Principles of Mechanical Measurement
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Module - 11
Motion Measurement
Lecture - 30
Basic seismic transducer

Good morning, everyone. Welcome to week number 11. Now, this is week 11; that means, we are approaching the end of this course and this is the penultimate topic that we have to discuss here. Over last three topics, we had quite intense discussion over three parameters or measurement of three parameters which are very common to our day to day application, industrial applications. Even layman also needs to have a knowledge about the values of these parameters namely pressure, flow and temperature where, we have discussed in detail about different possible types of measurement that we can have and accordingly different categories of instruments we have talked about; particularly the ones which are most common.

Now, in this week we are going to talk about the motion measurement because of the interest nature of the discussion in the previous weeks and also considering that we are approaching the end of the course. So, this week and also in the coming week I shall be keeping the discussions quite short and brief, talking only about the most important points or touching upon only the most relevant points related to the measurement of motion in this week and certain special topics in the next week, instead of going into too much detail of the relevant measurement systems.

Now, the term motion here I have to clarify at the beginning itself. Here we are definitely referring to some kind of velocity or acceleration measurement, but that does not relate to any kind of flow situation. Because the measurement of flow velocity or fluid velocity in a flow we have already discussed in the concerned week or during the concern module.

If you remember there we have talked about pitot tube. Now, pitot tube though it is actually a pressure measurement instrument or it actually senses pressure, but using pitot tube or to be more specific using the static pitot tube; we can actually get a direct measurement of the fluid velocity and that is the most common type of flow velocity measurement option that we can get. Similarly, we have talked about anemometers or mass flow meters where we definitely get the mass flow measurement, but if we have idea about the density of the flow or density of the fluid which is flowing and also cross section area of the duct we can calibrate the same directly in terms of fluid velocity.

Also, we have talked about very enhanced measurement systems like LDV – Laser Doppler Velocimeter or PIV – Particle Image Velocimetry or velocimetry. Both of which gives you a direct representation of the velocity vectors in a particular 3-dimensional flow field. So, the measurement of fluid velocity we have already covered both simple and complicated type of instruments.

Similarly, we are also not going to talk about the flow of or I should say the velocity of any rigid body motion; like only rigid body or particle is moving somewhere the measurement of it is velocity is primarily related to the displacement measurement. Because once you know the displacement with respect to time you can easily get velocity from there.

Like in one of the earlier weeks in week number 4 we have discussed about the circuits or data processing circuits like differentiators and integrators and from that knowledge you know that once we have suppose the measurement of displacement in form of an electric signal then we can use the easily use a differential amplifier or I am wrong differentiating amplifier I should say we can easily use a differentiating amplifier or just a differentiator which will actually give you the derivative of the input quantity that is the derivative of the displacement which is nothing, but velocity.

So, such particle velocity also can directly be measured using those instruments. Here our discussion will mostly be revolving around vibration peaking of vibration and measuring the magnitude of corresponding vibration velocities or velocity of the platform which is vibrating and corresponding acceleration.

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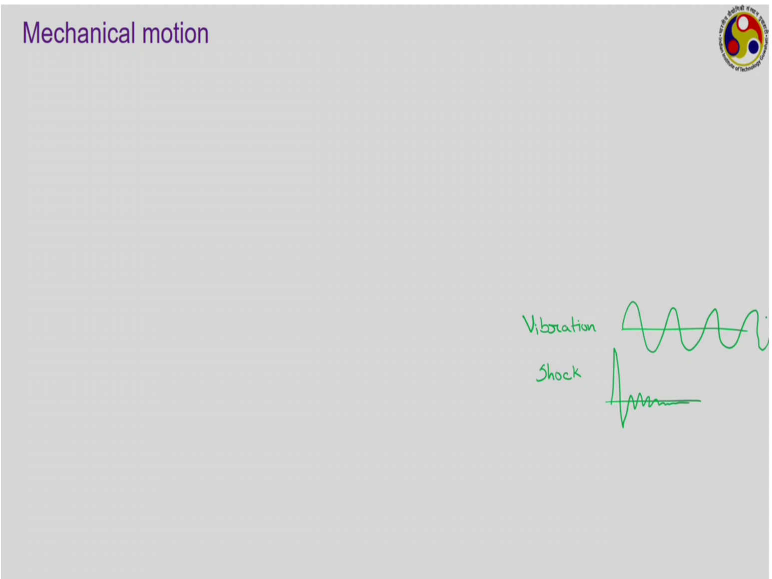

So, the term mechanical motion generally can have several kinds of applications means just the way you would like to visualize this accordingly all of them can come under this mechanical motion. But, before I move forward I have to clarify between two terms which are sometimes used quite interchangeably; one is vibration, other is shock.

