Principles of Mechanical Measurement
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Module - 01
Lecture - 04
Introduction to Measurement

Hello friends I hope all of you are well and I am sure you have already gone through the previous 3 lectures, I hope that you have understood the concepts that we have discussed there. If something is still not cleared if you want some more discussion to on a any particular concept please write to me in the portal I will try to respond at the earliest to them. Now, I was having a quick revisit to the earlier three recordings and I have found that I started a bit shakily in all the three lectures and settle down only after 10 15 minutes let me try to change the trend here bit in today's lecture.

My initial plan was to take 3 lectures on this first model on the or in this first week, on the topic of introduction to measurement, but as we are trying to add several or connect several different concepts or components related to this topic of measurement, in this particular week itself so, I decided to take a fourth lecture so, that we can have some more time and go through gradually in that if in different concepts.

Now, what we have discussed so, far like by their time as you have already studied the earlier 3 lectures, you are introduced to the significance of measurement you have learned about different levels of measurement like nominal ordinal interval and also the ratio levels of measurement, then you have learned about different methods of measurement different modes basically direct and indirect mode null difference and deflection mode.

And then we have discussed about the structure of a general measurement system and from there we know that any measurement system simple or complicated whatever it may be generally has three completely different or identifiable stages. There is one input stage which creates an some kind of interface between the physical device.

And your measurement system it receives some input signal from the physical device generally also receives another input signal from a so called standard. And then a different signal or so called transducer signal is supplied to the second stage. The second stage is signal conditioning stage, where the transducer signal go through one or multiple levels of alteration like amplification modification to some other form of energy etcetera.

Sometimes if some feedback kind of control is required, then a control signal is also sent back to the input stage from this signal conditioning stage itself, at the exit of the signal conditioning stage we get the deserved from the output signal. And finally, we have the output stage in the output stage it receives a signal from a signal conditioning stage and then creates an interface to the recording device and gives us the output in whatever form we want a display on a screen or maybe a printout of the observations or anything else.

Now, as any simple measurement requires the interaction between so, many different components there is always probability of having some kind of errors or inconsistencies or inaccuracies creeping into the final output that we are getting and like we have already seen examples in the previous lecture, even calculation on very simple quantities also may involved three four five different measurements.

Like just to calculate the power dissipation required by a domestic water heater or maybe a room heater, we all we have to measure maybe the current may be voltage by several resistances something like that or maybe just to measure the flow rate of water through a domestic water supply, again we may have to go through three for different measurements. And when we try to combine all these values in a suitable functional form to get the final output, then the inaccuracy small value amount of inaccuracy present in each of them can get multiplied and or can get combined into some suitable form followings of suitable pattern giving us a much larger amount of error which may be quite significant.

And that is why it is very important to understand the different kinds of errors that can be present in any measurement system and how to get an estimate of that, that is why instead of having just one I am taking 2 lectures on this error estimation thing, whenever you are talking about any kind of experiment any kind of measurement this measurement error analysis or so, called uncertainty analysis is a must. And that is why we need to have a clear idea about how to perform the uncertainty analysis and also how to identify the uncertainty present in the output of an individual instrument at the very beginning itself. Because in the next week onwards or rather in later weeks when we shall be talking about particular instruments, we always have to look for corresponding uncertainty values.

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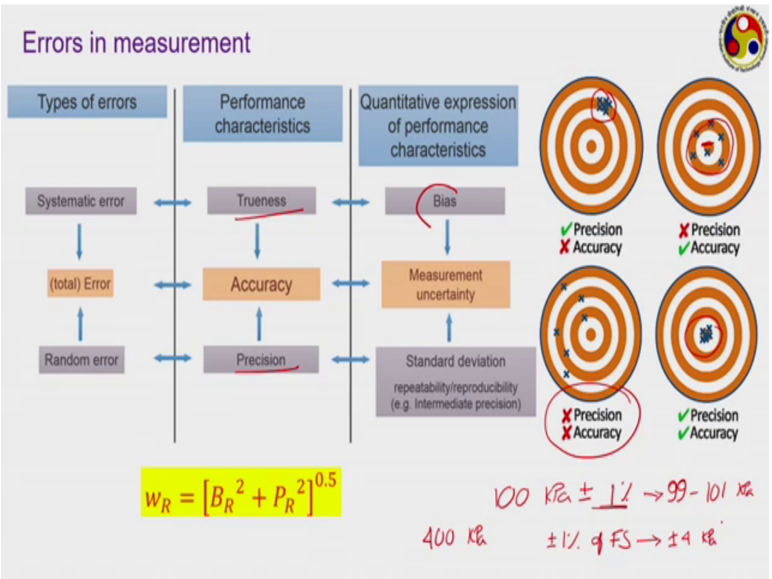

Unlike we have discussed in the previous lecture there are two broad kinds of errors that we may find in measurement. One is systematic error which generally is associated with very common terms like 0 bias error 0 drift sensitivity drift etcetera, measurement errors can be quantified we can easily identify their source and therefore, by the process of calibration we generally can identify or generally can eradicate or at least minimize the amount of systematic error from final measurement. The effect of measurement sorry effect of systematic error generally is to move the final recorded values into a certain direction or provide some kind of bias to the output.

And that is why they are quite commonly also referred to as bias errors. The other one is a random in a random error is difficult to identify from the source very difficult to quantify and they are as the name suggests a very much random in nature like using the same instrument, if we want to measure the same variable several times, because of the presence of random errors we may get different values. So, random errors basically affect what we call the precision that is because of the presence of random errors the different observations of the same quantity may get scattered around some kind of mean. They are by affecting the precision of the final output.

That is why random errors are also referred as precision errors. So, systematic errors are also called bias errors and random errors are also called precision errors, while systematic errors or bias errors affect the trueness of the final value random errors affect the precision. And these two combined affect the final accuracy of the observation, like the example that we have seen in the previous lecture when a system is having high precision that is very lower minimal random errors, but some kind of accuracy related issues issue or trueness related issue because of the presence of bias error, you may find all values are forming all output forming a nice clubbing, but actually away from the final readings. So, we are having some kind of systematic error present, but there is no random error.

Similarly, when this system is not having any kind of systematic error that is a highly true output that we are getting, but random errors are present. Then we may find that all the observations are well scattered and generally scattered around some kind of mean which may be somewhere here in this particular example.

Both the errors can be present simultaneously system and that is the generally most practical case, we always want our result to be both precision both highly precise and also true or accurate, but that is very ideal situations most often than not we have to deal with this particular scenario that is, it is neither precise not very true thereby affecting the final accuracy of the observation from both bias error and precision error point of view and that total error that can be present in a measurement is a combination of both.

We have already seen that there are several ways; we can represent the error present in a final reading. Say for example if we are using a pressure gauge which is giving us a value of say 100 kilo Pascal. Now, the inaccuracy present in this can be reported in several ways like one very one particular way of representing the error is the most common way is plus minus 1 percent. It indicates that the, what whether the while the instrument is reporting a value of 100 kilopascal, but actual value may range between may be anything between 99 to 101 kilo Pascal.

That is plus minus 1 percent of whatever we are getting actually whenever we are getting such kind of plus minus some percentage or some fraction be careful about what be careful about the details about this. Most often than not this one refers to plus minus 1 percent of the value, but there may be several situations where actually it is given as plus minus 1 percent of the full scale of your instrument.

That means, suppose the same pressure gauge can give you a reading up to 400 kilo Pascal. So, its full scale or the maximum where it can record is for 100 kilo Pascal and if it is having plus minus 1 percent error of this full scale; that means, actually the error that it may encounter is 1 percent of this maximum value that is plus minus 4 kilo Pascal and that is fixed for any kind of value that you are trying to measure.

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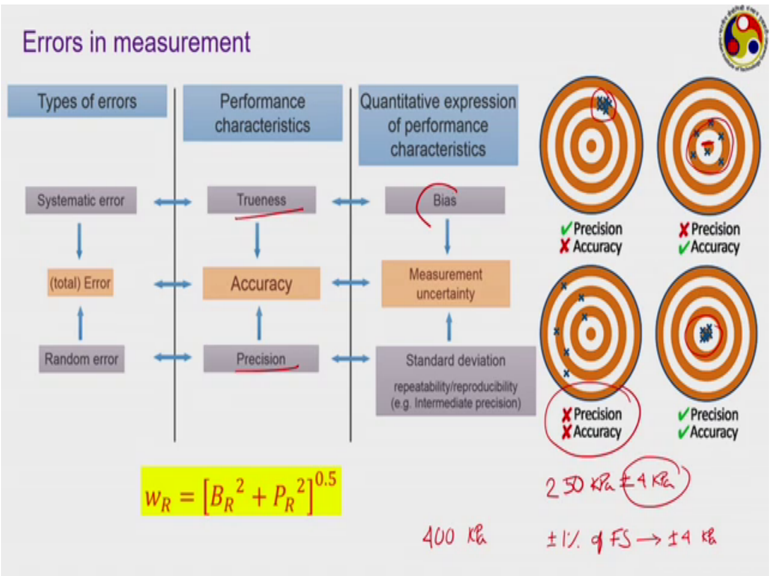

That means if you are trying to measure or if your output is something like say 50 kilo Pascal, then also there is possibility of having plus minus 4 kilo Pascal observing error or if your output is 250 kilo Pascal. There also you may have the same amount of error creeping in. So, when it is given as plus minus 1 percent of the full scale then we have to consider this fixed quantity of error.

