Principles of Mechanical Measurement

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Module – 02
Lecture – 05
System response to periodic inputs

Hello friends. Welcome back to our MOOC's course on Principles of Mechanical Measurements. I hope the first week has gone well. What do you feel about this one, I do not have any option to know that directly, but please keep on sending your feedbacks. It is only through the mails or the messages that you are sending on the portal, I am able to get the idea means how you are getting the lecture, what are you are feeling about the lecture.

Do you feel it was heavy a bit? because there are 4 lectures. And it is very unlikely that any of the other weeks will be having four lectures. Most of the weeks will be having three lectures, one or two may be having just two lectures. And in some of the weeks, may be just in one or two more where we may have to go for a 4th-lecture, just to have more time on the topic. The topic that we are going to discuss in this week, there we are not having any kind of plan to go for the 4-th lecture, I am hoping to finish this one in three lecture itself.

So, in this week number-2 our topic of discussion is the Response of Measurement Systems. As we have already discussed about quite a few fundamental concepts about the measurement system, like the basic schematic design, there are three different stages like input stage, console conditioning stage, and output stage.

We have briefly discussed about different types of input signals possible, we have discussed about how to process the output using statistical means to identify the random errors from there. We have talked about the importance of calibration to eradicate or minimize the presence of systematic errors from the output etcetera. But, now we are going to talk as about the system as a whole.

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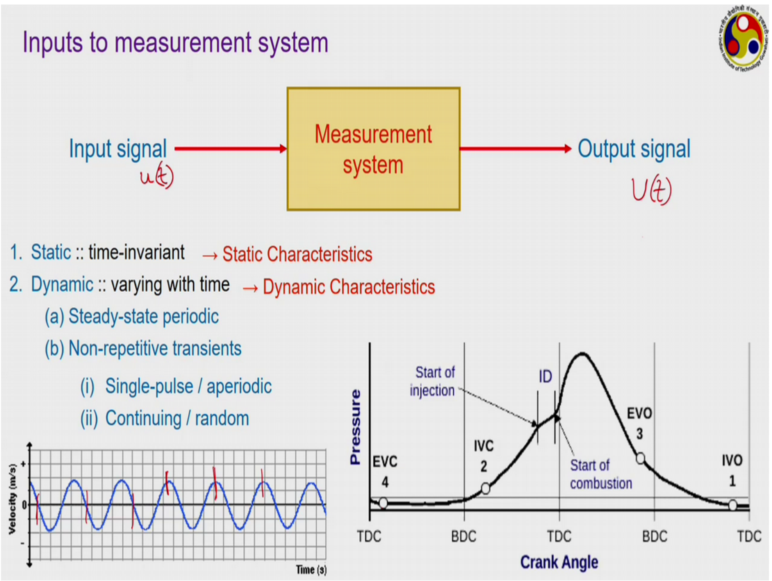

We now know from our discussion that in any measurement system, we generally have two kinds of interface. The input stage creates an interface between the physical system with your measurement device, where it collects the input signal from the physical system. Quite often that input signal is compared with some kind of standard reference to provide a transducer signal, which goes through the second stage the signal conditioning stage.

And the process signal, the processed output signal, which comes out of the signal conditioning stage is supplied to the output stage the 3rd one which again creates another interface between the measurement device and your recording device. Now, the recording device can be just a display, can be a graphical display, can be a numeral display or can be a recording device, can be a pain moving on a paper something like that. So, there are lots of different possibilities about the recording device. But, here we are just interested to know the interface or our interest is only to say that the output stage creates an interface between the device or measurement device and your output recorder.

Now, if we club all the three stages together and consider the measurement system as a whole instead of considering any of these components. Then we are just left with two signals on either side of it. One side we are getting the input signal coming from the physical device, and on the other side we are having the output signal directed towards a recording device.

Now, response of the measurement system here talks about what is going to be the form of your output signal depending upon the input signal, and also the measurement system itself. The nature of the output that we are going to get that definitely is going to depend upon the nature of the input that you are providing, and also it will depend upon the system that you are using, because inside the system it is possible for the input signal to go through several stages of transformation or modification.

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Like think about say you want to measure the length of this particular pen using a standard ruler or measuring tape. Now what we are going to do? We are going to take that ruler or measuring tape. We are going to place the ruler, and also this thing side by side. And then from the scale of the ruler, we are going directly going to get the length of this particular one.

Now, that is a very very simple kind of measurement system. Here we have just one device, they are basically no distinct three stages without having the input stage, which is getting the length signal if we can say from the measurement, which is pen here. And then the output stage, where the length or the final result of the measurement will be obtained from the scale, which is which is marked on your measuring tape. There is no signal conditioning stage, because there is no kind of signal conditioning required here.

Your input to the measurement device, the measuring tape here is the length of the pen and output is also the length of the pen. But, only in some very simple cases, we can get such kind of situation. In most of the common simple or complicated kind of measuring device, you will have some kind of transformation or transduction of the input signal to some other form of output signal that means, what I am trying to mean is that the nature of the input signal will get modified. And whatever energy you are supplying the form of input signal, may get transferred to some other kind of energy, which will later be converted to the desired form of measurement signal.

Like, think about the example of a thermometer, we have already talked about this. In the thermo meter the input signal is coming in the form of a temperature difference generally thermal energy, because the body if that is at a temperature higher than the thermometer will be supplying some heat to the body of the thermometer. And that amount of energy that is being added to the thermometer wall will be proportional to the temperature difference between the physical system and the initial temperature of the thermometer.

Now, that temperature difference is not the one which is coming as your output signal. Because, your output signal, we are talking about mercury thermometer like common clinical thermometer, which are generally called from proper mechanical meaning point of view in general call them liquid in glass thermometer means, we have a liquid which is allowed to move inside a glass capillary to get the thermometric measurement.

Now, the output in this case will be coming in the form of a volume change or a change in length, volume change of the liquid or change in length of the liquid column inside the capillary tube. So, your output signal is actually either a change in volume or to be more precise a change in length of a liquid column.

And now during the process of calibration itself that output signal is correlated to corresponding temperature, and we can have a temperature reading directly temperature scale directly mentioned on the body of the thermometer to get the temperature reading. But, what we are actually getting is a allowed as the output signal is a change in the length.

Think about a simple say tire gauge, a tire pressure gauge, there your input signal is a pressure commonly the difference of pressure between the desired location and the atmosphere, but that pressure difference is getting manifested in the form of the rotation of the indicator on an angular scale.

They think about say our common watch, say this wristwatch. Now, if we want to measure time here, what we are getting from your device, it is not time, it is rather the rotation of those hour or minute hands over a dial. And during the stage of calibration, the manufacturer itself has provided some scale in the form of time that is that angle of rotation has been converted to corresponding time scale, and it is mounted in or it is marked in the form of the time scale. So, we are getting a time measurement.

So, in most of the measuring device the form of the output signal will be different from the input signal that means, output signal will go through some stages of modification. And the amount of modification or the nature of the modification will be dependent upon both the nature of the input signal, and the measurement system itself. This is what we are going to study means the nature of this modification is the thing that is going to we are going to study in this week, in this particular module or about response of measurement systems.

Now, something we have already discussed last week depending upon their time varying in nature, we can classify the input signals into two categories static and dynamic. Static refers to time invariant, dynamic is something which is varying with time. Common examples of static signals can be fixed temperature of a body, may be mass or may be the length of something, unless it is going through some kind of expansion or contraction. These are common examples of static signal.

Dynamic signals there are infinite examples, like I talked about the restore just a few seconds back. The time itself is a form of dynamic signal, because it is continuously changing. But, sometimes we may have a particular pattern of this change in the dynamic signal. Accordingly, we can classify it a steady state periodic or non-repetitive transient. Periodic means when there is a particular pattern, and after a fixed time interval the nature of the signal gets repeated or whatever variation of the signal, we get over a particular time cycle that is going to get repeated over the next, and the next, and the next over all the cycles.

However, there are several situations like the temperature variation in the atmosphere over say one day, these are examples of non-repetitive transients. Like this is an example of the velocity of a simple pendulum, we know that the pendulum represents simple harmonic motion, which is an example of a periodic motion. So, over one cycle or over each cycle of operation is velocity gets repeated.

Like say at a particular time instant it is velocities like this, this is a magnitude is 0, and also it is directed downwards. And after the time period, after one time period, again it reaches the same condition, again the same delta t apart it reaches the same position, same velocity in both in terms of magnitude and direction. If you talk about any other point, so this is the maximum velocity.

So, after one time period it, we will again reach the maximum velocity. After another time period, it will again reach the maximum velocities this is what we refer as the periodic signal. But, and this is the example of pressure variation inside a typical IC engine, typical automobile engine. It is again a dynamic signal, but may well be our non-repetitive one.

Like here over one cycle that means, we are counting the cycle as the TDC position. The piston at the top dead center version of the cylinder going to the bottom dead center, and coming back to the top dead center. This is one particular stroke or I should say this is one full revolution of the crankshaft, and two strokes.

Now, you can see the pressure is varying pressure initially is almost flat between the first stroke that is during its movement from top dead center towards the bottom dead center, during actually the injection is taking place or I should say the supply of the fuel. Now, then we have the compression starting in, so accordingly the pressure keeps on increasing during the first stage and that is from this point to this point pressure remains quite close to the atmospheric pressure.

Then during the compression stage, as the piston is coming back towards the top dead center the pressure is increasing. Somewhere here we put the injection, and so there is a rapid increase in the pressure, quickly reaching the maximum pressure, which generally also corresponds to the maximum temperature slightly after the TDC location somewhere here.

So, this is the period over which though we are start though our injection that is a spark plug in case of petrol engine or injection in case of diesel engine that is done somewhere

here, the combustion reactions will start only for this particular point because of some pre combustion reactions, which may go on over this period.

Now, if we talk about this particular cycle, then it is not a periodic nature, it is a very much an a periodic variation over one cycle. But, if we get another stroke into picture, you will get the pressure is falling down. This is the expansion or power stroke that is going on, then somewhere here the examples exhaust valve exhaust valve opens, so there is a sharp reduction in pressure again reaching the top dates while it is achieve the top dead center position, during which it exhaust all the gases.

So, over this is a full cycle that is two revolutions of the crankshaft or four strokes of the piston in a typical four stroke engine, we have such variation of the pressure. And this is a very much in and of a periodic nature, but generally this four strokes keeps on getting repeated that means, once we are back at this particular position this point in a true engine or proper engine generally coincides with this particular point. So, the cycle will keep on getting repeated that is over four strokes, we can achieve a periodic measure of operation.

