Principles of Mechanical Measurement
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Module - 02
Lecture – 02
Response of Measurement Systems

Hello friends, welcome back to the second lecture of our second week, where we are talking about the Response of Different Measurement Systems. If you remember our discussion from the previous week, I am sure you remember that there we have talked about the characteristics of measurement system.

And primarily, there are two kinds of classifications we get. One is the static characteristics where we talk about both time invariant input and output, that means neither of the input and output changes with time both are perfectly in under steady-state condition. So, then whatever we get that we call the static characteristics.

Common static characteristics, we have already discussed last week like sensitivity, linearity, zero-bias, resolution, then drift, hysteresis etcetera, different kinds of static sensitive or I should say static characteristics not sensitivity. We have already seen different examples of each of those characteristics, and also from some given information how to harness information about each of the static characteristics, some of the exercise we have also done. I am sure you have done the assignments also where similar questions are there.

Now, one question that may be popping up in your mind that we are talking about both input and output to be time invariant. And the very common perception there may be if the input is a study one that is it is not varying with time, then you can expect the output from your measurement system also to be time invariant.

And then why we need to mention both input and output separately? The answer to this question is that it may be possible even if your input is under steady-state time invariant, your output that is the output that you are going to get from your measurement system may keep on varying with time. And we shall shortly be seeing a few examples right in this particular lecture.

And therefore, to ensure the static characteristics both input and output needs to be time invariant and then only we can talk about those characteristics like sensitivity, and linearity, and resolution, etcetera. However, when the both of them are varying with time or at least the output is varying with time, we talk about the dynamic characteristics.

And in this week’s content, we are primarily talking about the dynamic response of a system. And some of such characteristics we have already discussed in the previous lecture, where we focused quite a bit on different kind of inputs particular the time variant inputs in the form of periodic function.

We know that whenever we are dealing with a periodic function, however complicated it may be using the Fourier transform we can always convert that to a series of or a summation of a series of sine and cosines. And from there we can easily identify the amplitude and frequency, and if there is any phase lag at all. For each of those components, and we can represent them either as a times in signature or in the in terms of the frequency spectrum etcetera.

And when that kind of input signal periodic or sometimes may be time varying non-periodic signals are imposed, and measurement system. Of course, the output is also going to be varying with time. And whenever such kind of periodic signals particularly, we are talking about we there are several very common dynamic characteristics that we have to be careful of like the amplitude response, phase response and frequency response which are the most common used.

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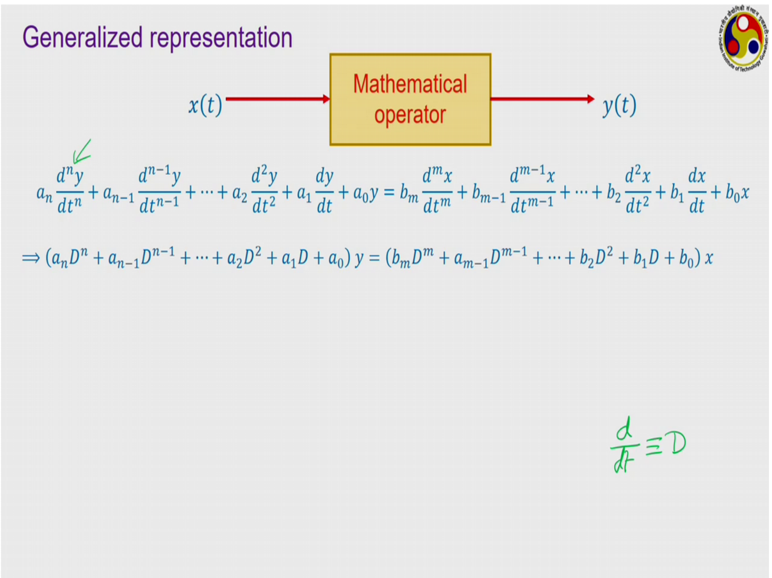

So, today's lecture we are going to discuss about 0th and 1st order measurement system, it is response to certain standard input signals. Now, in towards the end of the last lecture, I started developing a mathematical model of a general measurement system, but because of the paucity of time I had to quit very early.

And so I shall be starting that one from fresh, we have got the idea that the purpose of a measurement system or the operation of a measurement system can be viewed to be like a mathematical operation being performed on a input to get some output. Like as shown in this slide if x is the input that is given to the measurement system, we can view the system to be performing some kind of mathematical operations to give us, and output as y.

And of course, here we are restricted our self to single input single output systems, it is possible for the same system to have multiple inputs and have multiple outputs, but that will be too complicated to discuss in a course like this. So, we are restricting our self to a single input which is denoted as x, and a single output which is denoted as y.

Then the most common relation between this x and y in terms of differential equation can be a form like this, where the left hand side corresponds to the input or sorry corresponds to the output, where we have the y here a naught, a 1, a 2 all these are the coefficients. And this n or maybe this n, this is generally referred as the order of the system.

Whereas, on the right hand side we have the input part in terms of x, here again we have several coefficients b naught, b 1, b 2, etcetera to m n and m may be same or maybe different, but both are integer numbers. Order of the system, I repeat is given by the n and not by m. So, order of the system we refer in terms of this n.

And when n is equal to 0, we call that 0th order system; when we call to 1, we call it is 1st order system, and it goes on that way. Now, quite commonly in mathematics or any such kind of system analysis instead of using the differential notation which is d dt here, we like to represent this in terms of an operation capital D. So, if you introduced that capital D operator, it looks quite similar to an algebraic equation, but be careful here D is not any algebraic quantity rather it is just an operator the differential operator. So, this is the equation or relationship between y and x in terms of this operator D.

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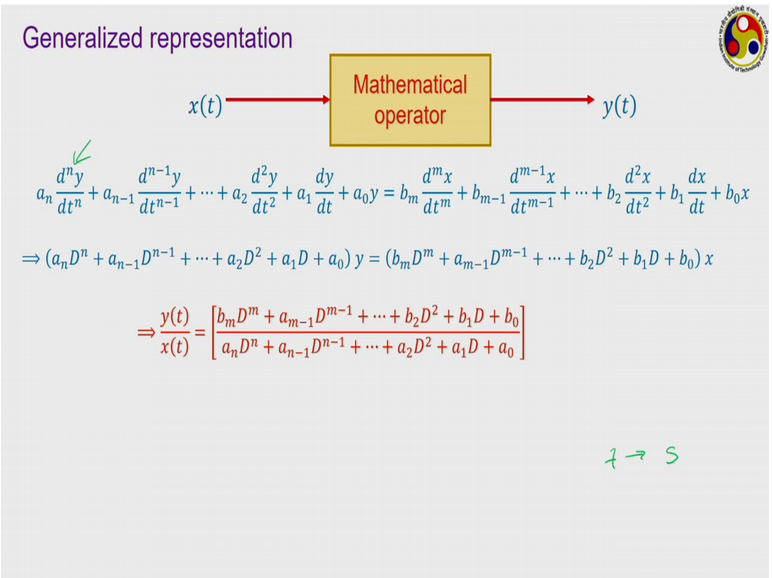

And so again drawing analogy to the algebra, we can represent them in a form like this. In the numerator contains the coefficients associated with the output, and denominator contains the coefficient associated with the input. Here we are getting then a ratio of the output to that of the input to the input both maybe time varying, maybe time invariant.

But, here our importance our focus is more on this right hand side, which is giving this ratio of output to input. Quite often here when we are dealing with a higher order equation that is n is a higher than n is a high in value or n has a large magnitude, then dealing with such ordinary differential equation may be quite complicated, and that is why quite often for solving such equations instead of directly solving that in the time domain we prefer to perform a Laplace transform, so that the instead of using this time t as the independent variable, we get that to converted to s which is the frequency domain parameter s is the frequency, which is actually a complex number.

And using the Laplace transform, we can get this conversion from this independent variable time to the independent variable frequency. And I have already asked you last week itself to take a look at Laplace transform just to revise your Laplace transform of exercises.

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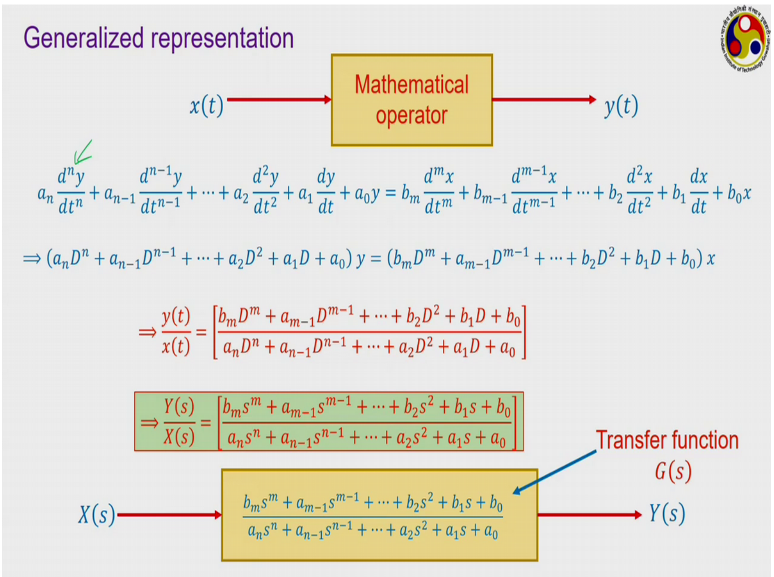

And from that knowledge, we know that the differentials can be directly related to s using the Laplace transform to have a form like this, where all this D operators can directly be replaced with this s to get a very similar algebraic operation like this. The advantage of this Laplace transform is that the ordinary differential equation that we have here with time as the independent variable gets converted to an algebraic equation with s as a primary variable.

And hence, we get this particular thing, where input capital X is the Laplace transform version of the small x input small x. And capital Y is the again the output represented in the frequency domain and in between we have the mathematical operator, which is being performed on X to get Y. This mathematical operator we often refer to as a transfer function. G is one of the common way of representing this, but there are several others ways also we represent this transfer function.

So, this concept of transfer function is a very important one in this topic of measurement system. Some of you may have already gone through some course of control systems, and their similar treatment we have already found. Actually, now if you have already done control systems, the entire content of this week you may be very much familiar with.