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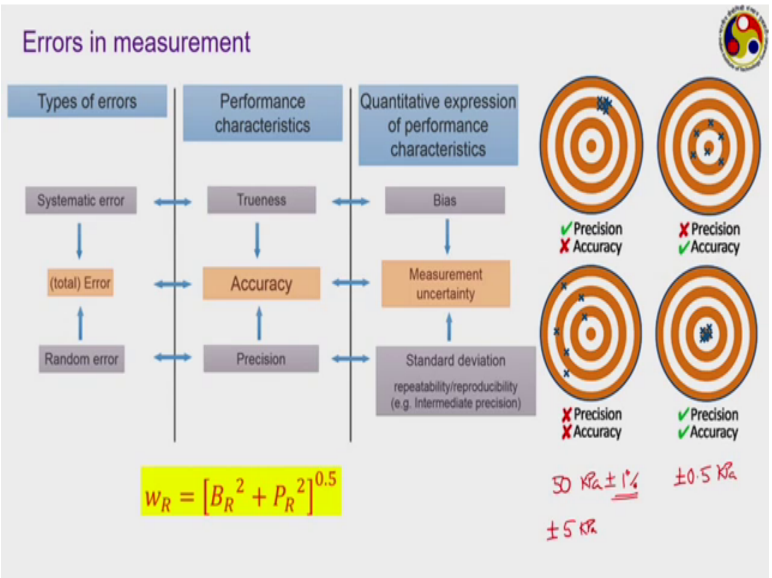

However in this full scale part is not mentioned, then the quantity of corresponding error will keep on varying with the output like when the output is 250 kilo Pascal, then it will be plus minus 1 percent of that that is corresponding error is 2.5 kilo Pascal whereas, if your output is in this 50 kilo Pascal it is 1, it is 0.1 percent of sorry it is 1 percent of that that is 0.5 kilo Pascal.

And the other kind of way of so, this percentage is one way of expressing the error rating of one instrument. And the other one is we have also seen like it is also possible into your instrument is having a fixed error of 5 kilo Pascal, where the error is given a in terms of absolute quantity.

So, any value you take that is always you have to consider this 5 kilo Pascal of error on either side of the measured quantities, it is quite similar to percentage of full scale; that means, there are three ways you can get the error specified like the examples.

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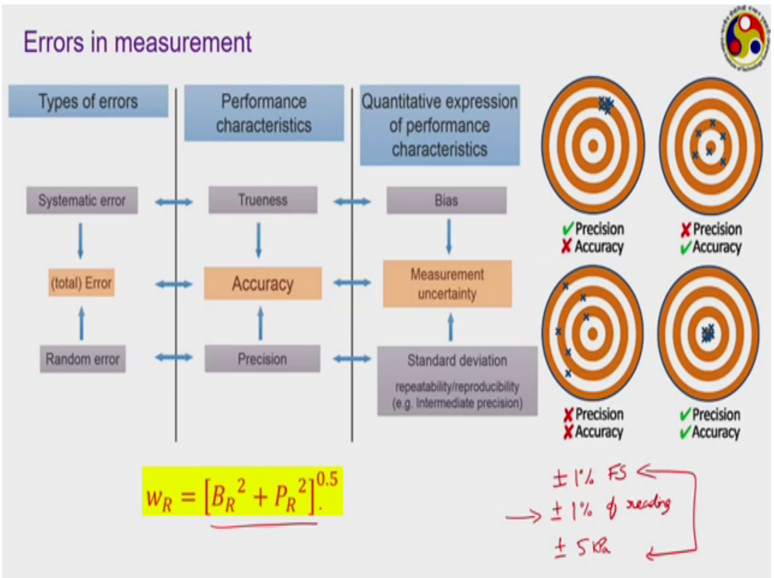

That we have discussed one possibility is plus minus some percentage of full scale which basically leads to a fixed quantity, whatever is a range of your instrument or the maximal output you can get this 1 percent of that is fixed, you can get plus minus 1 percent of your reading that is recorded value whatever you get.

So, it will keep on changing on what you are measuring or the other one plus minus 1 percent of an absolute quantity like say plus minus 1 percent of high kilo Pascal. So, both these two will lead to a fixed quantity of output in any measurement independent of the measured quantity being small or large whereas, this one will keep on varying error will be small for small output magnitude it will be large in larger output magnitude, but I come back to this particular one.

Now, in the previous lecture we have discussed about how to identify the uncertainty in a final measured quantity or I should say when we are trying to combine several measurement in to get one particular value, one particular calculation done. Then how to calculate the final error in our error in the final quantity.

There intentional I mentioned that the estimation of systematic errors, but I would like to correct that now, I intentionally did that just to avoid any kind of complexity actually what we did there that is the estimation of this total error this W R is a total error not systemic or bias a systematic or bias error or precision error separately, we actually estimated the total error present in the final calculation.

And also if you look carefully like the examples that we have discussed in all the cases we never talked about any individual instrument rather, we talked about getting observations from several instruments and then combining their to get some theoretical calculation done for a separate quantity, like calculation of power when we have the measured value of current and voltage available.

So, what we did there that is commonly referred as propagation of uncertainties in a final measurement, like we have some uncertainty present in the current value that the corresponding instrument has given. We have some uncertainty present in the voltage value that corresponding voltmeter has given and now we are combining them to give a measurement of power or a calculated value of power.

And following the method that we have discussed that will show us how the uncertainty present in both the individual measurements, like in power in like in current and voltage will affect the finally, calculated value of power. So, what we did there that is basically the propagation of uncertainties in the final value, we are going to see about how to estimate the precision error for a particular measuring device in today's lecture.

But before that this bias error or systematic error, we are not going to talk about because they generally appear, because of wrong calibration or wrong information provided by manufacturer or maybe you are having a correctly calibrated instrument, but with a long period of use or with wear and tear it may develop some kind of bias error. So, by repeated calibration or recalibration we can eradicate or minimize the systematic errors.

So, dealing with the systematic errors is a part of calibration process and we are not going to talk too much about that, but precision error or random error are related to the measurement that we get the individual measurements or a group of measurements. So, we are going to deal about this one today using some statistical knowledge.

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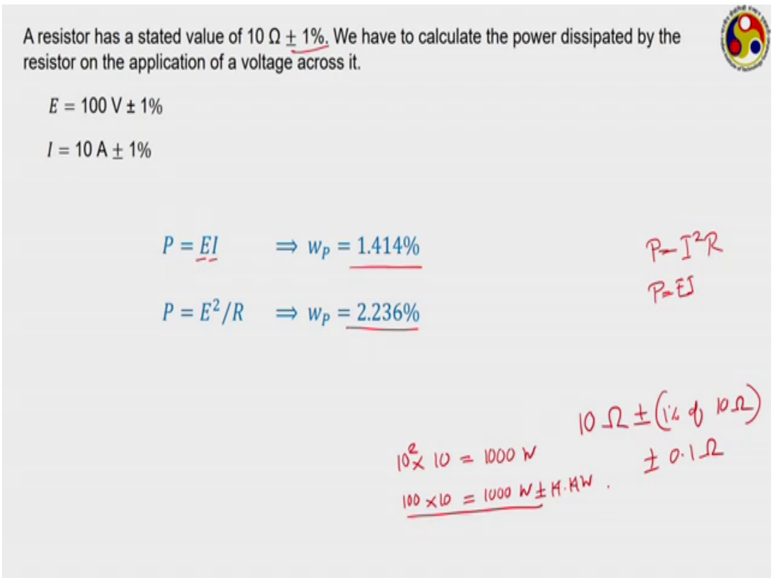

But before we do that in the last lecture I left you with one a couple of problems, this was one of them and there was another one related to the venture meter application have you tried to solve it. I request you please solve it if you have not done that yet, stop or pause this video here itself go back to the earlier video and try to solve these two problems.

So, that you have a better idea about how to do calculate the propagation of uncertainties or how to estimate the total error that may be present in a finally, calculated value. So, it is a request here if you have done that then please continue with this video, but if you have not done that then I repeat please pause the video here go back to the previous lecture and try to solve this two problems on your own.

So, here I gave you the answer here the problem was you have a register, where the values are given for these resistance, it is given a 10 ohm and plus minus 1 percent error associated with this it is not mentioned that this is of plus minus 1 percent of the reading or our full scale, if nothing is mentioned then we are going to consider this as 1 percent of the reading itself that is 10 ohm is the value of the resistance and so plus minus 0.1 ohm that is actual error that is associated with this resistance value is plus minus, it is 1 percent of this 10 ohm that is plus minus 0.1 ohm is the error that is associated this resistance measurement.

If it is full scale then that has to be exclusively mentioned. And there are two ways we can calculate the power dissipated across this, one using like we have discussed one where we can just measure the current that is flowing through this resistor and then we can calculate as I is equal to I square R. Other way you can calculate it is we can measure the current flowing through it and also the potential difference or voltage drop across this resistor and we can power is P equal to E into I.

So, finally, calculated value should be same like if we follow the first method what you are going to get in this case current is 10 ampere and resistance is given as 10. So, we are going to get following the first method it is 10 square into 10 and that is going to be thousand watt. Similarly if we follow the second method you have 100 as a potential difference into this sorry into 10 amperes current and that is again going to give you 1000 watt which is the nominal value of the final power output.

So, you may feel that both ways are equivalent we can go by any of them definitely, they are equivalent theoretically, but if you perform the uncertainty analysis on both the calculations in the first case when your following P equal to EI that is this particular one you will get a an uncertain develop 1.414 percent, I am giving you the value please calculate this and check whether you are getting this one or not, this is what I have got through my calculation, it can be around that one also if you are getting something between 1.3 to 1.5 that is fine, but any value either outside that range is a there has to be something wrong with your calculation.

Now, if you calculate following the other one P equal to I square that is that seems much easier, because we here in this case we have to measure both the E and I, but if we follow the second method we have to calculate only the current, but here we are getting a higher at least 1 percent more error or either possibility.

That means, while this first case P equal to E I is going to give you an error margin of something like plus minus 1.41 percent of 1000 watt. So, it is going to give you something like 14.14 watt whereas, if we follow P equal to E square by R oh sorry I have done as E square by R.