So, depending upon the time interval that you are considering, you may get same signal as an a periodic one or a periodic one's, we should be very careful about identifying a signal as a periodic or a periodic. And also a non-period repetitive transients, this signal can be of two types. One is a single pulse means just one signal and nothing else.

Like, suppose you think about an electrical line, there is a switch in the line we and so initially the switch is open conditions. So, there is no current flowing through this line. We just switch on this, so there is a rapid increase in the current, and then immediately switch it off. So, they suggest a single pulse, and then it will continue to be 0. This is something like single pulse situation, you shall be talking about this one later on a bit more. And we can also have the complete random signal, where there is no repetition or no pattern is possible to find.

When you are talking about the static signals, then we can also expect the output to be a time invariant that is both input and output are time invariant, and hence what we get that is called the static characteristic. Static characteristic refers to the relation between output and input, when both of them are time invariant.

But, when the input is a time varying 1, like say if your input signal is small u, it is a function of time, then we can expect our output capital u that to be also a function of time, and that kind of type of relationship between output and input is referred to as the dynamic characteristic, where time also is an important factor to be considered.

However, in static characteristic we can directly consider the importance or rather the relation between output and input this capital U and small u. However, in case of dynamic signal, we have to consider their time variation as well. We have already talked quite a bit about the static characteristics difference to properties of a measurement system from static characteristics point of view like resolution, like linearity, like sensitivity, hysteresis, drift etcetera.

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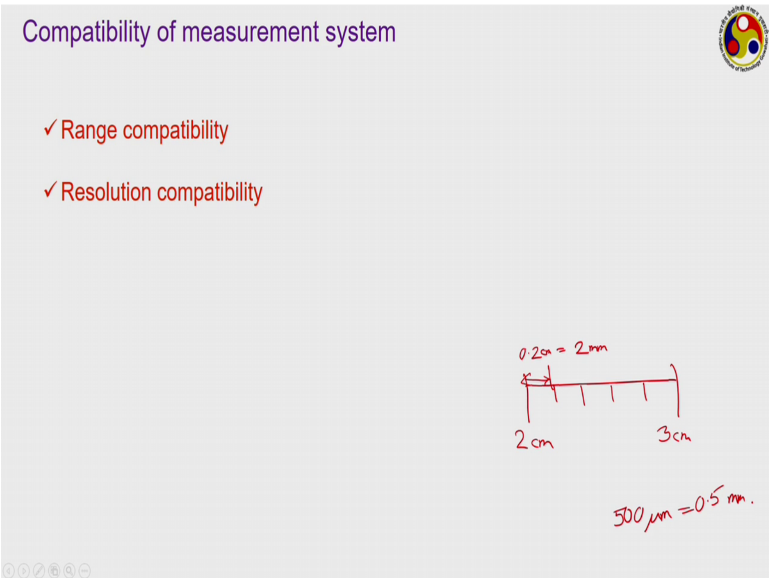

In this week, we are going to focus more on the dynamic characteristic, but before that we need to talk about the compatibility of the measurement system. Like suppose, I have given you one particular device, and you want to use it for measuring something. Let us say I have given you a length scale to measure the length of something. While you are trying to use this for a length measurement, you have to be sure that the measurement system itself, and your objective for measurement are compatible with each other.

There generally have five kind of kinds of compatibilities that we generally check for like range compatibility. Suppose, I have given you a common rule scale common ruler, which is having a length of 30 centimeter or 1 feet. And I have asked you to measure the length of a room using that. Now, the range of the ruler is 30 centimeter, and your room may be something like 5 to 10 meter long.

The ranges are not compatible definitely, so you have to use the ruler multiple times to get the same reading. Each time incurring some higher percentage of higher probability of some error in the measurement or like suppose I have given you a common clinical thermometer, and asked you to measure the temperature of boiling water.

Now, you know that the maximum temperature you can measure in the clinical thermometer is something like 42 or 43 degree Celsius. Whereas, under atmospheric condition the saturation temperature of water is approximately 100 degree Celsius. So, definitely you cannot use the clinical thermometer to measure this reading. Similarly, you cannot also use that for measuring the temperature of ice, which is expected to be at 0 degree Celsius. So, the range is the first thing that we have to check in from our measurement system, whether the value that you are going to measure falls within the measurable range of the instrument or not that is a first question we should ask.

Next is the resolution compatibility. If the range is sufficient, then we have to check the resolution. What is the resolution, I am sure you remember resolution is the smallest possible change in the input required to cause any change in the output. Let us say the length scale that we are talking about your ruler is having a gradiation mark like this. Here this is say 2 centimeter, this is 3 centimeter, and we have 5 marks in between.

So, what is the resolution of this scale, you can clearly see that smallest possible length it can smallest possible change in length it can measure is this much, which is 0.2 centimeter that is 2 millimeter. So, the smallest possible length it can measure small a smallest possible change in length that it can measure is 2 millimeter only.

Now, if I want you to change want you to measure the diameter of a micro channel using this ruler, which may be something like that list suppose I have asked you to write the measure the diameter of a micro channel, which have a diameter of 500 micron that is 0.5 millimeter. This is way smaller than the resolution of the instrument itself, and hence it definitely cannot give you a proper measurement.

So, resolution is the second thing that we have to check like a the common wristwatch that generally has a resolution of what is the smallest change in time that it can measure

1 second typically. So, common wristwatch as the resolution of 1 second. Now, if we want to measure a time smaller than this, definitely that is not possible. So, after the range, we have to check the resolution.

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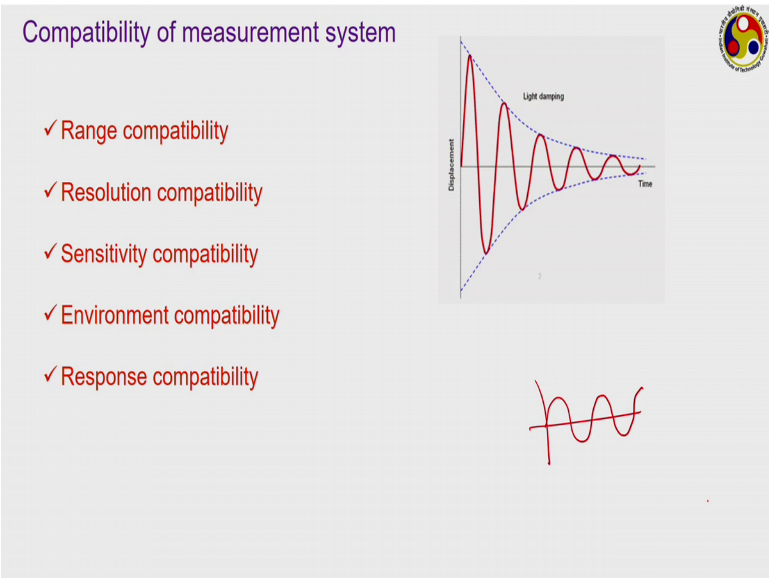

Next is the sensitivity. Sensitivity can also be important, generally we always prefer the instrument to be to have higher sensitivity that is the same change in input should cause a larger change in the output. And if the sensitivity is not sufficient or is very small, then we may not be able to get the correct output value or may not be able to note the correct output value, because on the scale the graduations may be very close to each other.

Now, environment compatibility. If we are looking to use the instrument in some harsh weather condition or some harsh environment, then we have to make it suitable for that. Like suppose you are trying to measure the temperature using a thermometer or using a thermocouple inside a power station boiler, inside the furnace. This is the furnace the temperature itself is very high.

So, whatever material that you will be using for your temperature measuring instrument that may not be able to handle or survive the temperature or suppose you are using in a location which is not having very high temperature, but something like inside a coal conveyor, where in the coal particles are generally pulverize to very small sizes something in the range of 75 micron.

And now if you are trying to measure the temperature they are using a common thermometer, then there is every chance that the coal particles will be striking the temperature measuring instrument and may cause decent amount of damage to that. So, we have to be careful under those kind of environment.

And the final one, which we shall be talking the most in this week is the response compatibility. Now, what we mean by response compatibility? Let us say in this particular situation, how this one looks like? This signal looks more like a damped seek version of the displacement of a pendulum.

The displacement of a pendulum the signal that we got in the previous slide, which is like a perfect sinusoidal kind of wave that is under when it is not at all facing any kind of hindrance from the surrounding here. But, if the surrounding here is putting significant amount of resistance to the motion of the pendulum itself, then that will act as a damping on this resulting in this kind of signals that is the displacement will gradually come down. Unless we are supplying some additional energy, the disperse will keep on becoming smaller.

And after a significant diversion of time, it can come back to complete standstill position think about a vehicle. In our car if we stop the engine, the car may still carry on running for certain distance, but as it is moving the road is putting some kind of friction on the tires that will lead to continuous reduction in velocity, and finally it will come to a standstill.

Now, these are of course environmental effects, but your instrument itself also may put some kind of frictional or registries kind of effect on the input signal itself or the source of the input signal. Thereby affecting the source of the signal itself.

So, we are going to get a response like this or sometimes say your input signal may be a perfect periodic one, but because of some inertia or certain other effects of the components present you say the measurement system, we may get an output like this which is definitely not a proper representation of the input signal. So, this is what we means refer as resonance compatibility means the output form of the output that we are going to get that should be able to reproduce the nature of the input.

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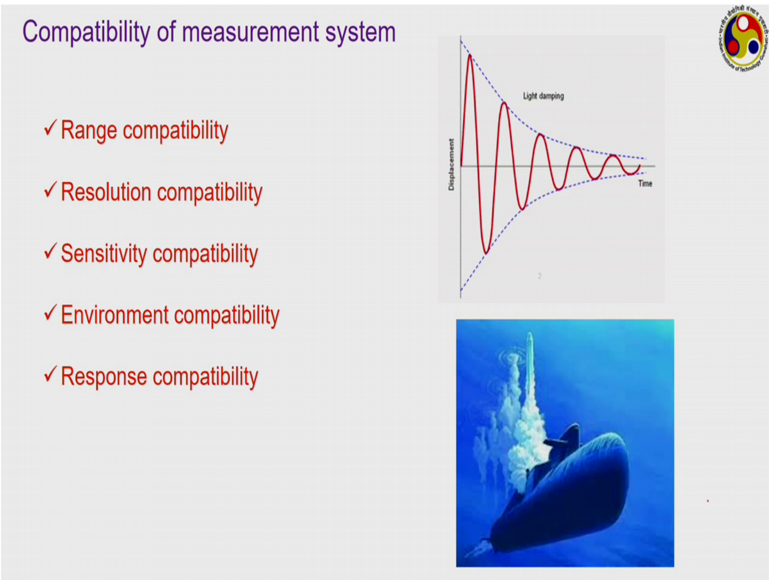

Like another example, here we have a submarine which is say when the submarine keeps on continuously going down from the sea level. Now, as it is going down the depth of water above it, keeps on increasing accordingly the pressure also keeps on increasing. So, if we are able to measure the pressure, then we can measure the depth that is the vertical location of the submarine or vice versa.

