particular one which is corresponding to tau equal to 1, even after this t equal to 4 seconds it is unable to reach the input signal and sometimes instead of plotting the curves this way we often adopt a non-dimensional representation.

In the non-dimensional representation on the vertical axis we plot y upon KA or I should say y as a function of time upon KA. So, that it becomes non-dimensional and on which we have also seen yesterday to represent the error, but on the horizontal axis instead of time we plot time divided by tau. That is, on this scale 1 will represent t equal to the time constant, 2 will represent the double time constant.

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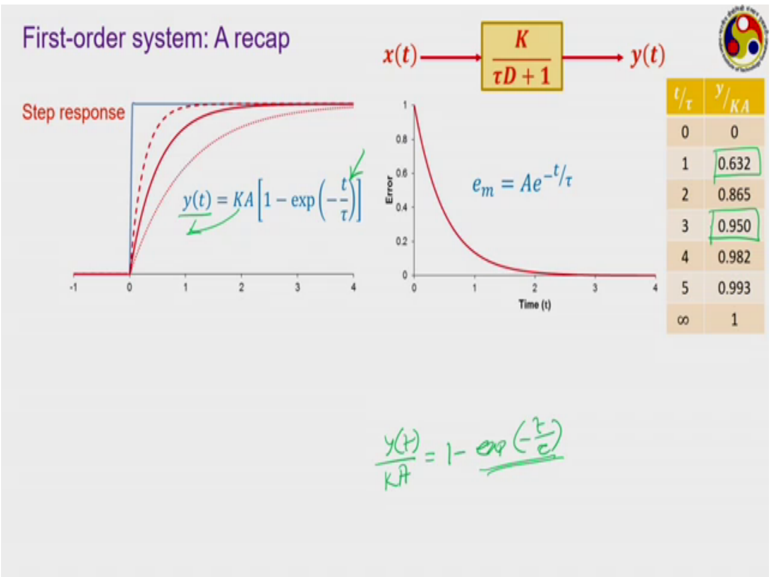

And, if we follow that particular standard just put t upon tau equal to some value here and take this K upon A in the denominator of this particular quantity then we are left with this for expression is K is equal to 1 minus exponential minus t upon tau.

And, as we keep on putting different values then we shall be seeing that is supposed to be something like this when t equal to 0, this particular power definitely corresponds to the error. So, at t equal to 0 error is one corresponding output is also 0 and as t keeps on increasing just look at this table when t becomes equal to tau the output will reach 0.632

times or 63.2 percent of the input whereas, when the time t is equal to 3 tau then it will be able to represent 95 percent of the output which is generally sufficient for most of the practical measurements.

And, if we take t equal to 5 tau, then it is 99.3 percent of the exact representation. So, though we have mentioned that the system theoretical records in finite time to reproduce the exact output value or exact input value in the output actually we can depending on we can depending upon what tolerance limit we can allow we can select some other time also.

For example, suppose if we are satisfied with high percent error then this one is sufficient that is 3 times of the time constant is generally the time that we have to provide. However, if we want less than 1 percent error, then we have to go for 5 times of tau and practically energy equal to 5 tau is the most common value that are preferred for measurement purposes with first order instruments.

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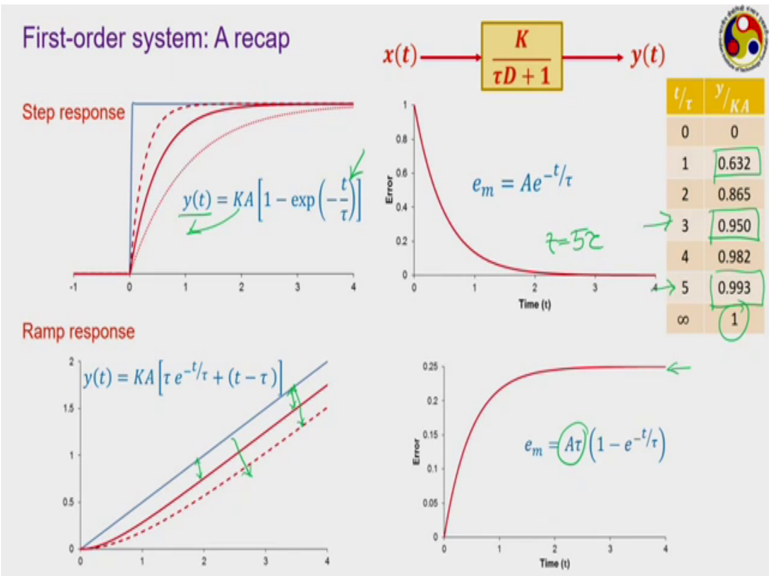

And, now if we have also discussed about ramp response where this input follows a linear profile with time and here we have derived the corresponding mathematical expression and here also we have seen that as the time constant keeps on increasing this is the direction of increasing tau, the response keeps on going further away from this. But, one difference here is that the error like in case of step input the error keeps on going to 0, as the timing keeps on increasing. However, here the error assumes a constant value after time like we have already seen the graph somewhere here there is a constant error that is maintained between input and output and the same error is maintained here as well between input and output. That means, the output will never be able to match the input rather as I input keeps on increasing output will keep on following that, but with a fixed amount of error given by this quantity which is often refer to the steady state error.

The steady state error is directly proportional to the tau; as tau keeps on increasing steady state error also increases. Like look at this diagram when tau is small this is the steady state error. However, at the same time instant with the larger value of tau this is the steady state error here tau are chosen to be 0.5 and 1 in these two examples, the dotted one corresponds to tau equal to 1 and definitely you can see that the it gives twice the steady state error compared to the previous case.

So, these are the step and ramp responses which are very commonly found in practice. Today we shall be moving on with another one known as the impulse response.

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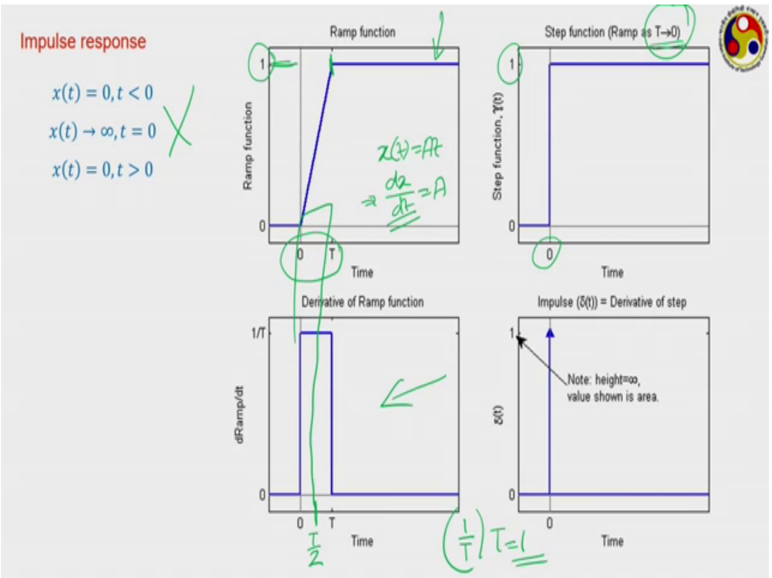

As we have discussed impulse corresponds to a very large magnitude of inputs applied over an infinitely small instant of time. Theoretically the magnitude of the input is infinite and the duration over which it last is just 0 or extremely small. Before going to that theory it is never possible to deal with such a mathematical function.

So, to find a more suitable representation, look at this. What do our ramp function says? Our ramp function corresponds to x is equal to some A into t, where A is some constant; that means, it follows a constant slope or if we differentiate this with time then we are going to get A, that over a period of time or ramp function or over theoretically an infinite ramp will follow this constant slope A and the magnitude will keep on increasing. But, practically speaking we continue to keep on increasing the input infinitely.

Quite often we deal with the structure shown here that it keeps on increasing and then at a certain time whatever value it assumes it keeps on maintains that value; that means, you can almost think of this is like a this is kind of profile is called a cartel ramp or sometimes called unit ramp and, because here we are using this value 1 here. So, you can almost think of this to be a combination of one ramp profile over the time t equal to 0 to capital T and when time t is greater than capital T, it follows a state profile or it just maintains whatever value it has attained at small t equal to capital T.

So, if we plot the derivative of this like if we plot this dx dt here, then over from t equal to 0 to capital T it will be constant at that capital A value and then it will become 0 because the value is not changing beyond this point, it is not changing. Now, what is the value of this height of this one, that is 1 upon T because that will depend if this particular height is 1 or the value is 1, then definitely will corresponds 1 upon capital T.