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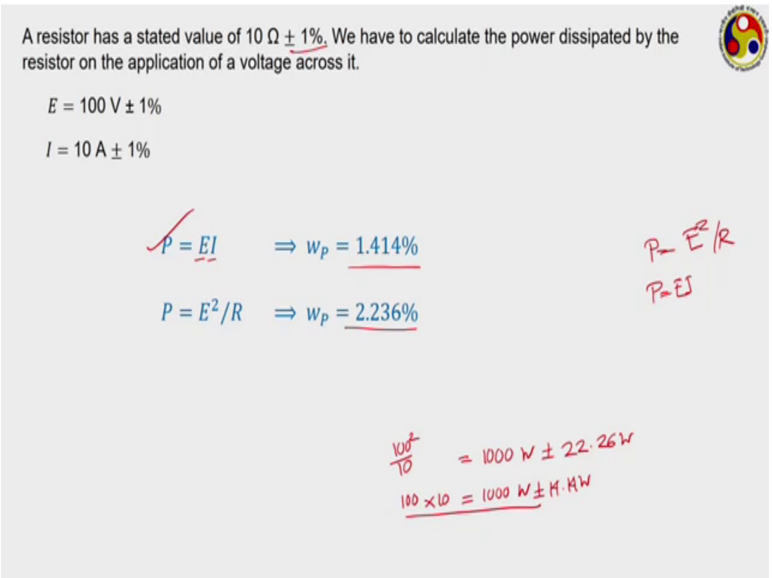

So, if we do E square by R it is only using the potential so, it is as E square by R then it remains as your E was 100 square divided by R again you are going to get 100 watt also, but corresponding error possibility is 22.36 watt.

So, much higher error value about 8 watt higher error prediction that you are going to get. So, your this way we can identify which way to follow despite we need to do two measurements, it is always better to go for this measure both potential difference and current and accordingly means of the power.

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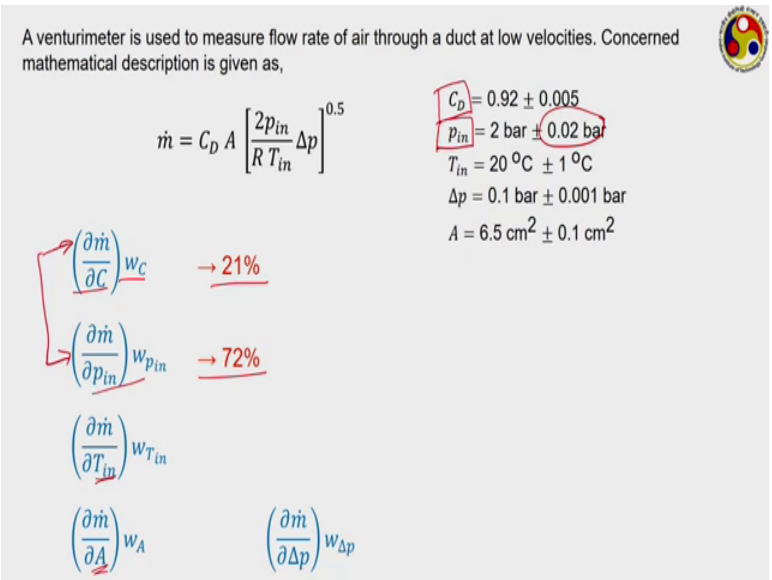

And this another problem was given for venturimeter again there are five different parameters which can five different measurements that we have to do to get the final value of mass flow through mass flow of air through the duct.

So, for we have to calculate basically 5 quantities, each for each cases try to calculate the product of sensitivity and corresponding uncertainty like for this coefficient of discharge, this is a sensitivity these is a corresponding uncertainty. Similarly we can get the product for all the 5 quantities calculate their value and do the and finally, calculate the uncertainty if you do calculations properly you will find that the measurement of this inlet pressure or the contribution of inlet pressure will be around 71 to 72 percent of the final value whereas, that for this coefficient of this is about 21 percent. That means, the final uncertainty that you are going to get 93 to 94 percent of that will be taken care of by only these 2.

Others are having much smaller contribution to the final error. So, that provides us a way of tackling or choosing on instrument. Like in the previous case the previous example you have seen a way of identifying the measurement method, despite we have to measure two quantities it is better to go for the both voltage and current measurement instead of only voltage measurement whereas, or only current measurement also.

Whereas, in this particular case you can see there are 5 quantities where some uncertainty we are getting, but instead of looking after or trying to improve all 5. If we look at only the this particular quantity that is if we look at reducing this particular uncertainty then we are going to get the largest gain in the final output. And once we are able to handle this inlet pressure measurement, then probably you should look at this, this cd the coefficient of discharge. And if we can reduce the uncertainty in both this quantities present at least to the half of the given value, there will be a drastic reduction in the final uncertainty present in final mass flow calculation.

So, there is no point going for improving the uncertainty this area measurement or this temperature measurement, we can always do that, but the effect on the final mass flow rate will be very small, because much more significant effect is coming from this inlet pressure and the coefficient of discharge and we should look into them first.

So, this way we can if I which are the most vulnerable instruments in a particular measurement system like. In this particular case the most vulnerable instrument is your inlet pressure measuring instrument. And we probably have to try a better instrument which has much smaller level of uncertainty please try to solve this problem, now let us move on to check the random errors.

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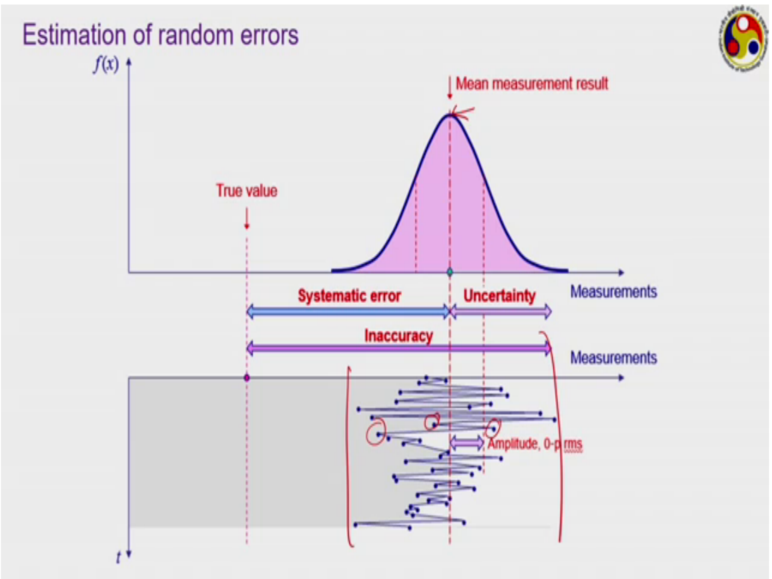

Now, the random errors as the name suggests again they are very random in nature, like in a particular quantity this is your true value like shown by the red arrow and you are getting the your measurements we restricted in this zone with a mean like this, then this actually is a measure of your systematic error.

The distance between the true value and the mean can provide you a way of the measuring the bias error problem is that we often do not know the true value of your measurement, if you are trying to measure an unknown quantity you can definitely get the mean from a set of readings, but we do not know the true value. So, we do not we cannot estimate the systematic error only if we know the true value then we can estimate the systematic error which is the process of calibration. So, by the method of calibration we can always eradicate.

Now, once we have the mean the precision errors will lead to this random fluctuations which you can see here, the readings keep on fluctuating on either side of the mean over a some suitable domain and ins you can just check any random value like this is one particular set of reading and immediate next reading we are finding somewhere here and immediate next one is somewhere here. So, there is over a over this range of data set that have been collected there is significant amount of random error that is present and because of this random nature we can treat them only because of i some kind of statistical measures.

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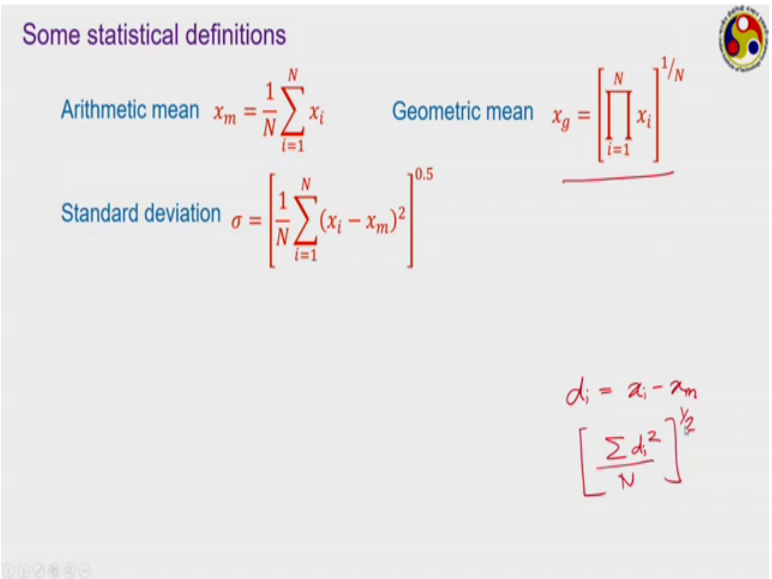

So, some various fundamental statistical definitions which all of you are definitely aware about. Let us say we are dealing with N number of data this capital refers to the total number of data that we have collected or we can collect in a certain situation, then arithmetic mean refers to the average of all these values whereas, geometric mean is refers to this quantity we shall not be using geometric means, I am not talking about this and the standard deviation talks about the average deviation. Like that deviation di in a particular measurement is x i minus x m where x i refers to the measured value and x m is the mean which is shown there.

So, standard deviation basically talks about the summation of this d i square divided by N the total number of reading whole to the power half, this is what a standard deviation gives us a way of measuring the total amount of deviation that can be present in a reading. But to use these definitions of mean and standard deviation or to use them for further statistical processing, we actually need to have a very large value of N. And practically in industrial measurements or in engineering measurements often we have do not have the option of going for so, many measurements rather what you do is sampling.