But, the temperature of the water is also changing as it is going down. And as the temperature is changing that may affect the way the pressure sensing instrument is working. So, we may get a different reading, different pressure reading or a wrong pressure reading from the pressure sensing instrument because of the temperature dependence of the corresponding instrument. Do we wish have to be careful about this kind of situations to deal with transient signal measurement. And that is what we refer as this resonance compatibility.

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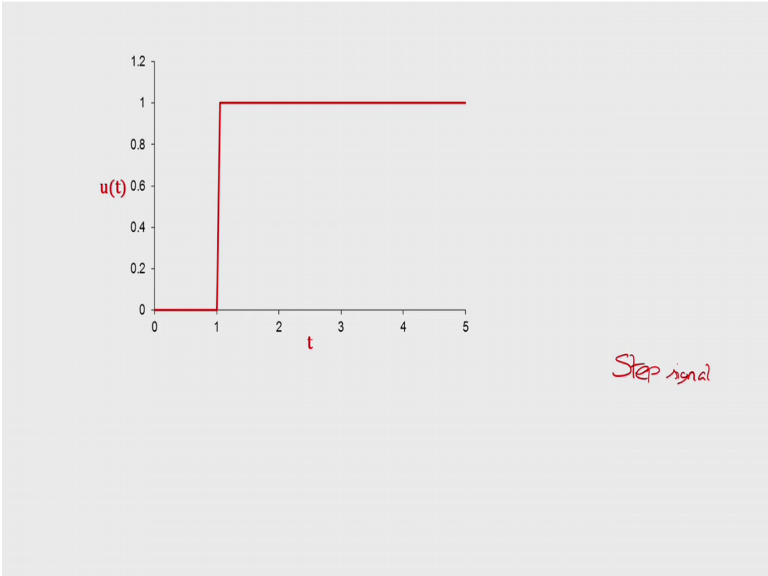

Let me take another signal. This is a particular type of signal, which we have to use quite a bit more. These are known as step signal or step input. Step refers to just a single step change in the input, here like the situation shown here till a certain time the input signal was 0.

And now at this particular time, we are suddenly providing a large magnitude or a large amplitude of a signal that is the signal changes from 0 to a certain value like 1 in this case, and then it keeps constant there remains constant there for a large period of time. This kind of signals are commonly known as step signal. Example of a step signal, suppose you have a weight measuring platform, you initially there is nothing kept on the platform, so the reading is 0.

Now, you want to measure the mass of a packet of sand. We are just taking the packet of sand, and putting that suddenly on the platform, then what is happening? Initially there is no load on this, so it is sensing 0 mass. Now, suddenly there is a large amount of mass that has input in.

So, if the mass of that packet of sand is 50 kg, so initially the mass input signal that is the signal was 0 to the system was 0. And now suddenly it is receiving an input signal of 50 kg. And that 50 kg is not changing over, next few moments of time moments and hence we are getting a step input to the measurement system.

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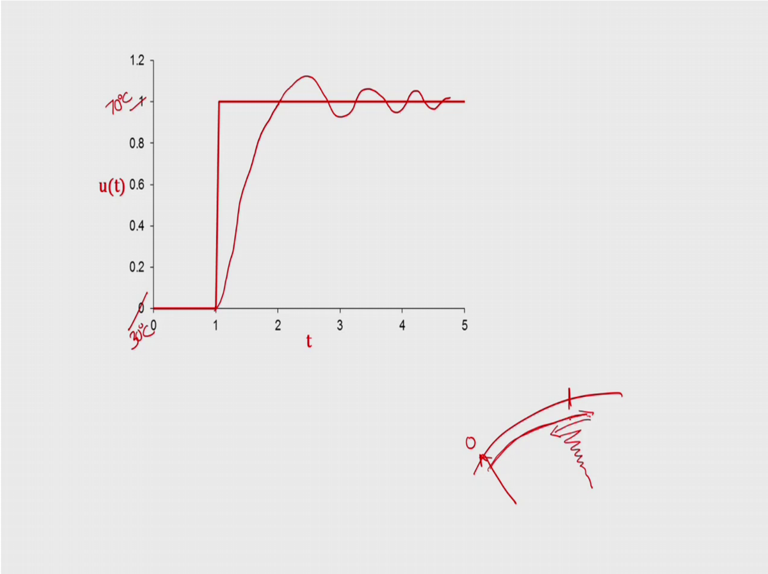

Another very common example can be your thermometer. Say your thermometer is initially kept in open atmosphere, which is at 30 degree Celsius. Let us say initially this instead of this 0, let us now write 30 degree Celsius, it is measuring 30 degree Celsius. Now, suddenly we take this thermometer, and immerse it into a pool of water which is maintained at 70 degree Celsius, the temperature of the pool is maintained constant at 70 degree Celsius.

Now, what is happening the input that was coming to the thermometer was 30 degree Celsius till that instant of time. And suddenly it is getting an input, which is 70 degree Celsius. Now, how your thermometer is going to respond. If it is say liquid in glass thermometer, then the mercury or whatever thermometric liquid that you are having that will take some time to respond to the situation, it is not possible that the corresponding volumetric expansion is are instantaneous. And immediately the thermometer column will reach the 70 degree Celsius mark, it will definitely take some time.

And you may very much expect that the temperature that is shown on the thermometer by the movement of the mercury column will be somewhat like this. It initially showing 30 degree Celsius, then it keeps on increasing over a period of time reaching that 70 degree over a sufficient duration after a certain duration of time. And it may not remain restricted to the 70 degree Celsius rather because of his own inertia, it may cross this one. And then it may come back again, and may oscillate a bit.

Just think about that measuring platform. Suppose, you are having a dial gauge to show the reading, this is the dial gauge, and this is an indicator initially mark initially showing 0. Now, because of this sudden load that you have imported, this indicator is going to move in suppose this is 70. Invisibly, we will say that indicator because of its own inertia will cross 70, then tries to come back to the 70. And in that effort it again crosses that on the other direction their weight oscillates around it for a period of time, finally settling down at say that 70 degree Celsius.

So, the final reading of 70 degree Celsius that we are going to get not 70 degree sensors in the measurement or rather in this measuring platform mass measuring platform point of view. But, 70 degree Celsius for the thermometer, so that 70 degree Celsius that we are going to get from the thermometer that reading may take sufficient amount of time.

And just think about what doctors advise while measuring the body temperature using a clinical thermometer. They generally ask us to keep the thermometer in contact with our body for a period of 1.5 to 2 minutes, so that will allow the thermometer column to settle down to the temperature of our body.

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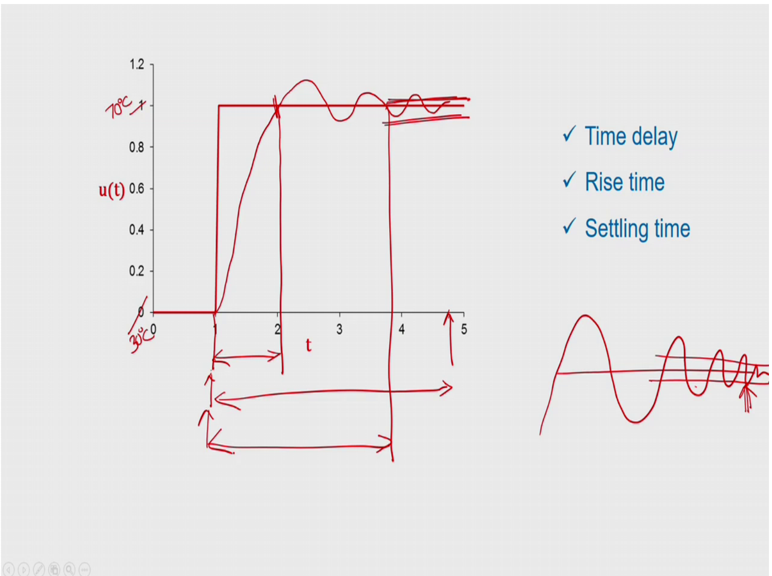

Now, that time the instrument requires to reach from its initial position to the first point, when it attains the output value, this particular time is known as settling time. We are seeing there is a significant amount of time delay between imposing the input signal, which is this is the point where imposing the input signal and you are finally getting the out signal may be somewhere here. So, this entire thing refers to the time delay of the risk measurement system.

The rise time refers to this particular one that is initial time. When you are imposing the signal and the first point, when it attains the output signal value. But, it is not staying constant at the output signal value rather it is oscillating around a bit. So, the next concept is settling time. Settling in efforts to that time within which it settles within a particular band around the output. I suppose this is the 70 degree Celsius, your output may be behaving somewhat like this, its one this one.

Then if we take a band of say plus minus 1 degree Celsius around this something like one here, and another here. Then this is the time where output signal or I should this is not correct, if I draw it once more. So, this is your 70 degree Celsius line, and this is the measurement that we are getting.

So, if we draw a plus minus 1 degree Celsius line, probably this is the time where it is settling down within that particular band. So, this is what is referred as a settling time or that distance between the time of initial impose meant of the signal this is here. And if your settling time it reaches somewhere here, this particular time is referred as the settling time.

So, rise time refers to the time required to reach the output magnitude for the first time. And settling time if the, settling time refers to that time recovered by the device or by the output signal to settled within a predefined band around the desired output. Settling time can be significantly larger than the rise time. And we have to be careful about a choice of the settling time or about our choice of the band that we are going to consider.

