Principles of Mechanical Measurement
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Module – 09
Flow Measurement
Lecture – 1
Bernoulli's equation in obstruction meters

Morning friends. Welcome to week number-9, where we are going to talk about the measurement of fluid flow in some channel or maybe in normal pipeline, which is of course a very common phenomena, which we frequently encounter in different kind of practical situations. Say it be just the flow of water in the domestic water supply line or maybe much more complicated situations of flow of some biochemical fluids through syringe, a hypodermic syringe something like a micro channel or even Nano channel or maybe the flow of blood through the arteries, so everywhere we have to encounter this flow situations. And therefore, it is very important to know the common methods of fluid flow measurement.

Now, the measurement of fluid flow of course can be done in different possible ways. But, before I start the discussion of corresponding measurement techniques, we briefly to know what do we mean by flow; flow of course here we are referring to the movement of some fluid. Then next question obviously is what we mean by a fluid. Fluid as you know, I am sure all of you have already on a basic course on fluid mechanics, so from that knowledge you definitely know that fluid refers to the substance which starts to deform continuously, under the action of action of some shear force. And that deformation itself is what we refer by this flow.

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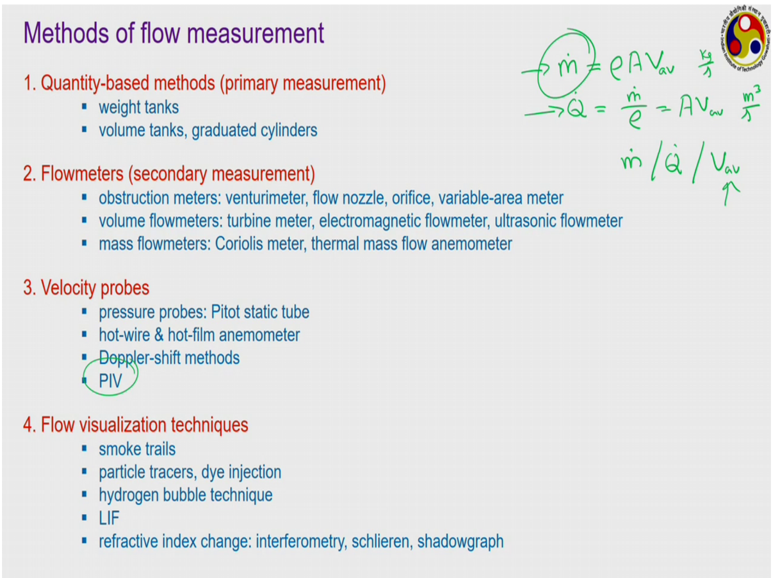

Now, the measurement of fluid flow can be done in different ways, because here not only the flow, rather whenever you are talking about the fluid flow measurement. We are also talking about the measurement of fluid velocities or maybe the mass flow rate as well and then trying to correlate the flow rate from there.

Because, we know commonly, say if we talk about the mass flow rate m dot for a fluid using just the one dimensional form of the mass conservation equation or the continuity equation, we can frequently write this one as rho into A into V, where rho refers to the density of the fluid, A is the cross sectional area of the flow conduit, and V is the average velocity. So, we should put a subscript V average.

And Q dot or just Q is mass flow rate divided by the density, so it rehearse A into V average. Now, what is this? This is the volume flow rate, where mass flow rate is having an unit of kg per second, this having a unit of meter cube per second that the SI unit of course that is the volume of fluid flowing through that particular area A per unit time.

And of course, V average is also there, therefore the measurement of fluid flow can relate to any of these three parameters, mass flow rate or the volume flow rate or maybe the average velocity or maybe just velocity profile across a particular channel. Depending on which parameter, we are measuring or what type of measuring method that we are following, we can devise different; we can categorize the different techniques for flow measurement just what I am going to show here.

Like, the primitive method is something like the quantity-based method which is defending the primary measurement, where we do measure the volume flow rate. Please note that, we are talking about a flow measurement which primarily refers to the measurement of this Q dot. But, quite often we are actually measuring this m dot or this V average or maybe a velocity profile over a channel, and then trying to calculate other parameters using simple mathematical relations, those are secondary measurement.

And those are the most prevalent one in the scope of fluid mechanics or maybe in the scope of industrial fluid machinery. But, still there do exist some methods for primary measurement, where you can directly get a measure of this volume flow rate something like weight tanks or graduated cylinders, but those are generally primitive in generally very primitive methods and hardly used in practical purposes.

So, what we use is the second one, for flow meters where the flow rate is transduced to some other kind of signal, and from there we get a measure of this flow rate generally this Q dot itself. Like there can be different options, we can have obstruction meters, where the flow rate is actually converted to pressure drop, it is transduced to pressure drop, and then this pressure drop is used to calculate the flow rate, examples can be venturi meter, flow nozzle, orifice meters, variable-area meters, we shall be discussing some of them in today’s lecture.