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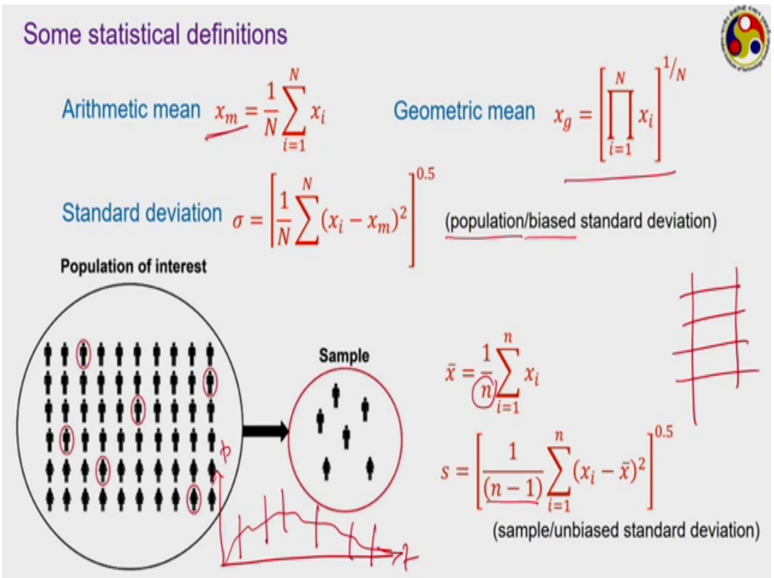

A sampling is this. Like suppose we want to know some characteristics of this huge population or huge number of data points that are possible, as it is not possible to handle with so many data. So, we just take some random samples from that and try to do all the calculations based upon that. And then whatever observations whatever conclusions we get from our statistical processing on this sample.

We try to get that back to the population to get some kind of conclusion about the population, this is what we measure no what we call as sampling. Now the definition of standardization that we have provided earlier that refers to very large value of N basically the number of sample number of parameters number of data points available in your population.

That is why this definition is often referred as population or bias standard definition; however, if we restrict our calculation only to the sample, then for the sample also we can calculate a mean and standard deviation problem here is the value of this mean for the sample x bar, what we are going to get that may not be equal to the arithmetic mean x m because that will depend upon your choice the choice of your sample. Say we are trying to measure the value of let me take an example. Suppose we are trying to measure the pressure of steam that is going x that is going out from a boiler and going towards the turbine.

Now, we are trying to monitor the pressure value and that we can do using any suitable measuring pressure measuring instrument we know that pressure we can be measured at every time interval. So, depending upon the resolution and time stability of your instrument, you can measure the pressure after every one second after every one millisecond every after every one microsecond and thereby providing an infinite set of data, but practically it may not be possible to handle so many measurements. So, instead of handle instead of measuring so, many data we decided to measure just 10 or 12 pressure values after every one second.

So, we have 12 values of pressure listed in a table like this, where we have a pressure value at every point. Now the value of this x bar that we are going to get depends upon which values that you are going to get, like suppose your pressure is varying with this way plot tip time on this axis pressure on this axis, pressure is varying in this way instead of taking such a continuous measurement which will lead to the population and infinite data set we are taking sample at certain points.

So, if our take you take sample one here another here another here, then what mean or what value of x bar that you are going to get P bar in this case the average pressure, if your choice of pressure is this one this one and this one your choice of sample then the value of x bar may be different.

That means the value of this x bar is dependent on the chosen sample and dependent on the total number of samples small in that you have collected. So, this x bar may vary with sample and as the number of samples that is the value of this small and keeps on increasing this small x bar approaches the actual arithmetic mean that is when small n approaches capital N, your taste to capital N x bar tends to x m.

Similarly the standard deviation definition of standard deviation also should include n, but instead of n we are using n minus 1, because here in the denominator we have the degrees of freedom while dealing with the population degrees of freedom is equal to the total number of data points, because the value of x m is independent of total number of data points, but while leaning in the sample the value of x bar is dependent on the value of small n. So, there is 1 degree of freedom that is less that is basically one kind of restriction that we are putting the number of data points.

And that is why we have degrees of freedom as 1 n minus 1 and this is called sample or unbiased standard deviation. So, while dealing with a smaller sample we generally go for this x bar and small s.

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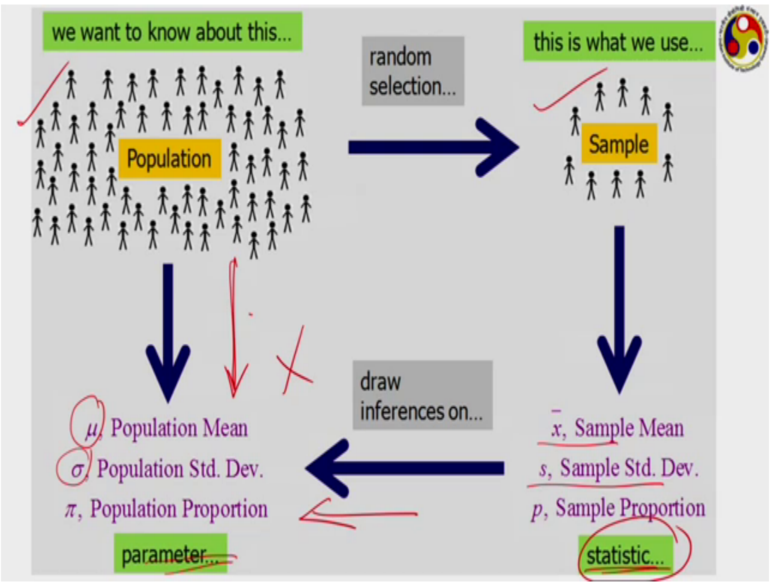

So, this is what is your population, if it is large data set from there we have randomly selected a small data set from the small data set we are calculating a sample mean the sample standard deviation, we are drawing some inference from there applying the statistical measures. And then we are trying to project that to get the population mean population standard deviation and some art and other parameters about which we want to get some kind of idea.

We could have got that directly from the population itself, but there you have to deal with a very large data set that is why you always do the smaller data set in the form of sampling. And in the choice of sample is proper then whatever you are getting from here you should get the same thing from here also; however, if the choice of sample is not proper then these two may differ from each other.

So, sampling itself is important which samples to take and also it matters which kind of statistics you are applying on the samples let us take one example.

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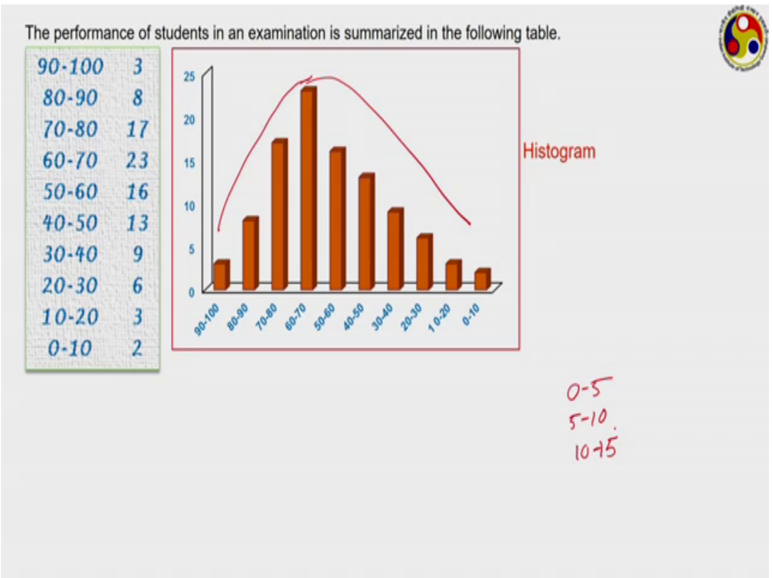

Here I have provided you a set of data for some students appearing in exam, there are a 100 students in a class and each of them has got some kind of marks in the exam, exam was conducted out of 100 as well and instead of dealing with individual marks of all the students, we have identified for 10 different groups the first group refers to students who have got marks between 0 to 10 we can see there are 2 students who have got that second group refers to students getting marks between 10 to 20 there are three students getting that similarly going above this way we can see between 70 and 80 17 students are performed which is quite good and between 90 to 100 the highest category the best category only 3 students have got.

So, we have got the of we have decided on 10 different groups 10 different intervals often referred as and we know the number of samples present in each of those intervals. Now, if we represent them in a column chart like this, where each of the column corresponds to one of the groups like this one corresponds to this particular group, this one corresponds to this particular group this number 13.

And accordingly we can say between from the chart also it is from the initial data table also it is shown between the 60 to 70 the maximum number students have got the marks. And on either side of this we can see a nice distribution that is the number of students appearing in a group is highest here, then is a decreasing on this side it is going now on this side as well. This particular kind of diagrams are known as histograms, which gives us a quite good estimate about the characteristics of our number of samples appearing in each interval here we have only 10 groups or 10 columns appearing in the histogram, but we can easily increase the number of columns using the same data set.

How we can do that like instead of taking 10 intervals if we take more number of intervals, let us say we decide on the groups like now how many squares have gone between 0 to 5 then between 5 to 10 then between 10 to 16; that means, this way instead of 10 we can have 20 intervals. So, we can have some numbers appearing in each of them it may be 0 in some it may not be 0 in others.

And accordingly we can again get more number of columns appearing the same histogram 20 column at least. So, even a better representation for this. And if we break it into even further that is treating each number separately how many student has got 1, how many student has got 2, then there will be 100 intervals appearing on the same thing. They by giving an almost continuous representation of this particular graph is not it, where we are just trying to plot the samples or the intervals on the horizontal axis and number of students appearing in each interval on the vertical axis, then as the number of interval increases it approaches the nature of a continuous curve.

Sometimes instead of dealing with the individual numbers we deal with fractions. Now, here what this table represents can you guess from the values like the first one that is up to 10 we can see 0.02 represents just 2 percent student have got below 10 or up to 10, then 5 percent student have got between 10 to 20 not is below 20. Similarly 72 percent student have got marks below 70 that is this is a cumulative sum that we are doing.