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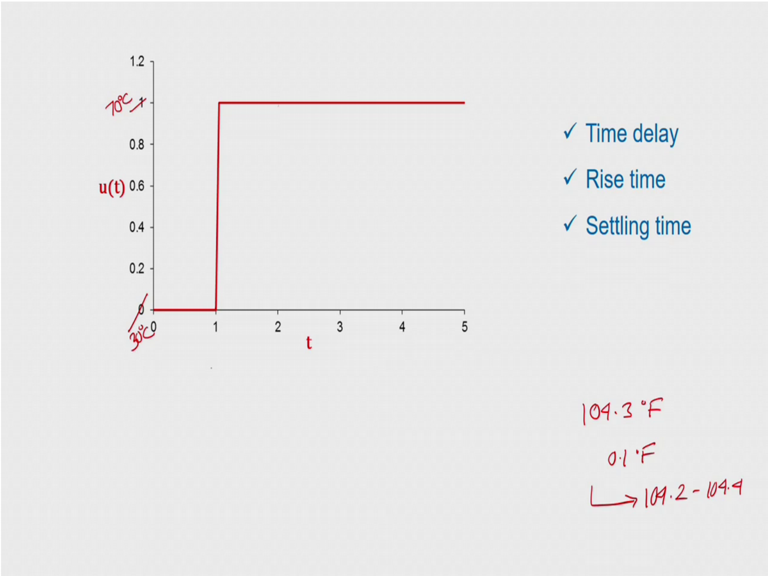

Like while measuring the body temperature, if the body temperature is say 104.3 degree Fahrenheit, then we may choose a band of something like 0.1 degree Fahrenheit that is was the time becomes restricted between 104.22 104.4, then we may say that it has raised the settling time. It has attained the settling time, it has settled in a 0.1 degree Fahrenheit band.

And now we can use this particular reading as our thought the further processing. So, this is a very simple device that we are talking about a measuring platform as measuring platform or a thermometer. But, there also you can see that the system is not able to respond instantaneously, because of its own inertia, because of a few natural phenomenon.

And hence we have to allow some time to the system itself, so that it can give us a proper output. Now, how much time it will require that will depend upon the characteristic of the system. So, the rise time is settling time on important characteristics to judge the time response or time stability of a system.

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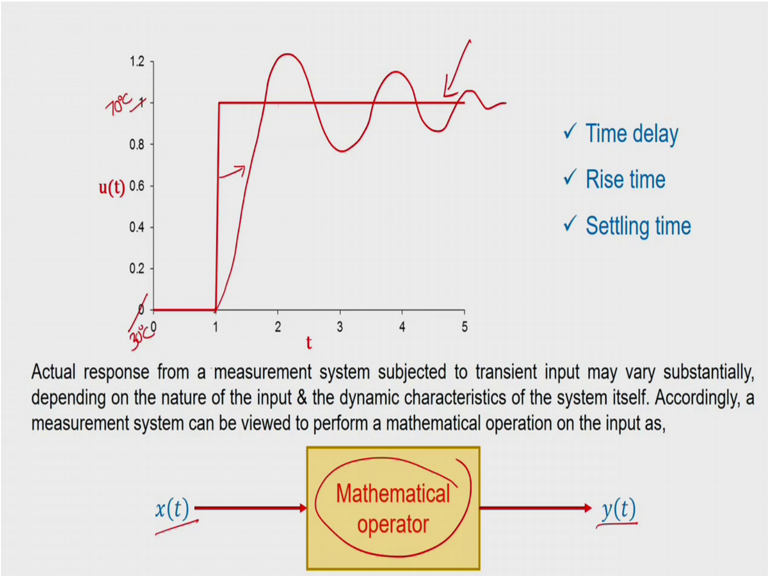

Actual response that we are going to get from a measurement system, when that is subjected to a transient response or transient input I should say that can vary substantially depending on the nature of the input that you are providing, and also the dynamic characteristic of the system itself.

Accordingly, we can think that the measurement system is taking a transient signal a time varying input signal, and then giving us another time varying output signal with a certain amount of modification. And hence we can think that the measurement system is performing some kind of mathematical operation on the input signal.

Like if x t your input signal, your output signal is y t that is something mathematical operation that done by the measurement system on that x t. Going back to the step signal example, this red line is your input signal means, I am talking about this particular one this is your input signal. If we plot your input signal, each time you are getting something like this.

But, how your output signal is going to look like, your output signal will look in somewhat like this a time varying signal, where the input is a step, your output is not a step signal, rather it will keep on varying with time over significant duration of time before settling out and the particular desired value. So, this conversion from this step signal to this particular curve that is being done by the mathematical operation imposed by this measurement system. And so we can often see the measurement system as a mathematical operator.

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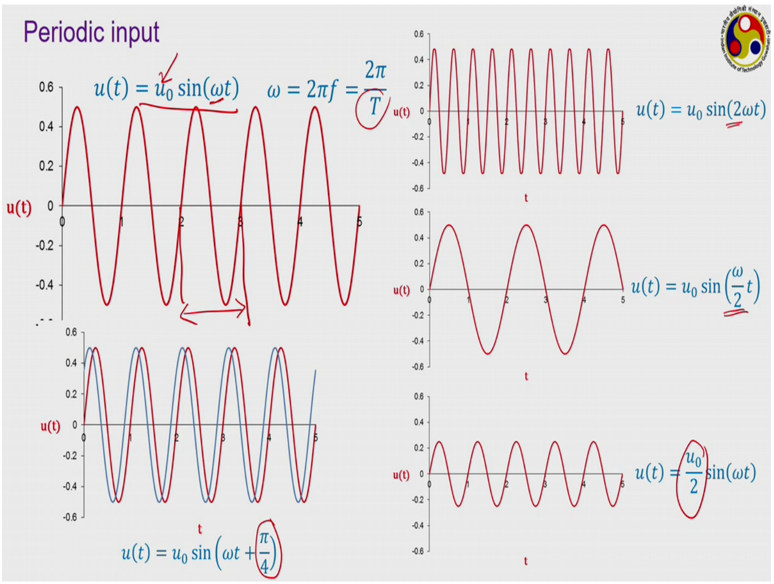

Now, as you are talking about time variation, let us just quickly talk about its a few standard signal. Of course, your system may be subjected to very random time varying signals, but those are not possible to analyze and that is why, we are still restricting ourselves to very common time varying signals, and the most common one definite is a periodic input signal.

Like the one shown here a very standard form u equal to u naught sin omega t, we know that this u naught is referred as the amplitude of the signal, which basically refers to the distance from this neutral point or I am very poor with the drawing on this.

This particular distance that is from your neutral position to the maximum magnitude in either direction is this amplitude omega is called the cyclic frequency or more commonly called the circular frequency. It is typically measured in radian per second that is we also call it the angular displacement or angular velocity should say, rate of angular displacement that is the amount of angular displacement.

The system has suffered over a period of time or I should say per unit time that is the omega I am making repeated mistakes today, whatever. We also know that we can relate this omega with the frequency in the form like this omega equal to 2 pi f is equal to 2 pi f capital T, where capital T is the time period of oscillation. Like in this case we can find that one particular cycle is repeated over this particular period. So, this is what we refer to as a time period for the example that we have here at T equal to 1 second. And accordingly frequency is also 1 cycles per second or 1 hertz.

By varying the amplitude and frequency, we can have different kinds of waveform like this example or amplitude the same, but frequencies trouble or time period is half of the original one, like we can see by this two here, which refer to double frequency. Like this one, here again amply the same, but frequency is half of the original which is omega by 2. This is another signal where frequency remains the same, but amplitude has become half of the original case. So, these two looks quite similar to each other, but with a difference in their magnitude.

Sometimes, it is also possible while dealing with multiple signals that signals may have the same frequency and same amplitude, but they may lead or lag each other where certain phase angle. And this is what we are referring to here the red one is the original signal, which is given by this particular relation. And the blue one is something which is leading it by this pi by 4 factor pi by 4 is a corresponding phase angle. So, this concept already you know, this is just to repeat this terms like circular frequency, time period frequency, phase difference etcetera.

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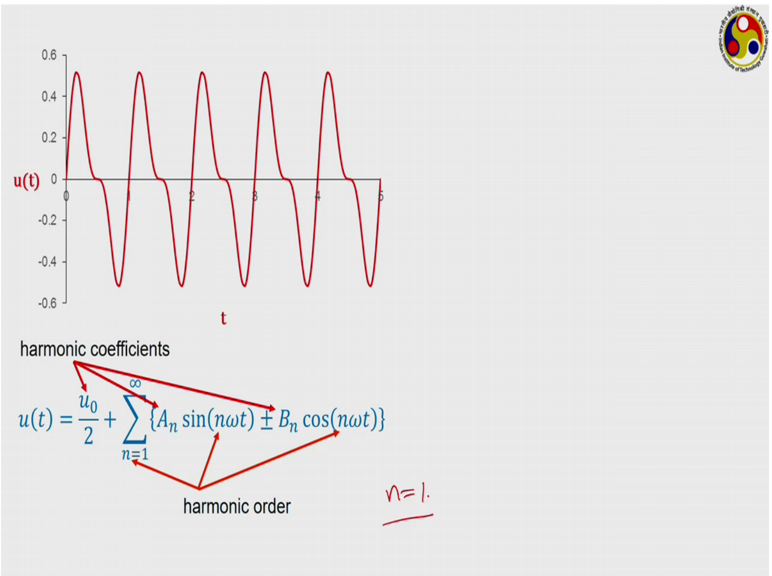

Now, look at this particular signal. This is definitely periodic signal, but it is no it looks somewhat like a sin wave also, but definitely is not a proper sin wave. Then how can we deal with such signals which are periodic in nature, but does not represent does not resemble any sin or cosine wave directly? In that case, it is possible to represent any periodic signal as a combination of sine's and cosine's who is something which is known as the Fourier transform.

So, this is a typical Fourier series of a periodic signal, where we have a constant part in the form of this, and in the periodic part here this constants that is u naught A n, B n, they are referred to as the harmonic coefficients. And the small n is known as the harmonic order. n equal to 1 refers to the fundamental mode of the first harmonics, and all other interior values of n 2, 3, 4 etcetera refer to the curse harmonics the 2nd, 3rd, 4th harmonics. A system may be subjected to a is signal, which is having just the fundamental mode or may be having multiple harmonics present in the signal.

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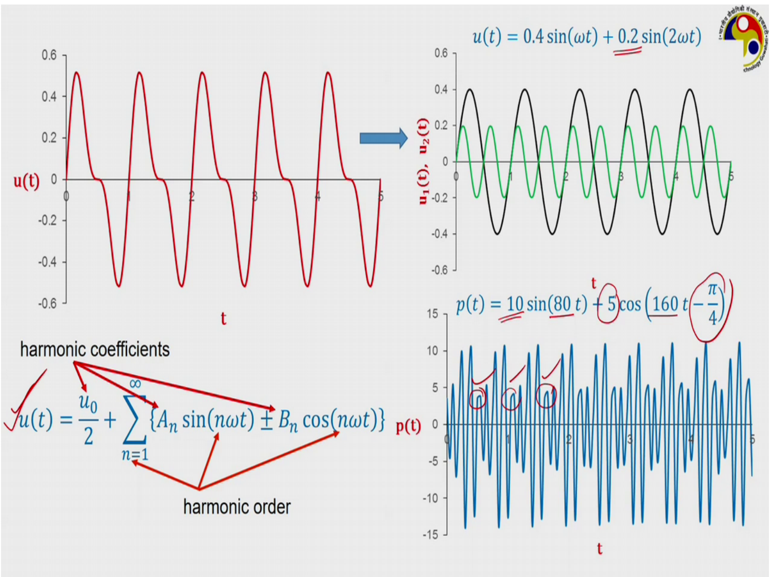

And any periodic signal can often always be represented in a form like this, like the example. The signal that we had here actually is a combination of these two signals. One is a red one, another is a green one you can clearly see their amplitudes are different.