Now, go back to the step function. A step function can be viewed to be a ramp when this T tends to 0. Means over an infinitesimally small amount of time we are supplying a ramp, so that the value of the input changes from 0 to 1 and then is maintained there. So, a step function can be visualized to be a ramp with T tends to 0 or a cartel ramp whose duration is 0 or extremely small and if we differentiate this function then what we are going to get? Over a very small period of time it is having an infinite amplitude because this capital T tends to 0 and nothing afterwards this is what is a an impulse function; that means, an impulse can be thought about to be a derivative of step.

Now, these are mathematical definitions. Let us think about much something more practical. Look at this particular profile again. So, here over a period of 0 to capital T we are providing you just think about over a period 0 to capital T we are providing one step input of this much magnitude. Now, over the, what is the area under this particular curve? It is definitely 1 upon T is the height multiplied by T it is 1. So, the area under this curve is one which sometimes referred as the strength of the signal which is one or strength of the input signal or maybe the energy that has been transferred with the input that is equal to 1.

Now, suppose if we want to transfer the same amount of energy over a shorter duration of time then what will happen? Let us say instead of we make the time to interval T by 2 if we make the time interval T by 2 while maintaining this one then this height will increase means this profile will be somewhat like will be something like this or instead of plotting here let me plot in a different way somewhere here.

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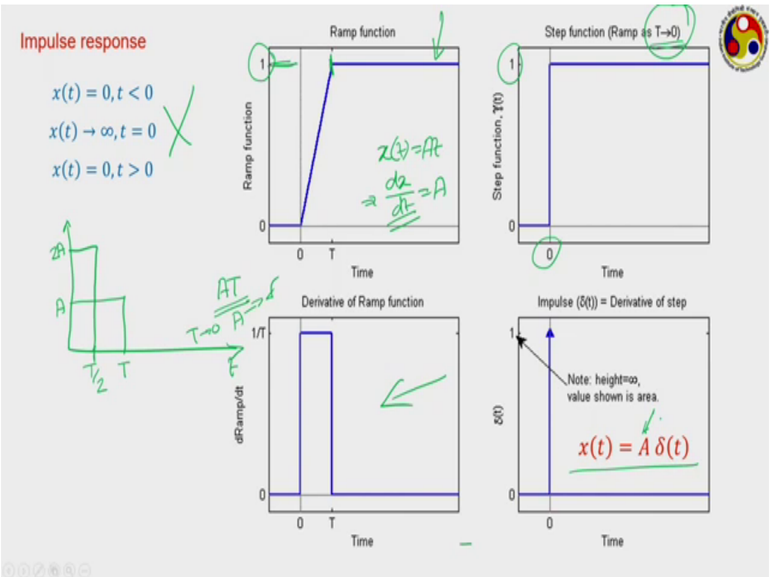

So, this is time. So, initially our profile was something like this what a period of capital T we are plotting it and it is achieving a height of say A, so that the total area under this curve is A into T. Now, you want to maintain the area the same, but we want to supply the same input or same amount of energy over a period T by 2 then how each signal should look like? Should look like this which is twice of A and this is T by 2 and this way if we keep on reducing the time while maintaining this the same then what we are going to reach when this T tends to 0, your A or whatever you will be are getting basically this ordinate that should tends to infinity.

So, then it will resemble a ramp signal, sorry resemble an impulse signal. That means, an impulse can be viewed to be a curtailed step signal applied over a very small period of time. Quite often it is represented by a form like this where del t represents the impulse nature and capital A is referred as the strength of the impulse.

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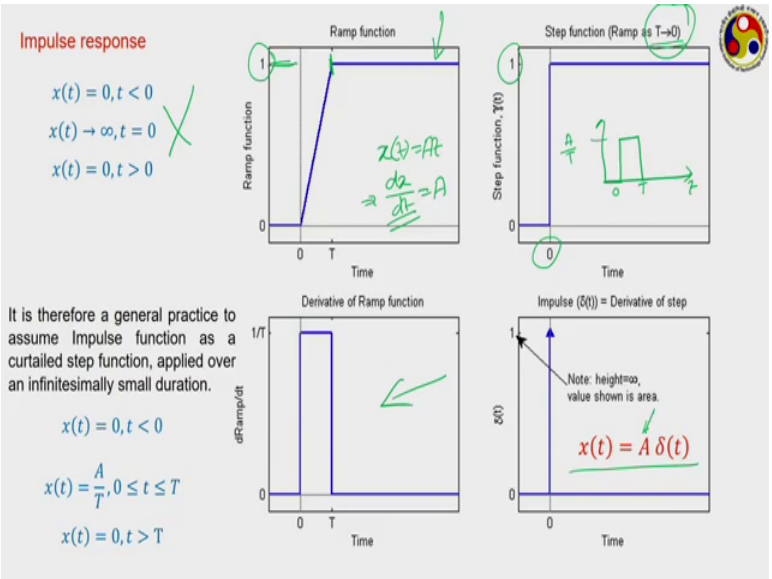

And, therefore, it is generally a general practice to assume an impulse signal as a commune curtailed state function which is applied over an infinitesimally small duration of time corresponding mathematical form can be like this. When t is less than 0 there is no signal then over a period of t equal to 0 to capital T we are supplying a signal of strength A upon T and then after that it is 0. Like if I plot the signal say let me plot it somewhere here your signal will be initially 0 then it will be something like this and then back to 0. This refers to t equal to 0 this refers to t equal to capital T and over this period the magnitude is A by capital T.

Now, when this capital T tends to 0, then we have got a perfect impulse function to deal with. So, the mathematical solution that we are going to go for that will also follow the same structure.

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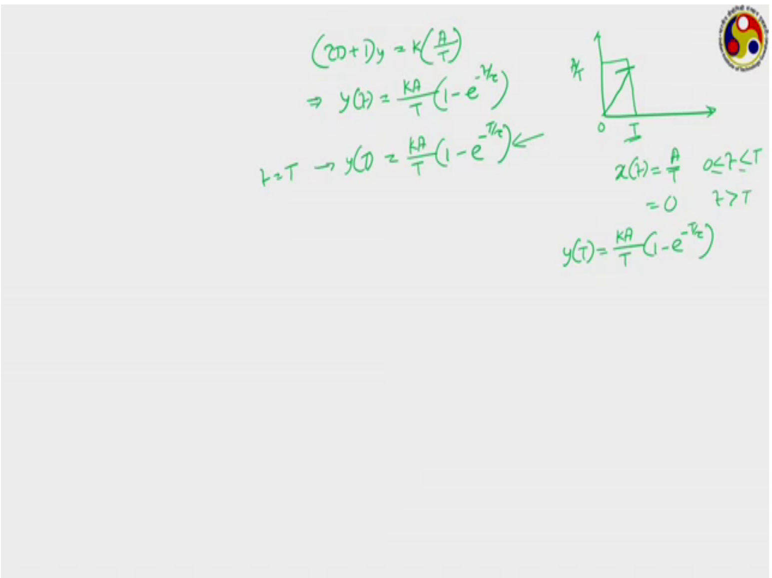

Now, following the impulse signal let me draw the impulse signal again. So, what we are dealing with is a over a period of 0 to capital T we are having a signal of 1 by t and after that it is 0; that means, we are going to follow x t is equal to A by T for 0 less equal to this to capital T and is equal to 0 when t is greater than capital T.

Let us go for the solution the for the first part what we are going to get this is a first order system. So, we have tau D plus 1 into y is equal to K into your x; x is A by T from the first order system equation this is what we are getting and we already know the solution of this one. So, following this solution you know that y t will be what just try to remember what we have already done in our previous lecture when you solve for this step function using the constant it will be 1 minus small t by tau. So, just they are also couple of slides back, where K by t is a right hand side and we are having the exponential negative exponential inside, tau being the time constant for this.

Now, when t becomes equal to capital T then y at capital T will be equal to what KA by capital T into 1 minus e to the power minus capital T upon tau. So, your input signal will start following this sorry, the input signal is already drawn here. If we draw the output the output will keep on increasing following the standard step response and the time t equal to capital T it reaches a value something like this which is given like here let us store this value. Y at capital T is equal to KA by t to 1 minus e to the power minus capital

T upon tau. Let me delete this entire thing and proceed with the remaining part of the calculation.

So, now, we have to solve for the rest when time is greater than t. When time greater than t there is no signal to deal with then what we have? Our equation will be tau D plus 1 dy is equal to 0.

