Principles of Mechanical Measurement
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Module – 01
Lecture – 03
Introduction to Measurement

Hello, everyone. So, we will come back to the 3rd lecture of our 1st module that is the 1st week where we are talking about some basic introduction to the topic of Mechanical Measurements. In this particular week we are just setting up different initial concepts which we shall be using throughout the duration of this 12 weeks and that is why we probably are going a bit slowly that is I want you to assimilate each of these concepts carefully and later when need arises you should be able to make proper use of that.

In the previous two lectures you have been introduced to a very basic design of a measurement system where we have seen that a measurement system or the components of the measurement system can be classified into three categories or any measurement system can be viewed as a combination of three different components or three different cater parts; one is sensors or input stage where the there is some kind of interfacing between the physical device and our physical system and the input side of your measurement system. Then there is a signal processing stage and finally, there is an output stage that is the signal which has been processed in the signal conditioning stage is supplied to some kind of output device through another interface.

We have talked about different properties of the measurement system in the previous lecture. We have seen that generally all the properties of a measurement system can be categorized into two categories depending upon their dependence on time like any kind of parameter which are dependent on time or any kind of input parameter which has a time varying nature, it is expected that the corresponding output is also going to be time variant in nature. Now, any characteristics of a system is generally referred as some kind of relationship between an input variable and a corresponding output variable that we can represent either in the mathematical functional form or maybe in a graphical form.

Now, when we are talking about a system characteristics which is or which relates one time invariant input to another time invariant output then we call that a static characteristic. And when both inputs are inputs and outputs or maybe one of them are varying with time then we call that dynamic characteristics. Dynamic characteristic will be discussed in detail in week number 2, but in the previous lecture we have already seen several kind of static characteristics of system; some very important one like property of resolution, sensitivity, linearity, threshold etcetera we have already discussed. Like we have done in the previous lecture also let us have a little bit of exercise to rehearse all those concepts.

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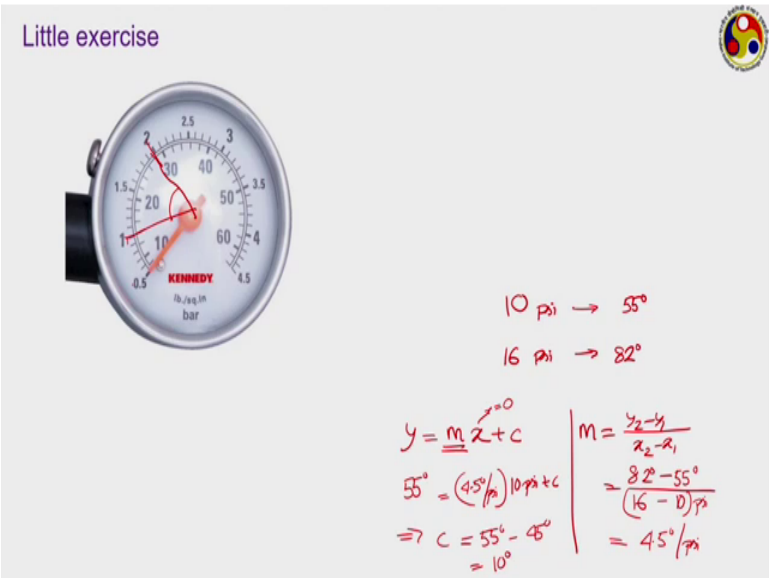

Let us take the example of a simple dial gauge a simple tire gauge I should say. So, once we connect this device to the location where you want to get the pressure measurement we get the reading in the form of rotation of this particular indicator over this particular dial. Now, let us try to identify the a few of the static characteristics of this particular tire gauge just from observation.

Now, can you identify the resolution of this? Resolution refers to the smallest possible change in the input required to get a response from the measurement system. I have already saved a few queries who is asking for the differentiation between resolution and threshold and here I can explain that through the example. Now, I repeat resolution refers to the smallest possible change in the input required to cause a change in the output.

So, just look at the gauge there are two scales, let us look at the one which is on the outer side which is what is given in the unit of bar. And now if you look at that what is a smallest possible value that we can measure like here this is 0.5 bar, this is 1 bar and in between you can clearly see there are 5 divisions; that means, each subdivision corresponds 0.1 bar. So, if there is a change in the input only by a quantity of 0.1 bar or more then only the indicator here will show some kind of movement.

Like suppose, in a particular situation your pressure value is 2 bar and now if you change the value of pressure from 2 to 2.05 you will not going to get any kind of observe reading on the scale. It will the indicator or this particular orange colored indicator will continue to be at this particular position only, but only if we make a change of 2.1 then it is going to move to this particular position. So, this is what we refer as a resolution; that means, we can clearly see that the resolution for this particular target is 0.1 bar. So, we need to go caused a change of 0.1 bar a minimum change of 0.1 bar to get any kind of response from the measurement system.

If we look at the other scale which is given in pound per square inch or so called psi here you can see here we have 10 in one case, 20 here and we have 1 2 3 4 5 divisions. So, each subdivision here responds to 2 psi. So, the for the psi scale the resolution is 2 psi whereas, for the bar scale it is 0.1 bar and threshold refers to the smallest possible input that can be measured with the device.

Now, you can see on the bar side 0.5 is the smallest possible reading that we have. So, if the system pressure is something like 0.3 bar, this gauge is of no use, it will not show any kind of reading. So, threshold refers to this 0.5 bar which you can also say it is the minimum value of minimum value shown on this particular scale. But, may not be always true, you should consider threshold as the smallest possible input that can be measured that is the smallest value of the input quantity whereas, resolution is related to the smallest possible change in the input quantity.

What about the linearity of the scale? Linearity refers to a linear relationship between input and output. Now, if you observe the scale properly we can say it to be linear if the angle covered by the indicator say over a range of 0.5 and 1 whatever the angle the indicator covers if that covers the same amount having l when traveling from say 3.5 to 4, then we can definitely call it to be linear. But, just from visual observation it does not look like in this particular case because as we are moving to higher ranges it seems that angle is increasing a bit like the angle the indicator is making between 0.5 and 1 and the one that is making between 3.5 and 4 does not look to be same of course, we need to measure it properly. If they are not same then this is this which it is not a linear device, but the linearity is changing as we are moving to higher values, but if this angle is same then definitely this is also showing a linear nature.

And the next is sensitivity. Now, how can we measure sensitivity of such a device? Sensitivity refers to change in output for a given change in input. So, sensitivity for this particular device can be measured by measuring the change or the angle covered by the indicator for a given change in pressure. Like say if we change the system pressure from 1 bar to 2 bar then how much is the change in the angle of this indicator? Like if we extend this to the center it will be 1 bar, this will be 2 bar then this angle is a measure of the deflection the indicator has suffered, then sensitivity will be given by this deflection divided by the change in the input quantities.

Let us take one example. Let us say your device is giving you or you know that you are currently at your device to a location where your pressure is 10 you know that the pressure is 10 psi and corresponding deflection that we can measure from here is 55 degrees. Now, we connect the device to another location where again we know the pressure to be equal to 16 psi and now, the indicator showing a deflection of 82 degrees. Now, if we assume the scale to be linear then how can you identify the linearity from these two given set of data.

Linearity means output and input will be related by a linear relationship. Like if output is y and input is x, then we can write a straight line equation of the form y equal to mx plus c to relate them, where this m refers to the resolution because this refers to a slope of the calibration curve and hence it is a resolution. So, if we compare the given data with the this particular form of equation then we can clearly say that m can be taken as y 2 minus y 1 upon x 2 minus x 1.

So, for this particular device your input that is x is pressure value, output is the angle of deflection for this indicator. So, if we put these values here y 2 refers to 82 degrees and y 1 refers to the initial angle that is 55 degrees, x 2 is the second pressure value which is 16 psi and the x 1 is 10 psi. Here both are in psi's. So, if we calculate the value we are going to get the resolution of this particular device to be 4.5 degree per psi.

So, this way we can calculate the resolution for any device, but what about c? If we put the c values into for any kind of x values. Let us compare for the first set that is 10 psi is equal to this 4.5 degree per psi into I am making mistake again here y side I have degree only. So, I should have written 55 degrees as y and x side I have 10 psi plus c. So, if we calculate c comes out to be 55 degree minus 45 degree is equal to 10 degree.