But, the idea of transfer function is to represent the entire mathematical operation performed by this measurement system in terms of a single operator, which is this transfer function. And once you know the transfer function of one instrument, then we can very easily relate the input and output by performing corresponding mathematical manipulations. And even more and even bigger advantage is that when we have multiple instruments are multiple components to deal with during measurement using this operators, we can easily connect different components to get an equivalent transfer function this.

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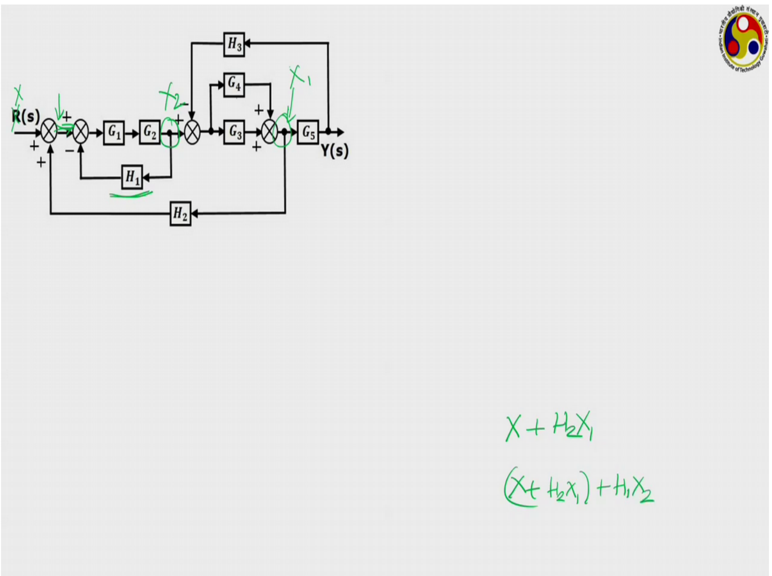

Let us see an example here. Here we have an example of a quiet complicated measurement system of where we are using series of operations. Here R refers to the input, Y refers to the output. Let us just instead of using R; let us just stick to our original notation X. So, X is the input, Y is the output. And here there are several operations being performed. Like, here we have several instruments having their transfer functions as G 1, G 2, G 4 etcetera. There are different components also you can see there are three H, H 1, H 2, and H 3 three components which refers to feedback.

I am sure you have some idea about feedback, which are primarily used for control operations. Like here whatever variable value that you have here that is being proceed to G 5 towards the component having transformation as G 5, but also that same signal is being routed through this particular feedback component to come get back to the original input position, which returns this which gives at this particular point instead of supplying the original signal X, if there is no feedback the original signal would have been X.

But, here as we are supplying something, then the original signal will be X plus H 2 times, if whatever we have here. Suppose, if at this particular point our variable value is X 1, then it will be H 2 times X 1. This is the signal that is coming to this particular portion through this line.

And then we are having another feedback. Here if the value of the signal at this particular point is X 2, then whatever being transferred here is X plus H 2 X 1, which was already coming plus this H 1 X 2, this is a signal that is reaching this particular component. This is the idea of feedback. And so using these components we can easily calculate the corresponding transformations.

Now, there are several components. If we count properly, there are we can see there are 5 plus 3 total eight components involved each of them having its own transfer function. So, somehow you have to combine them that combination process can be very easy. There are generally three kinds of possibilities we get.

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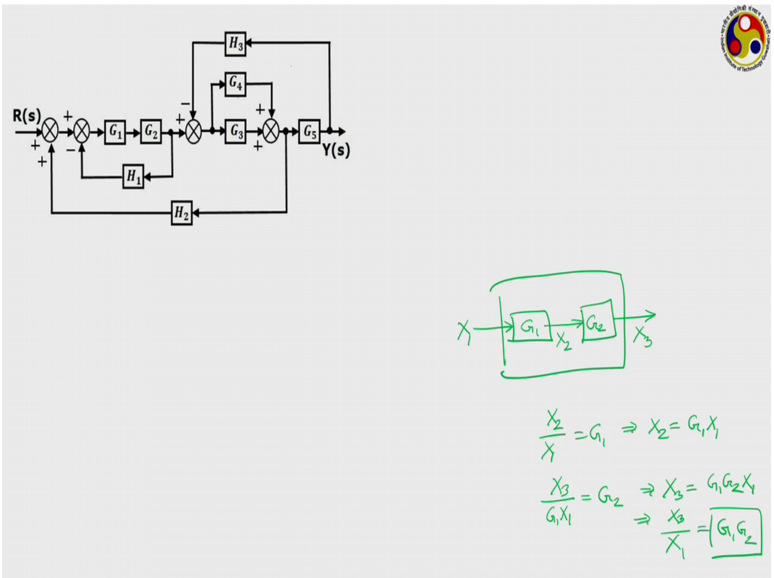

First possibility let me erase it make it a bit cleaner first possibility is where we have two components connect in series. Connect in series means, the output from the first component is directly fed to the second component. Like if the first one is having output G 1, second one is having sorry first one is having transfer function G 1, second one is having transfer function G 2.

And let us say X 1 is the signal that is being fed to G 1, its output is X 2. Then this output is directly fed as input to the component having transfer function of G 2, and giving a final output as X 3. Then how can you calculate using the notations that we have for the first component we know that output of an input if X 2 of a X 1 will be equal to G 1, which is giving X 2 is equal to G 1 X.

Now, look at the second component. Output from the second component is X 3, what is the input that you are giving that is G 1 into X, and that leads to our output as oh sorry G 1 X 1, so that leads to our output as X 3 output I repeat output from the second instrument is X 3. And its input is G 1 X 1 that will be equal to the corresponding transfer function, which is G 2 which gives X 3 is equal to G 1 G 2 X 1.

Hence X 3 upon X 1 is equal to G 1 G 2, which is the combine transfer function of these two components that means, when two or more components are connected in series their equivalent transfer function will be a product of their individual transfer function. We can see one such arrangement here in this case. So, this is possibility number-1.

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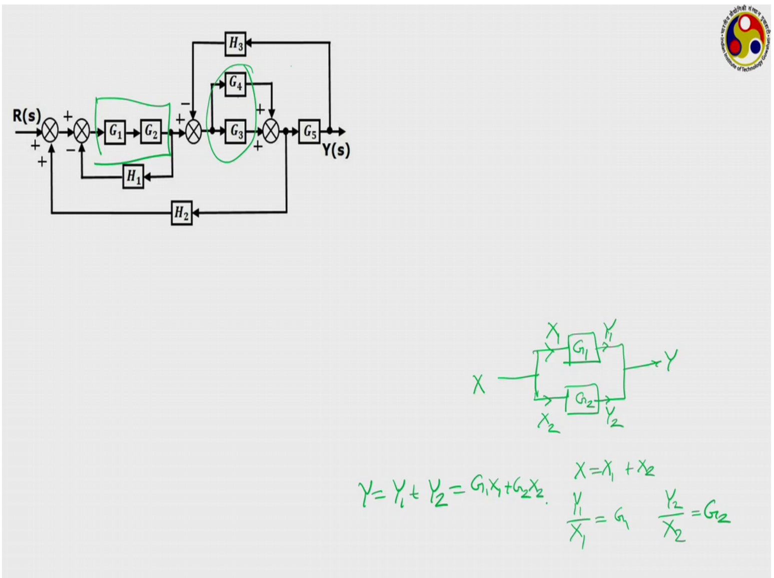

Let us move to the second possibility. Second possibility is when the components are connected in parallel, parallel means where they are not being subjected to same inputs rather something like here. Their both ends are connected to the same point. Something like see if X 1 X is the net output that net input that you are supplying A component of X, which is X 1 is going to the first component having transfer function G 1. And the remaining portion that is X 2 is going to the component having transfer function G 2.

So, from the first component, you are getting an output Y 1 from the second one you are getting an output Y 2, which is giving us the net output Y. Now, we can easily see that X will be equal to X 1 plus X 2. Now, what will be the relation between X 1 and Y 1 using the definition of transfer function, we know Y 1 upon X 1 will be equal to G 1. And Y 2 upon X 2 will be equal to G 2.

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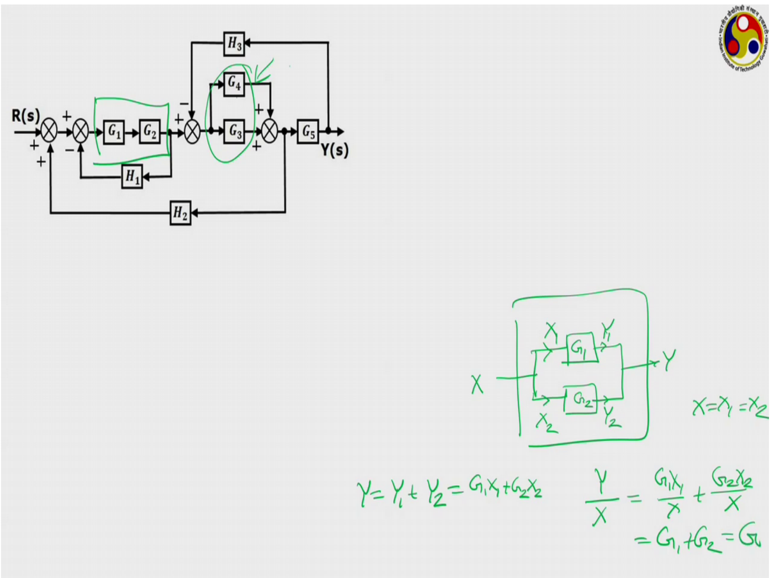

Hence, Y which is Y 1 plus Y 2 that will be equal to G 1 X 1 plus G 2 X 2. So, this is the net output that we are going to get and if our interest is to get the net transfer function or equivalent transfer function of this full assembly. Then we have to get a relationship between the net output Y by net output X, which will be of course G 1 X 1 upon X plus G 2 X 2 upon X.

Now, if once we are connecting them with a position like this, there essentially we have X is equal to X 1 is equal to X 2 that means, that the same signal is being directed to both the components. So, it leaves us with G 1 plus G 2 to be equal to the net equivalent transfer function that is G that means, when multiple components are connected in parallel then their equivalent transfer function will be the summation of their individual components.