Vibration basically refers to a motion which is some kind of repetitive nature. The amplitude may be changing, but it should be some kind of repetition of over this entire and there or so, there should be some kind of repetitive structure repetitive nature over the entire span of discussion. The like say if a system is forced to vibrate you may see an waveform like this with the amplitude continuously decaying finally, going back to the stable situation, but the repetitive nature is there which may not be the case in case of a shock.

In case of a shock, you may get a sudden disturbance in the flow stream and then it decays quickly to go back to the original situation, which is not truly either case in case of vibration. In case of vibration if there is not sufficient damping provided then you will get a continuous waveform moving continuously with the same amplitude and frequency over infinite period of time which is not the case in case of a shock.

So, we are going to primarily focus on this kind of nature where we can have a periodic movement over a long period of time which is we are calling as vibration.

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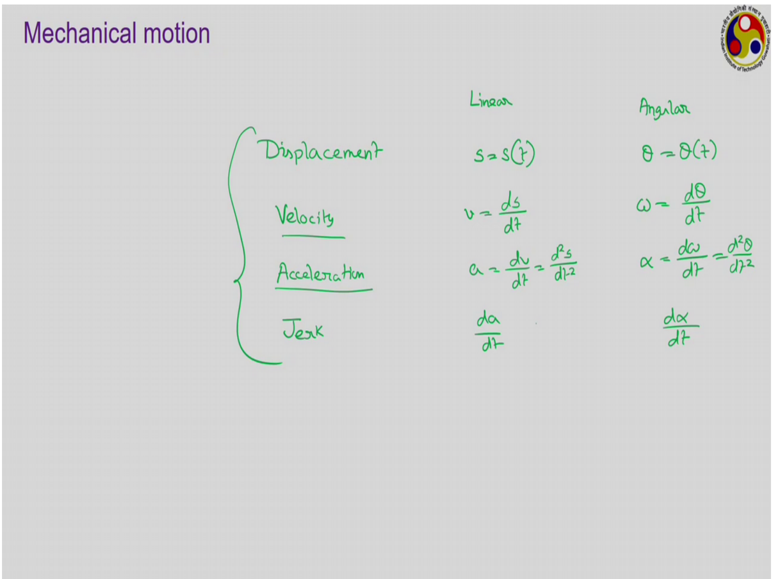

Now, as I mentioned mechanical motion can have different types of description. So, all these four can come under mechanical motion which I am going to talk about. Like if I say displacement, then we know that suppose in a linear motion displacement generally represent as S which can be a function of time depending upon the system that you are dealing with. Similarly, in case of an angular system or angular coordinate system if you have to deal with then we commonly use the symbol theta to represent this.

So, this displacement definitely is a kind of motion that you have to deal with. But, quite often instead of displacement we talk in terms of velocity v is nothing, but the differential form of displacement whereas, in case of angular coordinate system we generally use a symbol omega to represent the angular velocity. This mechanical motion can also relate acceleration. Acceleration a is again nothing but dv dt or we can write the second derivative displacement. Similarly, if alpha is a symbol that we use to represent angular acceleration this is just d omega dt or d 2 theta dt 2.

So, whenever you are talking about the measurement of mechanical motion we may be talking about measurement of any one of the three – displacement, velocity or acceleration or maybe more than one; means your system may be simultaneously giving a measurement of both the displacement and velocity. But, there is a fourth one also which we may need under certain situation which is called jerk. Jerk is the rate of change of acceleration in both linear and angular coordinate system which we are dealing with. So, sometimes in certain situations we may also have to deal with a jerk; particularly when designing mechanical or structures or large civil structures.