And what about the frequencies if we compare their frequencies, you can clearly see say for the black one 1-time period is this, this is one typical time period. What happened to the green one? This one is able to complete two cycles within the same period, and that is the time period for the green one is this much only that is the frequency of the green one is double than the red one. Actually, it is a signal somewhat like this, we can clearly see that the second one is having and frequency double than the first one.

Now, if we compare that to the standard form like this, then here the fundamental mode is having a frequency of omega. And is first harmonics is having a frequency of two omega, and there are no other harmonics present there is only the fundamental, and the second harmonic that is we are having n equal to this corresponds to n equal to 1 the fundamental mode, this is n equal to 2 the second harmonics. Sometimes it is also called a first harmonics, but to be consistent with this n equal to 2 notation, we are going to call it the second harmonics.

And hot both of them are having different amplitude, we can see the amplitude of the second harmonics is half of the amplitude of the fundamental ones. And accordingly, the magnitude of both of them are decided or the final signal has been decided. Sometimes, we can get much more complicated periodic signals also. But, with careful observation with some mathematical operations, we can always convert them to a combination of sine’s and cosine’s that is in the form of the Fourier series.

And thereby you can always identify the fundamental frequency as well as all the harmonic frequencies. Like see this one, this quite typical of the pressure variation inside an automobile engine. We may get such kind of situation, you can see there are several cycles getting cycle kind of operations, but each of them each of the cycles are having a different amplitude, then how can you identify their periodic nature.

If you look here fully see if we pick up this particular point, then somewhere where it is getting repeated this particular point is it is very very difficult to identify from the cycle that we have here. But, I guess it is getting repeated somewhere here giving us a period of something like this, but still quite difficult to spot, because it truly speaking if you follow like this is a one, and this one, this one all of them are having quite different nature. So, we should not identify this one as a proper cycle, whether the cycles are changing over this.

Now, can we convert this one to a Fourier series kind of form? We can it actually is having a quite simple form like this. You can see here, here we are having just two modes. The fundamental frequency is 80, and then its second harmonics having frequency of 160, whatever the amplitude of the second one is half of the amplitude of the fundamental mode.

And the harmonics is having a pi by 4 phase lag compared to the fundamental one, which is actually giving rise to this particular nature. Had there not been any kind of phase lag, then you could have definitely seen a much more regularised nature. So, this way we can always identify or we can always convert any periodic signal to such combination of sine's and cosines.

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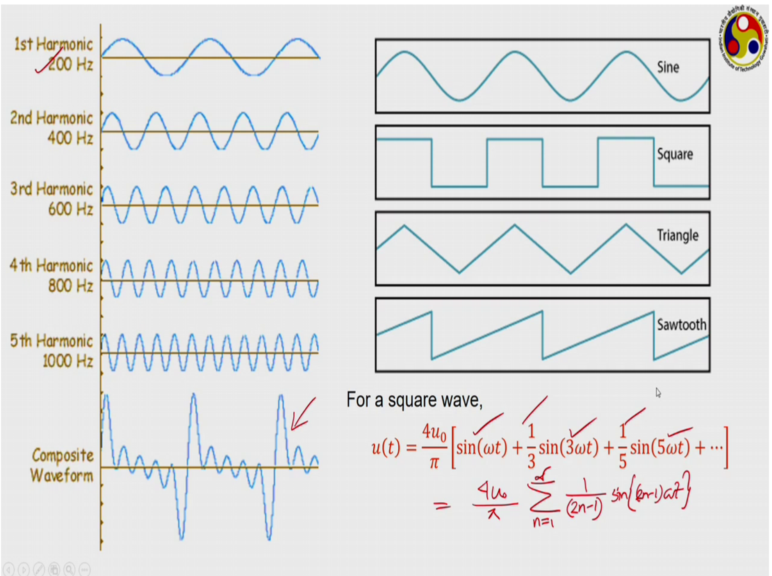

Another situation where we are basically combining five different signal this is a fundamental one is frequency of 200 hertz, and then for harmonics of that 2nd to 5th. And once we combine name, it is a very odd kind of form that we are getting a very complicated one. Even these signals also, which must forget the sin wave, but the rest of the three they are having very very different nature compared to the sin wave, but still they are all superior in nature. And hence, we can also come; we can also represent them in the form of Fourier series.

Like for the square wave, this is the equation that we can get. So, the this is the amplitude, you can see here the fundamental mode is having a frequency of omega that is no second harmonic, but there is a third one, and then there is a fifth one, then there will be the seventh one that is all the odd harmonics are present, but the even harmonics are not there.

And the amplitude of the harmonics are also continuously decreasing, like the third one is having an amplitude of 1 by 3 times, the fundamental one. Fifth one is having an amplitude of 1 by 5 times the fundamental one. And this way it keeps on reducing that means, we can combine this into a common structure, like if we write say this particular was 4 u naught by pi, which is the amplitude of the fundamental mode.

And then if we sum it up over n equal to 1 to infinity, we know that we are getting the signal as sin something. Now, we have an number to put like this 1 by 3, 1 by 5 those if it has to be coming in, and then there will be a sin omega t, but these are all odd numbers isn't it.

Then just seeing that odd number portion, we can write it as 2 n minus 1 omega t. So, when n equal to 1, this will give us 2 n minus 1 that is 2 minus 1 that is 1 omega t, which will give us this. Then when equal to 2, this will give us 4 minus 1 that is 3 omega t, which is this. When n equal to 3, this will give us this and divided by 2 n minus 1, which will lead to this 1 by 3, 1 by 5 etcetera. So, this way we can write this one into a common structure like this. And the gist is that, the square wave can be converted to of Fourier series kind of form.

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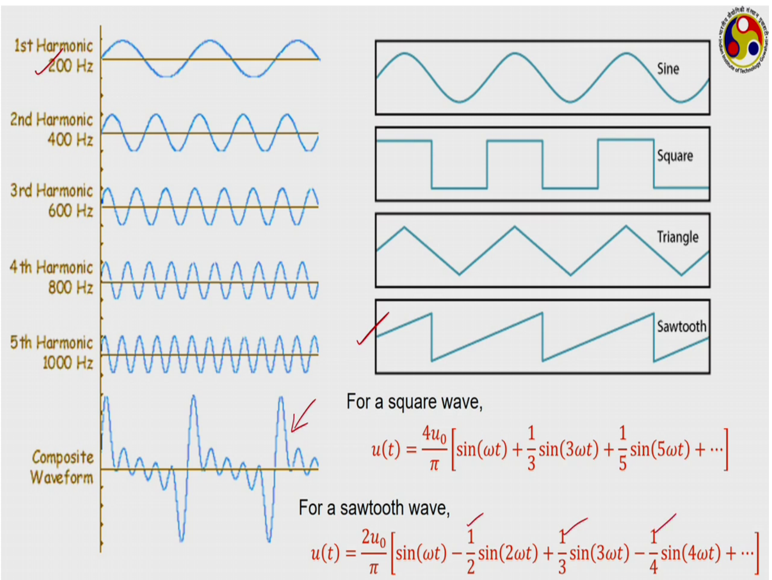

Look at the sawtooth one this particular I am talking about. This one also can be represented like this. Here all the modes are present, now both odd and even, but their amplitudes are changing. Like it is minus half for the second, then plus one-third for the third minus one-fourth for the fourth, and it continues that way.

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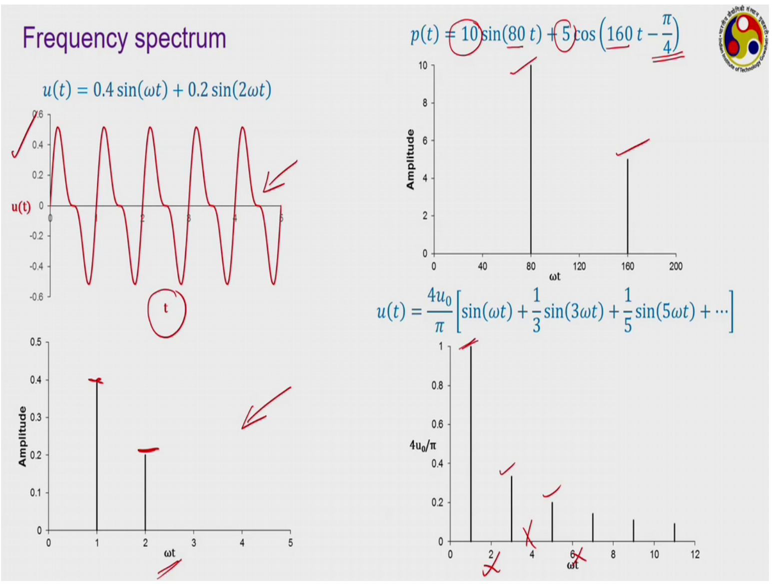

Now, as you can see that just going back to the example that we have studied. Here this signal is a combination of two sin waves, which are having amplitude is 0.4 and point when their frequencies are omega and twice of omega. Now, if we know these amplitudes and frequencies you can clearly represent this one both in terms of mathematical form and also in graphical form, and so while this remains the most common way of repressing the signal that is signal with respect to time.

There is another way of representing the signal, which is also called the frequency spectra frequency spectrum, where we use this amplitude and frequency information only. That is we plot amplitude on vertical axis and frequency on the horizontal axis, because it can be plotted in terms of omega t or in terms of sorry using the relation between omega and f, we can also represent this one in terms of f t or any other convenient form.

So, look what this diagram is saying, it is saying that when your circular frequency is omega, then we are having one signal with an amplitude of 0.4, and for twice omega we are having n a signal of amplitude of point 0.2, and nothing afterwards. This is what we refer to this, because frequency spectrum this is clearly giving us the information about that both the modes present their frequencies and their amplitudes.