Then we can have volume flow meter something like carbon meters, electromagnetic flow meter or ultrasonic flow meter etcetera. We can have mass flow meters also, where in fact or measuring parameter is this m dot. We get direct idea about the mass flow rate, and from there we can try to get back the volume flow rate using the information about the density of the fluid. Like we cariolis mass flow meter, thermal mass flow anemometer etcetera, can come under this category of mass flow meters. So, these are all our secondary measurement.

Then can be a third group also, where instead of mass flow rate or volume flow rate, we measure the velocity. The average velocity or maybe velocity at a particular point or maybe velocity profile along a particular line, something like that we measuring; these velocity probes. Options can be the pressure probes, which you have already learned in the previous week the pitot static tube, where actually measure the velocity using the pressure drop or pressure information. And from that velocity information, we can calculate mass flow rate or volume flow rate.

We can also have hot-wire and hot-film anemometers, we can have Doppler-shift based methods, we can have PIV Particle Image Velocimetry, we shall be talking about this one in later lectures. A fourth group of method can be a very interesting one a quite attractive one that is flow visualization techniques, where we can use some kind of smoke trails or dye injection, particle tracing, to indicate how the particles are flowing inside the flow, and from there try to get an idea about the flow pattern itself.

We can also have a very modern technique of hydrogen bubble, supply LIF. Another interesting fluorescence with techniques refractive index change instead of in terms of interferometry, schlieren, shadowgraph, all these are very interesting techniques. I shall be trying to provide you glimpse to some of these methods, just a description of the basic way of getting the flow visualization done. But, let us start from the very basic. So, these are these four category of methods that I am talking about, we shall be discussing briefly about some of these topics. Particularly, the ones which is which are most commonly used in industrial applications, and also in research laboratories.

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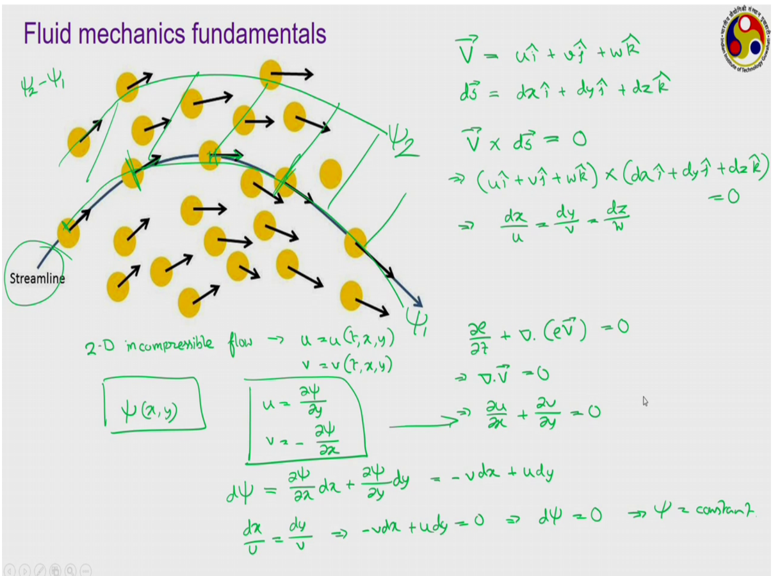

But, before I start talking about this measurement tools or measuring instruments, let us just discussed a bit about some fundamentals of fluid mechanics. Actually, in this particular semester, I am teaching fluid mechanics here itself at IIT, Guwahati at the second year level. And I personally feel, fluid mechanics is probably the most interesting subject that you can encounter in your entire undergraduate course of mechanical engineering that is why is quite tempting for me to go into the deep go into deep into this subject, but still I have to resist myself. And I shall be explaining or talking about only those concepts, basically reviewing those concepts which we need to make use of in this particular topic or in this particular week.

So, the first concept, which is very much important for different operation of different measuring tools, particularly the flow visualization techniques is the streamline. What do you mean by streamline? I am sure all of you already know the definition of a streamline. But, still for reputation purpose streamline refers to an imaginary line a hypothetical line, which is drawn in a flow field such that at every point the velocity vector at that particular point x is tangent to that particular line like if we take this point, the velocity vector will be tangent to this line.

Similarly, if we talk about this particular point, as shown the velocity vector is tangential to this. Similarly, if we come to this particular point, the velocity vector will be tangential to this. So, by joining these tangents or the tangential direction, so you can also plot a streamline.

Like we can draw another stream line, something like this if we take this particular one, then probably something like this. I am quite poor in drawing this particular situation, but the idea is to show you the basic definition of the concept about the stream line. So, stream line refers to a line such that at every point, the velocity vector is tangent to this.

Let us say V refers to the velocity vector in a flow stream, which is in a three dimensional stream can have three components. Let us say u, v, and w are the three components, three velocity components of this vector. And d s refers to an isotherm sorry if I simply small displacement, along the streamline an infinitely small segment of the streamline.

Let us say if we pick up this particular point, then this is the velocity vector v. And we are talking about an infinitely small flow segment like this or segment of the streamline, it should repeat. So, it is again a vector with components such as d x, d y, and d z. Then what is the relation between this V and ds, using your concept of vector mechanics, I am sure you will be able to tell as we know that this V and ds are tangential to each other, because the velocity V ds is a part of the streamline at a particular point, and V is a velocity vector that particular points, so velocity vector will be tangent to this.