So, your mechanical motion related discussion can involve any one of the four measurement of any one of the four or more than one. But, in this particular week we are focusing primarily on measuring the velocity and acceleration associated with a vibrating member.

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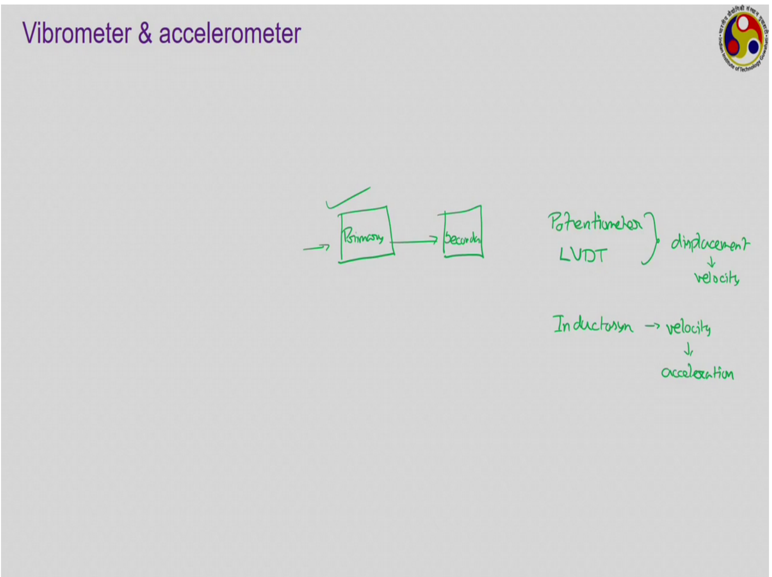

And, for that purpose we can use two kinds of instruments; one is vibrometer, other is accelerometer. Vibrometer actually is the one which relates to the measurement of velocity; accelerometer is a one which relates to the measurement of acceleration. Truly speaking a vibration pickup or vibrometer and an accelerometer has nothing different in terms of their working principle. Their major difference will be coming in terms of the secondary transducer that we shall be using to get the final output.

Like generally all these devices have two transducers in sequence. This is the primary one and this is the secondary one. Primary one we invariably will take the input and convert this one to some kind of electrical output which is going to the secondary one. Now, depending upon the nature of the secondary one the same device may act as a vibrometer or I should say the same primary transducer may act as vibrometer or accelerometer. Like, suppose you have already studied about displacement measurement. So, if I relate to that if your secondary transducer is a potentiometer or if your secondary transducer is an LVDT, then the output that you are going to get that is that is velocity.

So, if the secondary transducer is a potentiometer or a linear variable differential transformer, then you are going to get velocity as the output. However, if this secondary transducer is a variable reluctance type like something like an inductosyn then what you are going to get that is acceleration. So, by changing the secondary transducer it is the same primary transducer can be used to act either as a vibrometer or an accelerometer. Or I should say this may not be the perfect description let me be consistent with our earlier discussion.

Let us say the potentiometer and LVDT if you are using in the secondary transducer you are going to get displacement and then this displacement using because the potentiometer the true output is displacement and generally this secondary transducer is connected with a differentiating transducer or differentiating amplifier or a differentiator so we get velocity as a final output. So, truly speaking, a secondary transducer is going to the displacement which will subsequently be converted to velocity using the differentiator.

Whereas, if you are using an inductosyn you are going to get velocity and that will be scaled or differentiated to get acceleration. Whereas, we can have separate accelerometers also where the output is directly acceleration and we do not have to bother about the nature of the secondary transducer.

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Now, elementary vibration detectors we can talk about two different kinds of instruments or a techniques I should say which can give you very elemental identification whether there is vibration present in a system or not. But, before I mention about any technique the I should say the most common or most efficient vibration sensor that we have is nothing, but our skin or our touch our sense of touch. Human touch can sense vibration amplitude as low as 0.3 micron. This is just by touching something.