Now, what is 10 degree refers to? If we say put x equal to 0, then what value of y you are going to get? That definitely is the c, that is 10 degree. Now, what that refers to? This is the 0 error. So, this 10 degree is a zero error, even if we do not provide any kind of input your scale is showing a 10 degree deflection and hence once we have idea what this 10 degree zero error, then any kind of measurement cases we have to consider this and from the final value then we have to subtract this 10 to get the actual reading for this.

So, this is one example of how can we calculate any kind of pressure and temperature or for any other device any other measurement device we can calculate the resolution and sensitivity and also the zero error. Let me take the example of another one. Again, a very common example of a clinical thermometer.

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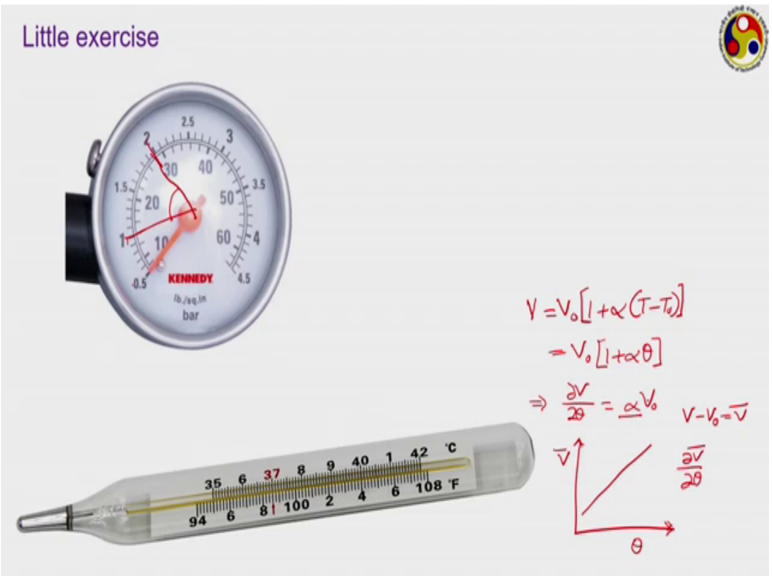

What can we say about this particular device? Again there are two scales, let us talk about the Fahrenheit scale only. So, in the Fahrenheit scale what is your threshold value? that is this 94. This is a minimum value that we can measure from this. How much is a resolution of this particular device? If we compare let us say this is 96, this is 98 and in between you can clearly see there are tens divisions 2 3 4 5 6 7 8 9 10 divisions.

So, here each the subdivision corresponds to 0.2 degree Fahrenheit; that means, it is a resolution of 0.2 degree Fahrenheit and for a thermometer kind of device we know that here the operating principle is based upon the volumetric expansion of the thermometric fluid. So, the volumetric expansion [noise] for any liquid can be related or can be represented as V is equal to V naught into 1 plus alpha into T minus T naught where T naught is a reference temperature, V naught is a volume of the fluid at the temperature T naught and alpha is the volume expansion coefficient.

Now, if we want to calculate the sensitivity for this particular scale then how can we do that? Let us represent this one in a slightly modified form 1 plus alpha into theta; theta refers to the change in temperature from the reference value. So, if we differentiate both sides with respect to theta then what we are going to get? We are going to get it to be alpha into V naught.

So, what is this dou V dou theta? If we plot a curve where I have the volume of the fluid on one side and change in temperature for the differential theta on the other side and if they are represented by a straight line or whatever kind of line, then this is basically a kind of calibration curve or the static irresistic because here theta or change in temperature is the input, the volume is the output. If we want to talk about the change in volume say something like V minus V naught that is change in volume from some initial value V bar. Then also we can easily plot the curve the same way by identifying dou V by dou theta and that will give you the sensitivity for this particular instrument.

Here one important point to observe is this alpha. Now, alpha is a property of whatever medium that we are dealing with. So, if the medium changes alpha is also going to change. If alpha is a constant then sensitivity of the instrument of this thermo meter also will remain constant. However, if alpha itself will vary like commonly such volume expansion coefficients or any other expansion coefficients of functions of temperature, they keep on changing with temperature. So, if alpha varies with temperature itself then the sensitivity of the device also will not remain constant rather it will keep on changing with temperature or with respect to the input value itself.

So, this way we can calculate the static characteristic of any device using whatever well information that are available. So, the range of the device will be from this 94 to 108 that is it is capable of giving you data over a range of 14 degree Fahrenheit. Now, let us move to the next topic of our discussion.

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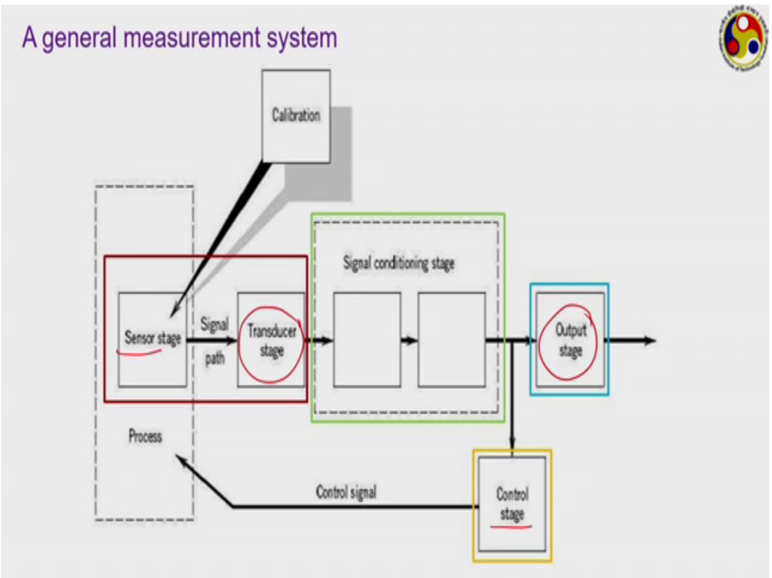

This kind of diagram we have already seen for a general measurement system we know that we initially have a input stage, then we have a signal conditioning stage and then an output stage. Now, in the input stage we have a sensor, which provides some kind of interface with the physical system and accordingly acquires some kind of signal which may get transduced depending upon the nature of the input signal also we just sometimes compare this with some kind of standard and the corresponding transduced signal which is the difference between your measured signal and the corresponding standard signal which is the signal conditioning stage.

In the signal conditioning stage we may have different kind of components; some just for amplifying the magnitude of this transduce signals, some for converting into some other kind of signal etcetera this signal conditioning part is individual to the type of instrument that we are dealing with and so, later on we shall be when you talk about any particular instrument we shall be talking about the corresponding signal conditioning. And whatever output that comes from the signal conditioning stage the output stage itself provides some kind of interfacing which takes this controlled signal to your output recorder.

If we are our objectives to get some kind of control issue then a part of that output signal can also be passing through the controls change and you may provide a feedback to the actual process itself. So, this is what a general system may have. But, the objective of showing this diagram again is that we can clearly see a general measurement system or any measurement system can have several components involved. And improper functioning of any one of the component can lead to error in the final measurement that we are going to get. That means, it is not that just having a sense a proper sensor or a proper output stage or maybe a proper signal conditioning sufficient we have to get all the components perfectly otherwise we may not get a correct output from our measurement system. Let us take a few example.