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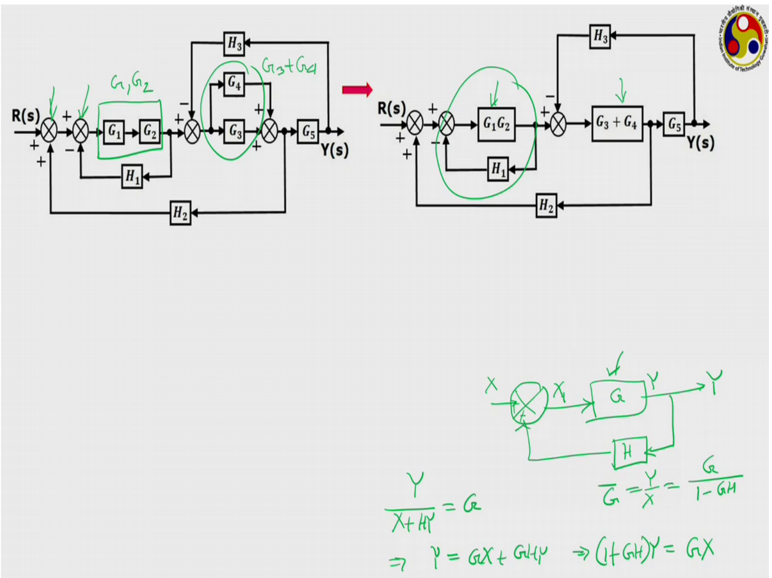

So, using this particular knowledge, we can clearly say that like say for this particular assembly our equivalent transfer function will be G 1 into G 2, whereas for this particular assembly our equivalent transfer function will be G 3 plus G 4, and that is what we have here G 1 into G 2 here replacing the G 1 and G 2 individual components and here again G 3 plus G 4 replacing the individual components.

Now, we have another situation here which represents or which involves one feedback. Now, the idea of feedback is that say let me clean it again idea of feedback is that let you have an input X being transfer to a component having transfer function G to get a net output Y, but you want to control the value of X itself. Then this output whatever you are getting this Y itself is routed back to a feedback controller generally denoted as H corresponding transfer function, and supply to this where there may be connected by suitable summation based components.

So, X is being supplied here, and also a modified portion of voice also being supplied here giving us this net. So, this one maybe now become X 1, and we are getting Y is the output from this. Then what will be your X 1, X 1 will be equal to X, which is the original input plus H times Y, then we are going back to the original component that is this one, but its output is Y. And what input it is sensing that is X 1, which is basically X plus H into Y that is giving us G. So, we are having Y plus sorry Y equal to GX plus GHY.

If we rearrange them, then 1 minus GH Y is equal to G into X, which leads us to a net transfer function. If you represent that as G bar, which will be equal to Y upon X as G upon 1 minus GH. Here of course we have assume that in this summation component, we have assume that both signals are getting added to each other.

But, that may not be the case like you can see in this particular one both are getting added to each other, but in this case they are getting subtracted from each other. If subtraction is there, then it will become 1 minus instead of 1 minus G I, it will become 1 plus G I. So, this way we can connect a feedback loop as well.

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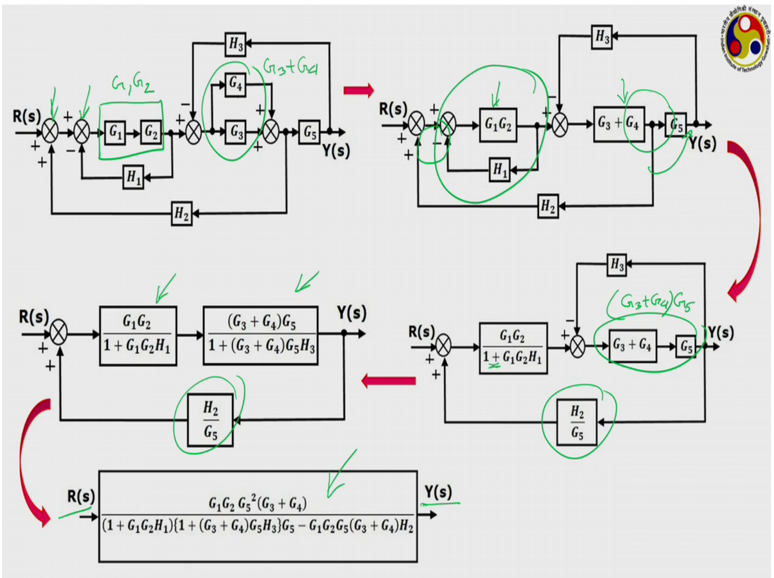

Let us see what we have here let me clear all litter from here. So, in this third step of modification, we have added the feedback components, look at what we have. Now, this G 1 G 2 combination has been added with H 1 to get G 1 G 2 by 1 plus G 1 G 2 H 1 look at where we have the minus sign here and which has led to this plus sign here.

And also we have made another transformation, this particular component has been shifted to this particular part to make it easy. I would leave to you about how to calculate this H 2 upon G 5 just same algebraic modification that we can do. And now this part is quite easy, this whole thing here we have G 3 plus G 4 and G 5, so the equivalance of this one will become G 3 plus G 4 into G 5, and that is a H 3 as a feedback component leading to this particular one.

And we already have this particular thing remaining, and this is a feedback from this which leads to this final transfer function. So, your input gets modified and to have this form output in terms of this function transfer function. And now if we know all the expression for each of this individual eight components, then we have the final transfer function.

So, the concept of transfer function mixes very easy to combine different components of a measurement system in a single transfer function. And once we have the single transfer function, we can delete with quite easily there are quite there are several standard mathematical procedure. So, today in the remaining part of this lecture, we shall be discussing about how to derived this transfer functions.

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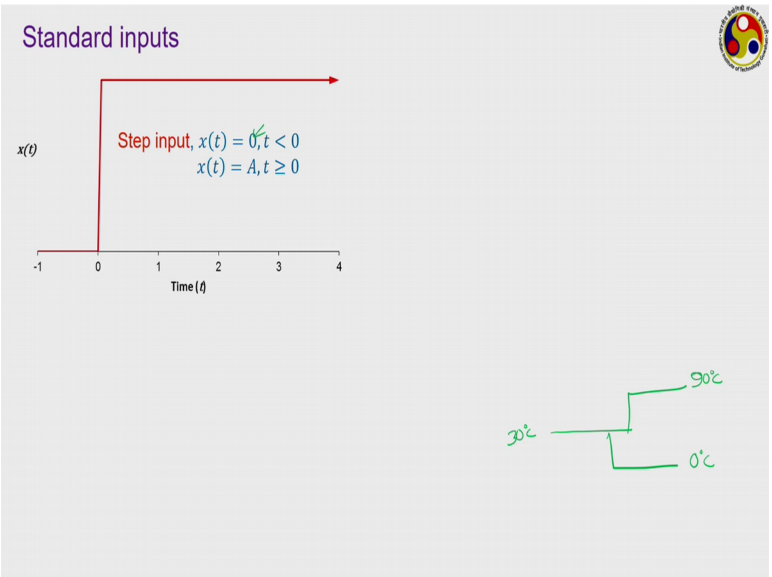

But, before that we have to talk a bit about the standard inputs also. Once you know the transfer function of a particular component, then theoretically it is possible to subject that instrument with infinite number of inputs, infinite types of inputs, but it is not possible to standardize the response for each of such possible inputs.

So, what is commonly done is we test the response of measurement system with known transfer function against some standard inputs. And then whenever you are dealing with an unknown or a different kind of input, we generally try to represent that as a combination of the standard input, like we have already seen in the previous lecture. Whenever you are having a periodic function to deal with, we can separate it into several harmonics component. And then the net response of the system should be the summation of the individual responses or the response to each of the individual harmonics.

Similarly, there are a few other types of also types of standard inputs also possible. Like this is the first one something like this we have already seen in the previous lecture. Here the input is 0 or constant in a particle particular instant of time. And then there is a sudden jump in the input, sudden change in the input to reach another constant value, and remain there for rest of the time. This is referred to as a step input.

So, it is x equal to 0 as per the diagram here till t less than 0, and for t greater equal to 0, it becomes a constant. Now, this constant can be positive or negative means it and also initial value this 0, it may have also some other value also. Like the example that we have discussed in the last lecture.

A thermometer was initially at a temperature of say 30 degree Celsius, and now suddenly you take it to an environment having temperature of 90 degree Celsius, then this is being subjected to step input. Instead of increasing the value, we can also subject it to suppose you take it in contact with a block of ice. Then it has again been subjected to a step input, but instead of being an increase in the value it is suffering a decrease in the value such inputs are called step inputs. Along with thermometer there can be several other examples also.

Let us say very arbitrary examples; you want to measure the mass of a pack of sand. So, initially your measuring platform is empty no load on this on this one, and now you suddenly drop this entire bag on top of this. So, its load was initially 0, and now suddenly it is being subjected to a static load a constant mass. If the mass of the bag is 50 kg, then suddenly the load on this one is increasing from 0 to 50 kg. And then that 50 kg is retained, so that what is referred as a step input.

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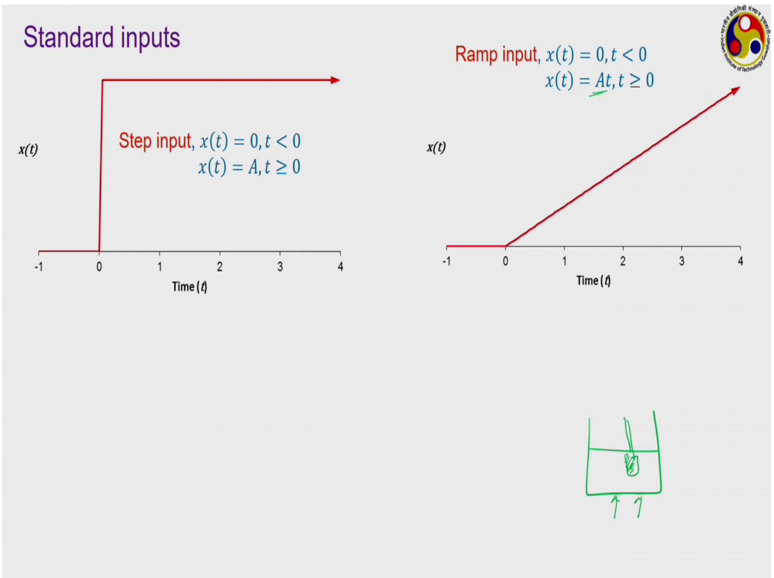

Next is the ramp input. Here it is the input signal is 0 till a certain time, and then it increases but I should say it changes, but it there is no drastic change rather its changes continuously which time following a linear path. This kind of inputs are referred as ramp input. So, it is input is equal to 0 initially, and from t equal to 0, t greater equal to 0 onwards, it is a linear function of time, where a is certain may be giving you the slope of this particular line.