So, accordingly, we can write the relation as V cross ds has to be equal to what has to be equal to 0, because they are tangential to each other. So, they are cross product, the vector product has to be equal to 0. So, we can use the components now to write that u i plus v j plus w k cross d x i plus d y j plus d z k that has to equal to 0. And if you perform the product, then we can see that, it will actually come as something like this d x upon u is equal to d y upon v is equal to d z upon w, which can be taken as the equation for a three streamline in three dimensional coordinate system.

Now, another concept that is quite frequently related to the stream line is called stream function. Remember stream line and stream function are two different concepts of course they are related to each other, but their origin is completely different, where streamline is an imaginary line which is having a physical significance in a way that the velocity vector always remains tangent to this. Stream function on the contrary is something like a mathematical concept.

Let us say, we talk about a 2D incompressible flow situation, we are taking a 2D incompressible flow. So, it is 2D an incompressible, so the velocity component u will be a function of time x and y, because we have not talked about steady, it is just two dimensional y are the two components, let us neglect the z dependency. And t is the time dependency, so we similarly leave a function of t, x, and y.

Now, what about the mass conservation equation? If we write the mass conservation equation, there is a continuity equation in general form differential form, so we can write that dou rho dou t plus red dot rho V will be equal to 0 that is a standard form of mass conservation equation. Now, we are talking about an incompressible flow situation means, density can be taken to be a constant leading to divergence of velocity to be equal to 0, which means dou u dou x plus dou v dou y to be equal to 0.

Now, let us define a function psi, a function of x and y such that it satisfies this two dimensional incompressible continuity equation. Then if this function psi has to satisfy this particular equation, then what should be its relation between with u and v. There are quite a few ways, we can relate them, but generally the most commonly of defining is u is equal to dou psi dou y, v is equal to minus dou psi dou x that is in early the most common definition.

But, we could have also taken u equal to minus dou psi dou y, and v equal to dou psi dou x. If you put this definition of u and v into this equation, then that definitely satisfies the continuity equation. So, this stream function psi is defined with this particular relation of u and v or definition of u and v.

Now, let us write d psi that is alone change in the value of the this psi is referred to as the stream function, but so far we are not related this one to the stream line, this is completely an unrelated concept till now. This is just a definition of a parameter psi which will satisfy the 2D incompressible continuity equation. And therefore, instead of using u and v as two different dependent variable, we could have used just a single variable psi as u and v can be defined in terms of psi itself.

Now, see this d psi which refers to a change in the value of the stream function, during a flow situation. Then what it will be, we can write this one as dou psi dou x dx plus dou psi dou y dy, now using the definition of u and v dou psi dou x is equal to minus v d x plus dou psi dou y is u, so u d y. And now look go back to the definition of the stream line. To the mathematical form a stream line in a two dimensional situation is d x upon u is equal to d y upon v that is equal to minus v dx plus u dy is equal to 0.

So, taking the definition of stream line, then d psi equal to 0 stream function which refers to psi equal to constant that means, stream function psi remains constant along a streamline. The value of the stream function itself may keep on varying across different stream lines, but it will remain constant along a particular line that means, the line shown here this particularly we have a particular value of stream line. Stream function similarly this part if we draw another stream line something like this, this will be having a different value of psi if say this one is psi 1; this may be equal to psi 2.

And what is the physical significance of the stream function, stream function refers to the volume flow rate that is this quantity psi 2 minus psi 1 will refer to the volume flow rate of fluid trapped between this particular space. So, this is the concept of stream function and stream line, which you need to make use of in subsequent discussions.

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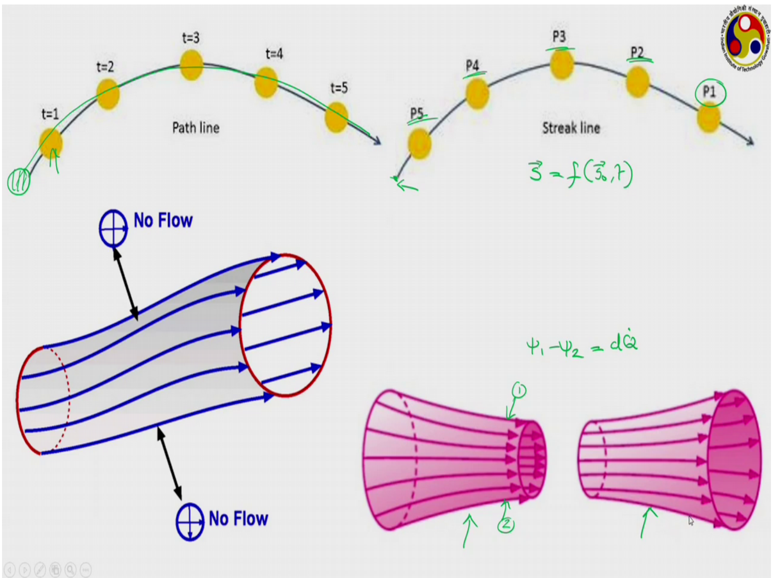

And, but two other terms or other definitions quite commonly come in conjunction stream line, those are path line and street line. It is important to understand the difference of these three types of lines. Path line all these three are imaginary lines, but while stream line is such a line, where velocity vector always you must transactional to that. Path line refers to a the trajectory followed by one particular particle over a period of time that is here we are keeping our focus on a particular particle, and just seeing how it is moving.