What about this one, here we are having two modes present. So, one frequency as at 80, another frequency is 160 corresponding to 80 or having an integer of 10 corresponding to 160 or having an amplitude of 5. So, if we plot that, this is what we are getting corresponding to 80 we are getting this as 10, corresponding to 160 we are getting this one as 5. Of course, this representation is not showing anything about this phase lag, there are other ways of representing that which we are not going to discuss.

What about this particular one, the square wave that we have just seen, where we are having only the odd frequencies coming in. So, if we plot this corresponding to omega, we are having this 4 u naught upon pi, then corresponding with nothing corresponding to 2; corresponding to 3, we are having one-third of that same nothing for one 4, then we are having for 5, nothing for 6, and this way we keep on continuing.

So, frequency spectrum is an alternate way of representing the time variation, we alternate way of representing such periodic signals. One option is this, and another option is this. We can represent in either of the way depending upon the need of the situation or depending on upon our convenience. But, another important thing that we are getting from here is that, while it is common to represent or select time as the independent variable to represent such signals. We can also select omega that is a frequency as another independent variable, which we shall be using later on.

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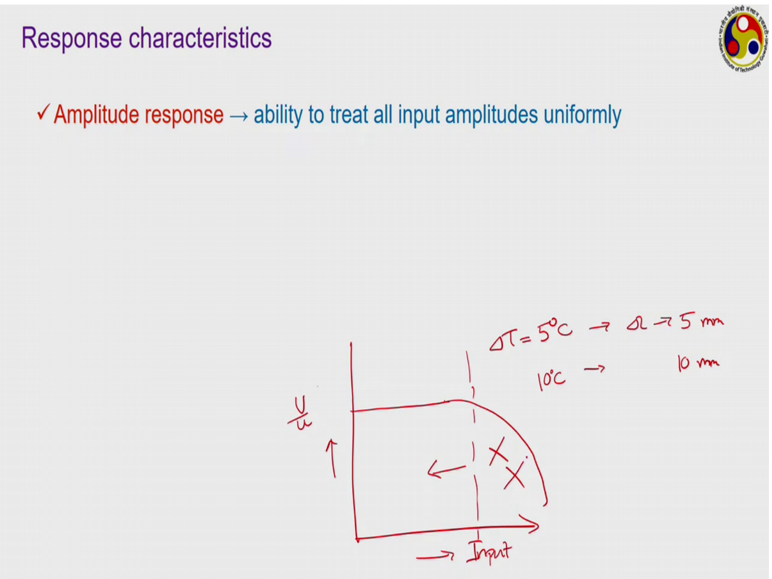

Now, the response characteristics. Input signal, so has a role to play particularly when the input signal itself is a quite complicated nature, we can expect complicated response from the output. And as we have already seen our input device may have some delay because of his own characteristics, which we shall be studying shortly.

We may have significant out of delay in representing the output, we may have a large amount of settling time to get the final output value. Accordingly, the system response will be dependent on the system nature itself. And response characteristics there are generally four terms or four definition that I would like to introduce.

The first is the amplitude response. It refers to the ability of the device to treat all input amplitude uniformly that means, suppose we have given a, we are going back to the thermometer reading. Suppose, the temperature rise is 5 degree Celsius our delta T is 5 degree Celsius, which leads to a change in length of 5 millimeter for this mercury column.

Then if our temperature reading is 10 degree Celsius, then it should show our 10 millimeter change in length on the column. This is what we refer as the amplitude response. Now, you may feel it is very very obvious, but it may not be true for the entire range of operation.

Like you may quite frequently have the situation, where say this is the input amplitude. And we are having this output by input ratio. Let us say capital U is output, and small u refers to the input. Over a large range of input we may get a constant value of this ratio, but when the input crosses a certain range, you may find this one to be dropping a bit. Then in that case then this device can be used only over this range, we cannot use for this range. Because, it is having a poor amplitude response in this particular range, but the amplitude response is very good in this range.

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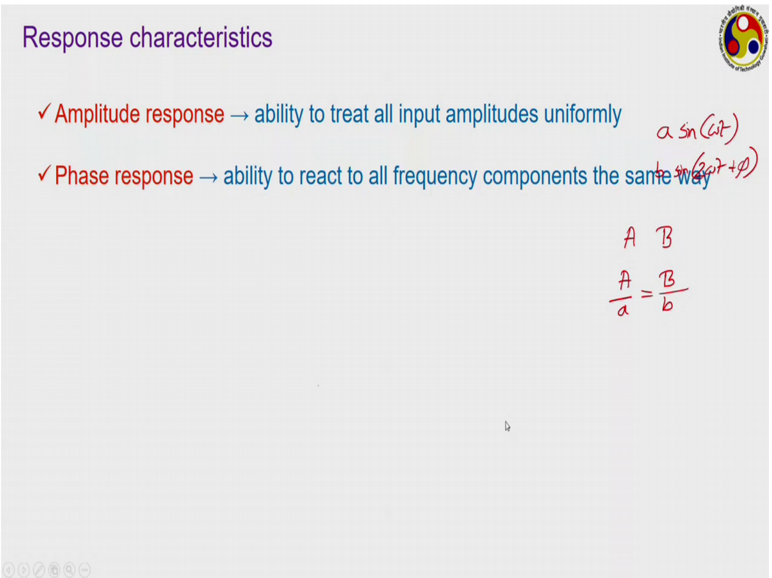

Think about two periodic signals, just we are repeating this similar kind of example. So, we have two periodic signal. One signal is having a form a sin omega t, and other signal is having a form a sin twice omega t plus some phi where phi is the corresponding phase difference.

Now, if the first signal is being represented on your device something like an oscilloscope by an amplitude capital A, and the second signal by capital B, then whatever is the ratio of capital A to small a that should be equal to the ratio of capital B to the small b that is what we refer as the amplitude response.

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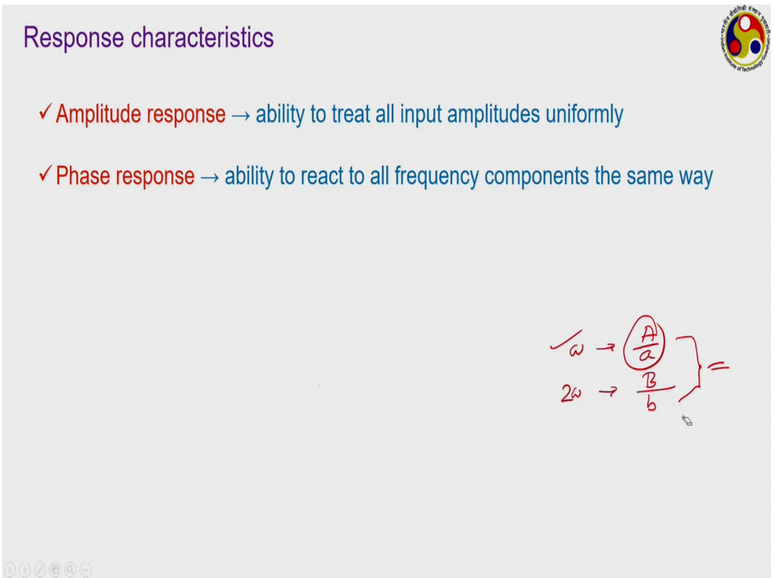

The next one in line is the phase response, it refers to the ability of the device to react to all frequency components the same way, and that means, what the example that I have just erased. For the first case, the frequency was omega for the first signal our frequency was omega, and it showed a ratio of capital A upon small a.

Now, when the frequency becomes twice omega, and the ratio is capital B of b upon small b. Then if these two are equal to each other, then we say that the system is a good phase response that is for responding to at least for these two ranges for because for both the frequencies, it is giving same ratio of output to input signal. This particular ratio is often referred to as the gain or the amplification of the input in terms of output. When the system is showing a constant gain for all the frequency that it is being subjected to we have an ideal kind of phase response, but similar to amplitude response that is also not always true.

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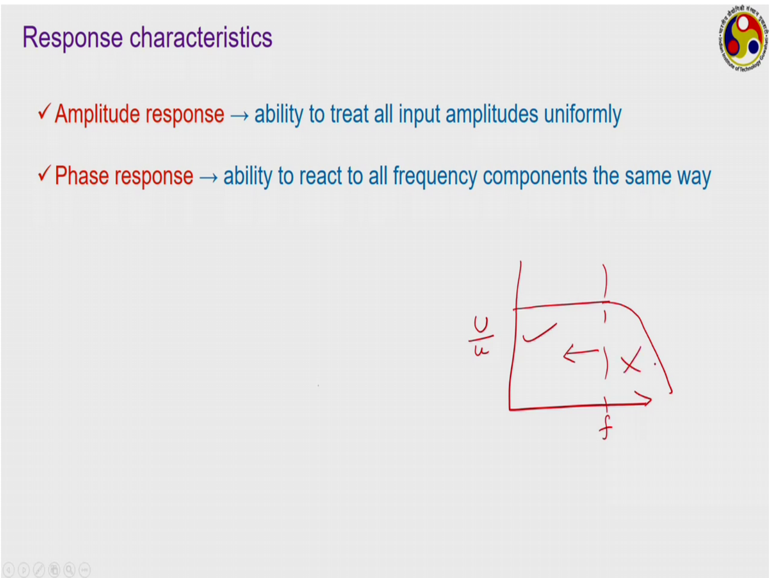

Like, suppose if we plot frequency on this axis is the frequency one axis, and this gain that is output by input on another axis, you may see that the device may have a good phase response over certain range of frequency. But a very inferior one after that that is so again we should use the device only in this particular range, but not in this range, because for this cross range the frequency response is poor.

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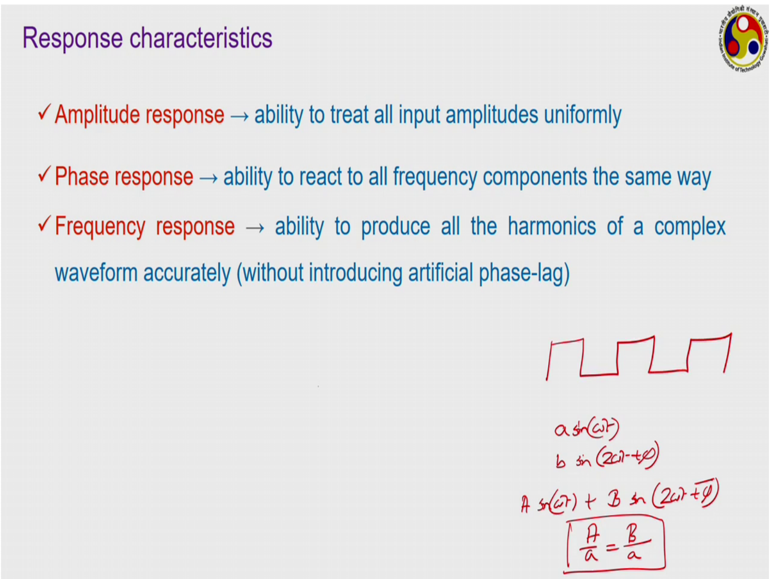

Third is the frequency response, sorry I you should you have used the term phase response in the example. Third is the frequency response, frequency response refers to the ability to produce all the harmonics of a complex waveform accurately without introducing any artificial phase-lag. Like again I am going back to the example that I had; I had one signal as a sin omega t, and the second signal is b sin twice omega t plus phi.