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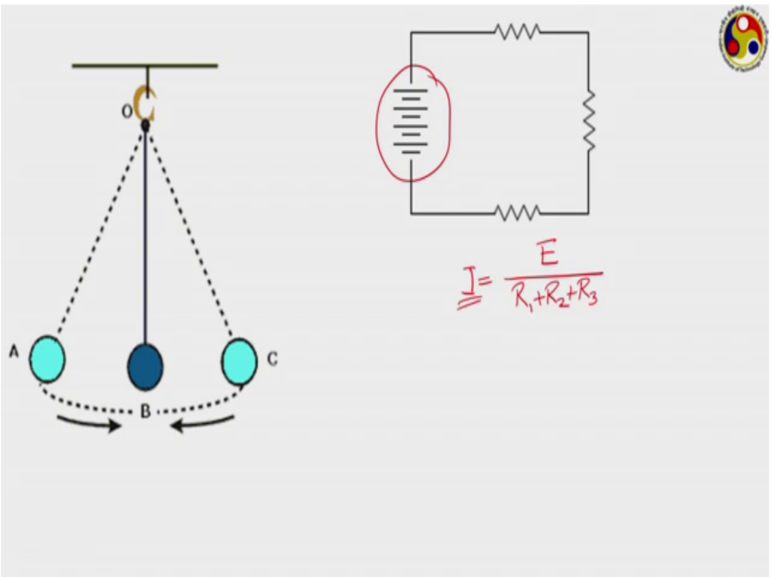

Look at this very very simple electrical circuit. Here we have a voltage source and three resistances. Suppose, we want to calculate the current which is flowing through this. Then how much into the current flowing through this circuit using our general electrical knowledge we know that current will be equal to the potential difference divided by all the three resistances; the net resistance which as the resistances are connected in series in this case that will be R 1 plus R 2 plus R 3, where R 1 R 2 refers to the three resistances.

Now, if we want to get a proper measurement for this current then actually you have to measure four quantities; one is the potential difference or the voltage provided by the source and three resistances. If any one of them is not correct or is has small amount of error then the final value of current that we are going to end up with also will have the same amount of error or may even a magnified amount of error.

Another example, another very common device which definitely you have seen at your high school physics laboratory. A simple pendulum which is commonly used for in the school life level physics in the school level sorry the school level physics laboratory for measuring the value of acceleration due to gravity.

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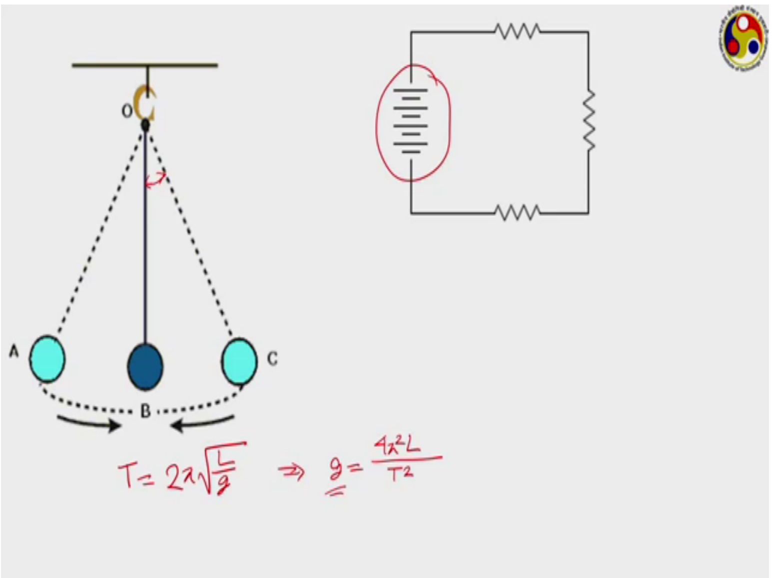

We know that as long as this particular angle is small generally limited to one radian then we know that the gravitational acceleration can be written as or I should write g equal to 2 pi root L by g. Here L refers to the length of the thread, g refers to the oh I am wrong sorry on the left hand side we have T, the time period of oscillation and g is the gravitational due to low gravitational acceleration and T is the time period of oscillation or if we rearrange the equation g comes out to be 4 pi square L upon T square. The mass of the bob of course, does not come into this picture.

So, to get a proper estimate of this g we need to have a proper estimate firstly, this L the length of the thread and secondly, this T time period of this oscillation. If any one of them is not proper then final value of g that we are going to end up with that will also be erroneous. And that means, in even in such kind of simple configuration we have every chance of making some kind of error. Let us move to a slightly more complicated relation.

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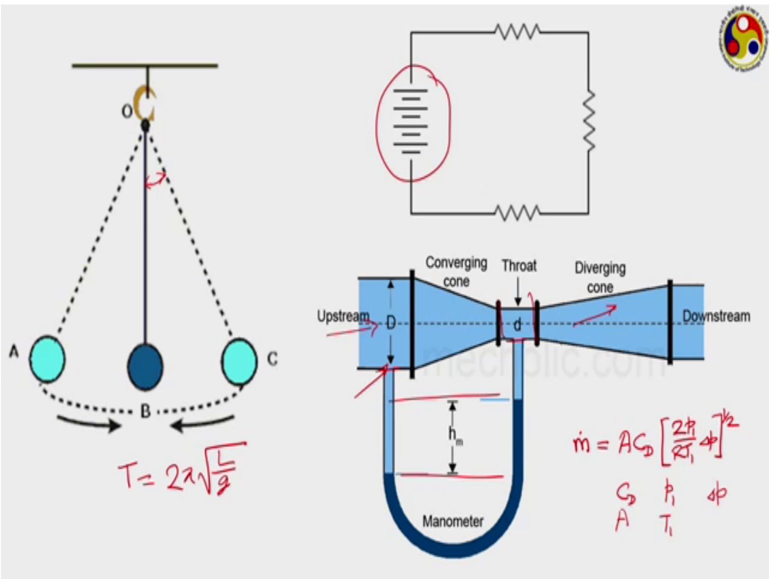

This is the diagram of a typical venturi meter or any orifice based flow measuring instrument. You may have already learned this in your fluid mechanics course if you have not do not bother about because later on in our flow measurement module we shall be talking about the working principle for this. But, in speaking very briefly here we have a converging diverging section like the flow which is coming in through this pipe, initially goes to a portion where the cross section area is reducing and then another section of the cross section area is increasing and in between you have a small portion of throat where the cross sectional area is the minimum.

And we learn using some kind of manometer or some other device we measure the pressure difference between this inlet portion that is before entering your venturi meter and this throat section. And that measurement generally gives us a measure of the total flow rate which can be written as if m dot refers to the mass flow rate we can write this as A into C D into C D power 2 p 1 upon R T 1 delta p whole to the power half. What the terms refers to here A refers to the cross section area of the this throat, C D is some kind of constant which generally supplied by the manufacturer of this venturi, meter p 1 and T 1 at the pressure and temperature corresponding to this particular location and delta p is the pressure difference missed by this manometer, R is the gas constant of the gas for which we are doing this measurement. Here the situation we have written assuming some kind of ideal gas flowing through the channel.

Now, look at this how many terms are there which may influence the final value of mass flow rate that you are going to get. One is C D, of course, it is supplied by manufacturer, but with aging with repeated use of the device there may be some change in the value of this constant. The next the area; we have to measure the area of the throat. We need to measure the pressure and temperature of the inlet section and also we need to get a measurement of this delta p using this manometer R is a constant that is a for a given medium R is a constant. So, you do not need to bother about this.

But, still we have five quantities here which we have to measure to get the final measurement of mass flow rate. And if any one of them is not correct then we may have some error creeping into the final calculation. And if there are several parameters containing error then their cumulative effect may give a significant amount of deviation in a value predicted value of this mass flow rate from the actual one. This is what we refer as the measurement error and that is the topic of discussion in today's lecture where we are going to get some idea about general types of errors and also discuss about how to estimate this error. We shall be coming back to this venture meter example at the end of this lecture to see some sample calculations.

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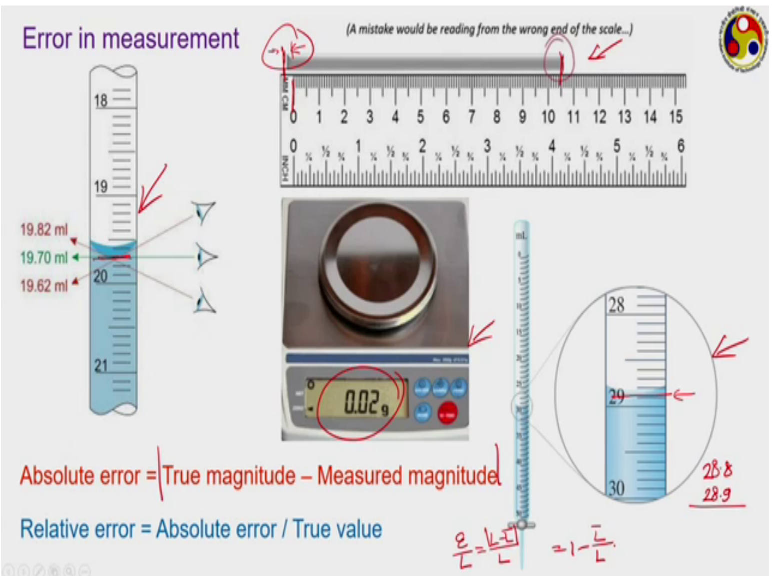

Now, some common errors; the one the picture that is shown here I am sure all of you are aware about, this is called the parallax error. Like if your objective is to get a measurement of the this particular lower elevation or sorry, the lower surface of this interface then depending upon our position of I we may get different values (Refer Time: 23:59). Its a very very common kind of error that generally quite come you know I would really miss during observation.