So, here say if we have the, this is the one is the particle, then you are just tracking the movement of this body. Let us at t equal to 0 particular somewhere here, then you are just tracking this movement of the particle over a period of time, and the locus of this one is the path line. So, this is the locus of this particle over this 5 or 6 seconds shown here, and that is the path line corresponding to this particular particle. It is in a way a Lagrangian way of describing the flow field. Because, here we are engaging our self a particular particle, and just moving with the particle to see which is the path that I am able to track that is the path line.

On the contrary, in case of street line, it is an oilarian description, where you are focusing on a particular point. Let us say we have a particular fixed point here, and then whatever part particles are passing through this particular point, we are joining them by an imaginary line, and that line is this streak line.

So, the first particle to pass through this one is this p 1, next is this p 2, next is p 3, next is p 4, next is p 5. Now, we if we join all of them at a particular instant of time, then what we get that is a streak line. So, streak line is available at particular instant of time, and they relate to a particular position through which all these relevant particles are passing through. Whereas, path line is drawn over a period of time, here we are focusing on a particular particle, and just seeing the locus of its movement.

So, these three lines particularly path and streak lines will be this coming back again, when you talk about different kind of flow visualization technique. Like for stream line, you have already seen a mathematical description in the form of stream function. For path line, however it is not possible to write any generalized mathematical equation, because it is just the locus of this particle movement or moving particle, and so that depends on the particle itself.

However, for streak line we probably can write the equation of the line as something like something like this, where s is the equation for this path line, s not refers to the fixed point. If this is the point with respect which you are drawing the steak line, then is not refers to a location of that particular point. And t is a time at which we are drawing this steak line, because this time t will decide, how many particles we have to consider for this. Under when you are talking about a steady incompressible flow situation, stream line, path line, a streak line, all coincide with each other. However, in unsteady flow situation, they are separate from each other. So, each of them may have a different existence. We shall be coming back to path and path lines and streak lines, during flow visualization.

But, another topic that we just now, we have to make use of that is called stream tubes. Stream tubes is nothing but a closed surface formed by a group of stream lines or a several stream lines, like just shown here. Several stream lines are joined together to form a closed surface, so that fluid is able to flow through this closed surface, and this is called a stream tube. As two stream lines cannot cross each other, so there is no flow possible across the surface that has been formed by this stream lines, and you can almost visualize this unlike a tube.

In fact, any pipeline can be visualized like a stream tube, because the valves of the pipes will correspond to zero velocity. So, correspondingly we shall be having some streamlines on valves using having zero velocity values, and no fluid is able to cross the wall of the channels or the pipeline, so that may also visualize a real stream tube.

So, stream tube is something that we shall be making use of now. Of course, the shape of the stream tube may keep on changing; the cross section you also keep may keep on changing depending upon the velocity values. Like the if the fluid is accelerating, then the stream this we can may have this kind of situation, how the cross section area reduces. Because, remember the streamlines or the difference each of the stream line has a different value of stream function, and if we talk about this particular stream from line, and this particular stream line, so you give this one name as 1, and this one has 2.

Then the difference between their stream function values will correspond in two that total volume float of fluid which is flowing through this stream tube. And as if the velocity increases, corresponding cross section area also will reduce accordingly, it will it will take this kind of shape. Whereas, when the velocity is reducing, then it lead to requires larger area to flow, to maintain a fixed volume flow rate, so you will get to this particular situation. The stream function or I should say the stream tube will look like a diverging section.

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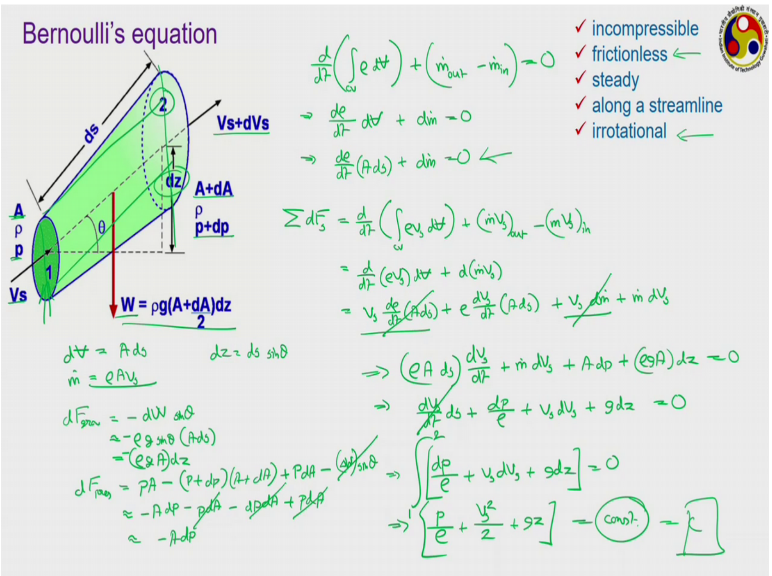

Now, with this idea of stream tubes and stream lines, let us move to something very well known to you that is a Bernoulli's equation. I am sure all of you know about the Bernoulli's equation, but not sure how many of you know how to derive this. Actually, there are several ways, we can derive the Bernoulli's equation, but we shall be showing just one of the ways starting from the fundamental conservation equations.