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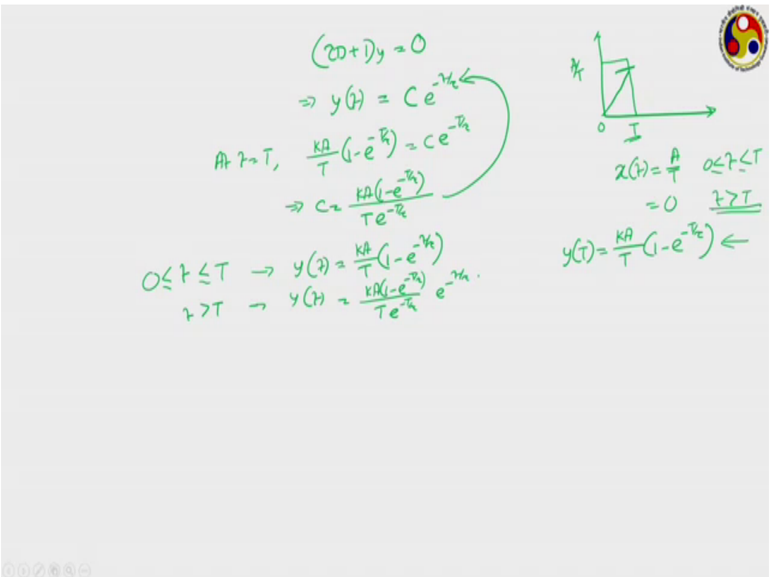

So, if you solve this for y 1 t what will be the solution for this? It is a very standard differential equation you do not have the particular integral part here basically you will be left with only the complimentary function C to the power minus t upon tau; C is the constant to find the value of C we have to put this condition. So, if we put that at t equal to capital T we already know it to be KA by T 1 minus e to the power minus t by tau to be equal to C into to the power minus capital T upon tau giving C is equal to KA 1 minus e to the power minus t by tau divided by capital T into t to the power minus t by tau.

So, this is the value of the constant and putting this constant back into this equation we now get the temperature profile beyond this. It is a negative a so, initial profile or sorry not here just putting it back into this we have the profile for this. So, what we are getting? When we are having the impulse part that is when you are having the step part time is between 0 to capital T our profile y t will be equal to KA by capital T into 1 minus e to the power minus t by tau and when time t is greater than capital T it will be given by just what we have written there KA into 1 minus e to the power minus t by tau divided by T into e to the power minus t by tau into e to the power minus small t upon tau. Let me erase all this.

So, the final response will be a combination of these two. I am using this just to make some space because we have to do something else also. How it will look like?

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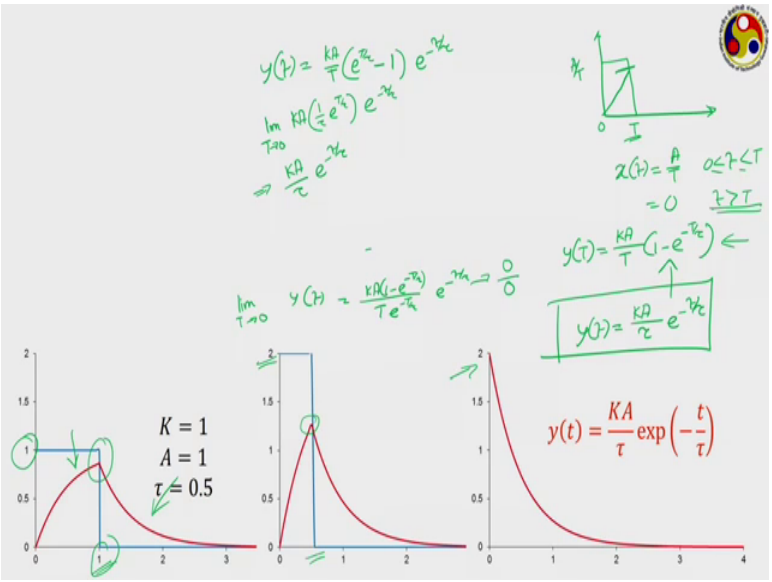

So, this is a profile the blue one is a signal which has been applied over a period of 1 and correspondingly now this particular part corresponds to this here it reaches up this particular value and then it in this portion it keeps on following this particular nature.

Now, here we have taken capital K the gain to be 1, amplitude A also to be 1 and tau we have taken to be 0.5 for the system. So, K and tau are characteristics of the system and here we have also taken A to be 1 for our ease of analysis which is given by this. Next if we make the interval half means instead of applying the signal over a period of 1, we are keeping the total energy of the system all the same, but making the time interval half of this, then what we are going to get? Here the total area under the curve even the same because here we have moved to 2, but here we have come to 0.5 and corresponding we are getting the same nature of the signal here there is some discontinuity that we can see in the graph which is because of the plotting errors nothing else.

And, this way if we keep on reducing the time of applying the signal this capital T basically then this should reach a perfect impulse signal. To reach that then what we have to do when we reach T equal to or T to be very small T tends to 0, then this part of the solution is of not of importance only are left with this particular part. So, let us try to see what we are going to get for this particular part if we put limit T tends to 0 over this then what we are going to get this should lead to a 0 by 0 situation which is a which is of course, not possible to deal with.

Then, let us try to simplify it a bit. We have y t to be equal to whatever we have we are taking the denominator in the numerator. So, we have KA by T into e to the power t by tau minus 1 into e to the power minus small t upon tau. And, now if we apply L S (Refer Time: 21:13) law on this that is we are trying to get limit T tends to 0 of this particular quantity differentiating both the numerator and denominator with respect to capital T, then we are left with K will be there. The denominator goes to 1 and the numerator is 1 by tau e to the power t by tau should be 1 minus small t upon tau. And, so, it reduces to K by tau e to the power minus t by tau; that means, the perfect response against an impulse input can be written as y t is equal to K by tau e to the power minus t by tau.

This is what we are looking for because here we have started with a ramp or a cartel ramp a ramp applied over a shortened diversion of time then that signal input signal going to 0 and from there by doing a mathematical operation that is this T tends to 0, we have got to this giving us the perfect impulse response just a plain negative exponential function. For a small deviation of time it the input increase is very high and then there is no input at all accordingly the measurement system is giving a high peak and then it decays to 0 each. This is the signal and of course, we have plot this using tau equal to 2 in this particular case.

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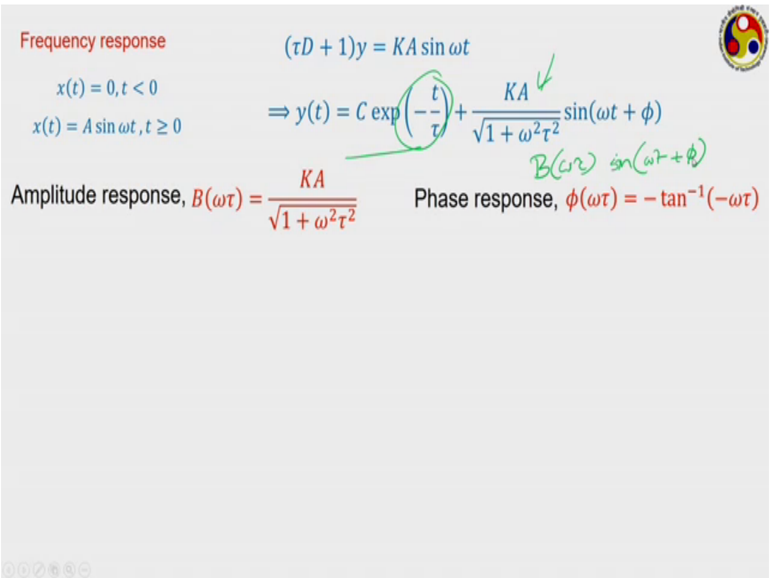

And, the final one that we have regarding the first order system is the frequency response when you are supplying or sorry subjecting a system to frequency some kind of periodic inputs. So, this is the periodic input correspondingly we are solving the conservation equation and I am not going for detail solution we are getting those two parts; we have any exponential part and we have a periodic part. The exponential part seeing from in the nature can clearly see at t keeps on increasing this goes to 0. So, our interest is only the exponential part.

Here this particular quantity is referred to as the amplitude response and the other part is referred as a phase response and accordingly is this part we often represent as B as a function of omega tau into sine of omega t plus sine omega t plus phi; phi again is a function of this omega tau.