Like, if this particular pen is vibrating and if I touch it very loosely even a vibration of the amplitude 0.03 micron can be sensed properly whereas, on the other hand if I grip it tightly it further we it is a much stronger grip in that case the amplitude vibration that we can pick up can be even smaller something in the range of 0.025 microns that is at least 1 order smaller than this. And, no instrument that we have at present can sense vibration efficiently with the same level of amplitude.

So, human touch is probably the most efficient way of sensing vibration, but practically we need some kind of options to measure or quantify vibration and therefore, there are several common kinds of instruments we can have.

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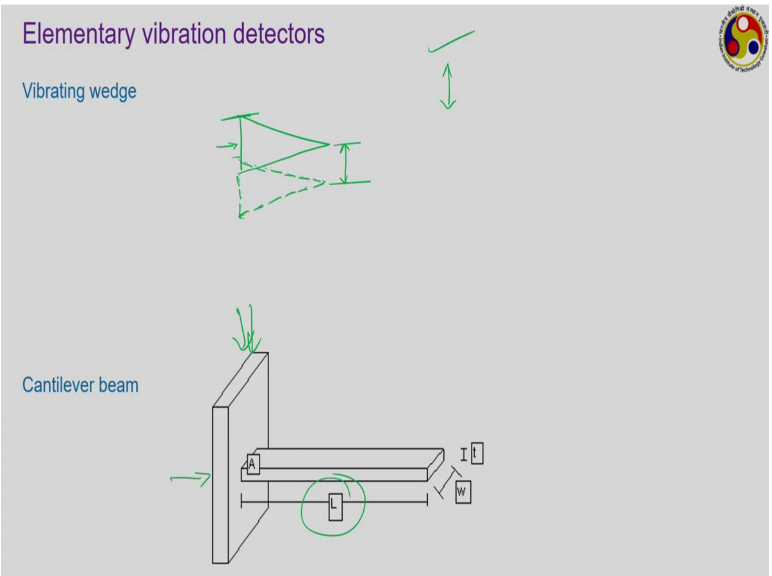

The vibrating wedge is nothing, but a triangular shaped wedge something like this which is mounted on a surface and this particular surface when it comes in contact with any vibrating member this particular surface, then this wedges keeps on vibrating and as the wedge keeps on vibrating its position keeps on changing continuously.

So, let us say at as the wedge keeps on vibrating over a range then let us say this is the extreme position of this topmost edge, the highest position this is the topmost wedge which can have and this is the lowest position. Then, if it is vibrating quickly; let me draw the lowest position of this wedge as well. So, this is the wedge when it is at the lowest possible position, it is fine. So, what is the distance between these two peaks? This is the one what this one signifies. This is definitely amplitude of the vibration.

So, if we measure this one the then we can or measure or get an idea about these two extreme positions or maybe get a photograph of this, then we can directly get and that peak to peak amplitude for this. Here of course, the direction of motion is this much. So, this mechanical wedge or vibrating wedge is a very simple way of giving you the direction of motion and also an idea about the peak to peak amplitude. However, we are not going to get the waveform from this and also not too much idea about the frequency of these oscillations because it is not possible to count the number of oscillations this wedge is having over a given period of time.

So, the most common method of measuring frequency can be a cantilever beam can be the use of a cantilever beam or a cantilever beam of variable length. Suppose, this is the member which is undergoing certain kind of vibration, then what we are going to do is that we are going to get this cantilever beam in contact with this particular body and then using some suitable technique we are going to change this length.

Now, the beam itself has some natural frequency and as the length changes its natural frequency is going to change. Now, if we keeps on changing this length slowly then we may reach a point where the natural frequency of the beam and the frequency of vibration of this platform, this member they becomes equal to each other. So, then there will be resonance happening and the beam will start to oscillate with very large amplitude. So, we can get an idea about the frequency of the beam from corresponding to that natural sorry frequency of the member, the object corresponding to the natural frequency of the beam with that particular length when we have very large amplitude oscillations.

So, these are very simple way of measuring or detecting vibration.