Another quite common error, like we are trying to measure the length of a some kind of bar using a scale very simple situations we have done this from our nursery class levels. But, see what we are doing when the end of the this one is somewhere here, this particular end should have been at this 0 level, is not it? Should have been aligned to this 0, but here this particular portion have been left out and so, the final value of the length of this bar what we are going to get that will contain this amount of error. This is again a careless and a kind of carelessness by the person who is doing this kind of measurement, but a very common one.

Another one see here the interface of the fluid it in this peep it is a curved one and now we are trying to get a measurement. Now, which one should we measure? If our objective is to measure the lower one or the upper one none of them are coinciding with any of the gradual scales that are provided on the device; that means, the resolution of the device is not sufficient to give an accurate reading. So, if our objective is to get this one let us erase this.

So, we know that it is going to lie between this 29 point ok, sorry, we are starting from the other side. That means we are sure that this reading will be something in between 28.8 and 28.9, but which value should we take it looks much closer to this 28.9 and we may be just guessing and taking a value of something like 28.88 like that, but that is a pure guess. That is a second decimal point that is coming here your device is not at all providing the second decimal point it is capable of measuring only up to the first decimal point and whatever we are adding in the form of second or third decimal that is a pure guesswork. So, here the instrument itself is not capable of providing a proper measurement.

Another very common error here we do not have anything on the weighing scale, but still we are having some value shown on the display. What we call this? This is a zero error. No input, but still some output is there. In this case it is showing 0.02 gram. So, whatever we are trying to measure whatever the equipment or object that we put on the tray, whatever final prediction that we shall be getting then on the scale, this 0.02 gram needs to be subtracted from there to get the correct reading.

So, this way there are several sources of error that may be present in any measurement and or their difference between the true value of any particular quantity which is being provided or I should say the difference between the true magnitude of a quantity and the value that has been provided or measuring instrument is called the absolute error. Our objective is always to keep this absolute error to ideally to 0 or at least to minimum, so that we can get the value of the measure end closest to the true magnitude. But, that depends on several factors like the instrument related issues like in this case or in this case or examples like this where actually it is an error done by the person who is performing the measurement.

So, absolute error talks about the difference between the true magnitude and measured magnitude. Like suppose we are trying to measure let us go back to this particular case again as let us say the true value of the corn particular bar is L and what we have measured is L bar, then this difference between L and L bar is what we are referring to as this absolute error. And sometimes this absolute error is divided by the value of this true magnitude and that we refer to as the relative error. So, relative error is the ratio of absolute error and true value or in this case it is L minus L bar divided by L that is 1 of minus L bar upon L. Generally, while selecting this absolute error we only take the magnitude that is we shall be putting a mod across this L minus L bar and then we should not do this particular calculation, we should just stick to this one only.

So, I repeat absolute error refers to the difference in magnitude between the true value and measured value of the object which is being subjected to some kind of measure end and absolute ratio of a solution and true value is called the relative error. So, if we have any idea about the absolute value of a quantity, then after getting the measured value can always calculate the absolute error and the relative error. But, generally we do not have any idea about the true magnitude because if we have any idea about the true magnitude then there is no point for going no point going for a measurement, unless we are trying to do some kind of calibration.

For the calibration stage we can definitely take some quantity with the known magnitude and then by seeing the whatever measurement that the system is returning we can calculate the absolute error and relative error from there to mostly to provide or to inform the user about the possible level of error that can be present. But, if we are dealing with an unknown quantity a new quantity then we do not have any idea about the corresponding absolute and relative errors.

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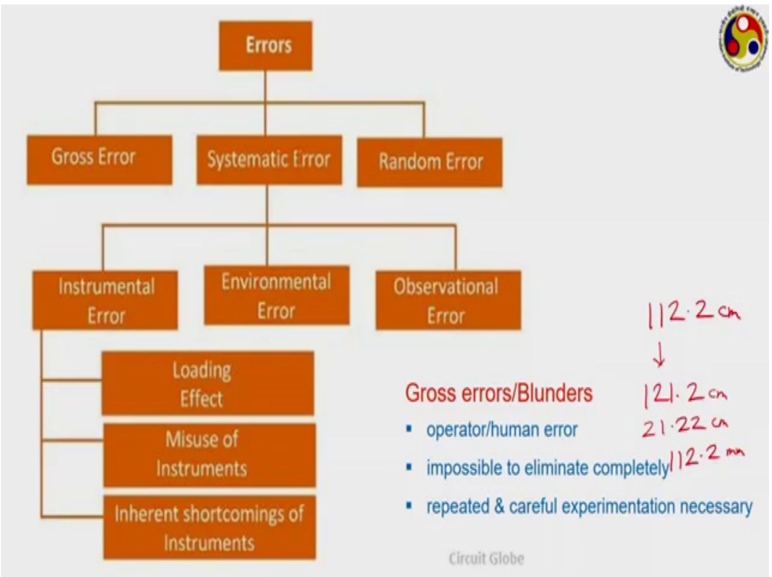

The errors can broadly be classified into three categories – gross error, systemic error and random error. Gross errors or blunders are generally associated with the human error; like the parallax that we have seen. Also we here generally refer to some huge blunder. Suppose, you have done some calculation and you have measured a length and your measured value is 112.2 centimeter.

Now, while reporting this there are several ways you can make careless mistake. Like suppose you have written 121.2 centimeter, so that is wrong. If you have written 11.22 centimeter, that is wrong and again if you have written sorry 112.2 millimeter that is also wrong. These are the gross errors these are impossible to eliminate completely, but we have to be careful while doing any kind of experimental measurement and generally by repeated experimentation by different observers experimentalist can sometimes eliminate or at least minimize this gross errors or blunders.

Next is the systematic error which is the most common type of error that can be present systematic error again can be of three types – instrumental, environmental and observational. Observational refers to something which is associated with the observation. Parallax of course, I have just mentioned that it can be a type of gross error, but commonly the parallax kind of error we put under observational errors because this can be eliminated or by measuring the angle at which the observer is seeing we can do some side of some sort of correction that is why it is quite often put under this category of observational error.

Environmental error related to different environmental factors like if there is some kind of measuring instrument which is open to the surrounding then it can be subjected to all the heat interactions with the surrounding air, the flow of wind, dust particles etcetera which can change the sensitivity of the instrument with our may lead to zero drift or sensitivity drift which we refer to this environmental error.

And the instrumental error inherent shortcoming of the instrument like we have already seen an example where the instrument does not have a sufficient resolution to be the measurement that refers to the inherent shortcoming and there can be some other problems inside the instrument itself. Misuse of the instrument suppose you are trying to measure some value which is outside the range of the instrument, that is a kind of misuse and finally, is a loading error now just take their term for granted for the moment. In the next chapter or in the next week we shall be discussing about this loading error while talking more about characteristics.

So, these are the different types of systematic errors which we generally can measure or get an idea about the corresponding magnitude and then can try to eliminate through calibration. However, random errors are is that a name suggests a random in nature, some errors coming from undetermined source or some from some completely unexpected source.