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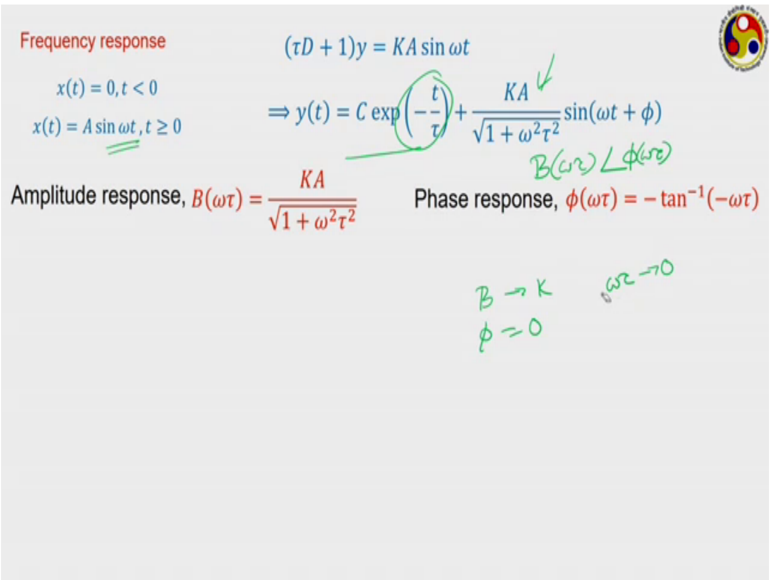

And, sometimes instead of just writing this way quite often we just write it as an angle and phi which is again a function of omega tau. Because sine omega t component is always there your input is having the sine omega t component have this phi refers to the phase lag that has been introduced in the output. For an ideal system we want our B to be equal to just K and phi to be equal to 0, because then we reach to a perfect zero order system. So, but practically that is not possible.

So, just take a look at the two expressions that we have when we can have this phi tends to 0 and B tends to K situation that is when this omega tau product tends to 0; that means, for a tau being a characteristic of the measurement system we want the system to be subjected to such signals which gives extremely small value of this omega tau product and then only your system may give you very favorable response.

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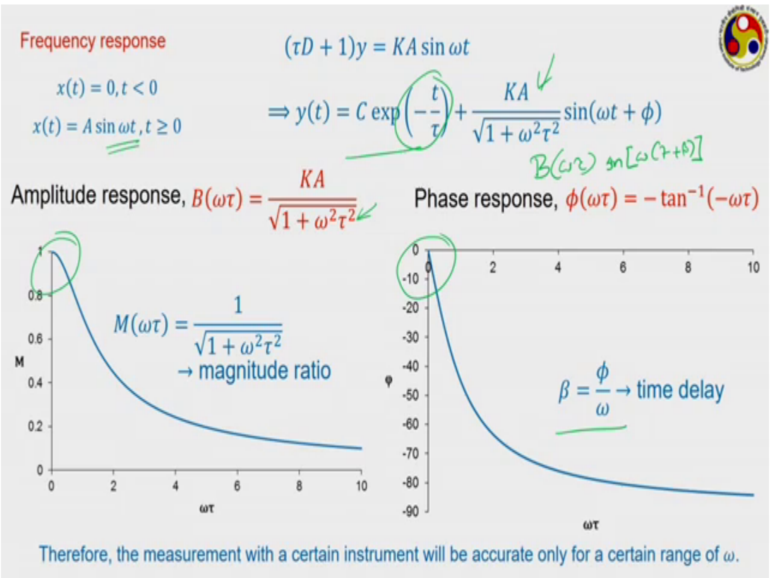

Just take a look at the responses we can get here. This capital name refers to the magnitude ratio of an it is called the magnitude ratio which is nothing, but just this denominator of the amplitude ratio. So, you can clearly see as this omega tau product that keeps on increasing the magnitude response also keeps on dropping. Only when it is very small, this magnitude response is our magnitude ratio is very close to one which is the ideal scenario.

Similarly plotting this phase response we can see with very small value of omega tau we can keep the phase lag within measure within allowable or manageable limits, but as omega tau keeps on increasing we have some kind of trouble; that means, whenever we are dealing with a first order system and frequency response or a first order system we have to choose the frequency of the input signal with care; that means, once we know the time constant of our first order system then we can put certain kind of limit above regarding the signals for which this or with which your system can be subjected.

Like if your omega can keeps on increasing we have to choose an instrument with very small value of tau and this is just an additional phase of information sometimes this phi upon omega product is also called beta which is just to give an advantage of this representation sometimes it is just written as this B omega into sine of omega into t plus beta. This is called the time delay and phi is the corresponding phase lag.

So, amplitude response definitely just changes the amplitude of your output, but phase lag probably is more important because as a phase lag gives introduced if you are dealing with just one sinusoidal wave then there is not that big issue. If you have idea about both omega and tau you can clearly calculate the corresponding amplitude response and phase response and you can put the required corrections.

But, suppose if you are dealing with a signal which has several harmonics to deal with then each of them will be subjected to some kind of phase response and whatever output signal that you are going to get that may be highly distorted thereby completely giving a wrong response or wrong impression.

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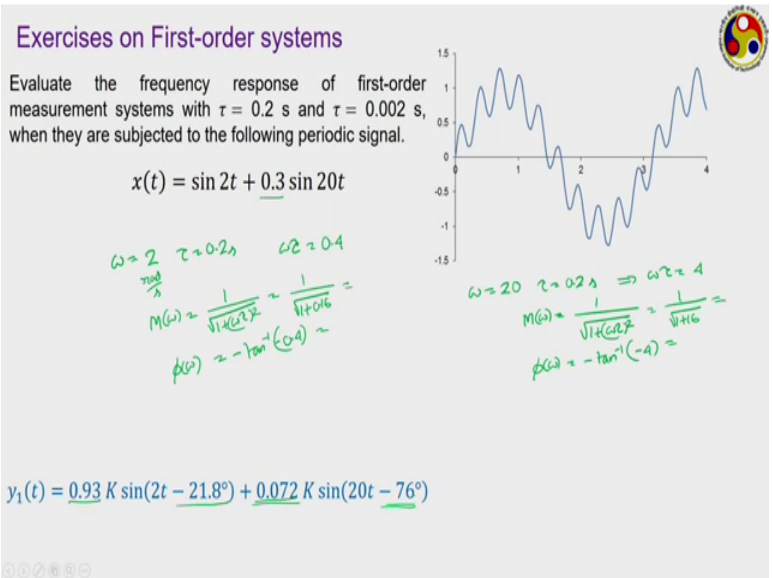

That we can check from this particular example. Here our objective is to check the frequency response from a first order measurement system with two different values of tau and that is being subjected to a periodic signal which has two components. We can see the first one is having an omega of 2, second one is having an omega of 20 and also it is a magnitude of 0.3.

So, there are the two signals or rather two components this is the standard input signal. We can apply the superposition principle; that means, we can deal with both of them separately we can calculate the amplitude response and phase response for both of them separately and then add them together to get the final response.

So, if we pick up the first one for the first signal omega equal to 2 tau equal to 0.2 second. So, your omega tau product is 0.4. Omega I should write omega is also unit that is general radian per second or second inverse. So, omega tau is unit less. So, your amplitude M omega m magnitude ratio root over one plus omega tau whole square that is 1 by root over 1 plus 0.16 some value will be coming I do not have a calculator here. So, I cannot calculate the value. Similarly phi omega or phi omega tau we can write will be equal to minus tan inverse 0.4. So, we will be getting the value corresponding to this some value in radian or degree will be coming out from this or I should write a minus sign here.

Similarly, if we. So, whatever we are going to get that is going to give you the signal corresponding with sine 2t. Similarly, with the next one where omega is equal to 20 and tau remains the same for 0.2 second, we have this omega tau to be equal to 4 accordingly M omega for the second component is 1 by root over 1 plus omega tau whole square that is 1 by root over 1 plus 16. So, we will be getting some number and phi omega will be equal to minus tan inverse of minus 4, again some number will be coming out from this.

So, if we calculate all these numbers we will be going to get this final this value is. Here K is the static gain which we have we do not know or the in that information is not given here. Let us assume K to be equal to 1, then we can see that the first signal is having an amplitude ratio 0.93 and a phase lag of minus 21.8 degree 8 degree. Whereas, the second signal is having this actually or 0.3 was already there. So, 0.3; 0.3 by point sorry 0.072 by 0.3 is the magnitude ratio and minus 76 degree of phase lag has been introduced or 76 degree of phase lag has been introduced.

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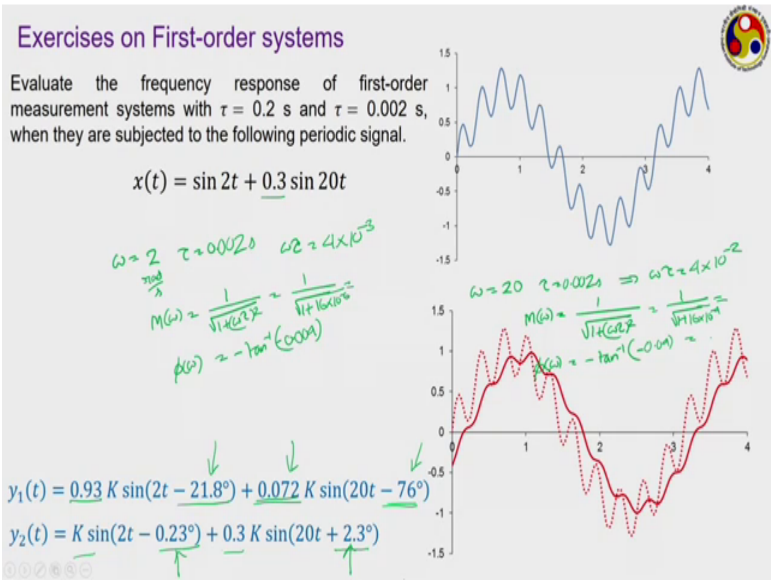

If we solve for the second system, in this case tau is equal to 0.002 second. So, accordingly your omega tau becomes 4 into 10 to the power minus 3. So, following the same procedure we can calculate this 16 into 10 to the power minus 6. All the calculations can be done the same way 004 and for the second component omega is 20, but tau has changed to 0.002 seconds giving your omega tau to be equal to 4 into 10 to the power minus 2.