So, the ramp version of the examples can be thought about say the example for the [team/ thermometer] thermometer] that we thought about. We initially you have a pool of water, and your thermometer is being dipped into this pool. This is the thermometer bulb dipped into this pool, both the thermometer bulb and the water at the same temperature.

Now, suddenly you take this assembly into a heater and start hitting the water at a constant rate. Then we can expect the temperature of water to increase also following a constant rate. And hence the thermometer will be sensing a continuous change units input value, input temperature value something resembling the ramp input.

If you talk about the pack of sand, instead of suddenly dropping the entire bag, we kept on adding sand on the measuring platform at a constant rate we just cut the cut open the bag. And then at a constant rate we keep on dropping the sand on this, so that is like a ramp input. Both step and ramp are quite common kind of inputs in practical application that we get, and hence it is very important to test the response of any measurement system against these two inputs.

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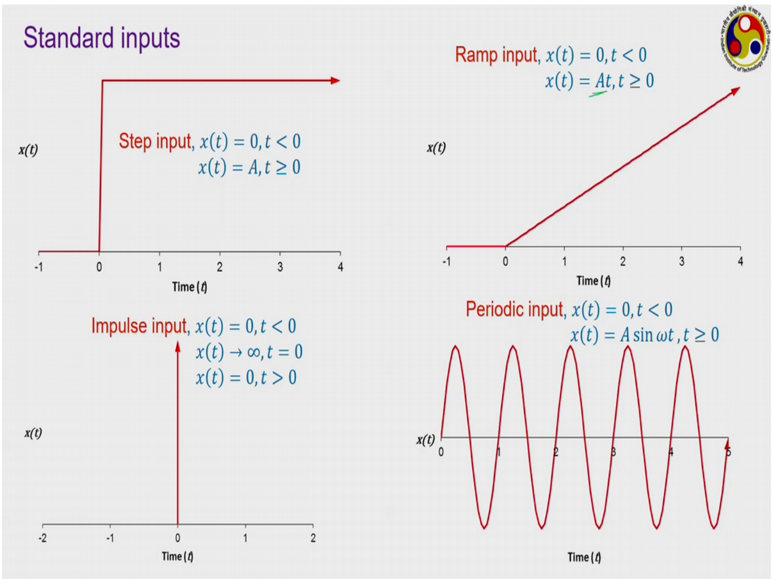

Another one we can have is called an impulse. Impulse means just think about that measuring platform again. Here we are not just directly drastically increasing the load by dropping a bag of sand or we are also not continuously adding the mass rather what we are doing, we are taking a hammer and striking the platform, just for a very very small fraction of time. Then what is happening for a very small duration of time? It is being subjected to a very large amount of load, before that instant there was no load after that instant there was no load, but only for a very small duration of time, it is being subjected to huge amount of load.

It is like so take the thermometer, now you in a somewhere you have a water kept at very high temperature, we just take the thermometer dip it into the pool of water for a fraction of second, and then immediately take it out that is something like an impulse input.

For time less than that instant, it was 0 for the next instant onwards again it is 0, but for a very small duration of time it will be infinite. Theoretically it is infinite, but there may be certain value for this. A more about impulse input, we shall be discussing in the next lecture, when we shall be subjecting systems to the impulse input.

And the other one is a standard periodic input, as we can represent any periodic function as combination of several sine waves. So, we generally test any measurement system against a standard sine wave, and test its response corresponding to the change in amplitude, and change in frequency omega, so that means, this A and omega both can both are generally valid to test the response of the system, and accordingly we can extrapolate it for any periodic function.

The response of a system against any standard input generally referred with the corresponding name. Like, when is when we X site one measurement system using a step input, we call is a step response. When we use a ramp input, we call it a ramp response, similarly you call it impulse response. And when you are subjecting this by a periodic function, we generally call it a frequency response; the term which was already introduced in the last lecture.

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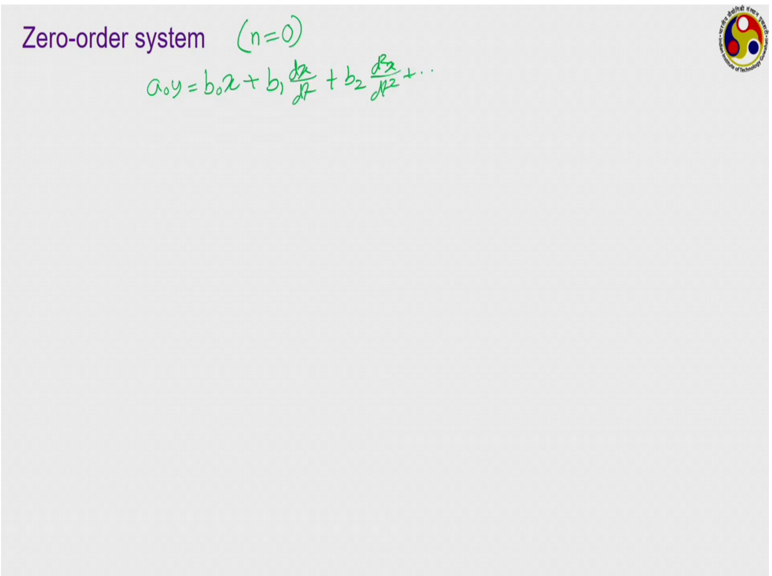

So, the first one in line is the zero-order system. Zero-order system is when the order of the system that is that n that we have referred is equal to 0, so what will be the form for this, it will be a naught y is equal to b naught x, here we are talking about n equal to 0. We are not putting any restriction on m.

So, instead of having b naught x, we can have several other terms also that is b 1 d x d t plus b 2 d 2 x d t 2, and go on like this. But, a from our practical experience you have seen that we never need to consider this higher order terms on for the input, it is just this b naught x itself is sufficient regardless of what type of system or what order of system you are dealing with. So, we shall be restricting our input side only to this b naught x component, but on the in output side we are taking a zero-order system that is you are putting n equal to 0.

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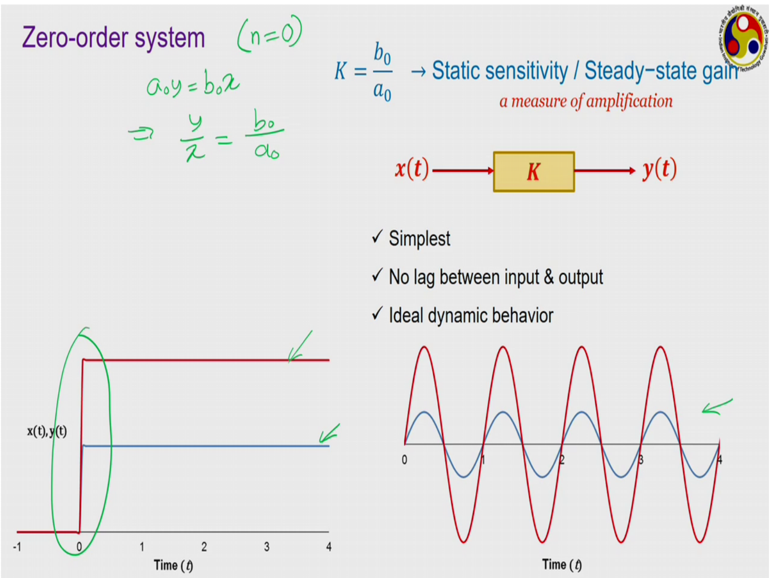

And once you have putting n equal to 0, then your transfer function that is y upon x will be equal to b naught upon a naught. This particular component b naught upon a naught is referred as a static sensitivity or steady state gain of an instrument. What it suggest or what it is looking like, it is just a this static sensitivity is just a ratio of output to input something very similar to the amplification or gain that we have talked about earlier that is the static sensitivity is nothing but an amplification in the output.

It is giving a ratio of input to sorry output to input, and if both x and y are time invariant, then what will happen the static sensitivity reduces to the sensitivity that we have defined in the previous week that is why, it is wrong this name is actually drawn from there. It represents the slope of a static calibration curve, and if the instrument is subjected to a steady-state input, then it will also give you the amplification this.

Zero-order system is therefore is characterized by the static sensitivity, which is a measure of the amplification, it is the most ideal system that we can have because, here our transfer function is just a single K just a one para meter K none of the differential operator appearing.

And so once you know the value of this K, the static sensitivity we can easily correlate the output and input. And it is the simplest possible measurement system we can that we can have, there is no lag no time lag between the input and output means whatever input you provide, you will be instantly getting the output. And the value of input and output will be differing by this static sensitivity K. It is the ideal dynamic behavior that we expect from any measurement system.

Like, suppose if we are subjecting the instrument to step input, this blue line represents the input, then depending on the value of the K, it is instantaneously going to give you the output given by this red line. Depending upon the value of K the output, magnitude can be higher than the input magnitude or can be less than this or K equal to 1, both the red and blue line will superimpose on each other.

But, there is no time lag that is you can see here as soon as you provide the input your system is able to respond, it is immediately giving the output value which is what we want means, we do not have to wait to get any measurement, we have get the measure value immediately. And by varying the value of this K, we can also amplify the signal means we can increase it in magnitude or we can even reduce in magnitude also to whatever range that we want.

Even if they even when the, it is subjected to some kind of periodic signal, just look at this. Here as the output the red one again it is showing. The output in the output we are again getting another sine wave having the same frequency, but maybe a different amplitude depending upon the static sensitivity of your instrument.

So, zero-order system is the most ideal system that we want, which we say which is able to provide instantaneous response to your system like look at this case. Here it is there is no phase lag, if you are dealing with zero-order system in there are suppose several three harmonics present in the input signal, it will not impose any artificial phase lag be the harmonics that you are going to get in the output, so that is the most ideal behavior that we can expect.