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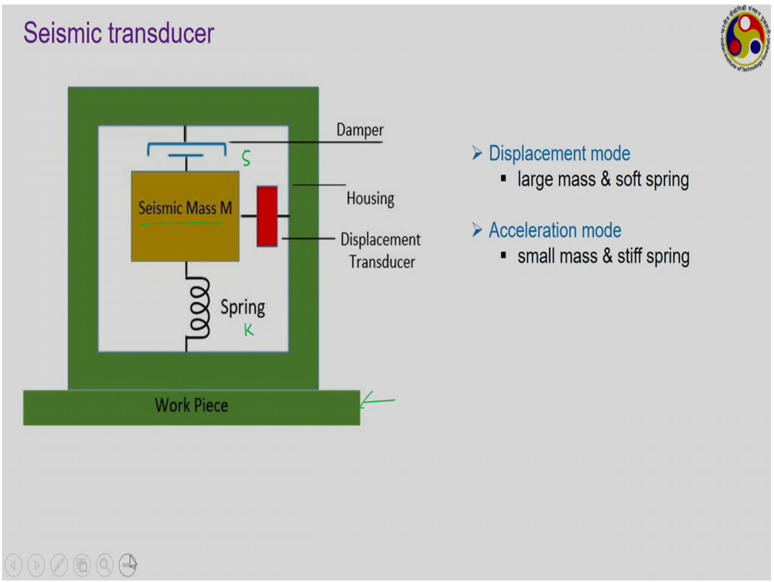

But, the most common way of doing this is use the use of seismic transducers. A seismic transducer is nothing, but a spring mass damper system here we have a large mass generally called a seismic mass given by M. There is a spring, let us say the spring is having a constant K and we are also having a damper let us say the zeta is the damping coefficient. So, this entire assembly is how in located inside a house and we are having this work piece which is the one that is undergoing oscillations.

So, from this particular assembly we can easily get an idea about the vibration or the entire nature of vibration amplitude, its oscillation frequency and also the nature of the wave form using a simple arrangement like seismic transducer. Depending upon the configuration of transducer it can be used as vibrometer or accelerometer like if you are looking for velocity measurement, then it will work in a displacement mode when we need to have a large value of M and a soft spring.

Whereas, if you are looking for acceleration measurement then the mass should be smaller and the spring should be much stiffer for this, but working principle remains very much the same.

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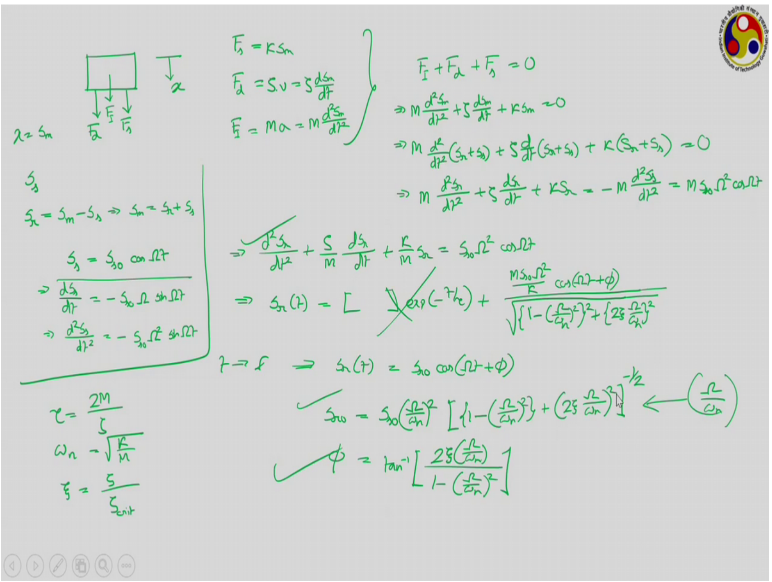

Let us quickly check the working principle. Let us say this is our mass M and we are drawing a free body diagram then under certain situations what are the forces acting on this? Let us say this direction indicates x. So, under a certain situation the spring force is acting there or if I say this is the spring force, it is also the damping force can also be there. Of course, the direction of both the forces will depend upon the direction of motion and there will be an inertia force also. This inertia force is the one that is opposing the motion of this against the vibration. So, under equilibrium condition the summation of all these three forces has to be equal to 0.