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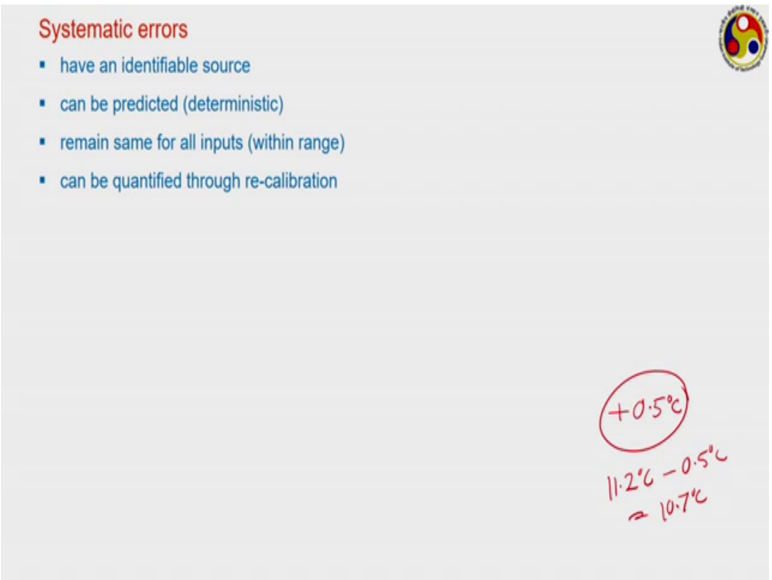

Systematic errors have always an identifiable source like the zero drift of sensitivity drift that I am talking about. We can always recalibrate the instrument and get a measure of the corresponding change in sensitivity or if there is any shift in a zero bias. So, we can get the idea about the corresponding source. Like the example of that venturi meter that we have seen. There are five different inputs required to calculate the final value of mass flow rate. Then there are five ways some kind of systematic error may get introduced. So, we can deal with each of them carefully and can it is possible to minimize the error.

The systematic error as I have just mentioned can be predicted. So, it is a deterministic kind of error. It remains the same for all input within range; that means, suppose a temperature measuring instrument has gained over a period of time is zero drift off something like 0.5 degree Celsius then in any temperature reading that it is going to show all the values will be containing this and if we have idea about the corresponding magnitude of zero drift, then what we have to do they say we have got a measurement of 11.2 degree Celsius from the device, then we have to subtract this zero drift from there and actual reading will be coming out to be 10.7 degree Celsius.

So, this way the magnitude of the error should remain the same or at least always go to the same direction and hence we can somehow eliminate them and also they can always be quantified through recalibration and recalibration generally is the on the most suitable process of eliminating or taking care of such kind of errors.

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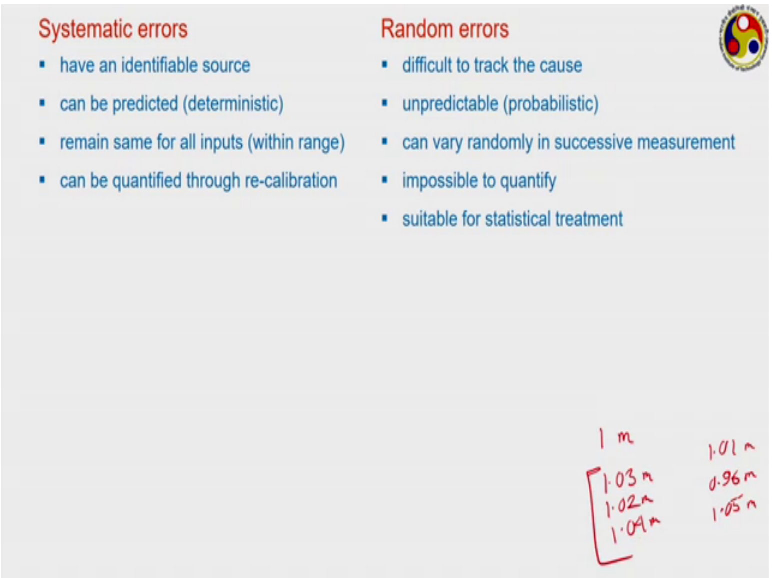

Random errors; however, are very difficult to track very difficult to identify the source of such kind of error they are very very unpredictable often leading to random change in the successive measurement like in three successive measurement we can get complete three completely different value going in three different alternate directions.

Again to give an example suppose we know that the length of a device is 1 meter or some bar is 1 meter. Now, if we are dealing with an instrument or trying to measure this length with an instrument which is having a systematic error we should get the inners all in same direction. Like suppose one device measure 1.03 meter another measurement give me give you 1.02 meter, another may give you 1.04 meter that is all in the same direction, but for random errors they can be completely random. You may get a 1.01 meter, you may get another 0.96 meter, you may get another 1.05 meter, very difficult to compare successive values just from the magnitude and so, very difficult to get an idea about where the from where the error is coming in.

Whereas, in this case we can we can somehow guess that their probably the problem with the zero-bias and we have to recalibrate the instrument to eliminate any kind of zero drift which may have appeared, but the random nature of the random errors have made the corresponding analysis more probabilistic in nature as it is not possible to get an a quantified value for them rather we have to go for some kind of statistical treatment. The statistical treatment for random errors we shall be seeing in the next lecture whereas, in this lecture we are focusing more on the systematic errors.

So, here I would like to introduce two terms; one is precision, other is accuracy. Both are very common English terms; however, have very very different meaning in the context with measurement systems. The term precision refers or deals with the random errors.

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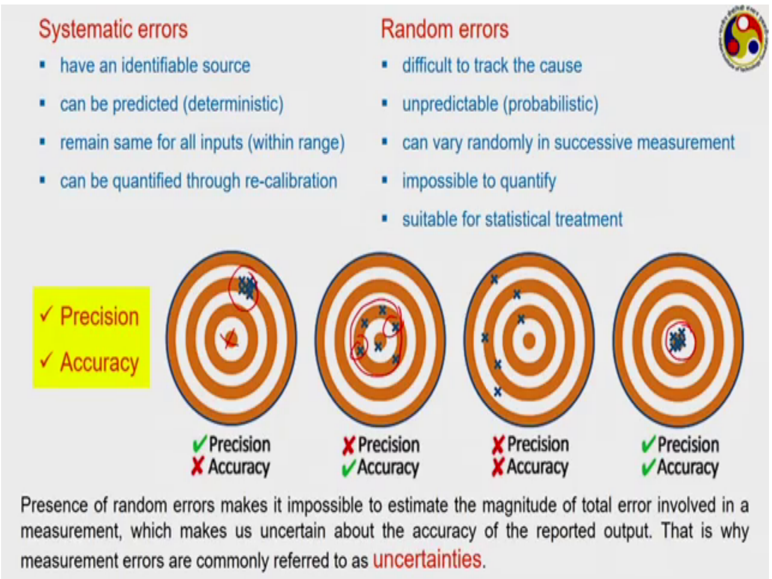

A device can be said to be precise if the random errors are very small or negligible whereas that of accuracy generally deals with the systematic errors a devices accurate means it will have very less systematic errors.

Let us take a look at this diagram. The look at the first case. Here several readings have been taken and all are coming in a nice bunch, but that is far away from our actual target which is supposed to be here. So, we can say that there is hardly any difference between successive readings and so, there is not too much random error so, that device is precise, but it is definitely not an accurate one.

Whereas, look at the second case. Here all are quite close to the desired value, but they are quite a bit scattered someone like this one is on one side, this is on the completely other side of the target. So, it is much more accurate compared to the previous case, but it is not precise; that means, the systematic error may be quite low, but there are random errors in successive measurement.

Look at the third case here all a scattered and none of them or most of them are far away from the actual reading. So, this is neither precise nor accurate and this is the most desirable situation here the they are all in a very nice bunch. So, that is precise random inertia less and also this bunch is very very close to the desired value. So, systematic errors also less that is it is accurate as well.

So, the term precision deals with the absence of random errors or elimination random errors whereas, the term accuracy deals with the elimination of systematic errors. While we have seen that systematic errors can be quantified, they are deterministic in nature and hence we can get a get some idea what their magnitude, random errors are very much probabilistic in nature and so, it is not possible to calculate their values.

Now, the total error that we get from any kind of measurement is a combination of these two from where the well the systematic part we can get some idea if t is not possible to get any idea about the random one which makes us very much uncertain about the final value that we have got. Whether for an unless we have done a large number of simulation we do not know whether our measurement is precise or accurate or neither or both.