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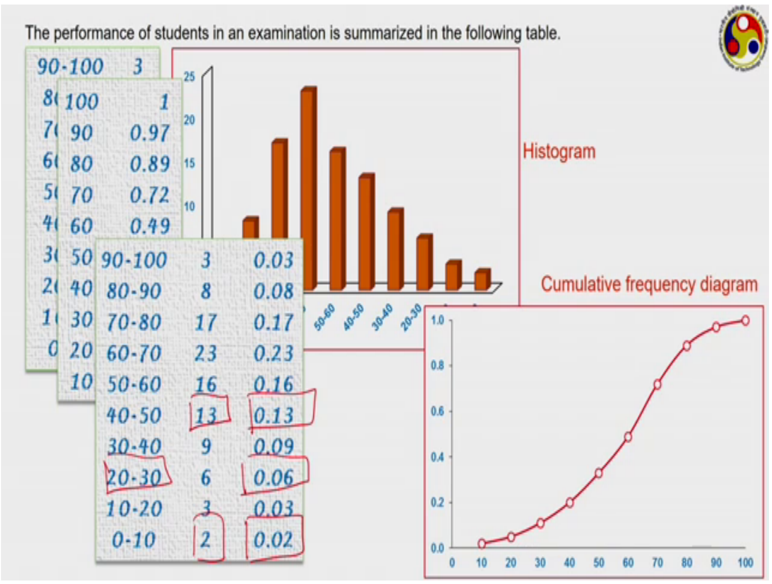

And if we plot that with respective of samples, then we can see a nature like this which is known as cumulative frequency diagram, this represents the cumulative frequency that is appearing like if you look at the diagram if we plot a vertical line from here, then on this side the area under the curve here or instead of drawing an arrow.

Here in the vertical axis if we plot the numbers of students then we have this particular area will give you the total number of students, we have got marks less than sixty here as we are plotting fractions, then this is going to give us some idea what number of students who have got marks less than 60. Similarly when you reach 100 if we delete all this if we reach 100, then all the students are inside this; however, the area under the curve is not going to give you total number of area of the curve under this curve is not be equal to 1 because that keeps on varying it is a cumulative frequency only that we are dealing with.

So, instead of with cumulative frequency generally prefer in dealing with individual frequencies. So, we are going back to the original table and just adding one column to that, like here and there are two students in the bottom most group and these are percentage they are appearing, there here we have 13 students appearing in the group 40 to 50 and that refers to 13 percent of the total data.

So, here the third column the values that if another third column that represents the frequency of a student appearing in the corresponding interval, like the frequency of a student frequency of students appearing in a frequency of sorry a student appear in the interval of 20 to 30 is 0.06 or just 6 percent.

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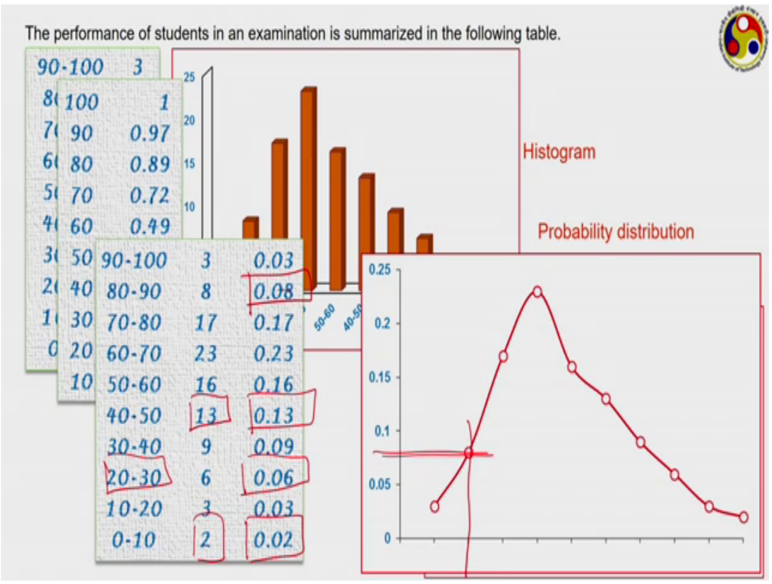

So, once we plot that with respect to the individual intervals, then we get a distribution like this it clearly shows at which interval how what is the frequency of appearing like, if we see this then this refers to something like this actually I have actually missed the bottom column, but or the horizontal axis here, but I guess this data refers to this particular one probably sorry it refers to this particular one correct.

That means, if we have this horizontal axis properly denoted here, then we can clearly see within this particular interval which is 80 to 90 in this case and the number of students appearing or the probability of a student appearing in this is in this particular interval is 0.08 or 8 percent, this particular diagram is called a frequency diagram or often called a probability distribution, because this is giving only the probability of students appearing in a particular interval or probably of a data point appearing in a particular interval.

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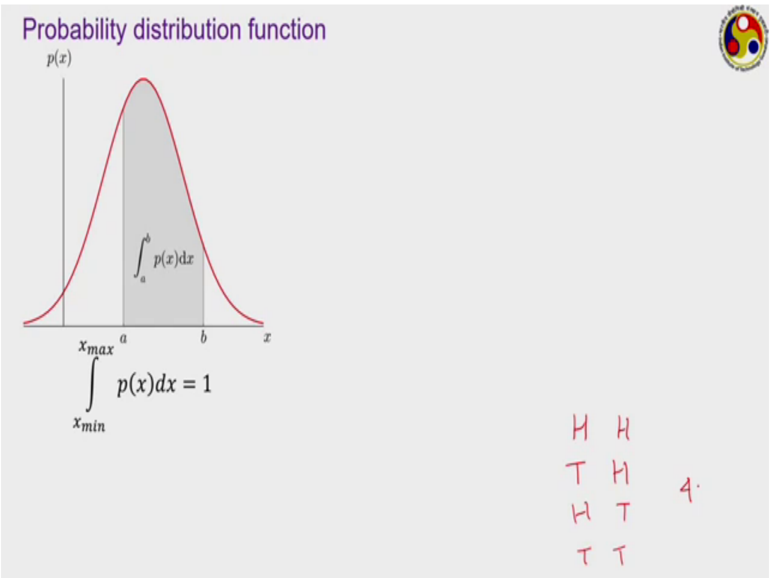

So, that takes us to something known as a probability distribution function, quite often probably solution functions are having a form like this, which are having a peak somewhere and generally decreasing on either side p is a prioritization function, then the total probability of a data appearing between this a and b is just the area under this particular curve. And so the probability of all the data points appearing under this within the limit of x minimum to x maximum that is a range of x has to be equal to 1.

Because it encounters all the possible probabilities or all the probabilities that something can a phenomenon can happen, like you can think about say you are given with a coin an unbiased coin. Now, if you are tossing that coin, then what are the possibilities you can either get one head and or you can get a tail. So, there are only two possibilities. Now, if I provide you 2 unbiased coins and you are tossing them simultaneously, then what results you are going to get? You are going to get either 2 heads or tail in the first head in the second or head in the first tail in the second you can get 2 tails. So, there are only 4 possibilities.

So, this way when you are tossing number of coins, we can combine all of them to get the probability of a particular combination appearing this. And the area under the curve will give you the total number of probability.

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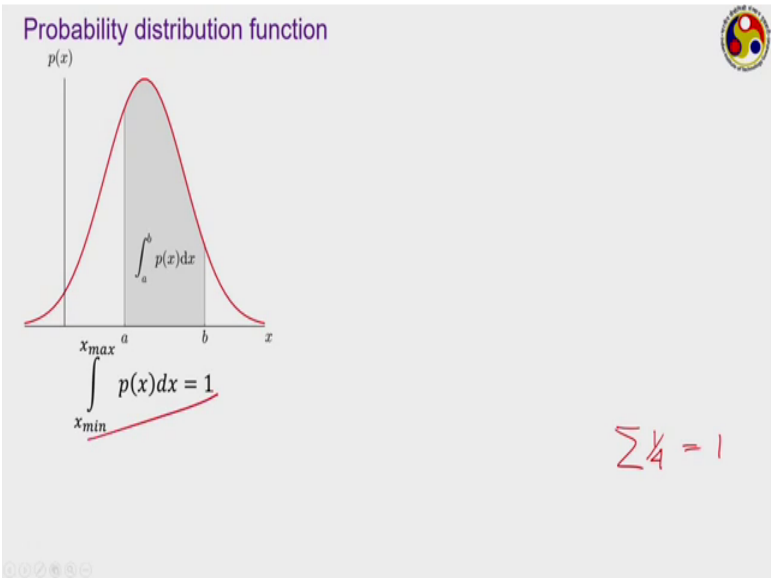

Like here there are 4 events that are possible and the probability of each of them appearing is just 1 by 4. So, if we sum it up over all of them that has to be equal to 1 which is given by this particular thing. There are several kinds of probability distribution function that can be defined, there are several that are used in statistics also, but in measurement purpose the one that is most commonly used is the Gaussian error distribution form.

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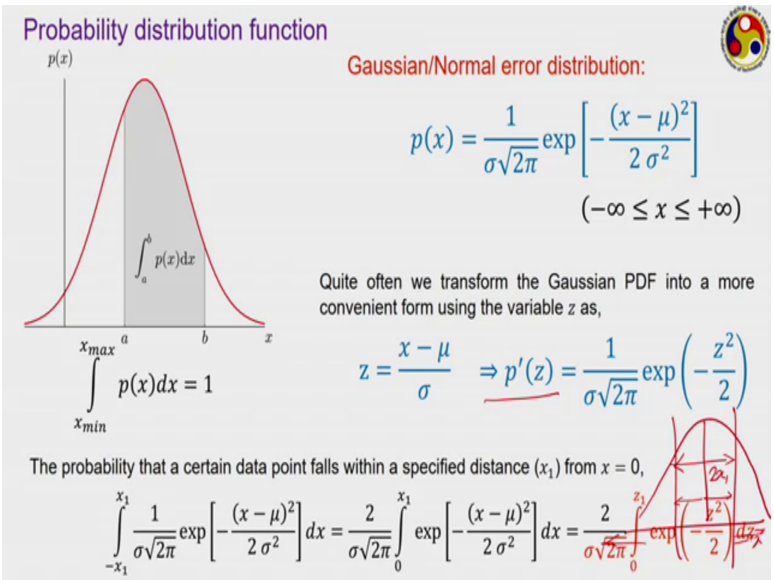

Which is also called the normal error distribution from, because several natural phenomena also follows this particular kind of friend, it is a function of a form like this here sigma refers to the standard deviation the population standard deviation because here we are performing over a very large set of data, or when x tends to minus infinity to plus or ranges from minus infinity plus infinity; that means, our data set is an infinite 1. And here this mu refers to the x m I actually I used x m earlier this refers to the mean the arithmetic mean or based on a population.