Now, what are the magnitude of these forces? The spring force if x is the amount of displacement the body is having, then the spring force will be equal to K into x. What about the damping force? The damping force will be, the damping coefficient into velocity that is we can write damping force into dx dt. Now, instead of writing S let us say x is equal to S m which refers to displacement of the mass. Then, let us replace this x with S m S m refers to the displacement of the mass. And, what about the inertia force for this mass? The inertia force for this mass will be equal to M into its acceleration that is we can write M into the second derivative of the displacement.

So, if we put all of them together, then what we are going to get? Putting all of them together we have F I inertia force plus the damping force or viscous force plus the spring force is equal to 0, that is the inertia force M into d 2 S m dt 2 plus d S m dt plus K S m is equal to 0. This is very much the equation of a second order system, but S m is the displacement of this mass M. But, we are not interested to know the displacement of this mass M rather we are interested to know the displacement of the member which is having this displacement.

Like, if we go back to the previous slide this is the mass M that we are talking about what our interest is about this work piece. So, let us say S m refers to the displacement of the work piece. Then let us say S, s refers to the displacement of the subject or of our object because S m already we have used. Then if S r indicates the relative displacement between the two then what will be S r relative displacement of the mass with respect to object then S r will be equal to S m minus S s or S m will be equal to S r plus S s.

So, if you take this here S m is getting replaced by S r plus S s. S r plus S s K into S r plus S s is equal to 0. Now, the member is not at all affected by the spring or the damping force or the subject and accordingly our equation gets modified to 2 S r dt 2 d S r dt plus K S r is equal to minus of M d 2 S s dt 2.

Now, let us assume something. Let us assume that the member is undergoing a simple harmonic motion. Of course, simple harmonic motion is not possible in practical situations and so the one final value or expression that you are going to get that will be application of that will be quite limited, still that can give you some important insight into what is going to happen. It is quite similar to the way we have analyzed the generalized second order measuring instrument in the second week during the second module, but as that was quite a few weeks back or couple of months back at least. So, I am doing the exercise again and also to show you the relation between this member displacement or object displacement with the mass displacement.

So, we are assuming that this mass or object S s is undergoing a simple harmonic motion of the form say S s o into cos omega t where omega is the frequency of oscillation and S s o is the amplitude. So, if we put it back here or before putting back here if we differentiate this, so, it will become minus S s o omega sin omega t and if we differentiate it once more minus S s o omega square sin omega t. So, putting it back here we are having this as M into S s o omega square cos omega t.

So, this is a second order system that we are dealing with. A very much a second order system and we have to get a solution for this when it is subjected to this kind of simple harmonic motion. We could have also chosen a instead of choosing this particular waveform we could have also chosen a step input or something like this, but as we are talking about vibration. So, the simple harmonic motion is most idealized input that we can have for a system.

Now, the solution for this one is quite standard and I am sure you already know the solution for this particular system to simplify this. Let us divide this entire equation by capital M. So, that we have d 2 S r dt 2 plus by M S r dt plus K upon M S r is equal to S s o zeta square cos omega t.

So, it is a very standard second order ode which we can solve using any standard boundary condition, but before if we from there we can easily write that the standard from S r t will contain two parts; one, exponential part and another transient part which is of importance generally, where the exponential part will be having some constant. I am not writing the detailed expression. You can go back to week number 2 or any standard textbooks where it will be having minus t upon tau where tau is the time constant and in this particular case your time constant will be equal to twice of M upon zeta and the one that is of major importance to us is the second part.

In this second part we are having M S s o omega square upon K into cos of omega t plus phi, where phi refers to the phase angle divided by root over 1 minus omega whole square this whole square plus 4 sorry, plus it will be equal to 2 zeta by omega whole square. Here this omega refers to the natural frequency. We can also write this one to be omega n just to clarify that we are talking about the natural frequency and omega m, can you identify the omega n from this equation this particular equation that we have written from there?

It is very simple it will be root over a naught by a 2. So, from there it becomes root over K by M. And, so, if we are talking about when t tends to infinity or t is the last time this exponential term fall drops out and we are left with S r t to be equal to some S r 0 into cos omega t this omega is wrong this should have been the capital omega which is the frequency of the vibrating member at t plus phi. Here S r 0 is the amplitude of oscillation of this relative displacement.