And, putting these numbers here we are going to get 1 plus 16 to 10 to the power minus 4 some number and 0.04 some number here, putting this, this is a signal. In both the cases magnitude ratio is actually if you calculate the numbers both cases magnitude ratio will becoming very close to 1 and the first signal is having 20 0.23 degree phase leg and second one is having 2.3 degree phase leg.

So, clearly as the tau has decreased by two orders the magnitude ratio has become very close to 1 in this case magnitude ratios are quite different here the magnitude ratio is 1 in both the signals and more important effect has come on the phase lag, while the first harmonics was suffering if a syllabic 21.8 degree that has reduced to this 0.23 degree whereas, for the second harmonic which are suffering 76 degree phase lag it has reduced rustically to as 2.3 degree.

What is the effect of the output signal? Let me just erase this. Here both the signals are shown the continuous line refers to the first case that is tau equal to 0.2 second. Look at the structure and just compare with the original one. It does not, does it look the similar? It is looking completely different the output has completely been distorted and it hardly give any idea about the actual input.

When you are doing a real life measurement when you do not have any idea about the actual input then you are going to go out to the completely wrong impression. But, look at the dotted one which corresponds to tau equal to 0.002 second. Then what you have here? The signal quite meticulously follow the actual one. This dotted signal has excellent similarity with the actual one and that is evident from its mathematical form also where we have magnitude ratio to be equal to 1 for both the harmonics and also very small phase lag for both of them. So, the output meticulously follows our input.

Therefore, it is very very important to have a small value of tau for your first order measurement system particularly when you are dealing with periodic inputs.

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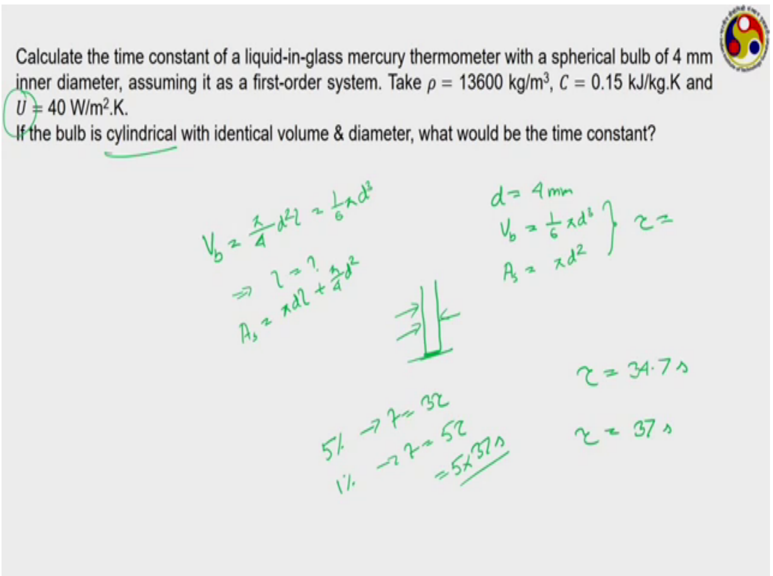

Here of course, we have drawn it K equal to 1. Here is another example not for periodic in, but we have not solved any probability to step input just of an idea or just to give you an idea about this.

This is the problem corresponds to a liquid-in-glass mercury thermometer which is the information about this one is given. So, I want to you to calculate the corresponding time constant. Now, how to calculate the time constant? Here all the information's are given.

So, it is a spherical bulb of 4 mm inner diameter; inner diameter is equal to 4 millimeter, then volume of the bulb will be what? 1 by 6 pi d cube and the surface area of the bulb will be pi d square. So, we know the volume and the surface area of the bulb and in the previous lecture we have derived the mathematical expression for tau relating these parameters, other properties like rho, C and U are also given. So, you can calculate the value. I am leaving it to you, please try to calculate the number from this.

Next question is if the bulb is cylindrical with identical volume and diameter what will be a time constant. Now, it is mentioned that the diameter is same, but it is cylindrical same. So, the volume for the cylindrical one will be what? It is a cylindrical one. So, pi by 4 d square into l should be equal to that 1 by 6 pi d cube for the spherical one and from there we can calculate the length of the cylinder. So, once you know the length of the cylinder then the surface area can be calculated as serve pi d into l.

Now, there is one question. This is the cylindrical bulb of it. So, while calculating the surface area pi d l definitely gives you this peripheral surface, but what about this particular portion? This small laser sorry this lower wall that may be also in contact with in contact with the system where you are measuring the temperature, then while calculating the surface area you have to measure the peripheral area plus this area of this lower portion which is pi by 4 d square. So, using this you can calculate the both the cases I am giving you the final numbers tau equal to will be 34.7 seconds for the first case which spherical bulb and for the second case tau will be equal to 37 second. So, quite similar to each other, only small difference.

And, just now think about if you want to use this particular thermometer for measuring your body temperature, then how much time we should allow the thermometer to be in contact with your body? Hopefully you remember if we can allow 5 percent time a 5 percent error rather than our time should be 3 times the tau whereas, if we want error to be less than 1 percent then t should be 5 times the tau. So, if we want our error to be less than 1 percent with the cylindrical bulb then we have to allow 5 into this 37 second, this much of time for the thermometer to the thermometer in contact with our body for a correct measurement.

One thing here is this U is given to a 40 watt per meter square Kelvin, but as we have already seen U is not a constant rather it depends upon your actual environment. Like in stagnant fluid whatever will the value of this heat transfer coefficient in flowing fluid the value will be different. Value in water and air or liquid and gaseous medium they will be also be distinctly different because of their different thermal conductivity.

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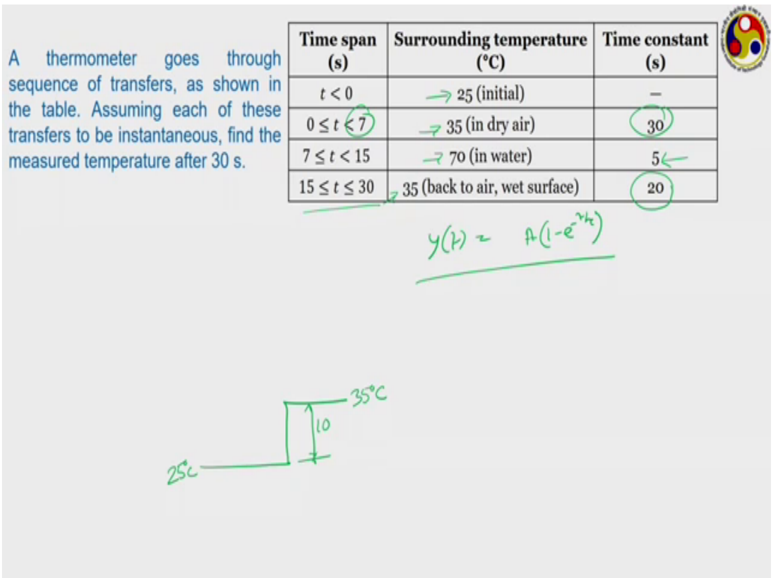

So, let us take another similar example here our subject is a thermometer which goes to a sequence of transfer. Initially it is kept in here at 25 degree Celsius, then we are dipping this for a period of 7 second into another container of die here where the temperature is 35 degree Celsius and corresponding time constant is this. Here directly the time constant values are given. Remember they are the thermal time constant for the thermometer depends upon the geometry the properties and also the value of U. Of course, the geometry of the thermometer is not changing, the properties of the thermometric fluid those are also not changing it is only the U, the overall heat transfer coefficient that is changing.

So, after 7 second, we take the thermometer into water at kept at 7 degree sorry, 70 degree Celsius corresponding see the drastic reduction in the time constant. Initially it was air, now we have taken into contact with the liquid which has much higher thermal conductivity leading to one sixth value of time constant and after that we again take the thermometer back to that container with here 35 degree which was at 35 degree Celsius and maintain therefore, 15 to 30 seconds. But, here now the surface of the thermometer is wet which hinders the heat transfer because it has come out of water accordingly it is value of heat transfer sorry, overall heat transfer co efficient sorry his value of time constant is different than what we had originally and the question is how to solve it.