So, this is the Gaussian error distribution quite often, we transform the Gaussian pdf in a more convenient form using a variable z such a z equal to x minus mu upon sigma. So, if we put the expression of z there then we get a modified function p prime z which is having a form like this. Range of z can be defined based up on x itself, but as x ranges from minus infinity to plus infinity z also will be ranging from minus infinity plus infinity.

Now, the probability that is certain data point falls within a specified distance x 1 from 0, before doing that if we see the Gaussian distribution as the range of x is minus infinity to plus infinity the plot of p x will be somewhat like this. This is your x we are going to minus infinity on this side plus infinity on this side and this is x equal to 0 when you are having the peak of this curve appearing. So, if we are trying to identify one point whether it falls within a certain distance x 1 from x.

So, your x 1 can be measured in this direction and also in this direction; that means, we are talking about a band of thickness of with twice of x 1 that is x 1 on positive side and x 1 on negative side. To identify that what we have to do we have to integrate this particular p x over this interval that you have to integrate it over minus x 1 2 plus x 1.

And if we integrate this over this interval, then it being this particular curve being a symmetric on we can take this two out there by changing the integration limits from 0 to x 1 and then using this transformation of z it comes to a form like this, 2 upon sigma root 2 pi integral 0 to z 1 exponential minus z square by 2 dz.

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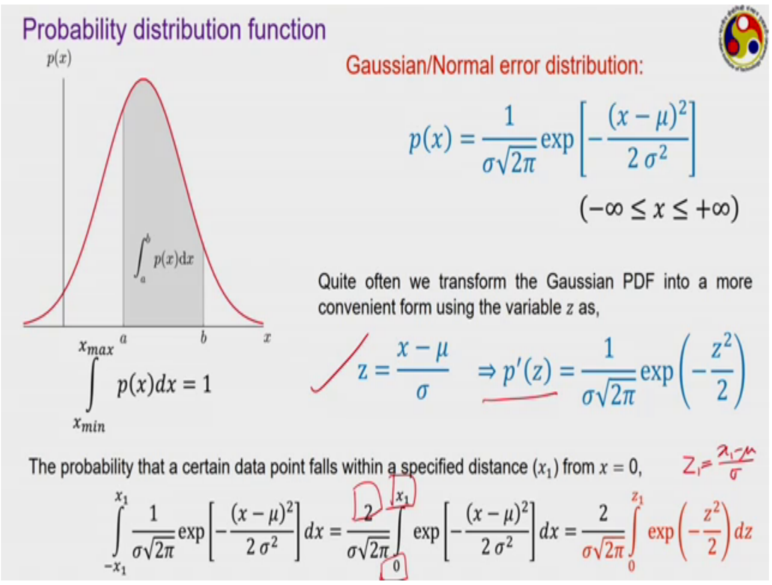

Where this z 1 is nothing, but x 1 minus mu upon sigma quite often z is also combined with sigma itself or we get directly the measure of z on open sigma. Now, here I have marked out this particular portion in red separately because of course, for any value of z starting from 0 to infinity we can perform this integration easily. And we can get the value of this integral, but it is not required because people have already done this and there are standard tables available for this Gaussian distribution who had the magnitude of this red port point portion is already available like shown here.

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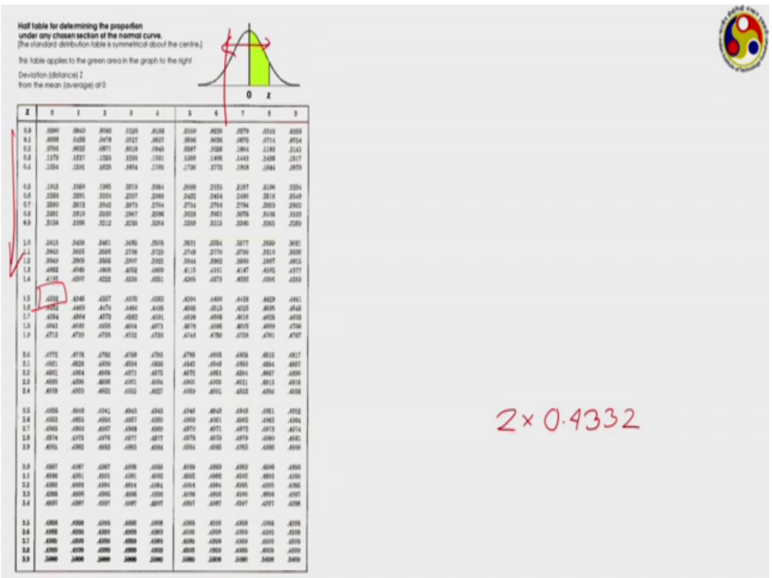

Here we can see the values of z given here that z refers to just integration between 0 to z. So, we can perform integration in three possible ways, if our trial we are trying to identify the other value rise only on the right hand side, then what is shown here. That is the one if our interest is to identify the probability of a data point appearing on the left hand side within a distance z, then we just have to do the same thing, but on the other side the probability also will remain the same, but if our interest is to identify not left or right, rather whether just what we mentioned earlier whether it will lies within a certain distance z.

Then we are basically trying to identify over this total quantity to z and hence the any value within this has to be multiplied 2. Like suppose if you are trying to identify whether your data point lies within a distance z equal to 1.5 from the point z equal to 0 or x equal to 0 then for z equal to 1.5 this is the data probability we can see that probability is 0.4332 and this being a 2 sided probability.

So, we have to multiply this one with 2 thereby giving the final probability that we are looking for as z keeps on increasing this one approaches 0.5. So, that the two sided probability approaches one as you can see for z equal to 3.9, it is equal to 0.5 and; that means, all the points will come under this.

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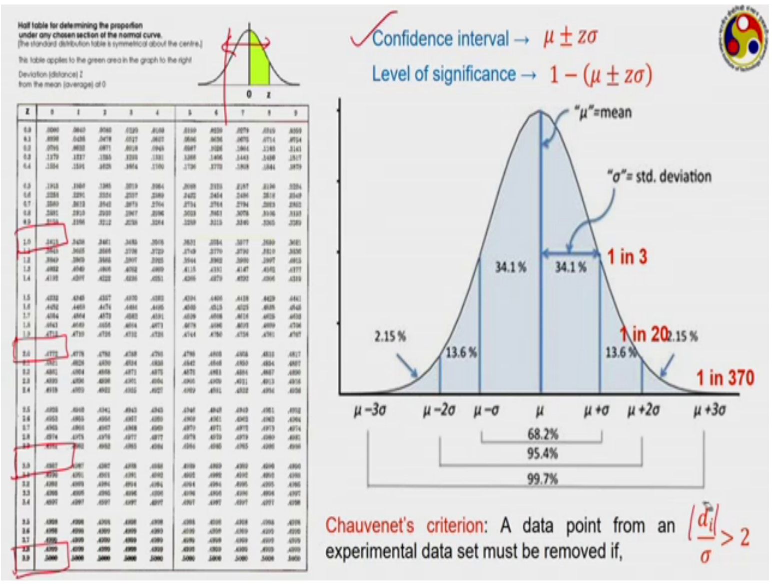

That means we can say the peak of the curve which is the mean once we can identify the z, we can clearly say that the data point will be lying within mu plus minus z into sigma where the z is something that will be coming from the, you are coming from the measurement values that you have. This particular thing is known as the confidence interval and what is left out is called the level of significance.

This is something that where we have first pick z equal to sigma itself, that is the standard deviation itself, if you perform the integration I am going back to this particular curve this is z equal to sigma you can say this 0.3413. So, 34 percent 34.1 percent of the total data will lie between your mean and mu plus sigma another 34.1 percent will lie on the negative side.

So, that is a total of 68 points something percentage of data will lie within a plus minus sigma distance from the mean, if we expand this to twice of sigma, then if we go back to the table twice of sigma is giving a 0.4772. So, if we multiply with 2 we can see that 95.4 percent of all the data will lie within a plus minus 2 sigma of your mean, we can keep on expanding this like if we go to 3 sigma, it is 0.4987 that is 99.7 percent of data will lie within a distance of plus minus 3 sigma distance.

That means, if your measured values are following this Gaussian distribution, then we can clearly say that or we can confidently say that out at least 68 percent of the all the measured data should lie within plus minus sigma distance of the mean. And 95.4 percent should lie within plus minus twice sigma distance from the mean, there is another way of representing the same sometimes they represent in terms of odd like as 68.2 percent datas are lying within plus minus sigma distance; that means, the probability or the odd for a data to lie outside that in just 1 upon 3.

Similarly 95 percent data is lying within plus minus 2 sigma distance. So, the probability for a data point to lie outside that plus minus 2 sigma range is only about 5 percent that is an odd of 1 out of 20 and that is for plus minus 3 sigma limit it is 1 out of 370, that is once we expand our range to plus minus 3 sigma, then hardly any data will be left out. Now, the question is which one we should choose that is depends on your choice of this confidence interval, depending upon how strictly we want our data to be considered we have to determine the range of observation.

Like if we see that if we decide that two sigma is the one that we should go for, then we can see that 19 out of every 20 data should be lying within our range and then we can take this plus minus 2 sigma or as our limit of operation or any z value also we can calculate. So, once a dataset is given then we can calculate the z corresponding to the large corresponding to all the points by preferably to the points which are showing the largest deviation from the mean, we can calculate the z.

And then we can check the z with all this limits sigma or 2 sigma or something and then depending upon our choice whether should we at all consider the data or not, we have to make some kind of decision. One particular criterion was specified by chauvenets just a very strict criteria his idea was if any data point is showing a deviation more than 2 sigma, then that has to be some kind of blunder or some kind of random points that is coming in so, that should be discarded during the final processing.

So, any data point which is showing this particular thing that is di per sigma greater than 2, or I should say the absolute value of di that is mode di upon sigma greater than 2, then we know that 95 percent of the data should be should have d upon sigma less than 2, but only 5 percent data can show this kind of criterion and should discard them thereby causing a change in both your value of x min and your stand corresponding sample standard deviation and thereby providing a much better measurement.