For that we are considering one stream tube. Here V s or s is the direction, the sum of can often called a stream wise direction. The stream tube is having area A at one side, as density of the fluid is rho. And pressure at this particular point is P. On the other end area is A plus dA, pressure is P plus dP. And we are assuming it to be incompressible, so that density remains constant at that rho.

Here velocity is V s at in the stream wise direction, the velocity is V s whereas, the at the other end let us that (Refer Time: 22:53) 2 the velocities V s plus d V s. The x is or the stream wise direction is making an angle theta to the horizontal, accordingly the weight W of this total fluid volume bound within the stream tube is given by this, and total length of the stream tube is ds.

So, if we write the mass conservation equation for this particular stream tube, then in general integral form what we can write, we can write the general mass conservation equation as d dt of integrated over cv rho dv, where this if I write it properly, why this V which striped out V refers to the volume, and V will refer to the velocity. So, cv is a total control volume, which is the stream tube only plus the mass flow rate which is moving out minus mass fluid which is coming in is equal to 0. This is a standard definition or standard form of the mass conservation equation, just the differential version of the shown we have used two slides back, to introduce the concept of stream function.

Now, if as we are talking about a fixed stream tube, so the volume is constant. Accordingly, we can write this one as d rho dt dv plus dm dot equal to 0, where dm dot is nothing but the same dot out minus m dot in basically refers to relate sorry net outflow of mass from this control volume. And what is dv, dv can be taken to be something like A into ds. Of course, area is changing from A to A plus dA, but dA can be quite small compared to s, we can approximate this one to something like. So, we can approximate this one to be something like is equal to 0.

Now, we go to the momentum conservation, but before that we can write that mass flow rate at any particular point can also be written as rho A into V s, which is corresponding to that particular point A refers to area that point, and V s refers to velocity at that point.

Now, we write the momentum conservation equation. So, as per the momentum conservation equation summation of all the forces in the stream wise direction, you have to remember that momentum is a vector. So, you have to leave it specify the direction, and we are writing it here along the stream wise direction should be equal to d dt of rate of change of momentum which is rho into Vs into d of volume plus m dot Vs out minus m dot Vs in.

And again dv remaining constant, we can write this one as dt of rho Vs dv plus d of m dot into Vs or if we break this one to two parts, we can write this one as Vs into d rho dt into A ds plus rho into dVs dt to Ads plus breaking the other one can write this one as Vs into dm dot dt plus m dot into dVs dt.

Now, if we compare this one with the mass conservation equation or sorry just a mistake here Vs into dm dot to dVs is now if we compare with the mass conservation equation that is this particular form, you can see this particular term, and this particular term, they actually cancel each other. So, we remove them with two terms remaining.

Now, there are several forces that can act on this which can constitute the part of this left hand side. So, here we put our second assumption, which is frictionless or inviscid fluid that is the shear stress can be neglected. So, if we neglect the shear stress along the periphery of the fluid motion, then we are left only the pressure force and the gravitational force.

Now, how much will be the gravitational force, the gravitational force already shown on this, so this should be equal to minus d w into sin theta, where d w refers to the mass of the elemental mass which is bound to in this. And the elemental mass shown by this expression, but again assuming dA to be extremely small compared to A. We can approximate this one to be as rho into g sin theta into the volume which is Ads.

And accordingly, we can write this one as rho g A d z, where d z refers to the height in the direction opposite to gravity or the change in elevation from the inlet to the outlet opposite to gravity, dz is equal to ds sin theta which gives us this situation. Of course, the minus sin I have missed and the other force corresponding to the pressure.

Now, the pressure is acting on all the phases. So, what are the pressure force that is acting in the stream wise direction, if we talk about the first phase first phase, phase number 1, so the pressure force acting on this is P into A. On face number 2, the pressure force acting you remember pressure is always compressive once, it is acting always in the inverse direction. So, it is P plus dp into A plus dA minus we have to consider now the pressure which is acting on the periphery longer sides.

But, one thing we have to be careful here, here the actual pressure force that is acting if we consider in a cylindrical core, and then an extended area outside of this conical section, then the effective pressure force on the side, in the stream wise direction will be coming only from the extension of this area d A. And So, if we take that out, correspondingly the area will be pressure acting on this extended area d A minus this d W sin theta or I should not say d W sin theta, I should write delta W sin theta.

Here this delta W refers to the additional mass that may have come in because of the extension of the cross sectional area. Now, this one can be neglected in practical purposes, because it is quite small. So, if we now expand these two terms, and the product of dp dA can be quite small. So, we can write this one as minus A d minus A d p minus P d A minus d p d A plus P d A. And so from there so from their P d A cancels out, and d p d A can also be taken to be negligibly small leaving only minus A d p.