This each of these cases can be thought about as one step response. Just think about the first one. We know that for any step function our input is something like KA by T into 1 minus sorry, I am still stuck with that impulse case. So, it is KA e to the power minus t by tau. Here nothing is given about K, so, let us take K equal to 1 which goes out of this. What is A? A is the height of the step. So, initially the thermometer is at maintained at 25 degree Celsius and now input changes to 35 degree Celsius. It is 25 degree and it is 35 degree Celsius.

So, height of the step is this much. This is this 10 degree Celsius. But, you also have to be careful that initially this expression we have derived by assuming initial value of y t be equal to 0, but here y is kept at 25 degree Celsius.

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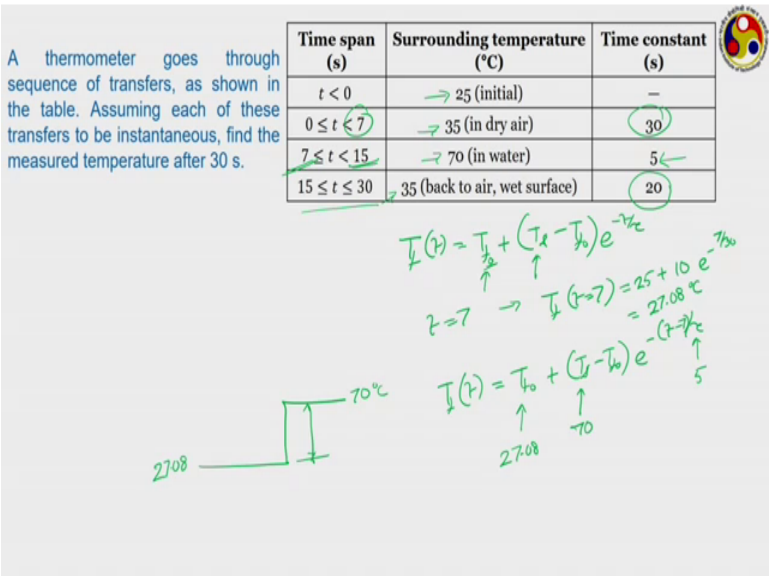

So, if we take that into account then our expression for temperature of the thermometric fluid can be written as T f at any time t will be equal to T f at t equal to 0 or let me use some other notation T f 0 plus T infinity minus T f 0 which is the height of the step into the power minus t upon tau. So, in the first case T f 0 is 25 degree; this is 25 degree, T infinity is 35 degree or this T infinity minus T f 0 is 10 degree and accordingly tau equal to 30 and accordingly we get some profile for this temperature. And, at t equal to 7 second the temperature of the fluid will be T f at t equal to 7 to be equal to T f naught 25 plus the height of the step is 10 to e to the power minus 7 by tau to be equal to 30. So, you can calculate the value I have pre calculated this one to be equal to 27.08 degree Celsius.

Now, come to the second case. In the second case our initial temperature is this another step we are providing with an initial temperature of 27.08 and this to be 70 70 degree Celsius. So, height of the step is 70 minus 27.08 and tau is equal to 5 second, but while doing this calculation do not forget to reset this time 0, because here we cannot start with t equal to 8, 9 or something. The entire step has been provided after 7 second.

So, if we are writing this then you should write this as T f at any t should be equal to T f naught which is actually this 27.08 plus T infinity minus T f naught. Here T infinity is 70, T infinity is 70 into e to the power minus t minus 7 by tau and tau in this case is equal to 5 and t minus 7 the 7 is coming because it has started after 7 second of operation.

Accordingly we can calculate the temperature after 15 at the instant of 15 second and once we get that then the same we can calculate for the rest of the part. So, our objective is to calculate the temperature at t equal to 30 second and if we plot this then you will get a representation like this.

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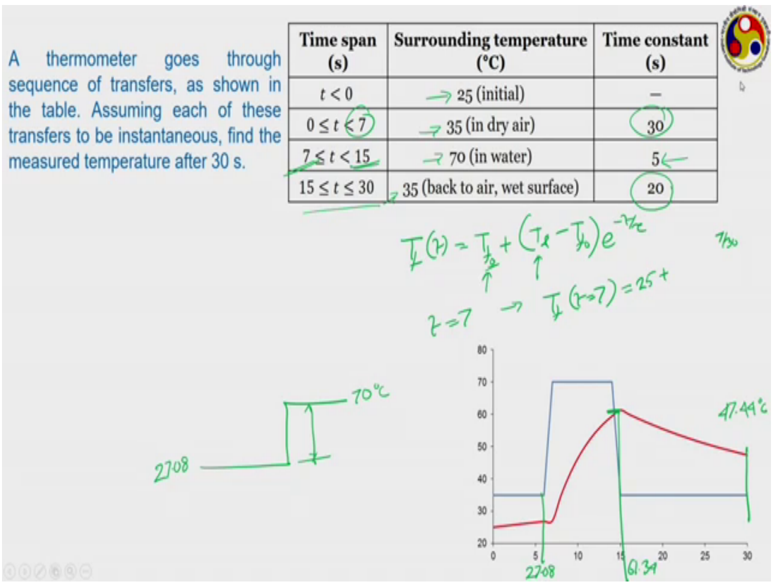

This here at this particular instant of time the temperature is that 27.08. Then when the second transformation comes is takes place here the temperature I have noted the value to be equal to 61.34 and finally, here at this particular instant answer will be 47.44degree Celsius. So, this way we can calculate a temperature at now at any point using this thermometer.

So, this takes us towards the end of our discussion on first order system. Please try to solve similar problems from their books and we shall be you will be able to solve the assignment problems as well.

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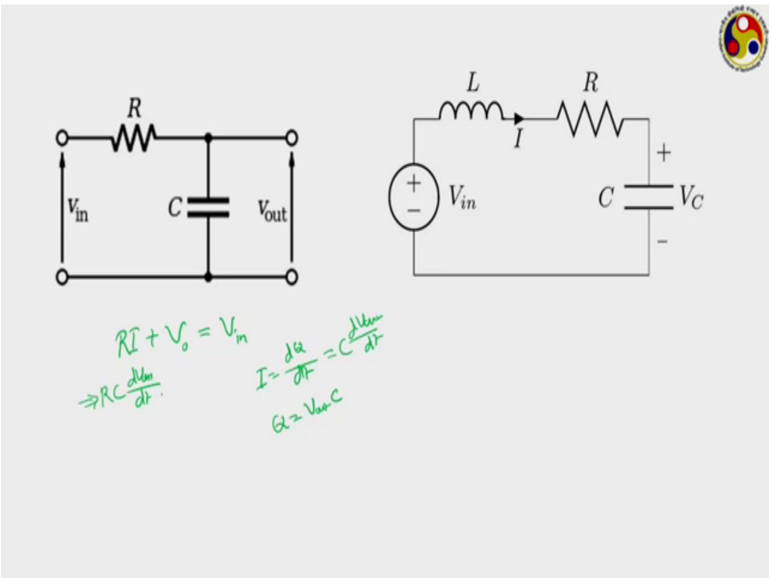

Let us quickly move on to if you another example of a first order system. Look at this. What we have here, an electrical circuit where we have a register and one capacitor.

So, if we apply our Kirchhoff's principle on this then what we have for this circuit? We have R into I, that is the voltage drop in the register plus V out to be equal to V in. Now, what is I or how we can relate this V out to I? We know that I is equal to we know I is equal to dQ dt; Q being the charge and what is Q? Q is equal to V out into C, that is the voltage that is applied across the capacitor and C is the capacitance of this.

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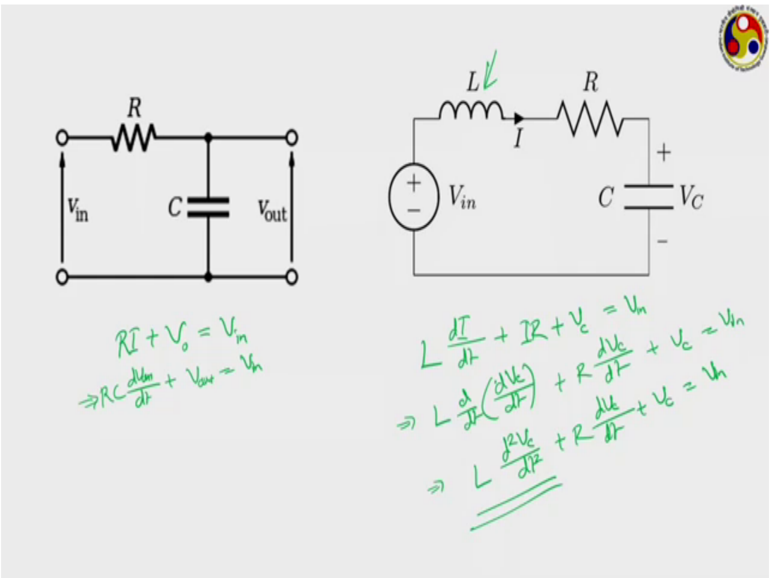

So, if we use that then I becomes equal to C into dV out dt, putting that we have RC dV out dt plus V out is equal to V in; a very much a first order system representation. So, you can easily calculate the K and tau from this and we can proceed with our calculation.