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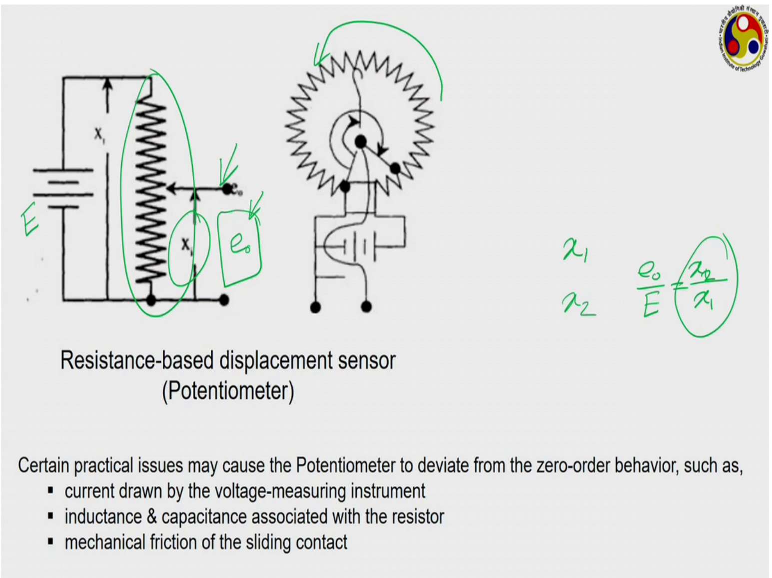

Something that very closely resembles the input is the resistance based displacement sensor called the potentiometer. In the concerned model, we shall be discussing a bit more about potentiometer. But, the principle of potentiometer is that here you have a resistance like this which is being subjected to which is being excited by a constant voltage source kept here.

Let us say the height of the resistance is this X i or say x 1 is the original height of the resistance. And your the displacement that you want to measure or the displacement of the device following of the system which you want to measure that is being connected with this particular needle or and this needle as the system is moving, this needle keeps on moving up and down along this register.

So, as the system in moving up and down accordingly the voltage available at this side that also will keep on varying and suppose at if the total resistance, if resistance we take as proportional to the length of the resistor, then x 1 is the original length. And at a particular instant of time your indicator is connecting this x 2.

Then if E is a voltage that is being imposed, and small e refers to the voltage or small e naught this refers to the voltage that your indicator is showing what will be the ratio, it will be the output is e naught input is capital E that will be a straight forward relation between these x 2 and x 1 that is a straight line relation, it is a perfect zero-order instrument that we can get. Instead of being such a linear scale sometimes, we can also use an angular scale to measure the angular displacement, but regarding potentiometer assembly talking about a bit more.

Another system which another very common system which you already know about which is also quite can be it thought about is a zero-order system, can be a spring gauge or a spring balance I should say spring balance, which is generally used for measuring the weight of something.

As the system is as the system or the measurand is connected to the hook of the spring gauge, immediately you will find a deflection of the spring indicating which will cause the deflection of the indicator on a scale which will move on a scale, and you get that measured weight; weight of the measurand directly from the scale that is also quite close resembles of a zero-order system.

But, practically there may be several issues. For each such examples, which may limit the zero-order behavior. Like in case of potentiometer, there can you several issues we can force it deviate. One can be the here to measure this e naught, we have to connect some voltage measuring instrument maybe a voltmeter, maybe an oscilloscope, which is going to show the output.

Now, that itself requires some current for its own operation. And therefore, it is going to make a some change in the value of this e naught itself, thereby causing some distortion in the or some change in the final output. And then secondly the resistance that you are talking about here, this resistance you are representing by the in terms of its length, but that is true only for a pure resistor. And there is nothing like a pure resistor in nature. Every resistor will always have whatever small may be some inductance, and capacitance associated with this. And those inductance and capacitance will lead to some amount of voltage leakage from this.

Another very common reason the indicator or this particular indicator, which is moving over the resistor that itself may suffer from mechanical friction that we have some quanta friction associated with it, may have some inertia associated with it means once the system suffers some displacement the indicator itself may take some time to move over the indicator, move over the resistor rather, and so those lags those mechanical lags, mechanical friction will also cause some change or some distortion in the final value.

And all such reasons may force the potentiometer or any such zero-order instrument to deviate practically from the ideal zero-order behavior, but still zero-order instrument is the most preferable one for any kind of measurement. Unfortunately, we have very few examples for zero-order instrument, but there are several examples of a first-order instrument.

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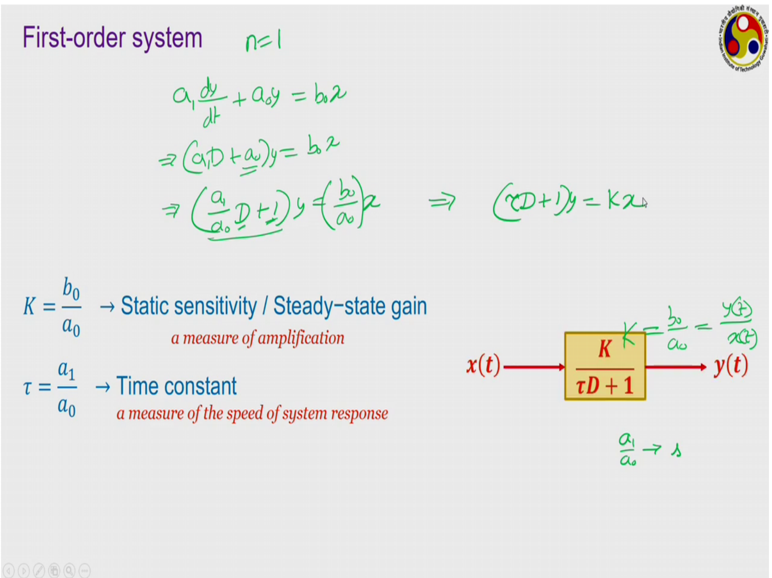

So, for a first-order instrument now, we have n equal to 1. So, if we go back to our original equation what we have, our equation now is a 1 d y d t plus a naught y is equal to b naught x, remember we are not taking any term after the first one on the right hand side. So, going back to our differential representation, we can write it is a 1 D plus a naught into y is equal to b naught x. So, we have three coefficients to deal with a 1, a naught, and b naught, all other coefficient are zero for first order system.

Truly speaking we do not need three coefficients. We can always define divide the other two by any one of the coefficients, thereby reducing the required number of parameters to just two. We can either divide everything by a 1 or a naught, but just to be consistent with what you have done for zero-order system. Let us divide everything by the a naught, we are dividing everything by this a naught.

So, if we divide everything by a naught, we have a 1 upon a naught D plus 1 y is equal to b naught upon a naught into x. What is b naught upon a naught, we already seen for the zeroth-order system that is the static sensitivity or steady state gain, but we are having a new term here. This a 1 upon a naught, a 1 upon a naught what do you feel, what will be the dimension for this quantity ok. Before, that what is the dimension of the static sensitivity that is this K static sensitivity on this if we just go think about the zero-order instrument itself, there K which is b naught upon a naught is directly your y t upon x t.

So, the unit of K will be your unit of the output by unit of input. Like if you think about say one thermometer liquid in glass thermometer, there your output is the displacement of mercury inside the column. So, y will be having a unit of meter or some corresponding length scale. And x is the input which is the temperature change in temperature, so it will be say degree Celsius or degree Kelvin. So, output to input whatever is the unit the same will be the unit for this K.

But, what about this quantity a 1 upon a naught, look at this particular equation carefully. a 1 upon a naught whatever may be the nature of your x and y, you will always find this ok. Once more you just take a look here, you have one involved here, which is dimensionless quantity. And you have a differential operator involved here for this.

So, if you look at carefully your this a 1 upon a naught will always have a dimension of time, you shall be seeing one example may be next slide that is why, it is often referred to as a time constant tau which you gives a measure of the speed of the system response.

And accordingly, we can represent this first order system as tau D plus 1 y is equal to K into x or this is the corresponding form in transfer function K upon tau D plus 1 is the transfer function for a first order system, it is different from the zeroth-order, because here we have been a denominator involved, and we cannot have just a straight one to one correspondence between output into output and input. Rather we have one differential operator involved in this D.

And also like in zeroth-order system, there is only one cauterizing parameter in form of static sensitivity; here we have another one to deal with which is this time constant. So, time constant I repeat is a measure of the speed of system response, and it is always desirable ok. Let us wait to see whether we want a larger value of tau or a smaller value of tau, we shall be seeing through some examples.

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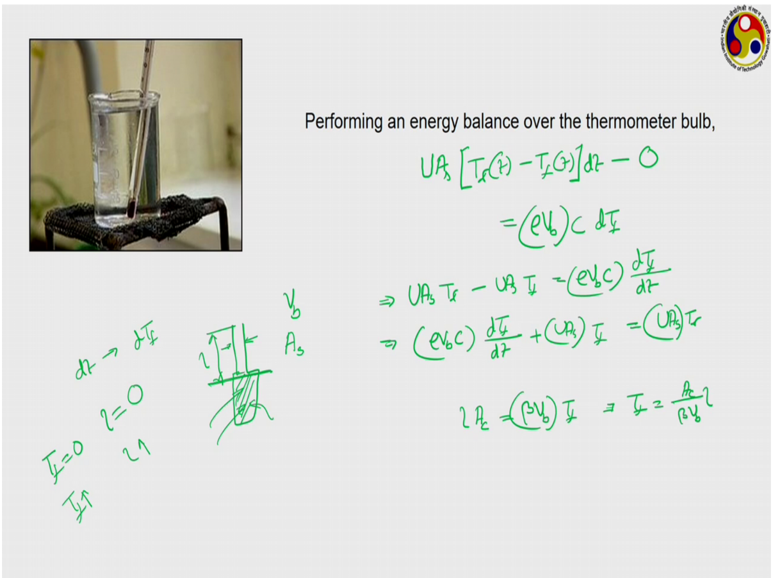

Now, example for first-order system, there are several most common one is a liquid in glass thermometer. There can be other examples like one capacitor discharging current through a resistor, there can be one tank of water leaking through a wall, these are all very common examples of first-order system. We shall be seeing a few more examples later on, let us just concentrate on this one.

You have or we have a common thermometer which is being dipped into a pool of water. Let us perform an energy balance over the bulb of the thermometer. Now, if we think about the thermometer bulb, say this is your thermometer bulb maybe a very bad drawing of that, so this is the bulb the hashed portion, what is bulb maybe having a volume V b, and it is having a surface area of A s.