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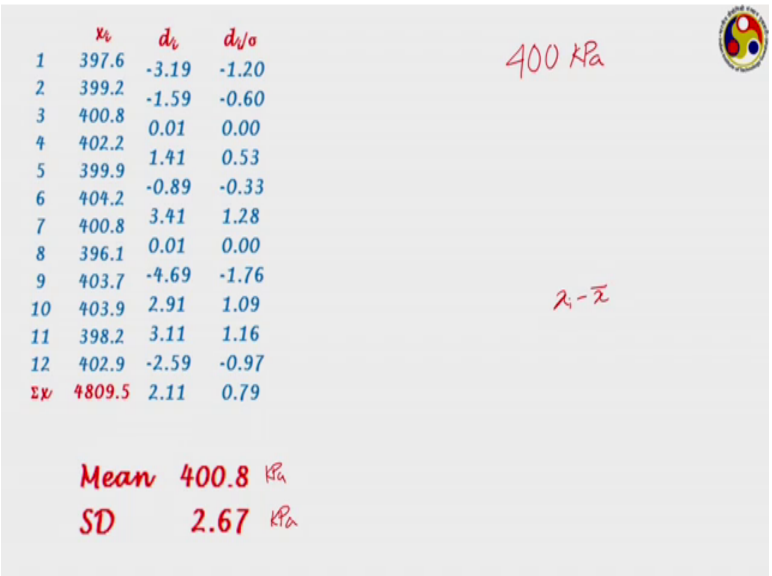

	x_i	d_i	d_i/σ
1	397.6	-3.19	-1.20
2	399.2	-1.59	-0.60
3	400.8	0.01	0.00
4	402.2	1.41	0.53
5	399.9	-0.89	-0.33
6	404.2	3.41	1.28
7	400.8	0.01	0.00
8	396.1	-4.69	-1.76
9	403.7	2.91	1.09
10	403.9	3.11	1.16
11	398.2	-2.59	-0.97
12	402.9	2.11	0.79
Σx_i	4809.5		

Mean 400.8 KPa

SD 2.67 KPa

Here we take a sample like that pressure measurement example that I mentioned about here, we have measuring the pressure 12 times correspond. And our objective is to maintain a pressure of 400 kilo Pascal, but from our reading we are seeing that on the for all of the 12 we are not getting four 100 any of them, but they are fluctuating on either side.

So, we calculate the mean and standard deviation mean is coming to be 400.8 kilo Pascal standard deviation is 2.67 kilo Pascal I have not mentioned the units here, but I should have standard deviation is 2.67 kilo Pascal. So, we can plot the Gaussian curve to check whether there which ranges they are following, but which point is having the largest deviation let us check whether any of the data point need to be discarded during the final calculation or not that we first have to check.

Now to check that which point is having the largest deviation, let us calculate the di values we have calculated di values that is x i minus x mean di refers to your x i minus x bar because here we are talking about a sample. So, we are having this d i and once we get the di and then we are we are estimating d i upon sigma. So, this is the value of d i upon sigma that we have these are table of the i upon sigma that we have. And we can it got shifted a bit actually all the data points you please try to address a correspondence like this one should correspond to this particular case and all got shifted in a way probably some problem with the software that I am using for this data processing.

Now, which one is having the largest amount of deviation if I go through it, this one is having the largest deviation of 4.69 largest division is 4.69 kilo Pascal.

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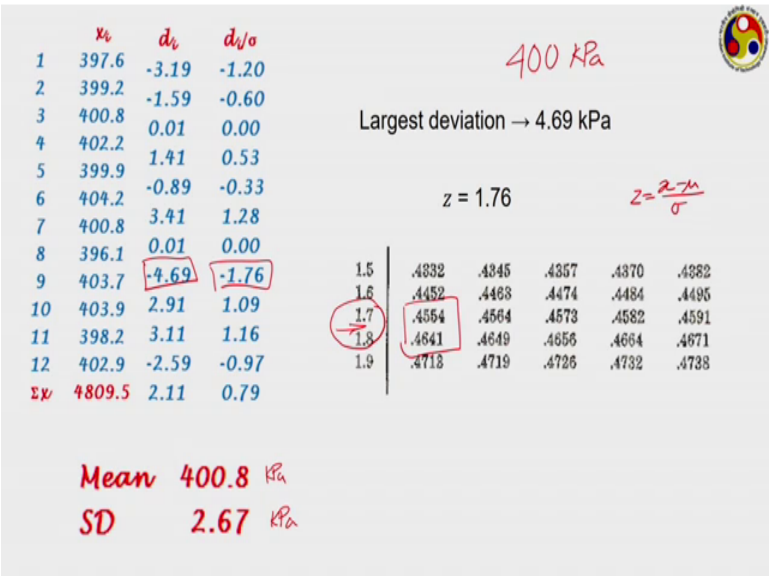

So, let us compare this even for this one also we can see that the d upon sigma value is less than 2. So, there is no point discarding this data we can continue this particular data. And now using this we can calculate the z from this z it comes to be 1.76 what is the relation for z, for any particular value of z how he calculate it z was x minus mu upon sigma. So, from there we can calculate z to 1.76.

Now, we try to identify the corresponding value of the probability z is 1.76. So, in the first column of our table we have z and here we do not have 1.76, but we have 1.7 and 1 and 8 so, we have to do kind of interpolation or 1.7 is allowing something in between. So, the probability will be something in between these two.

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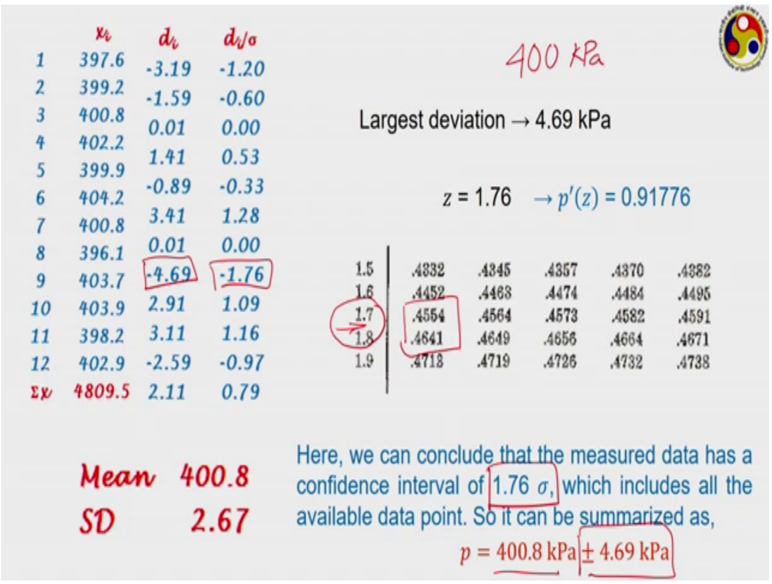

We have to do an interpolation between that and correspondingly. We are seeing that the probability is coming out to be 0.91776. So, which is a quite good value to have. So, where we can conclude that the data has a confidence interval of 1.76 sigma that is all the data point that we have accumulated during the measurement lies within plus minus 1.76 sigma distance of the mean and there is no need of discarding any data and finally, we can represent all this data somewhat like this, this is a mean sample mean and this is the corresponding error that we can expect.

So, the table of data that we have correspond comprising of 12 pressure readings that is having a mean of 400.8 kilo Pascal and that is having an error of 4.69 kilo Pascal. And at this in particular case this particular this particular example this error of 4.69 kilo Pascal is not a very significant one, because it is coming only within that plus minus 2 sigma limit.

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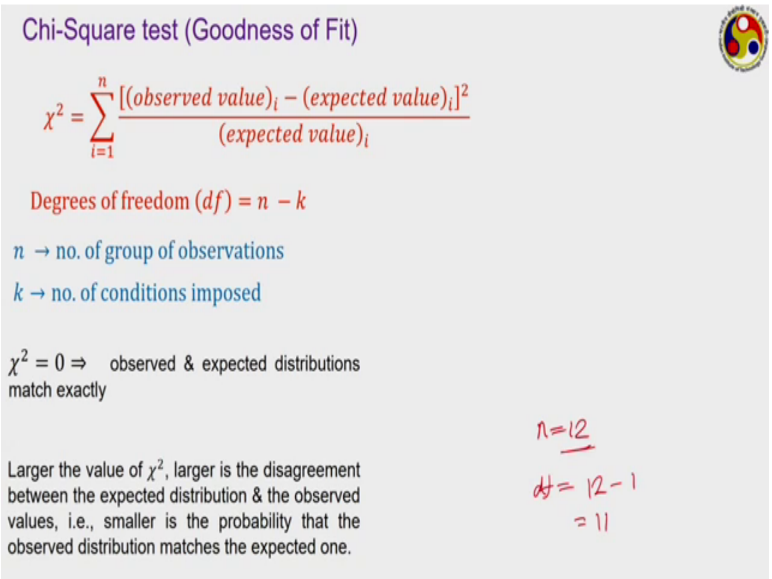

Next another test that I will cover very very quickly of course, we can compare any data set with the Gaussian distribution this way, but we have to know whether this data set at all fits any Gaussian fits the Gaussian distribution or not. And that is generally given done by the chi square test which is also a test for goodness of fit here, this chi square is given as the summation over the observed value minus expected value whole square divided by expected value here small n refers to the total number of samples we have and another term is defined as a degree of freedom which is n minus k here n is the as I mentioned total number of sample we have and k is the number of conditions that you are imposing.

When chi square is equal to 0 then your observed and expected distributions match exactly like, if we expect our data set your observed will be coming from whatever our measurement device is giving expected is what we want. Like suppose the if we go back to the example of that pressure reading our expected values 400 kilo Pascal for all of them, but observed we are getting something else.

So, accordingly over this set of 12 data that is small n equal to 12 we can calculate the chi square value, if chi square equal to 0 then that is an exact fit as the chi square keeps on increasing that shows an increased discrepancy between the observed value and the expected value. And if the chi square is larger than a certain predefined limit, then we can not go with this particular set of observation, but before the degrees of freedom as we have used in the previous case, like when we are having we are deciding on n equal to twelve there itself we are putting on restrictions in number of in terms of this total data set.