And this uncertainty generally allows us to define a new term which is called uncertainty and this is the term which is more popularly used in color in correspondence to measurement errors and from now onwards instead of errors we are going to talk more about uncertainties. We are using this term uncertainty because it may be possible like suppose we have got a situation somewhat like this in one case. If our true value is not known we are still not sure whether we have got a correct result or not. Only if we keep on doing several experiments or you use some other device etcetera then we can we shall be sure that we have got a very precise and accurate measurement.

This is the uncertainty which hmm which keeps us in some kind of doubt despite they are not being any kind of error in the measurement. That is why instead of using the term error that amount certainty is more popular or more common.

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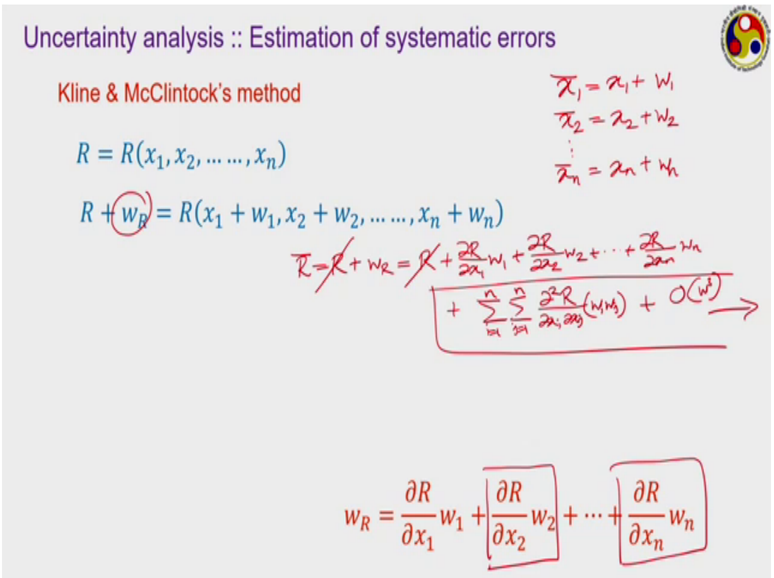

Let us try to see how we can estimate the systematic errors and we shall be seeing the method proposed by Kline and McClintock's which is a very popular and followed everywhere. Let us say we have an output R which is a function of several possible inputs x 1 to x n, there are n number of inputs that we are supplying to a device and it is giving an output R.

Now, each of these input may have its own error like say x 1 bar is the value of that instrument actually sensing which is different from the true value of x 1, x 1 is the true value x 1 bar is the measured value of inaccurate value or the uncertain value and the difference between the two is hmm W 1 which is the uncertainty involved with the value of x 1. There is a value of x 1 that we are getting that is having this amount of uncertainty. Similarly x 2 bar that we are getting from measurement device that is containing an error w 2 and same for everything till x n bar refers to the value of x n that we have got with an uncertainty of W n.

So, each of the values are continuing his own uncertainties and hence what we are going to end up with is not the true value of R rather with that will be being added with some kind of uncertainty the w R. Think about the venturi meter situation again. Here this x 1, x 2, x n may differ to all those five variables that is if we consider n equal to 5, then x 1 may refer to the area x 2 may refer to that C D, x 3 may refer to the delta p etcetera and final R is the mass flow rate. So, this is a function that we have got.

Now, if all these uncertainties are quite small then we can allow we can expand this following the Taylor series with respect to that fixed point x 1, x 2 up to x n that is if we expand this R bar which is R plus w R following Taylor series what we are going to get? We are going to get as R plus dou or dou x 1 W 1 plus 2 W 2 plus this till the n-th term plus the second order derivatives.

The second order derivatives will be of the form dou 2 R dou x i dou x j, where i varies from 1 to n j also varies from 1 to n into corresponding omega W i W j plus other term of the order of omega q let us neglect all the term from second order onwards. And if we neglect all the terms from second order onwards then we can eliminate R from this side. So, what we are going to end up with is W R is equal to this. This gives us some kind of idea about the error that can be present.

Here this w R or dou R dou x 1 this is called sensitivity of R to the variable x 1. Similarly, dou R dou x 2 refers to sensitivity of R to x 2. This way each of these differential refers to the sensitivity of the output to the corresponding input and W 1 W 2 etcetera the uncertainties present in each of the measurement.

Now, once we have got some idea or we have done this mathematical exercise then how can we get the maximum possible error that can be coming in? So, each of these terms the product of sensitivity and corresponding uncertainty is giving a measure of how much maximum amount of error the constant variable can import. Like the variable number 2, the maximum amount of error it can impart is this particular quantity the product of corresponding sensitivity and its own uncertainty like the variable x n can impart this much of error the product of its sensitivity and its corresponding uncertainty.

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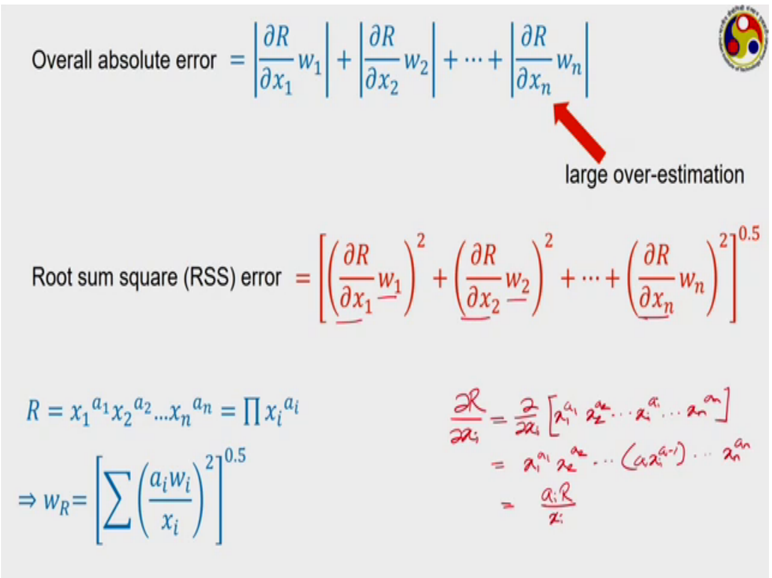

And from there we get the overall absolute error which can be considered as the summation of the absolute value of each of these errors or where each of this each of these contributions which basically corresponds to the product of the concern sensitivity and uncertainty.

However, in practical measurement it may very much possible that the error contributed by one particular component gets negated by another one and so, this overall absolute error is going to lead to only very large overestimation. So, a more realistic approach is to go for root sum square approach or RSS approach, where we shall we taking the square of each of this contribution, adding them up and then finally, we shall be taking the root of this particular summation.

This is generally the accepted formula for analysis of error. So, whenever we are trying to calculate the error in any kind of final measurement first we have to calculate the sensitivity of each of the inputs; like this dou R dou x 1, dou R dou x 2, dou R in the x n then we have to multiply each of the sensitivities with corresponding uncertainty like duo R dou x 1 will be multiplied by W 1 dou R dou x 2 will be multiplied by W 2 etcetera. Then you have to get the square of all these products and then add them together finally, this summation we shall be taking the square root of that to get the final root sum square error or which is conventionally called the uncertainty in the final value of R.

A couple of very simple example like one case suppose all the variables are appearing in a product form. So, x 1 to the power a 1, x 2 to the power of a 2, x n to the power a n, then if we want to calculate say uncertain sensitivity dou R dou x i for the i-th variable, then basically we are trying to take this to the power a 1, to the power a 2, x i to the power a I, x n to the power a n. So, if we get this product then what we are going to get x 1 to the power a 1, x 2 to the power a 2, dot a i x i to the power a i minus 1 x n to the power a n.

Because all the x variables are input quantities and they are independent of each other and hence we can write this as if we multiply this as x i and then divide by the same it is going to lead us to a i into the R itself divided by x i and finally, if we get the uncertainty multiply each of them is corresponding W 1 whole square and we are going to lead up to this particular thing.

So, this is what we are referring to as the uncertainty in the final value in such a situations where each of them are or they are appearing in a multiplication rule.