So, if we take it back, in the momentum conservation equation on the left hand side, so we have left with, on the right hand side we are left with rho A d s into d V s d t plus m dot d V s plus the forces coming from the pressure part, we have A d p plus coming from the gravitational part rho g A d s or sorry rho g A d z is equal to 0.

If we divide everything by this rho A, so we have d V s d t d s plus d p by rho plus remember m dot is given by rho A V s. So, from there, it is just V s d V s plus g d z is equal to 0. At this particular portion, we introduce our third assumption that is steady. So, as you assuming steady, so this part goes out of the equation, we are left with d p upon rho plus V s d V s plus g d z equal to 0.

If you integrate this entire term over a particular stream line that is from this point one to this two, then what we are going to get? Rho being a constant, we are going to get this as p by rho plus V s square by 2 plus g z is equal to constant which is the most celebrated Bernoulli’s equation. Actually, a steady assumption is not a very strict one, we can use the Bernoulli’s equation in our steady flow situation also. As per the derivation that we have done, where just we have to keep this last term that we have negated. However, general we apply Bernoulli's equation to steady flow situation also.

A fourth assumption which we have which is inherent there, here we are doing this along a streamline, because the final integration we have performed along a streamline from this point 1 to 2 along a particular streamline. And another assumption which is not strictly required for Bernoulli's equation that is irrotational, this constant it is often referred as the Bernoulli's constant.

Now, the value of this Bernoulli's constant C that will vary from one stream line to another stream line, because what we have here is a summation of three quantities, which actually refers to the energy. The first one refers to a friction energy, second one refers to the kinetic head, and third one refers to a potential head. So, this C actually the summation of the energy content of the fluid along a particular stream line which remains constant, if we are talking about journalist flow situation.

However, the energy content may vary from one stimulant to another. But, if we are putting this irrotational assumption, then this C is constant everywhere. So, this irrotational is not a strict assumption for one Bernoulli's equation, if it is rotational flow, then we can still use the Bernoulli's equation, but the value of this Bernoulli's constant will vary from one stimulant to another.

However, in case of irrotational flow, this will remain constant over the entire flow domain. But, one being assumption that we are putting in here is this frictionless. Incompressible is also a quite restrictive assumption, but for most of the liquids that is valid, and that so we can confidently use the Bernoulli's equation for liquids under most of the operating conditions, but may not be for gaseous, because gaseous are highly compressible. But, frictionless is a quite idealistic assumption, because every fluid has some viscosity on its own. So, they are will be shear stress, and there will be shearing losses. So, to take care of the shearing losses, quite often we have to ads in a corrective factor into this equation for this, we may see examples for that very soon.

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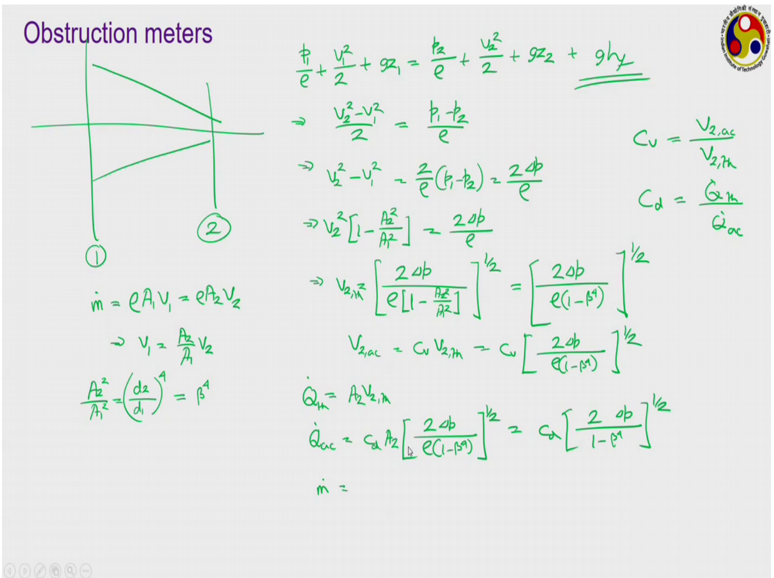

But, let us now move to our discussion on flow measuring instruments using this concept of Bernoulli's equation. And for that first we have this obstruction meters. Now, under the obstruction meter, we are talking about the measuring tools, where actually you are putting some kind of obstruction in the flow field. Thereby causing some kind of artificial pressure drop or some additional pressure drop and then measuring that pressure drop to get an idea about the flow velocity.

Let us say, we have a flow channel like this of changing cross section area this or section 1, this is section 2. So, if we apply Bernoulli's equation between these two sections under a steady incompressible flow situation, then what we have? For section 1, we have p 1 by rho plus V 1 square by 2 plus g z 1 is equal to p 2 by rho plus V 2 square by 2 plus g z 2.