So, bulb is having a volume V b, and a surface area of A s, then what it is doing if whatever may be the initial temperature, if that is different from the temperature of the water surrounding water, then it will experience some heat exchange. So, let us assume that the surrounding water surrounding fluid is at a higher temperature. Then heat will be flowing into this, what is the mode of heat transfer here, correct it is convective heat transfer.

And the result of this heat transfer will be a change in the energy content of the fluid the thermometric fluid here. So, using this let us write an energy balance for this thermometer bulb. So, how much energy that is being supplied in the form of convection that will be U, which is the overall heat transfer coefficient into the surface area, which is this A s into the T of the surrounding fluid, let us say T infinity refers to the surrounding fluid minus T f refers to the thermometric fluid. Both can be function of time, of course T infinity itself can be a function of time, and the fluid temperature T f definitely is a function of time.

Let us neglect any heat loss from this. So, this the amount of heat addition, there is no loss. This entire quantity will lead to a change in the energy content of the system. Now, how we can calculate the energy content of this or how much is the energy change in this energy content. Let us perform this energy balance over a small interval d t, then this is the total amount of energy that has been received by a thermometer bulb minus there is no loss.

And if this over this small period of time d t, let us say there is d T f amount of change in the temperature of the thermometric fluid may be mercury. Then how much will be the change in total energy content that will be the mass of the thermometric fluid, which is rho into V b into specific heat into the change in its temperature this here C is the specific heat, rho is the density both corresponds to the thermometric fluid, V b as mentioned.

And so we simplify this, so we have U A s T infinity minus U A s T f is equal to rho V b C to d T f d t or write in more formal way rho V b C d T f d t plus U A s of T f is equal to U A s t infinity remember U A t infinity itself can be a function of time. So, how this equation is looking like very similar to your first-order equation.

Like here this one is your a 1, this one is a naught, and this one is your b naught. T infinity it represents the x, T f represents the y, and time is a variable for this, but think about the thermometer itself. Our output is it coming in terms of T f, because actual system output is coming only in terms of expansion of the fluid through the capillary tube.

So, we have to somehow convert this T f to the expansion of the field in the capillary tube. Let us assume that when T f was equal to 0, all those mercury or all the fluid was restricted inside the bulb. And so there was the length l of fluid column in the capillary, it was 0. As T f keeps on increasing l also keeps on increasing.

Then how can we relate this T f with l, if of course the length or let us writes at a certain instant of time. If l is the length of this thermometric fluid column in the mercury, what is the volume of that it volume is definitely l into cross section area of the capillary here A c refers to the cross section area of this particular capillary.

So, l into A c is the total volume inside the total volume of mercury that is there inside the capillary, and from where this is coming this is coming because of the thermometric volumetric expansion of mercury which was there inside the bulb originally. So, if beta refers to the volumetric expansion coefficient, then beta into the original volume V b into T f that gives a relationship between l and T f or we can write T f to be equal to A c upon beta V b into l.

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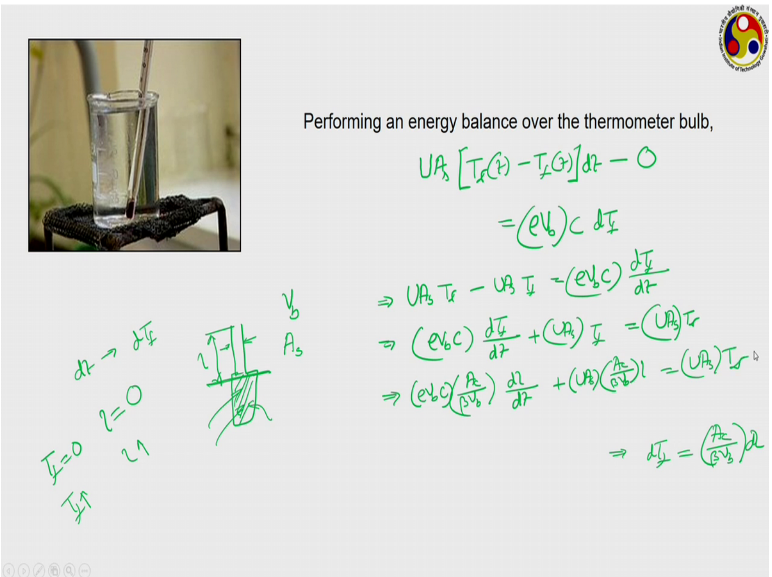

So, let us replace this one in the earlier equation. So, if we go back to the earlier equation, then we have row V b C into from here we can also write that d T f will be equal to A c upon beta V b into d l. So, putting it there we have A c upon beta V b into d l d t plus original equation at U A s into AC upon beta V b into l of course is equal to U A s T infinity.

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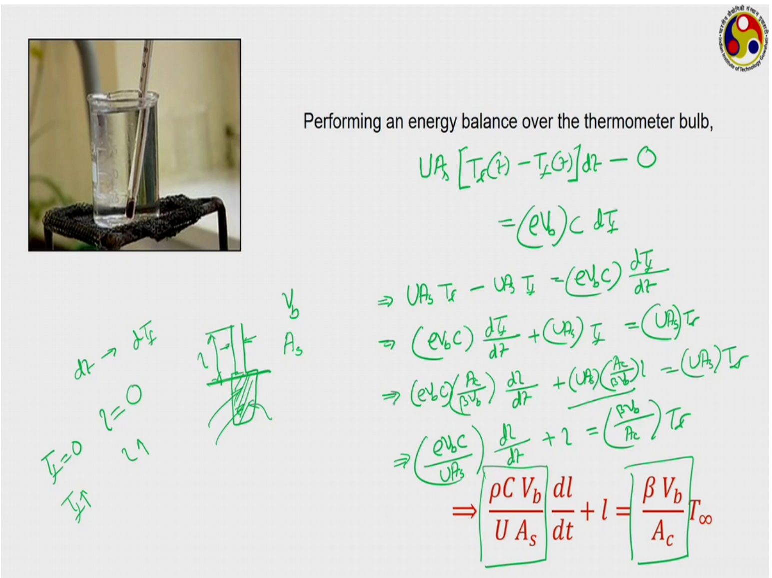

So, the original equation we have now converted in terms of T f, it was there now you have converted that to the terms in terms of l. And l is the output, and T infinity is the input. And now if you divide everything by your a naught that is this quantity, then what you are going to get? You are going to get rho V b C divided by U A s that is going to be equal to d l d t plus l will be equal to beta V b upon A c terms of T infinity.

So, what we have? This is the final equation just written in a much clear fashion. So, this one what is this one beta V b by A c, this is a static sensitivity of steady state gain. And this row C V b upon U A s, it is what? It is the time constant for a system. Of course, wild so we have a situation of a first-order system where you already got the expression for both static sensitivity and the time constant.

And once you know this parameters, like you can just look at this V b A s and A c are geometric parameters. V b refers to the volume of the bulb, A s it is the surface area of the bulb, and A c is the cross section area of the capillary tube. So, once you are having a thermometer, you have all this information known.

Row C and beta are properties of the fluid that you are using mercury or whatever. So, those are also generally well known. And only problem is with this U, U is the overall heat transfer coefficient the U may depend on the surrounding or general depends on the surrounding like if the thermometer is dipped into stagnant full of liquid, whatever will be your heat transfer coefficient.

If it is dipped into a flowing pool filled of liquid, then we can expect U to be much higher. And also if it is dipped into institute of liquid, we keep it in air, then U will be much lower. So, U is not truly a system parameter, we need some information about the surrounding as well to get a measure of U.

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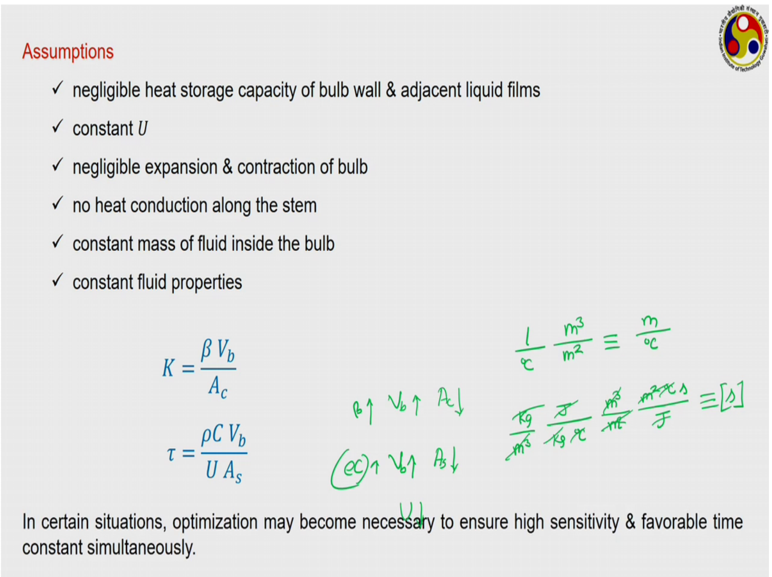

But, like in case of zeroth-order system, there are also several factors which may cause the first order system to deviate from its first order behaviour. Like there are several assumptions, we have already taken in this derivation. First we have neglected any heat storage capacity of the wall material like the glass. We are assume that, that is not storing any heat, and also there may be a small film of liquid out on the outer side of this, and also small film of thermometric liquid on the inner side of the bulb. We are neglecting the heat storage capacity of those.

We have assumed U to be constant, but truly speaking U itself may be a function of temperature, because properties of the fluid both thermometric fluid, and the exterior fluid both can vary with temperature. We have neglected the expansion and contraction of the bulb that is not a bad assumption truly speaking. Like glass kind of material has very small expansion coefficient, and generally are negligible, as long as the temperature change is limited.

We have neglected any heat conduction along the stem to the upper side of the capillary tube. Again a if you choose a material with very low thermal conductivity, this is not a bad assumption. Constant mass of fluid inside the bulb that we have assumed, whenever there is an expansion taking place some fluid is moving out of the bulb during contraction some fluid moving back into the bulb. But if the diameter of the capillary is very small, then the amount of mass that goes out of the bulb that can be very small almost negligible and finally, we have assume the fluid properties to be constant.