So, this is equal to M by K we can represent as or K by M can be written as omega n square. So, this becomes S s o into capital omega by omega n whole square into 1 minus omega upon omega n whole square plus 2 zeta omega upon omega n whole square whole to the power minus half and this phi is the phase difference that is going to get introduced because of action or presence of all this mass and damper and the spring and so, it will be equal to tan inverse of 2 zeta omega n divided by 1 minus whole square.

Here of course, this zeta is the ratio of the damping coefficient to the critical damping coefficient that you may have system. Truly speaking I should have made another change also where this term should have been and also here, this should have zeta only this ratio this. So, this is a very standard mathematical practice to get the waveform for this corresponding the relative displacement, now we have the amplitude of this one and frequency for this one. But, you have to consider that here you are talking about when you have progressed a between time so that the exponential part has dropped out and we are left with only this so called near steady state or stabilized part of the displacement.

Now, here there are several points to look for like look at the expression for this S r o. Now, here the natural frequency and the value of this zeta they are all or they are both can be altered at the design level of the seismic instrument itself. So, omega n and zeta are given then everything comes to the value of this capital omega that you are introducing or in a way the ratio of omega upon omega n. Everything becomes dependent on this frequency ratio of angular frequency are just normal frequency it becomes dependent on that for both amplitude and phase response.

Now, we have earlier studied the standard amplitude and phase response for instruments, for secondary instruments more a repetition of the same thing.

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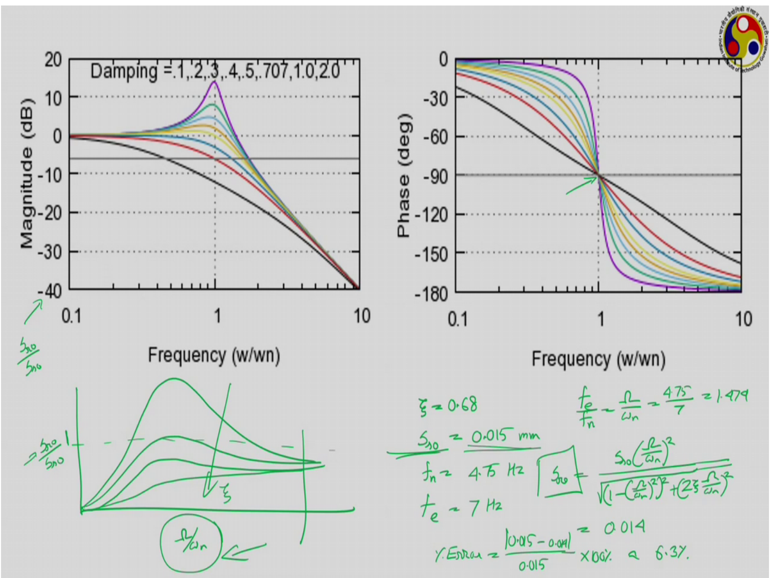

You can see as the frequency ratio keeps on increasing the magnitude or the amplitude you can say this one can be viewed to be a representation of S r o divided by S s o; actually I should have given a plot of S r o, S s o directly. If I represent that in this way say S r o by S s o and divide it here upon omega n separately, then you will find that for small value of zeta you may have a profile somewhat like this, or it should not be such sharp line, but as theta keeps on reducing it becomes more moderate this for very high value of zeta it is just trying to approach this and this particular level is 1.

So, interesting part is as the omega keeps on increasing frequency keeps on increasing typically when omega capital omega is about 3 times for this then everything is quite close to 1 and so, that is something quite desirable. And, these are the zeta values zeta keeps on increasing in this direction. So, we have to choose a damping also accordingly because damping has a critical role to play. Similarly, the phase difference for frequency omega upon omega n equal to 1 we are having my 90 degree phase difference between the two and this ratio of these two frequencies this particular thing has to be taken into consideration.