Now, suppose our device is having a good amplitude and phase response that means, your output signal will be capital A sin omega t plus capital B sin twice omega t plus phi, where the ratio of capital A upon small a is equal to capital B upon small b when this is satisfied, this signifies a good phase response, because for both the frequencies it is giving the same maintaining the same ratio of amplification or gain.

However, there is very much there is a large possibility that in the output signal that we are getting this omega will not be conserved, rather it will acquire a different value depending upon the internal characteristics of the system that means, the nature of the output signal that you are going to get that will not be the same.

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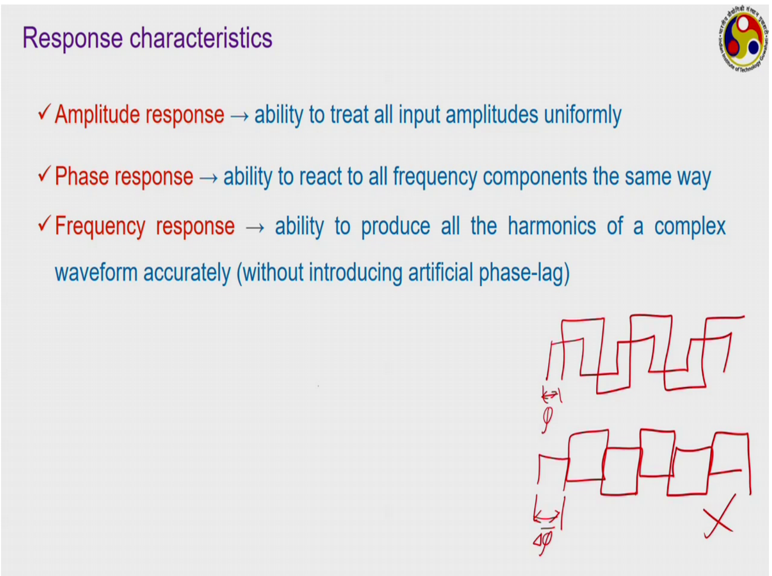

Suppose, your input signal is a square wave kind of thing. Now, if the signal is not having a proper frequency response, then it let me give you another example. Suppose, your input signal is a combination of two square waves, this is one, and the another square wave which is lagging this one by a certain distance, this is the lag.

But, if the system is having a pool frequency response in the output, this is what we are going to get for the first one, and the second one may lag it by a larger fraction. So, in the output signal what you are getting, it is showing this much of phi or this much of phase leg this, but in the true input signal you are having only this much of phase lag.

So, you are getting a wrong representation from the output device, this is what we refer to as the frequency response. This is again it dependent upon the frequency of the input signals, quite similar to the amplitude response, and we have to be careful about the ranges over which we can preserve the phase lag.

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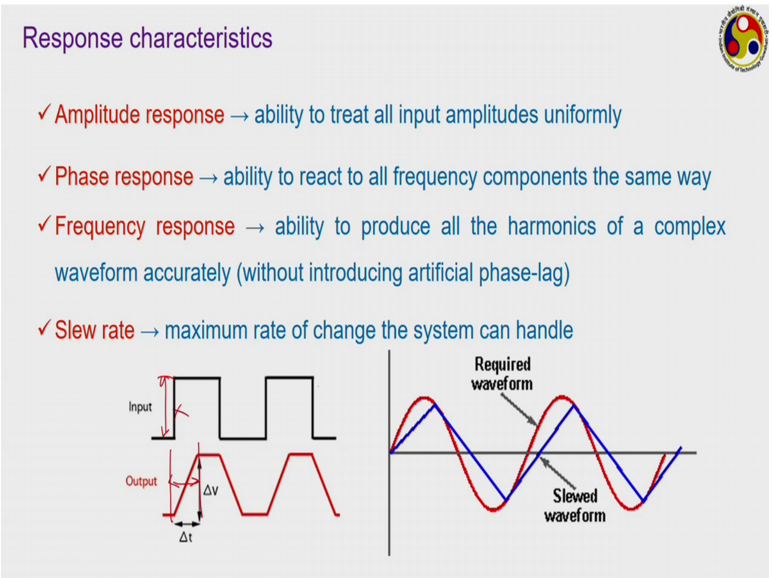

The last one is a slew rate, this is a maximum rate of change the system can handle, every system because of its own characteristic has certain rate of change that can be handled properly. Like one example, we are providing a square wave, but the system cannot handle such large change in input. So, it is showing the response that is low like this that is over this period, it is having a kind of lag.

And hence the vertical line, this particular vertical line is lost here. Another the situation here we are having a sin wave as the input signal, but the one that you are getting because of this limited slew of the system it is showing more a triangular kind of representation. This is what is slew rate representing yes.

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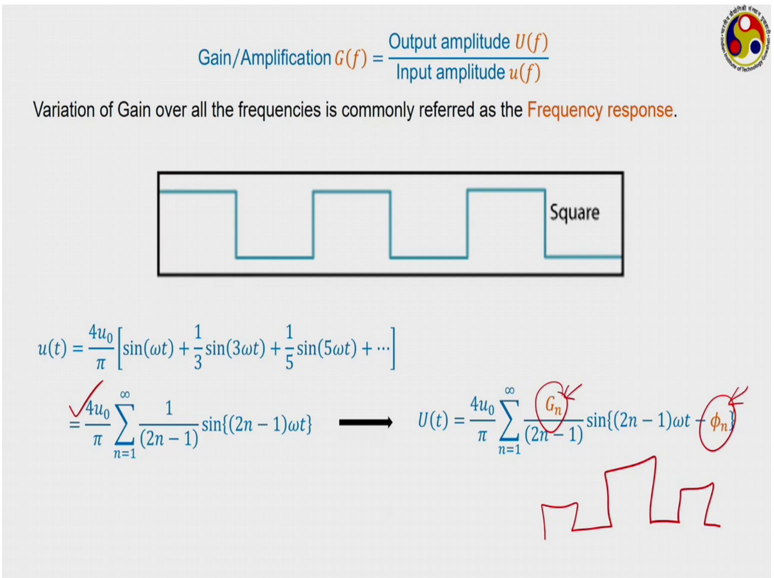

Now, as I mentioned gain or amplification is referred as the output amplitude to in fluid this ratio. Variation of again over all the frequencies commonly referred as the frequency response. Like and let us say it is a square wave, we know that the square wave is represented by a signal like this or we can sum it up over this particular form, which we have already seen in earlier slide.

Now, suppose the output device is putting a gain of G n, and a phase lag of phi n. If this G n, and phi n remains the same over all the frequencies, then in the output also the square will be retained. But, if this phi n keeps on changing with the frequency that is keeps on changing with n itself, then the square wave can be retained and or I should say the each of the successive signals will the square I should say the square of can be retained.

And similarly, if this G n is also varying within, then one signal may be something like this, next one came something like this, next one may be something like this, this or it will keep on changing thereby completely distorting the output signal. So, we should be careful about the about the range of frequencies over which we can have a good response of the system response from the system in terms of all of them, that is the amplitude response, gain response or I should say a phase response, and frequency response, and also we have to be mindful about the slew rate. These are the four characteristics of a dynamic system that you always have to consider. I repeat amplitude response, phase response, frequency response, and slew rate.

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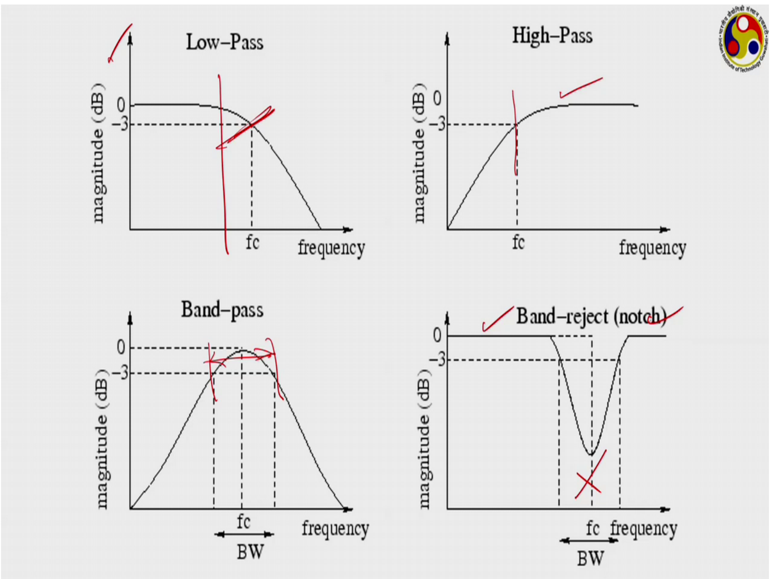

Just one example about how sometimes we can use the frequency response to our advantage that is what we have done in filters. Here this side we have the gain, and this on the (Refer Time: 58:20) of the frequency. Now, over this particular range the system is showing a constant gain. The gain keeps on changing, and after this there is a drastic drop this.

Now, we can use them sometimes as a filter that is when the amplitude drops below this particular value, the signal will not be allowed to pass through the other one. See here it is a high pass filter, when the gain is constant only after a certain frequency range like this. So, below all the frequencies below that are filtered out only these frequencies are allowed to pass through.

Even a combination of these two a band-pass, here we are having a decent gain only over this particular range. And so we are allowing only these frequencies to pass in others are all discarded or the opposite of that where we are having a constant again in this zone and this zone, but not in this zone, and we can easily filter out these situations. This way we can make use of the amplitude response or the variation of the gain with frequency to our advantage in certain situations. This kind of filters we often have to use in measurement systems.