Now, look at the same circuit is there, but here we have an inductance coming into picture. If we apply our principle on this then what we are going to get? The voltage drop across the inductance will be L dI dt these are straightforward electrical principle and applying the Kirchhoff's law it will be this plus IR plus V c that is a voltage is across the applied across the capacitor should be equal to V in and it is a principal from the previous case it is d dt of d V o dt plus R dV o sorry, not V o mixing up with the notation sign here. So, d V c and dV c dt plus V c is equal to V in.

That is Ld 2 V c dt 2 plus R dV c dt plus V c is equal to V in; that means, just when the circuit has just one register and one capacitor basically one capacitor discharging fairest we have a first order system, but as soon as we are having this inductance coming to picture or inductor then we have this particular term appearing into the system which leads to a second order system.

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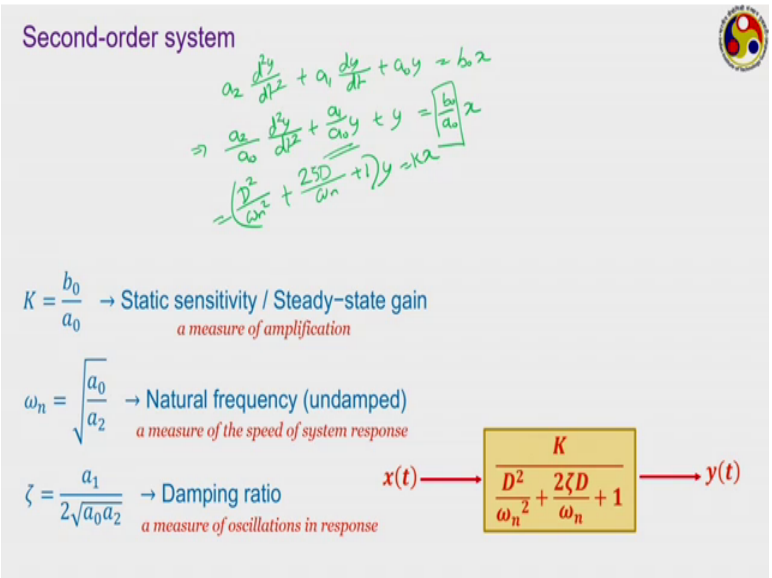

We shall be discussing very very briefly about the second order system. From our original mathematical structure we know that a second order system will have n equal to 2 giving d 2 y dt 2 plus a 1 dy dt plus a naught y is equal to b naught into x or following our previous nature d 2 y dt 2 plus a 1 upon a naught y plus y is equal to b naught upon a naught into x. But, instead of using the here the b naught upon a naught remains the same static sensitivity, but instead of using the concept of time constant here we have to use something else because here we have not 2, but 3 parameters to deal it.

We have is this a 1 upon a naught here and also a 2 upon a naught here and in context with this we introduce these parameters first one is K that is the same static sensitivity which is a measure of the amplification. Next we introduce on omega n. Omega n is defined as the root over of a naught upon a 2 and called the natural frequency or un damped frequency which gives a measure of the speed of system response.

Omega n can be viewed to be something coming here this can be viewed to be D 2 by omega n square plus this one we write as 2 zeta D upon omega n plus 1 into y is equal to K into x, where the zeta is called the damping ratio defined as a 1 by 2 to root a naught a 2. It gives a measure of the oscillation in the response or how the oscillations are getting suppressed in the response. We shall be seeing that very quickly.

So, this is the corresponding transfer function that we can get. D square upon omega n square plus K by D square upon omega n square plus 2 zeta D omega n plus 1. So, like in zeroth order system we have just only one characterizing parameter which is a static sensitivity, for first order system we have static sensitivity and time constant here we have three to deal with K, omega n and zeta simultaneously.

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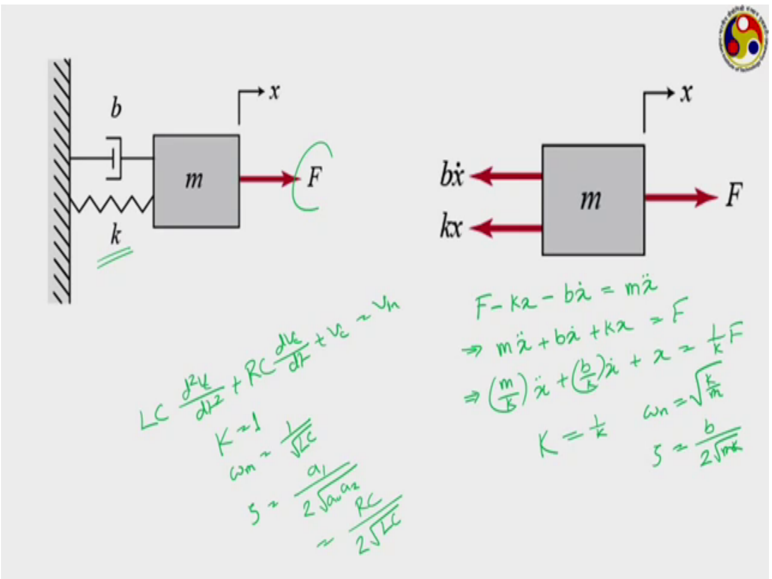

We shall be this one example can be this mass spring damper system. Here we have a spring which has been subjected here for sorry, we have a mass being subjected to this force is pushed by this force, but we have this K the spring and also on damper.

Now, we know that the force of the spring is proportional to the for the force experienced by the spring is proportional to the displacement whereas, the force experienced by the damper is proportional to the velocity and both of them will be acting opposite to this F. So, you feed from this free balanced a free body diagram if we write then we can write that F which is acting in the positive x direction minus kx minus bx dot should be equal to m into x double dot martin mass into acceleration following Newton's second law of motion. So, m x double dot plus bx dot plus kx is equal to F, it is a perfect example of a second order system.

If we compare with our earlier notation plus b upon k x dot plus x is equal to 1 by x F. Then what is your capital K, the static sensitivity? Sorry, it is not one of x it is 1 by this small k. So, it is 1 by small k is the static sensitivity, omega n will be equal to root over k by m and what will be your choice for zeta? Zeta will be equal to b y twice root over mk. So, this we can get the expressions for natural frequency and damping ratio.

If we compare that with and the inductor capacitor registered system that we had. There we had this LC d 2 V c dt 2 plus R or should write this RC dV c dt plus V c is equal to V in. So, how we can get the expressions? If you compare here what expressions you are going to get here definitely K equal to 1 and the natural frequency omega n will be equal to 1 by root over LC and zeta will be equal to what expression for zeta we are going to get from here? Zeta equal to a 1 by look at the previous expression that we have developed we know that zeta is equal to a 1 by 2 root over a naught a 2. So, here a o1 is RC 2 root over a naught a 2 and your a 2 is LC. So, from there we can get the expression for this zeta as well.

We shall very quickly we shall be discussing about step and frequency responses of our second order system. We have already discussed about first law a first order system in detail, so that can be extrapolated where second order system are much more complicated because of the presence of this damping issue.

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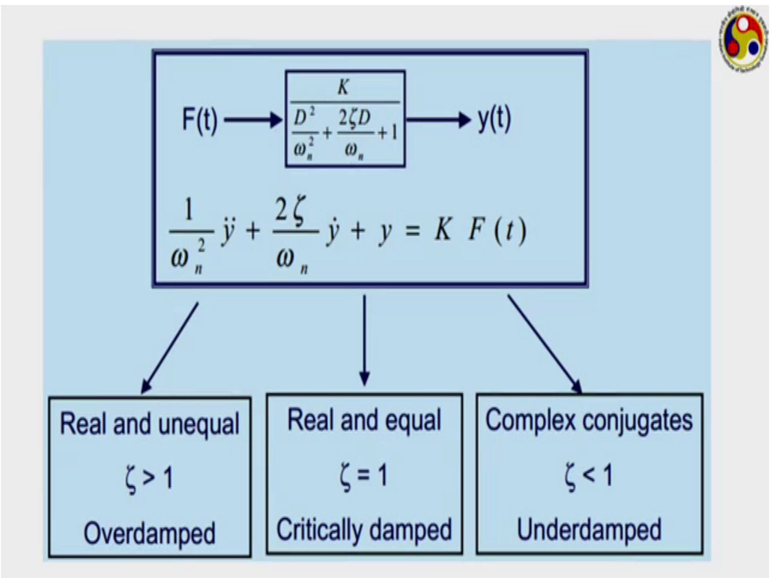

Because here for any given input we can get three kinds of solution depending upon the value of zeta. When zeta greater than 1, we get real and unequal solution, we call it an over damped system. When zeta is equal to 1, we get 2 real and equal roots and it is called a critically damped system; whereas, when zeta is less than 1, we get complex conjugates of the solution and we call it under damped.