So, your degrees of freedom in this particular example will be 12 minus 1 that is equal to 11. So, we have a data set from where you can calculate the chi square we can also calculate the degrees of freedom to be 11. And now we have to refer to that chi square table this tables are very standard I shall be trying to provide you some standard table what you can get this tables on internet itself.

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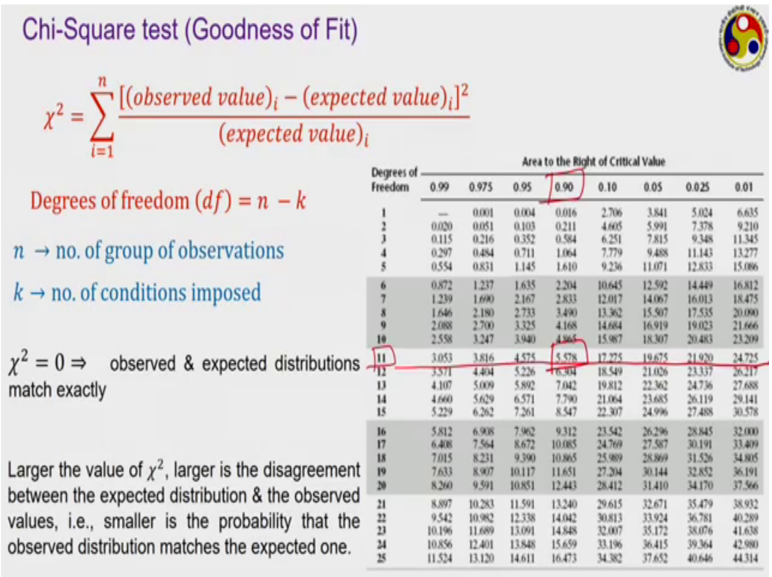

Here is a chi square table here first try to see what we have on the first column we have the degrees of freedom and in the table you have the chi square values and on the top horizontal axis we have the level of the confidence interval, which we want look at this particular situation in our example we have degrees of freedom equal to 11.

So, we have to take a look at this particular x square values or chi square values. Now, we have to decide what degree what level of confidence that we want, if we want our data with a confidence level of 9.9 that is we want our data to fit the Gaussian distribution 90 percent or not I should not say Gaussian distribution fit the expected distribution 90 percent, then the chi square value that we are calculating that should be smaller than this one, I repeat some chi square that we have calculated from our measurement we have to compare that with the value that is given in the table. For that first we have to identify the degrees of freedom like in our case degrees of freedom is 11. And then we have to decide on some kind of confidence interval what level of match we want.

So, hero 0.9 is the confidence interval that if we decide on then corresponding chi square value give another table chi square limiting values 5.578, if the value that we are getting from our dataset is coming smaller than this, then we can go for the decided distribution if it is larger than this, then we have to identify some other distribution. So, I would request you to try to calculate these values expected from a 400 kilo Pascal and also with Gaussian distribution and see what values you are getting.

Now, one final test there are infinite statistical analysis that we can show, but I have a limitation of time and there is no need for also from the point of view of our course. So, just one final thing where the standard deviation of the mean.

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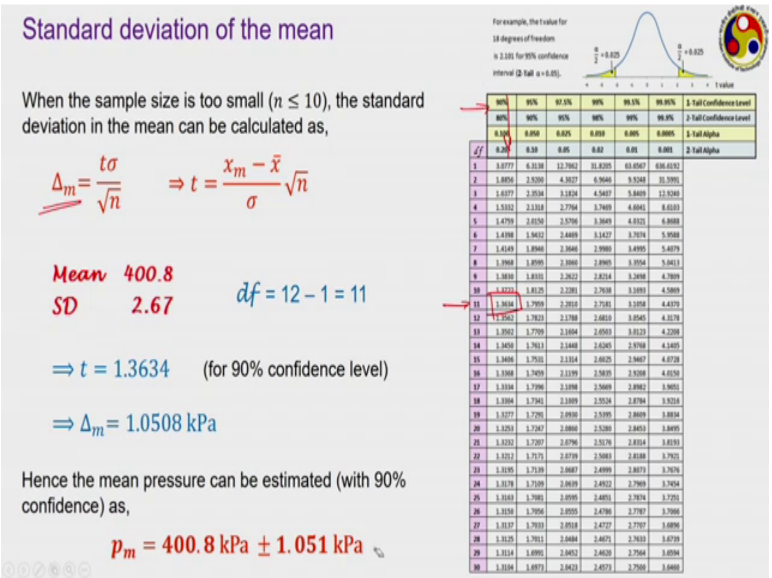

As you have seen depending upon our choice of sample the mean itself may have some kind of variation, when the sample size is very very small the standard deviation is given as a formula like this, where sigma is standard deviation corresponding to your population small n refers to your number of data in the sample. And this is the standard deviation in the mean itself t is a parameter which is given by another statistical test called t test.

So, t is equal to your x m which is the standard deviation sorry which is the mean that you are getting from population that we also use a symbol sorry, we have also used the symbol mu earlier, I should have used the same symbol, but as I miss this up please note that x m and mu are same. And x bar refers to the mean that you are getting from our population from a sample x m refers to the population x bar refers to the sample and using this t test we are trying to get an idea about the standard deviation that is present in the mean itself.

This is particularly useful when you are dealing with a very small sample of data like n less equal to 10, which is very relevant in several practical cases, because generally in practice we do not have the option or time or even requirement also to go for very large sampling.

So, we can deal with a very small level of samples here we have again a data set like this, here what your t provides is again the degrees of freedom on the first column and your confidence interval on the axis on the like just take a look at this particular row, we have the confidence interval and then we have the corresponding t values. If we go back to our example there we had mean as 400.8 and standard deviations 2.67.

So, if you put that then and degrees of freedom was 11 as we have already discussed. So, if you put that degrees of freedom is 1. So, this is the line if we decide a 90 percent confidence interval. So, this is 90 percent in ninety percent confident interval our t 1.3634 so, that gives us a deviation of the mean of 1.05 kilo Pascal.

So, here the mean pressure can be estimated with 90 percent confidence is 400.8 kilo Pascal's which we have already done plus minus 1.051 kilo Pascal's. So, this t test allows us to allows a way of calculating the mean itself. And once we know the mean then we can go for the recalibration process to identify the standard bias present there this, all are related to the randomness or precision error that can be present in the measurement and this mean itself is associated the standard bias.

So, in a sampling process following the t test we can also identify the error that can be present in this mean itself, now from this co measurement course point of view you may not need to go for. So, many statistics these are more from your information point of view whatever we have covered today it is important just to know the concepts like the Gaussian distribution.

How to get that what are the needs of performing the chi square test or t test, but I would not ask you to solve any problem on this apart from very simple examples like we have done here. And from the just to give you some idea about us to get some idea about how to deal with the random errors, but in this course from the instrument point of view we shall mostly be working with this W R only which is a total which combines both precision and random errors.

So, from this point onwards that is from the next lecture onwards we shall be dealing only this total error, we shall not be talking too much about the bias error and precision error separately, whatever error values that will be given either in percentage or in absolute those will always refer to the total error, but the of the discussion that we had today these are how to identify the values of those W R. So, this takes us to the end of our first week. So, in the module one we have learned about the significance of measurement.

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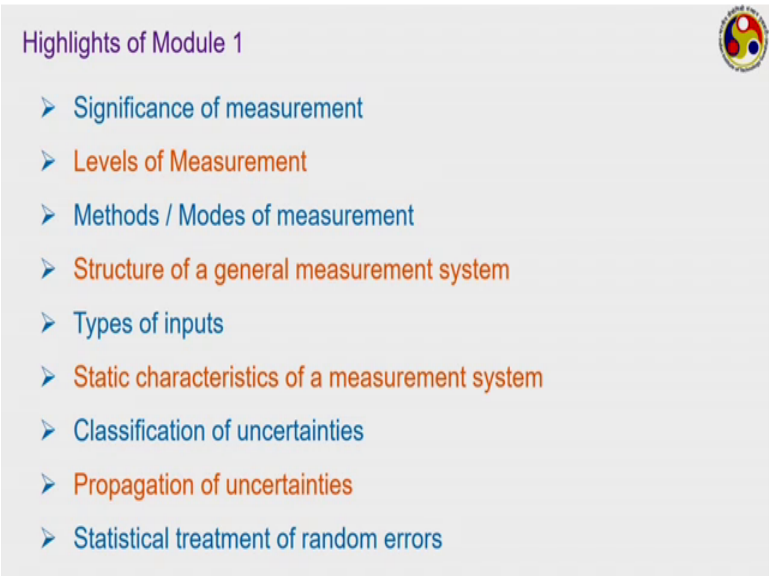

Then we have talked about different levels and methods of measurement we have learned about the structure of a general measurement system is different stages three different stages basically we shall be discussing about each of these stages later on. We have talked about the different types of inputs, the digital inputs is something of very importance very large importance we shall be talking about the digital import inputs and its procession processing in the in the third module.

And the some of more discussion about the output about how to process the output we shall be talking in 4th week. We have talked about the static characteristic of measurement system, there are several static as you have been introduced with the resolution linearity are generally the most common one and something actually I forget to mention hysteresis one property which actually is associate with a random error.

So, that is another static characteristic which can lead to some error in a measurement dynamic characteristic is something that we shall be discussing in the next week as a part of module number 2. We have talked about the classification of uncertainty, uncertainties and how to deal with the propagation of uncertainties. And finally, we have discussed about different statistical treatment to get the idea about possible magnitude of random errors that can be present in a in our final measurement reading.

So, that is the end of first module thanks for your attention and assignment is already there on the portal. So, please try to solve the assignment whatever queries you have I repeat please send to me immediately. So, that I can respond my teachers also will be continuously keeping a track of the portal, to respond to your queries in the next model we shall be talking about the dynamic response of a system.

And one topic that you probably can revise a bit before starting that that is the Laplace transform, which is definitely you have it has been covered in your mathematical course in first or second year please devise the Laplace transform a bit so, that in the next week we can start the dynamic characteristic of response systems.

Thank you.