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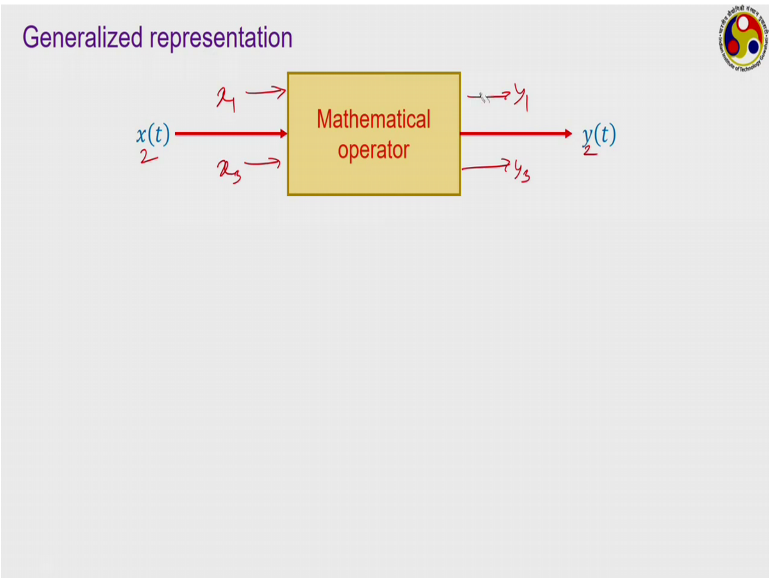

So, we know that our measurement system is acting like a mathematical operator on our input signal. They are by providing depending on the input signal itself and also about what kind of ethical operation it is doing, we are getting certain kind of output. It is possible that your system may be subjected to multiple inputs, like we are having an x 1, x 2, x 3, and we are getting multiple outputs like y 1, y 2, y 3 all being time dependent.

But, this kind of system analysis extremely complicated, and beyond the scope of the present course, so we are restricting ourselves to single input single output devices, where or we shall be considering this kind of cases effect of each input at a time. So, x is the input that we are considering, and y is the corresponding output that we are going to get.

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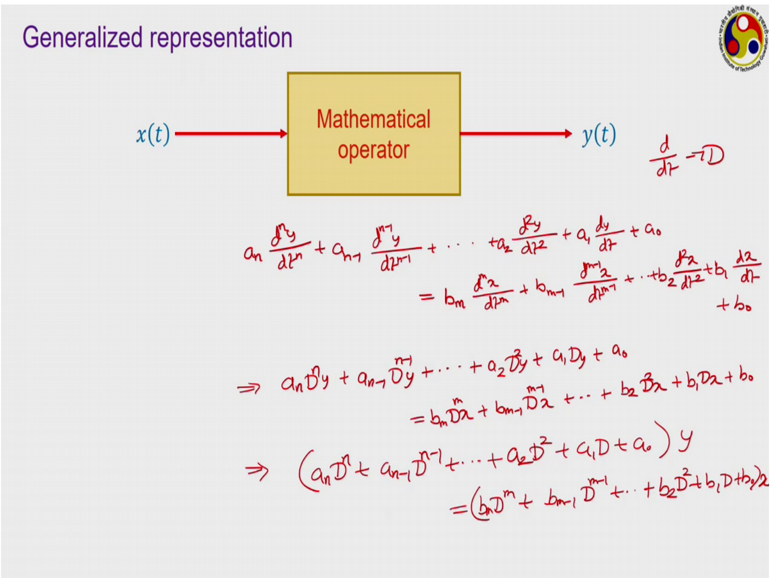

Quite often the mathematical operation that is being performed by your measurement system on this x to get the y can be represented in terms of a general ordinary differential equation, which can take a form like this. We can write some like this is the left hand side, this should be equal to the corresponding transformation on the right hand side on x, which can be let us take this to be as b m kind of form like this. Here n and m both are integers. They can be same, they can be different also depending upon the nature of the system.

Quite often we find it more convenient to represent or to replace this d dt operator as another operand capital D. If we introduce this capital D, we can write this one as a n D n y plus a n minus 1 D n minus 1 y a 2 Dd 2 y plus a 1 D y plus a naught is equal to b m D n x plus b m minus 1 D m m minus 1 x plus b 2 D 2 x plus b 1 D x plus b naught. Here this a n, a n minus 1 etcetera. And all this b m, b m minus on all these are coefficients, which generally are constant. If they are dependent on x or y or they can be dependent on x, but if they are dependent on y, we have a non-linear system to deal with.

Now, sometimes just taking analogy with the algebraic equations, we can write this one as is equal to b m b m minus 1 D m minus 1 b 2 D square b 1 D b naught into x. If we move forward with this, then what will happen? Just you have to keep in mind that here D is only an operand, this is the representation of or a modified form of this d dt, it is not an algebraic variable which we can find the value for. But it is just a common way of representing this particular system of equation or this particular symbol, but just I am going back.

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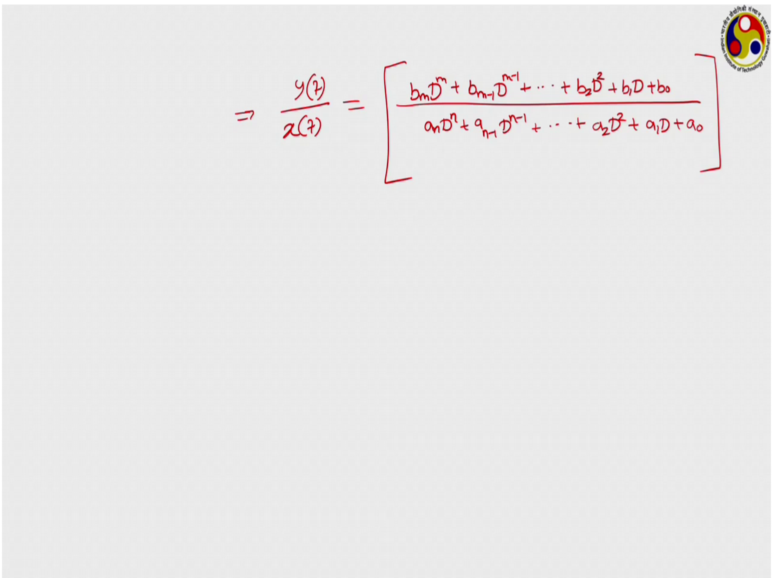

So, if we write this as y function of time divided by x function of time, then drawing analogy from the previous case what we can write? Here we have b m D m plus b m minus 1 D m minus 1, and in the denominator we have a n D n sorry a n minus 1 D n minus 1 plus a to d square plus a 1 D plus a naught. So, this is some kind, this is the representation of the operation that is being performed. Because, you can see here we have the output upon input, which is the amplification or gain that we are talking about. And on the right hand side we have is some kind of operation that your system is imposing.

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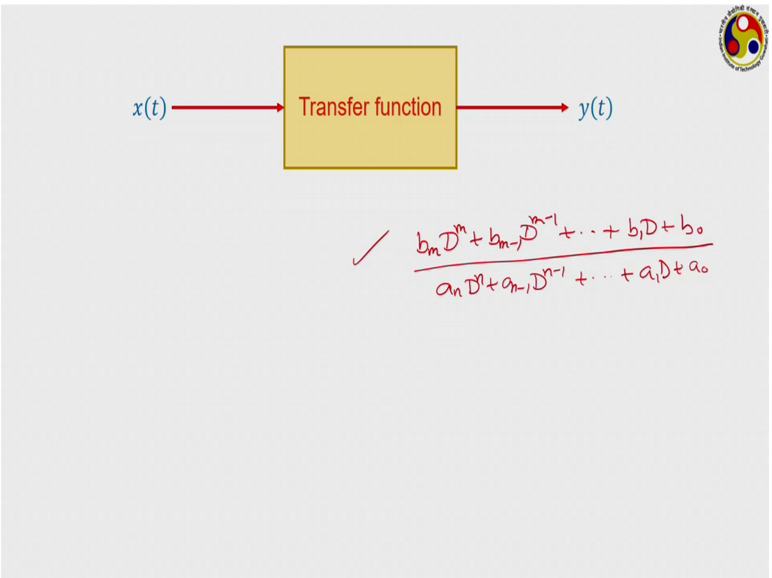

And this is what we refer as the transfer function. If I delete this entire thing, then I am sorry for this mistake, because this got superimposed with the previous one. So, in this particular example then what is your transfer function? Your transfer function is the right hand side that we wrote earlier, where we have this; these this are the numerator, and this is the denominator. This is the transfer function for this particular system.

We shall be taking it forward, I shall be repeating this particular part in the next class itself. This is just to give you a touch, and as I suggested earlier you please take a quick look at the Laplace transform, because for solving this kind of system of equations or ordinary differential equations, we have to make use of the concept of Laplace transform. It is not possible for us to analyze such detailed kind of equation, but we shall be talking about some very simple systems often referred to as a zeroth first and second order system. For this order refers to that we shall be talking in the next lecture.

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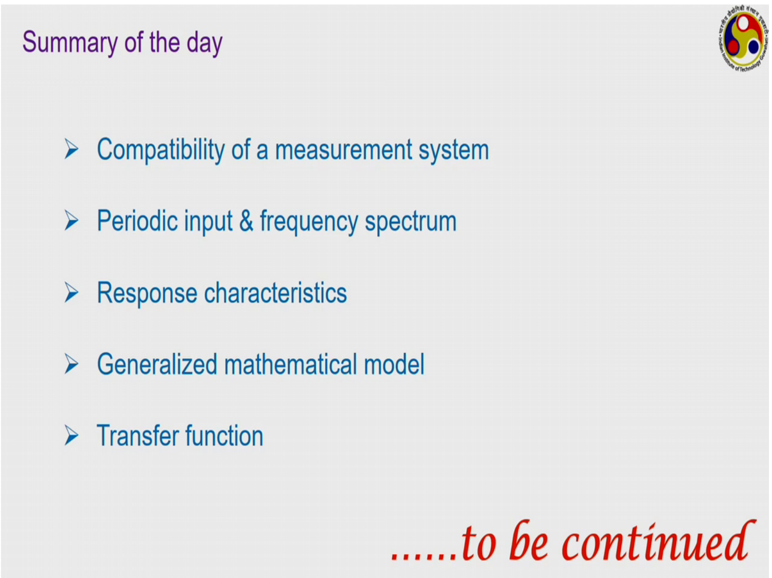

So, what we have done today? Today, I have talked about the compatibility of the measurement system. There are five kinds are comfortably that the system needs to be on the operation is to be looked at while selecting a measurement device. We have talked about the periodic input corresponding frequency spectrum, and there are multiple signals lumped into one.

We have talked about the response characteristics a system must possess. Then we have derived about the, we have briefly talked about the general mathematical system, and the transfer function. So, I shall be taking it forward in the next lecture, we shall be talking more about this mathematics and analyzing 0th and first order system in the next class.

So, thanks for your attention, see you very soon.