And if there is any frictional losses, quite often we add h f which is the or a loss of pressure, pressure head in a way sometimes not truly pressure head. If we somethings like g h f term can we add where h of f refers to a pressure loss head or some kind of loss coefficient, but here we are not going to talk about this (Refer Time: 37:09) for now at least.

So, in this particular situation, if there is no change in elevation that is z 1 and z 2 are equal to each other, then what we have is V 2 square minus V 1 square upon 2 is equal to p 1 minus p 2 upon rho that is V 2 square minus V 1 square is equal to 2 upon rho into p

1 minus p 2 or 2 into delta p upon rho a delta p refers to a change in pressure between these two cross section.

Now, under steady state condition, mass flow rate remains constant. So, mass flow rate at section 1 will be equal to rho into A 1 V 1, and at section 2 it will be rho into A 2 V 2. Remember, we are considering steady and incompressible, so rho remains constant. Accordingly, V 1 is A 2 upon A 1 into V 2. So, if we replace that, it becomes V 2 square into 1 minus A 2 square minus A 1 square is equal to 2 delta p upon rho or V 2 is equal to 2 into delta p upon rho into 1 minus A 2 square upon A 1 square this whole to the power half.

In this channel is of circular cross section area, then A 2 square upon A 1 square can be related to s d 2 upon d 1 whole to the power 4, often refer to as beta to the power 4, where beta is just the just this density ratio. So, accordingly this one can be written as 2 delta p by rho into 1 minus beta to the power 4 whole to the power half.

However, the expression for velocity that we are getting here that is theoretical one. For this what do we need to know, generally rho is the density of the fluid, which remains constant along the flow direction, beta is a geometrical information. So, once we know the dimensions of this channel, we can easily get this value of the beta or beta to the power 4. So, only thing we have to measure is this delta p.

And the delta p measurement, we can do using any kind of common pressure measuring instrument commonly a manometer. So, with that pressure measurement, we can directly calculate this value of velocity. But, as I have just mentioned briefly this V 2 value that we are getting that is only theoretical one, because we have neglected this loss term which practically is present.

Now, if we want to consider this one, then the value velocity value that we are going to get that will be lower than this theoretical limit. So, accordingly we define a coefficient to take care of the value of this or this change in velocity or because of the frictional losses or a way of comparing the theoretical and actual velocity for that we define a coefficient C v called the coefficient of velocity, which is defined as this V 2 actual by V 2 theoretical.

So, V 2 actual will be this C v times this V 2 theoretical that is C v times this expression which we have just derived. But, our interest is in flow rate, so the flow rate Q dot will be equal to theoretical flow rate will be equal to rho into V 2 theoretical. And again to compare the theoretical and practical flow rates, we define a coefficient called C d coefficient of discharge which is again the ratio of Q dot theoretical by Q dot actual.

So, Q dot actual will be equal to C d into rho into V 2 theoretical which is 2 delta p by rho into 1 minus beta to the power 4 whole to the power half that is equal to C d into 2 rho delta p by 1 minus beta to the power 4 whole to the power half. And if your interest is mass flow rate, then mass flow rate will be sorry I made a mistake here actually, this is not correct because it should be equal to A 2 into V 2 theoretical, so it just comes A 2 here. And we can of course, consider this we by making a change such that beta to the power 4, we can go back to the original expression which is A 2 square by A 1 square.

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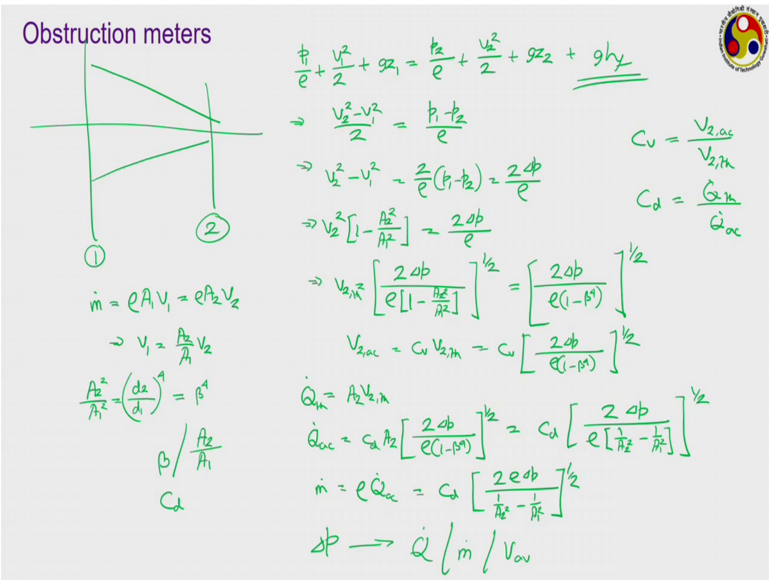

So, accordingly it becomes C two into 2 delta p by rho into 1 upon A 2 square minus 1 upon A 1 square and if you are interested in mass flow rate that is exactly, when the row comes into picture, so rho into Q dot actual. So, as rho goes in it will be the coefficient of discharge into 2 rho delta p divided by 1 by A 2 square minus 1 by A 1 square whole to the power of half or you can remain continued to be in terms of the coefficient beta or the ratio beta, yes.