So, with this we have got this two expressions for K and tau, just look at this what will be the unit place for this K and tau. We do this unit calculation from the right hand side, beta is a volumetric expansion coefficient what is its unit, do you remember, it is rate of change of volume per unit volume per unit temperature.

So, its unit is 1 upon degree Celsius, where we are sticking ourselves to Celsius as the unit for temperature. V b is volume, so it is meter cube A c is area, so meter square giving the unit of this quantity to be meter per degree Celsius. Like, we have discussed earlier meter is the unit for your actual output which is length degree Celsius is the unit for your actual input, which is the temperature of the exterior fluid.

Come to the time constant. Here you have row, so row is SI unit is kg upon meter cube, C is specific heat. So, it can be taken as joule per kg degree Celsius, then you have V b upon A s V b is meter cube upon meter. And U volumetric expansion coefficient watt per sorry user overall heat transfer coefficient watt per meter square degree Celsius. As we have already having a joule left, so instead of watt we write it as joule per unit time. Now look at this, so this meter cube goes of sorry here we have meter square this corresponds to the A s, so the meter square goes off degree Celsius joule, and also kg goes off leaving only this second, which is the time constant for this.

And also another thing we can look for if you want to use increase the sensitivity of your thermometer, what we have to do? Firstly, we have to choose an instrument, which is sorry choose a fluid which is having high beta, and we also have to design it such that this ratio V b upon A c ok, V b upon A c will come not into picture.

We have to ensure that the volume of the bulb is high, and the cross section area of the capillary is very small. Similarly, if we think about time constant, if you want high time constant rho C should be high rho C product, again V b should be high, and surface area should be low, whereas heat transfer coefficient again it should be as low as possible.

So, a high value of V b gives us high value of K, and also high value of tau. High sensitive is always desirable, but for time constant we do not know yet. Let us check it with respect to one standard input. In certain situations, however the effect or the parameters tau with effect K, and tau they may be contrary to each other. So, you may have to go for certain kind of optimization to identify both high sensitivity and favorable time constant.

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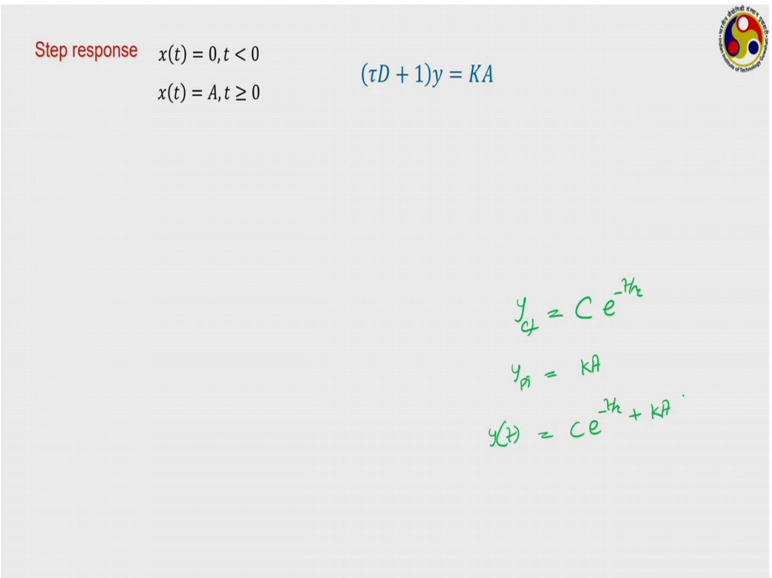

So, first you check the step response, we know this is the step response. So, how to do it? We know for a first-order system, this is the way this is the standard form here K is the static sensitivity, and A is the input that you are giving for any value of tau or sorry any value of time greater than 0. So, we have to solve it.

Any ordinary differential equation of this kind where we can find the solution in two parts one is called a complementary function, other is a particular integral. So, if I am just directly giving to the solution, if you solve it for in this particular case the complimentary function for y will be some constant C into e to the power minus T upon tau, whereas the particular integral in this case will be the right hand side itself the KA. So, the actual solution for y will be a summation of this two C into e to the power minus t upon tau plus KA, but C is an arbitrary constant.

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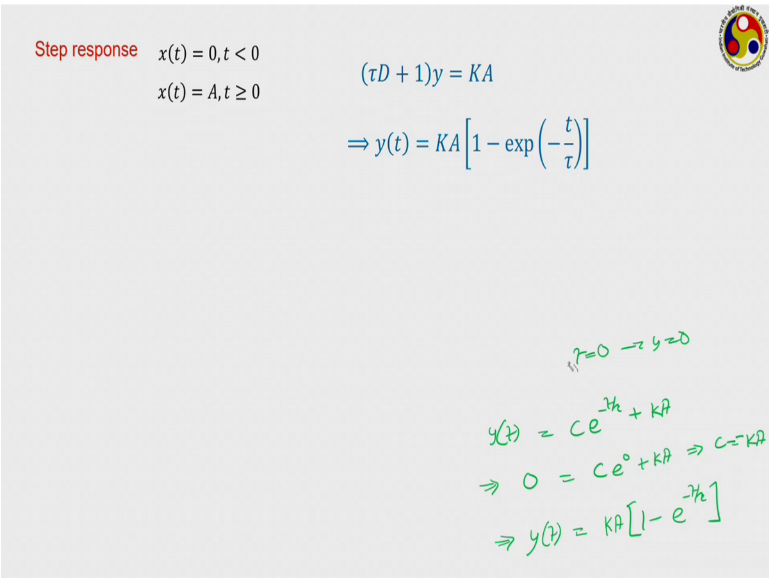

So, we have to get the value from our boundary conditions or I should say initial conditions, what condition we know. We know that t equal to 0 your y is equal to 0. So, if you put this value 0 will give you C into e to the power 0 plus KA leaving to us C is equal to minus of KA.

So, we put it together final expression for y t comes to be equal to KA 1 minus e to the power minus t upon tau, which is this where K and tau are the properties of your measurement system, A is the amplitude of the step input that you have provided, and t is the time variable which changes from 0 and can go to infinity.

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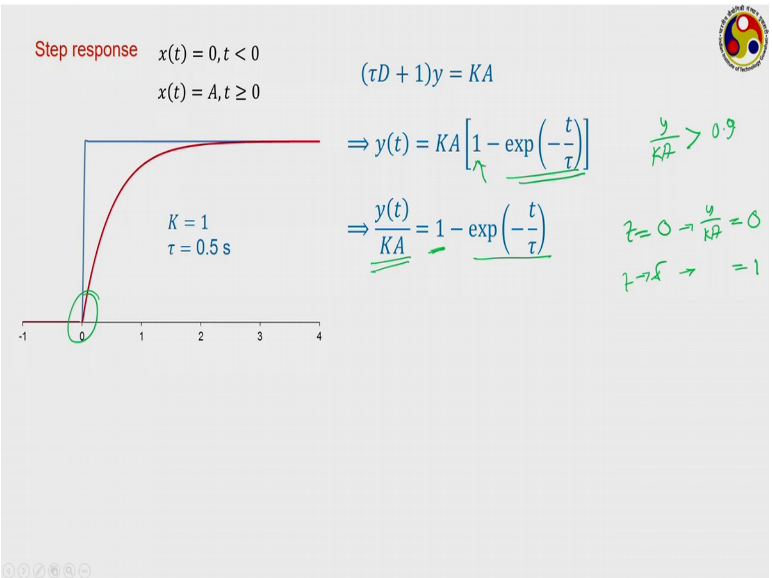

You can clearly see, there are two parts of this solution. One is this 1 which is a constant value, and other is this particular quantity. We can explain it bit more, if you represent the solution in this form. Here we are dividing the output by K into A thereby making it non-dimensional. Look at this here one this term is 1 which represent the constant, and what about this term this is an exponential term which an exponential term negative exponential term.

So, when t equal to 0, this term goes to one giving leading y equal to or I should not say y giving the left hand side that is y upon k into a to be equal to 0. But, when t tends to infinity, then what happens then this exponential term goes to 0 leaving this to be equal to 1 that means, this actually is a measure of the error that can be present in the system.

As we move with time, this exponential term this error that keeps on reducing, and for a very long duration of time this goes to 0, thereby giving ideal response from your measurement system. This is one example where we have taken K to be equal to 1, and tau equal 0.5 second.

Just see here the blue line is the input your step input, the red one is the output that you are getting. You can see like in zeroth-order system, we are getting an immediate response which we are not getting here. Of course, the system start to respond from this point that itself, but it will take some time to reach the actual signal.

Theoretically, it will require infinite amount of time to reach the actual signal, and so it is rise time. I hope you remember the rise time, rise time refers to the time the measurement system reaches to match the input value. For this y upon KA quantity the time it requires to reach this value of 1, theoretical it is infinite.

But, practically instead of rise time we use the settling time. Setting time means, the time the measurement system requires for this output signal to reach within certain band say within 5 percent or 10 percent of this value; like if we take a band of say 90 percent, then we are taking about this y upon KA value to be greater than 0.9. Once this y upon KA value becomes greater than 0.9, we can see that the output has reach with in a within 10 percent of your desirable value.

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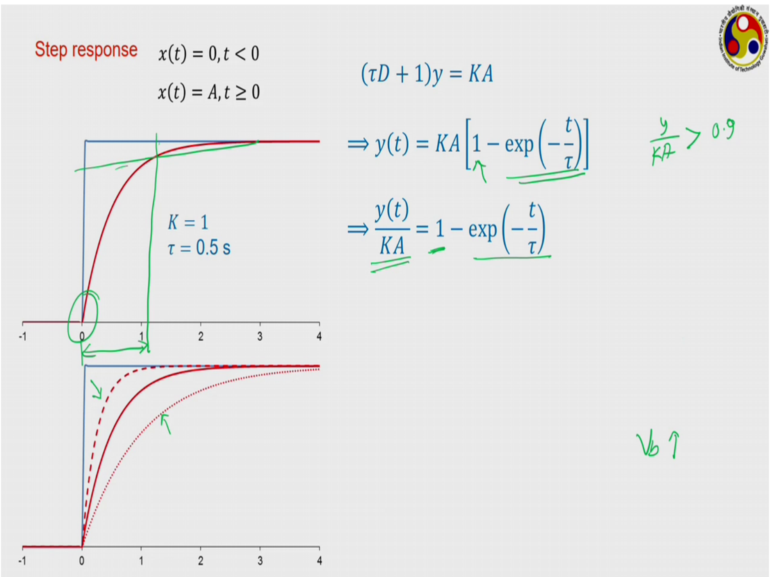

Let us see this is the time if this is the range, then we can see this is the point where your system enters that band. So, this particular period can be referred as your settling time. But, let us now check what can be the effect of your time constant. Here we have compared three different time constant. The continuous red line is the same, when this particular line refers to tau equal to 0.25, and this particular line refers to tau equal to 1, what we can see there.