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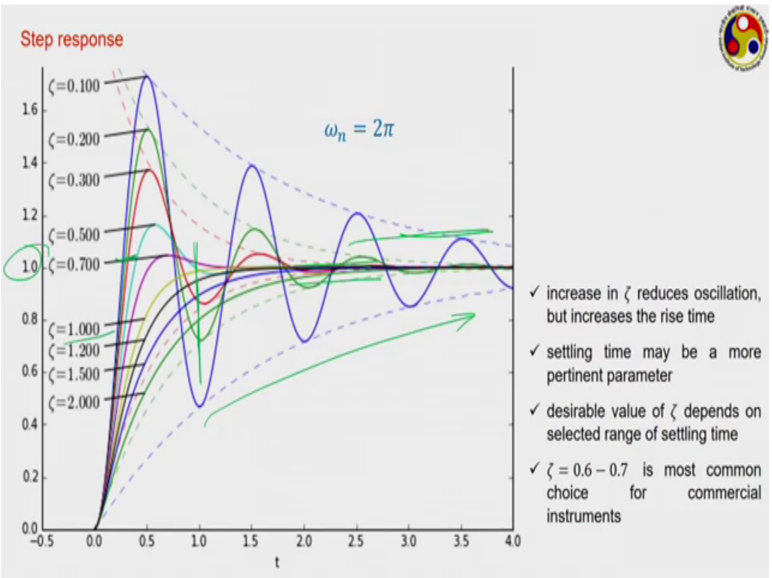

This is the step response of such a system look at when zeta is very very small we are getting lots of oscillations. Look at this particular graph. When zeta becomes is equal to 0.3 we have mean zeta has increased 3 times, but still quite small. We are having oscillations, but the magnitude of oscillations are coming down. You can just follow this envelope. This is for zeta equal to 0.1. So, quite large what I saw zeta equal to 0.3, this is the envelope you can see that is continuously coming down the end. The it is trying to settle down within a reasonable range much quicker; when zeta equal to 1, just follow this particular curve. It keeps on increasing and then there is no oscillation at all rather it follows that one value quite quickly.

So, as the value of zeta is increasing the oscillations are getting dampened. But, also the time required to attain the first time into for the first time to attain the value of one that also keeps on decreasing. Like for zeta equal to 0.3, it is reaching somewhere here; whereas, for zeta equal to 1, it is reaching somewhere here, but that hardly gives an any information because if it keeps on oscillating there is no point talking about when it is for the first time attaining the value, basically there is no point talking about the rise time.

Here everything has been drawn for the natural frequency of 2 pi. So, the rise time keeps on increasing with a decrease in zeta, now sorry with increase in zeta, but it is much better to talk in terms of settling time because that sense in much more part in parameter. Look at for zeta equal to 0.1, it is still very high after crossing about 4 seconds or tau t equal to 4 in non-dimensional unit it is still very high, but if you are talking about say settling period of 10 percent of range then within this period this tau equal to points here settle down and tau equal to 0.1 settle down from may be somewhere here itself.

So, but if tau keeps on increasing again the settling time is much more because rise time keeps on increasing accordingly settling times also keeps on increasing. So, we may have to go for some kind of optimization. Depending upon what range of settling time, we want we have to select the tau accordingly we can modify the damping component in your system. Commonly systems uses it m theta value of 0.6 to 0.7 in commercial instruments.

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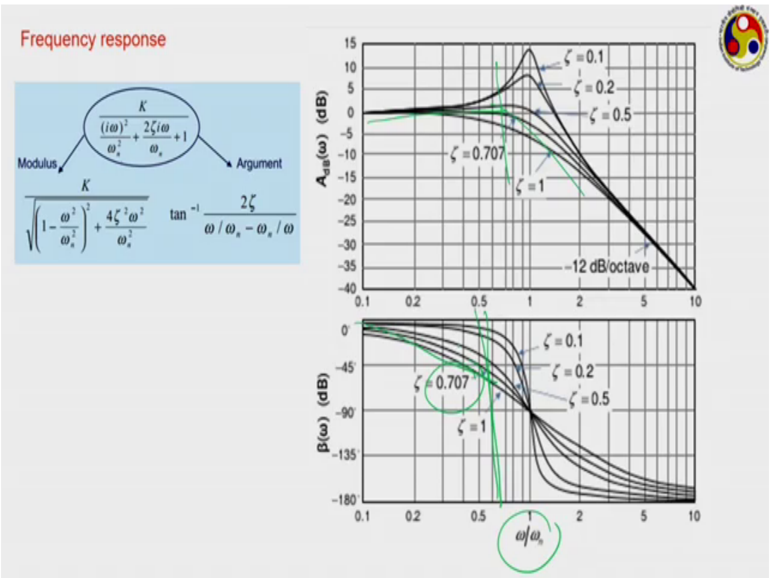

And, if we subject a second order system to a periodic input then this will be the corresponding response you can see when tau is equal to 0.7 or 7 over a large range we are getting amplitude ratio to be equal to 1 or a very static response and then it keeps on dropping after this. That is why 0.7 on 7 is generally a quite common choice as a zeta for second order system subjected to frequency response. You can see for the phase response the phase lag is always there as this omega upon omega n ratio keeps on increasing phase angle keeps on increasing or phase lag keeps on increasing fought for 0.7 on 7 you are getting almost a straight line representation till a reasonably high value of omega upon omega n.

So, this is generally a quite common choice for zeta. This 0.6 to 0.7 is generally a favorable choice.

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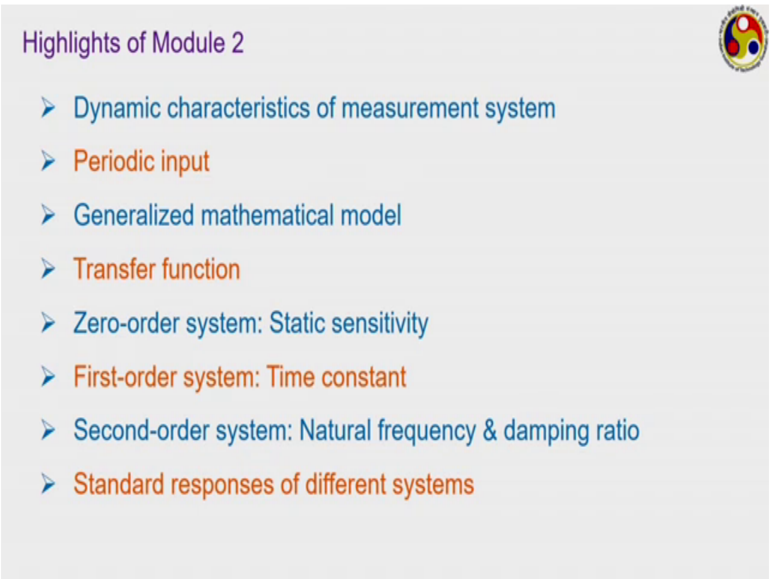

This takes us to the end of our second module. We have discussed about the second order system very very briefly because we do not need to go into any more detailed. But, those who are interested into this can refer to the textbooks and people who are working on control systems generally read them need them much more.

So, in this module in this week we have discussed about the dynamic efficiency cover measurement system, we have talked about periodic input in quite detail in our first lecture here, then we have developed a general mathematical structure using which we have got the concept of the transfer function. We have discussed about zero, first and second order system for zeroth order system the concept of static sensitivity came into play.

For first order system we have got the time constant and second order system gives the natural frequency and damping ratio the catch raising parameter and we have subjected these two systems to some standard responses to get their idea. Zeroth and first order we have discussed in detail, second order we have discussed very briefly because we can easily extend on discussion a first order to any kind of second order response. In measurement a point of view generally they were need to go to higher order systems. As we shall be discussing about different kind of measurement system or different specific measurement system in later weeks we shall may have to go back to this first order or second order systems. We shall be seeing several such examples later on.

So, till that moment you can just stick to whatever we have discussed here. This takes us to the end of our second module. There will be an assignment, please follow the assignment and whatever queries you have please refer to the textbook and also write to me. I shall be very happy to response. So, by for this week, next week we shall be talking about the very exciting field of digitalization and analog to digital conversion.

Thank you, very much.