Like, we can check one numerical example. Suppose, let us say for certain situation for a certain seismic instrument we have zeta given to be equal to 0.68 and it is given that the amplitude of this S s o is something like 0.015 millimeter extremely small oscillations it is having. The natural frequency has been measured to be 4.75 Hertz whereas, the excitation frequency is given as 7 Hertz. Now, we have to estimate how much error may get introduced if we directly check this S r o to be equal to S s o. We have for that we need to because this is the actual amplitude oscillation that is happening, but your S r o is slightly different.

So, now what is the frequency ratio here? Here f e upon f n can be taken to be equal to using suitable conversion factor we can we take this one as omega upon omega n is equal to 4.75 by 7. I have pre calculated a number to be 1.474. So, if we calculate now S r o using the expression provided in the previous slide, so, we have S s o into omega n whole square divided by root over just referring back to the previous slide look at this expression.

So, we have this thing root over 1 minus omega upon omega n whole square this entire thing to be whole square plus 2 zeta and omega n and again this entire thing to be equal to whole square and if we put this number you are going to get S r o to be equal to 0.014 something like this. So, how much error that gets introduced? So, percentage error, if we take this one to be the correct reading instead of taking this one then the percentage error that is getting introduced is mod of 0.015 minus 0.014 divided by that true one it is 0.015 into 100 percent. This will roughly be coming around 6.3 percent approximately.

So, about 6 percent error gets introduced because we are taking the amplitude of this relative displacement to be equal to the amplitude of the true member displacement, but if we have proper idea about this particular omega upon omega n then we can easily do this correction to get the final value properly means we can easily use this value of S r o to get the value of S s o.

So, this is the mathematical base for any seismic instrument. As I have promised these lectures I shall be keeping quite short and that is why today I am going to finish it here itself. In the next lecture, I shall be discussing slightly more about a few other vibration related instruments.

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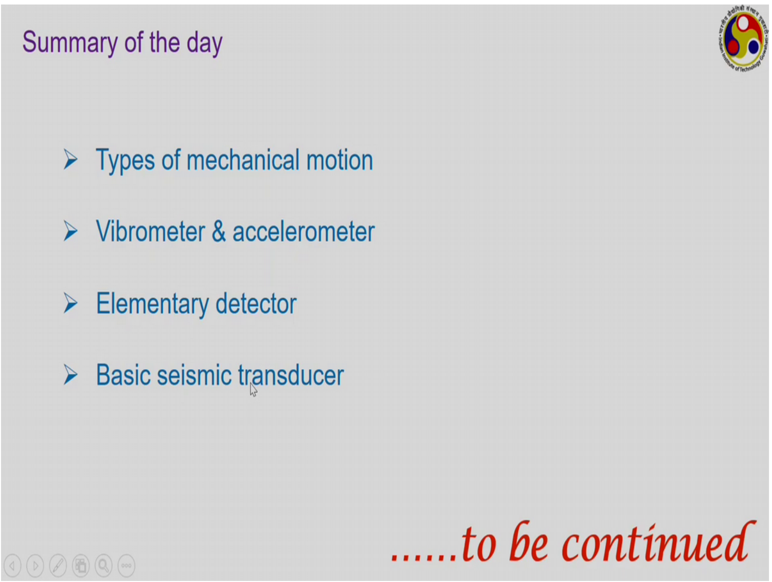

So, we today talked about different kinds of mechanical motion. We have seen the displacement, velocity, acceleration and jerk. All comes under this category of mechanical motion. Then, we have talked about vibrometers and accelerometers where difference is mostly in terms of the secondary transducer that is used. Like, with the same primary transducer if we use an LVDT that will act as a vibrometer giving a displacement and subsequently getting converted to velocity, whereas, if you are using an inductosyn we can get velocity which can be converted to acceleration.

Then, we have talked about the elementary detector in two forms. The simple wedge which generally gives the amplitude of oscillation, but not frequency; to get the frequency we can use a simple cantilever beam of module changing length. And, then very basic discussion about seismic transducer we had today, where we mostly focused on mathematics. Most part was a repetition of what we discussed earlier about the response of a second order transducer against simple harmonic inputs.

So, that is it for the day. I would like to discuss a bit more on this in the next lecture; till then good bye.