As the time constant is increasing, system requires more time to reach the actual output or the settling time keeps on increasing that is why, it is always better to have system with smaller time constant that is logical also because, time constant gives you the speed of system response. Smaller the value of time constant higher is the speed of system response, so we always prefer system to have been having a smaller time constant.

Now, if you just refer back to the example of earlier we have used, there we have seen that when the V b increases, sensitivity increases, but time constant that also increases. Therefore, we may have to go for some kind of trade off, because higher volume of this bulb will give you higher sensitivity, but at the expense of so lesser time constant. And time constant is very precious, because we cannot wait infinite amount of time to get your output from the system.

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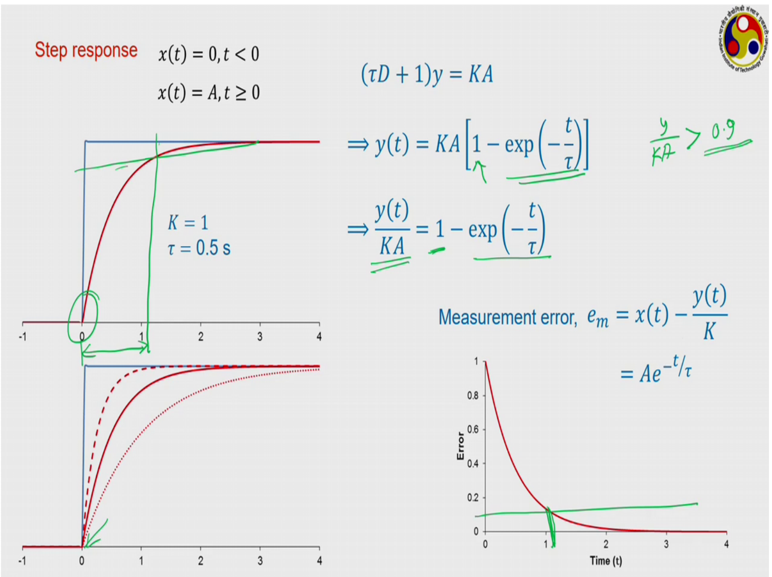

The measurement error is often referred as the input minus this y upon K quantity. So, in this case it will be A into e t upon tau. And if we plot that with time, you can see that initially your measurement error is 1 which is referring to this particular point. And as the time goes on the error keeps on reducing.

So, again if we are sticking to a 9 percent sorry 90 percent band, then we can allow a maximum of 10 percent error measurement. Then something here somewhere sorry somewhere if 0.1 value is here, where the system enters that band. And so from this point onwards, we can take the reading.

Like the just think about how is the clinical thermometer, we take it and put it in contact with our body, but we do not take the reading immediately, rather we wait for some time.

And we take the reading after say one and half minutes or two minutes, because it has a certain time constant it requires certain time to enter the available error limit or for this error to become smaller than the available value and then only we take the limit. Higher time constant is not desirable, we always prefer an instrument with is smaller time constant.

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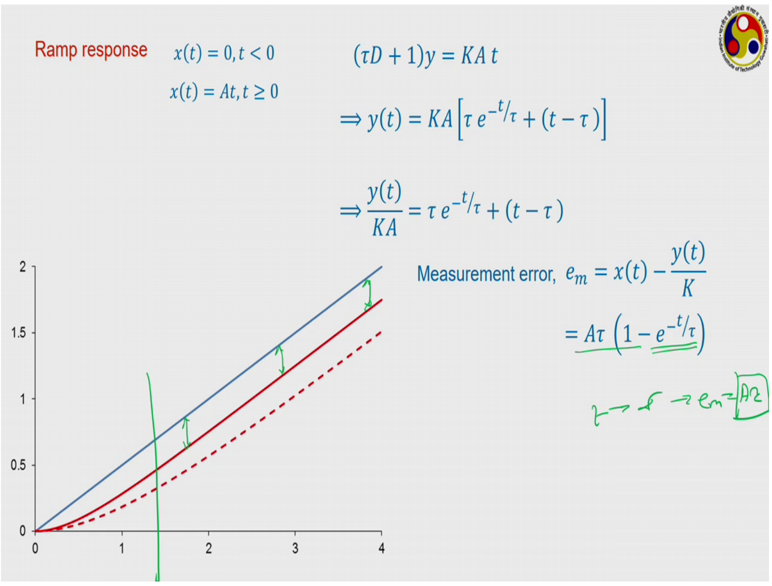

We shall be checking other response today that is the ramp response we know this is the form of the ramp. So, we this is the corresponding function of functional form. And if you solve this on following again the complementary function and particular integral, we are going to get a solution of this particular form where we again have two parts. Both cases the tau is involved.

And take out the measurement error then, we having a form like this now look carefully. Here in the error in the previous case, we had only one part which is an exponential term A e to the power minus t upon tau with step input, but here we are having 1 minus that. This term of course as time goes on this 1 goes to 0, but what you are left with.

When this tau sorry when your time t tends to infinity, your measurement error will become A upon tau that means, it attains a constant value that means, in case of ramp response your system will always have some constant error, which is often referred as a steady state error. And it will never be able to reach the actual input value. As the input keeps on increasing, the output will keep on following that, but with a constant gap just just follow this.

Here again the blue line is the input, the red one continuous one is the output. So, in the initial period over some period somewhat like this the nature of the output also keeps on changing, but once it reaches this, you can see here there is a consistent gap that is maintained between input and output, which is this a tau quantity this steady state error.

The system will never be able to recover this error, rather you will always have as long as the input keeps on increasing, output will keep on falling it, but with this amount of error. The dotted line is a something with a different time constant, again a different time constant higher that smaller the time constant value lesser is the amount of error in this case.

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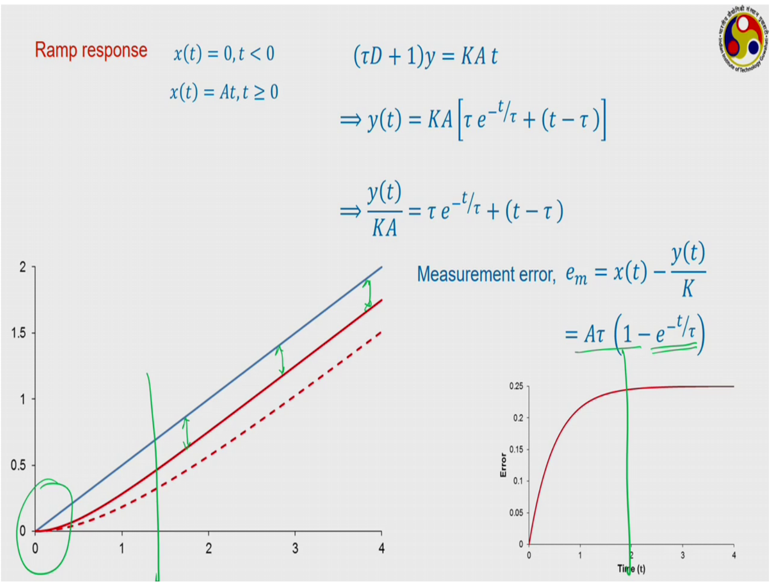

So, this is the corresponding error, you can see the error initially is 0 here. And then it keeps on increasing, and then becomes constant beyond a certain time for this. So, this is a step and ramp response, we shall be talking a bit more on them in the next lecture, and we shall also be checking out the response of a first-order system in terms of impulse and frequency response.

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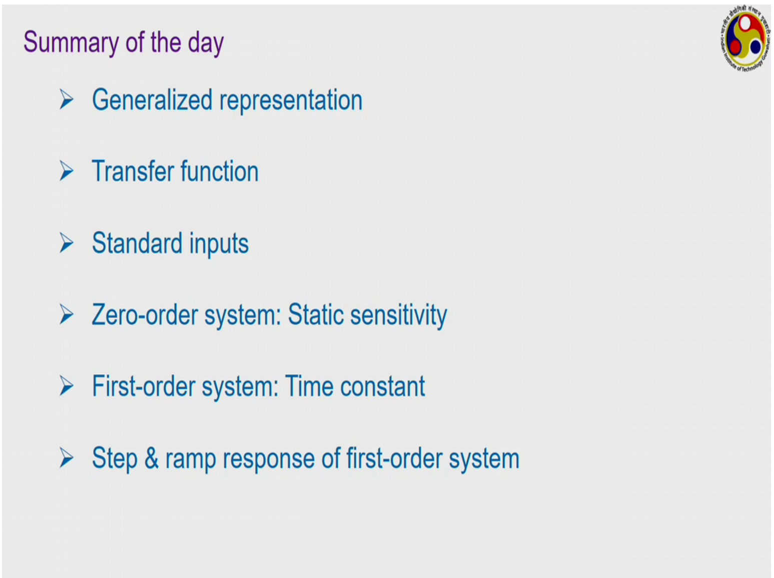

And then we shall be moving onto the second-order system. So, just to sum up what we have learned today. We have talked about the generalized representation, we have developed generalized mathematical model, then transfer functions was introduced, and we have also seen how we can combine transfer function.

And then we have talked about different standard inputs step, ramp, impulse, and frequency input or periodic input. Then we have talked about the zeroth-order sensitivity, which gives the concept of static sensitivity zeroth-order system which gives the static sensitivity of the cauterizing parameter. Then the first-order system introduced the time constant. And then we have discussed about the step and ramp response of first-order system.

So, in the next lecture which I am expecting to be the last in this week, we shall be discussing a few more example of first-order system, response of first-order system against impulse and periodic inputs, may be solving a few numerals, and then we shall be talking about second-order system. Generally, we do not have to go beyond second-order system, because all common measurement system can be classified either as zero, first or second order system.

So, that is it for the day. Thanks for your attention. We shall be back soon with the next lecture of this module.

Thank you.