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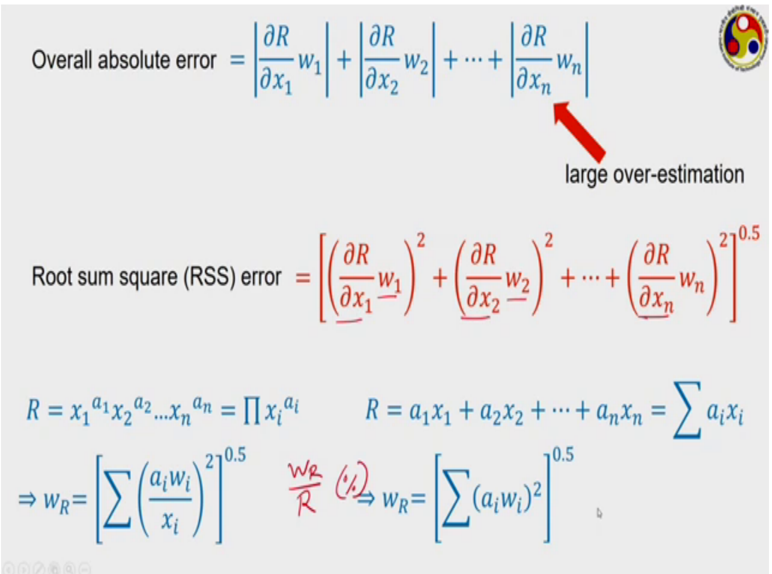

But, this is the absolute error if our objective is to get the relative error then of course, we have to get this W R divided by the true value of R which often we represent as a percentage one.

Another example where they all appear as a summation then the same way you can calculate corresponding uncertainty as a i W i whole square summation of that whole to the power 0.5.

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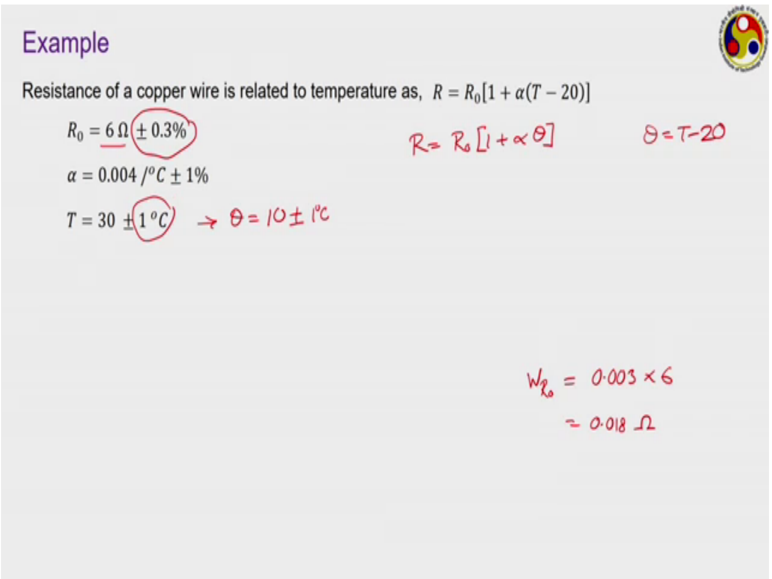

So, let us take a few examples numerical examples and try to see the application of this uncertainty analysis. So, we have the resistance of a copper wire related to temperature given by this relation where R equal to R naught into 1 plus alpha T minus 20; 20 is the reference temperature and here all the values are given. So, just take a look at the values that are given here. The way on here said R naught the resistance is given a 6 ohm plus minus this quantity. This refers to the uncertainty that is present in a value of this R naught.

The plus minus symbol is put to indicate that error can be on the higher side can be on the lower side also and this 0.3 percent refers to the error is actually a percentage of the value that is 6 ohm itself; that is the where error in this W R naught is actually 0.003 into this 6. So, ohm that is 0.018 ohm that is error in W, before proceeding anything as there is a T minus 20 factor let us convert that as R equal to R naught into 1 plus alpha into theta, where theta is equal to T minus 20.

So, accordingly here this becomes theta equal to 10 plus minus 1 degree Celsius. Look at the way the uncertainty is given for this temperature here there is no percentage just exact value; that means, in any temperature reading we may had plus minus 1 percent error or sorry just we may have plus minus 1 degree Celsius error.

So, from this relation let us try to calculate the sensitivity in the final R.

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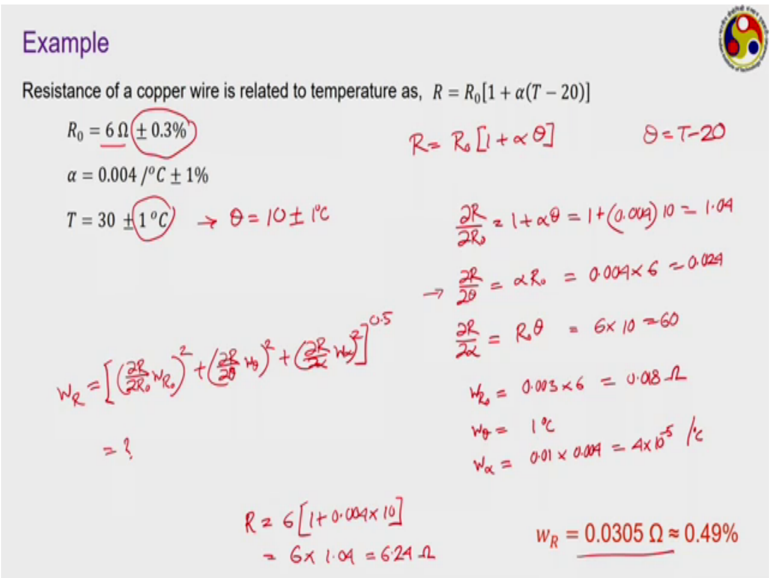

So, dou R dou R naught will be equal to 1 plus alpha theta, dou R though theta will be equal to alpha R naught, dou R dou alpha will be equal to R naught theta. Now, how much is the uncertainties present in each of this measurement? Okay, let us first calculate the values maybe. I do not have a calculator here, but, I have noted down the values. So, I can use them. So, if we put the values 1 plus, what is alpha? That is 0.004 into theta in this case is 10.

So, the sensitivity of R to R naught is coming to be 1.04 and for this theta it is coming to be alpha is again 0.004 into R naught is 6. So, 0.024 and R naught into theta 4 with respect to alpha it is 6 into 10, that is 60. I am not writing the units, but each of them has its own unit. Like in the first case it is unit less both numerator and denominator of the same quantity, but in the second case like this one what should be the unit? Numerator is in ohm, denominator is in degree Celsius, so, the corresponding sensitivity is ohm per degree Celsius and the same way we can get for the third one also.

Now, here W corresponding to R naught is given as 0.003 we calculated earlier also to be equal to 0.018, W corresponding to theta is equal to 1 because it is absolute value that is given, W corresponding to alpha is given as 0.01 that is 1 percent of the value that is 0.004. So, it is coming to be 4 into 10 to the power of minus 5. It is an extremely small value and we should write their units also. So, it will be in ohm this will be in degree Celsius and this will be in per degree Celsius.

Now, how much should be the contribution from each of them? We have to multiply the each of the component sensitivities the corresponding uncertainties, then get the square and that will give us the final value. So, final W R will be equal to the sensitivity with respect to R naught into W R not whole square of this plus dou R dou theta W theta whole square of this plus dou R dou alpha W alpha whole square of this to the power 0.5 and by putting the numbers you are going to get whatever is the final value of uncertainty.

And, if we do it we are going to get 0.0305 ohm and but this is the absolute uncertainty and if our interest is to get their actual uncertainty or sorry relative uncertainty, then we have to get the true value of R. How to get the true value of R? There we have to use the values that is given. Like true value of R will be equal to when you put the value of R naught that is that is 6 into 1 plus 0.004 that is for alpha into theta that is in 10 this is often referred to as a nominal value. So, it will be 6 into 1.04 and whatever value you are going to get is say 6.24 ohm. So, while the actual measurement should be 6.24 ohm we are going to get 0.0305 ohm uncertain in that which is about 0.49 percent which is very very small.

This way we can calculate uncertainty.

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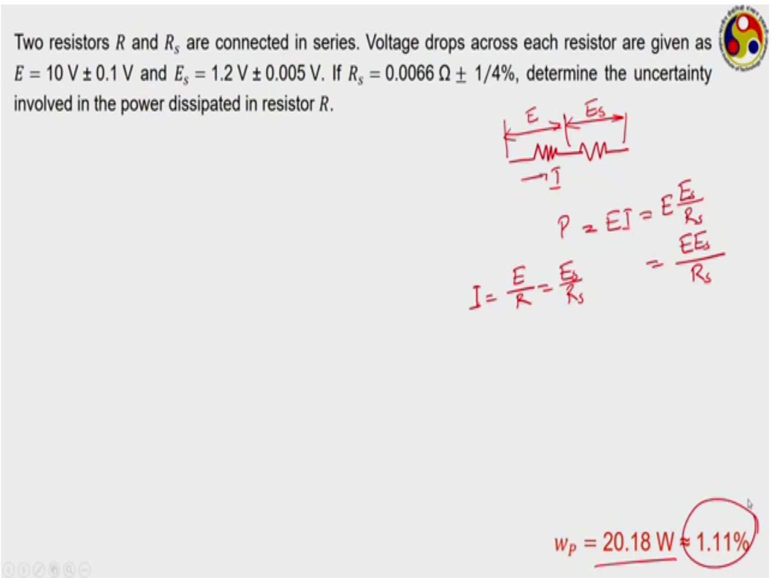

Just another example for you to check. Here we have two registers R and R is connected in series, voltage drop across each register given by E and E s and then R s values also the uncertainty is given. So, we can say we have one register here, we have another register here and we are having say this much of voltage drop E here and this much of voltage drop E s here, a current I is flowing through this. So, if you have to calculate the power that is being dissipated in the register R, the corresponding power will be equal to E into I.

Now, we know that I is equal to E by R and again E S upon R s, so, if we put it back here it will be E into E s upon R s and or I write. So, we have a relationship between the desired quantity which you want to calculate and the three values for phase values our measure and uncertainties are given and this is the corresponding power error that you will be getting 20.18 watt or 1.11 percent uncertainty. I would request you to do the calculation. I will leave you with a couple of problem exercises which you please try to do till the next lecture.

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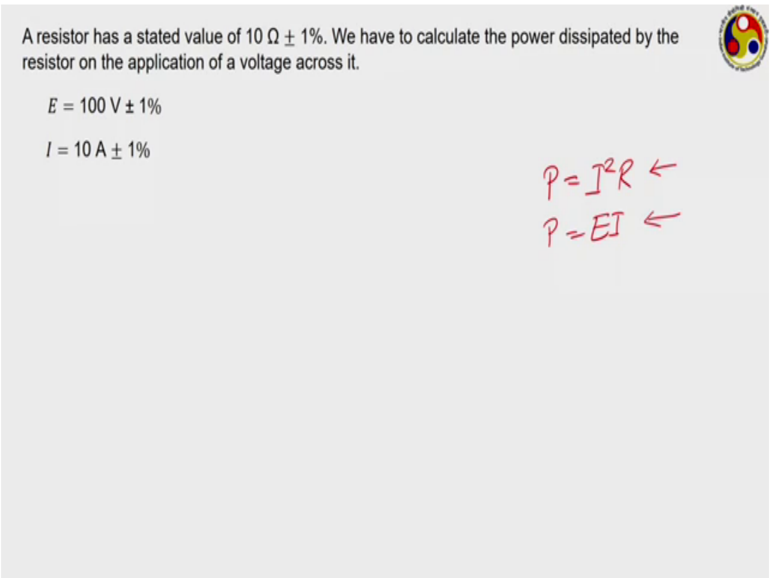

In one case I have given you a register which is a stated voltage of 10 ohm plus with an uncertainty of 1 percent. We have to calculate the power dissipated by the resistor on the application of a voltage across it. Both value and uncertainty in corresponding voltage and current are given.

Now, power we can calculate in two ways. We can calculate power as an I square R term we can also calculate power as EI term. In one case, like in the first case we have to measure only the current. In the second case you have to measure current and voltage both. Try to solve for both the cases and see which one is giving how much of uncertainty.

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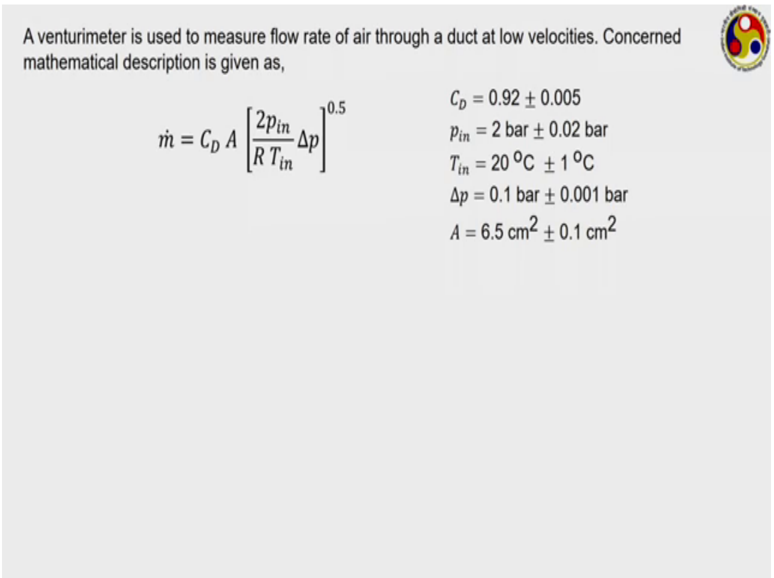

And, the final problem, where the problem of venturimeter that I mentioned earlier the relation is given and all the five parameters with our values are uncertainty has given. Try to calculate again the final uncertainty, but another thing you please try to check from here which one is going to give you the highest contribution among all the five.

In the next lecture, I shall be coming back to this problem to see how can we insulate the instrument. Like in this case if their uncertainties are different you may get an idea about which process to follow. Again after getting your answer, I can proceed with this and same in this problem try to see the product of sensitivity and uncertainty for each of the components and compare those values to see which one is having the largest contribution in the overall uncertainty.

So, I am leaving with you with this.

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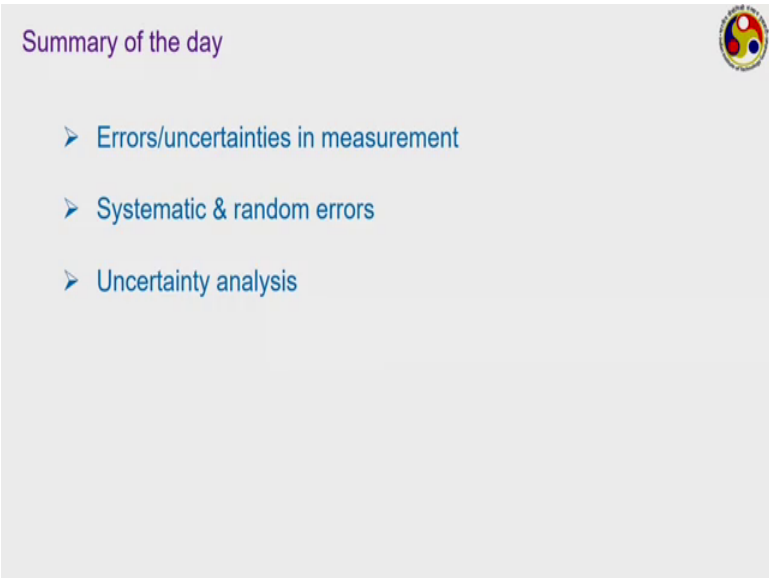

In today, we have discussed about different possible errors and uncertainties present in measurement. I have talked about systematic and random errors and then uncertainty analysis in terms of systematic errors. In the next lecture, which will is going to fill the last one in module – 1, we shall be talking about the random mirrors and corresponding statistical statement, statistical treatment should say.

So, thanks for your attention. Please solve those two exercise problems. Try to solve each of them that we have discussed, so that before you start the next lecture you have a good idea about how to deal with the systematic errors.

Thank you.