So, what we are trying to do in this obstruction meter is transduced the pressure signal in terms of this delta p get a measure of this delta p, and then from there calculate this Q dot or m dot or the V average whichever is of our interest. But, for that purpose, we need to know a few information about the parameter about the machine that you are using about the instrument such a geometric information something like beta or the area ratio, whatever is of interest A 2 upon A 1. And also you need to know the coefficient, particularly the C d the coefficient of discharge. It generally comes from other manufacturer, but with repeated use there may be change in the flow condition, the C d may also be depending upon the fluid that you are using the operating condition of the fluid itself. Accordingly, we may need to repeatedly calibrate the instrument.

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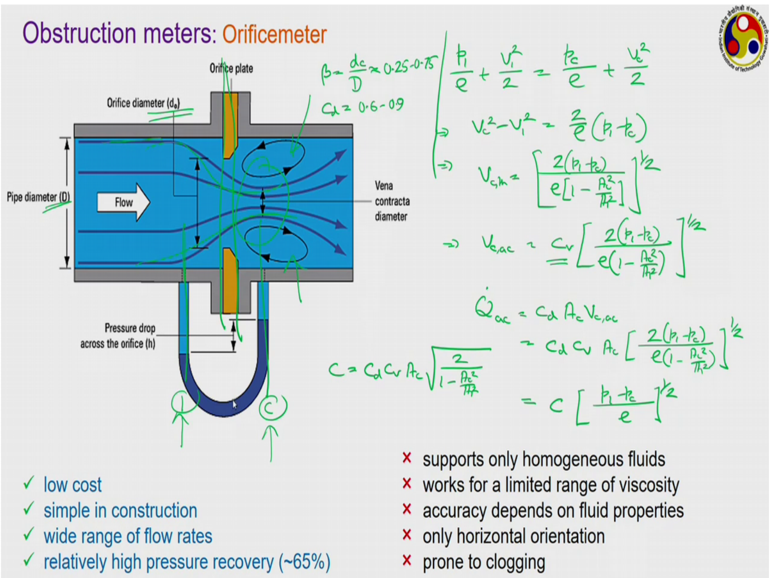

Now, there are quite a few different kinds of obstruction meters with working principle remaining very much the same like shown here. The first one the simplest one is an orifice meter. Here the idea is you have a flow channel, and in a particular section on the flow is suddenly have incorporated an orifice, just like shown here.

In fact, the diameter of the orifice shown here is quite large, practical cases this orifice diameter d naught can be much smaller compared to the actual parameter d. It is just like all blocking the flow blocking the entire flow, and then leaving a small hole to for the inter flow through past flow for passing through.

As we are putting this, then the fluid flow has to adjust itself, accordingly we will find the fluid which is flowing smoothly will contract initially, and reaches the smallest possible area somewhere here exactly, not exactly an orifice, but a bit downstream to that which is known as vena contracta. This particular thing is called the vena contracta. The location of vena contracta will depend upon the flow velocity or maybe the Reynolds number. And after that vena contracta, the fluid again starts to expand, they are recovering and the pressure.

So, there is a drastic reduction in fluid pressure, when it is passing through because of the drastic change in the flow direction. However, as the fluid expands on the downstream direction, it recovers the pressure a bit, but may not be able to recover fully because there are large a deformation in these zones, leading to certain losses.

But, we have to measure the pressure drop across this. So, there will be two pressure tapping put I on either side of the orifice plate, their position may change depending upon the configuration of your orifice meter. So, in case of an orifice meter, we generally go for the measurement based upon the C v.

Like if we just see this particular configuration, where one tapping is put in this particular position one, and other tapping is put in this vena contracta positions equation C. Then we can apply the Bernoulli's equation to write as p 1 upon rho plus V 1 square by 2 is equal to p c upon rho plus V c square by 2 assuming there to be having at the same elevation.

Accordingly, V c square minus V 1 square is equal to 2 upon rho into p 1 minus p c. And just we have done earlier V c becomes equal to or V c theoretical becomes equal to 2 into p 1 minus p c by rho two 1 minus A c square by A 1 square whole to the power half. Subsequently, we see actual will be equal to C c into C c by rho into 1 minus A c square by A 1 square whole to the power half. And we can then keep on using the value of we can just by measuring this p 1 minus p c and from the knowledge of the C c, we can get an idea about this actual velocity.

And from there, we can measure this Q dot. So, Q dot actual will be equal to the coefficient of discharge into A c into V c actual which will involved C d C v, here it should have been C v actually the coefficient of velocity into A c 2 into p 1 minus p c by rho into 1 minus A c square by A 1 square whole to the power half.

And quite often, we combine all these terms into just a single parameter C, you can C into A c or sorry C into p 1 minus p c upon rho whole to the power half, because rho is something that depends on the fluid that you are using and p 1 minus p c will come from the measurement, but rest of all generally are coming from the manufacturer, yet this C is equal to C d into C v into A c into root over 2 divided by 1 minus A c square by A 1 square.

So, this C is something that we need to know. For most of the common flow situation or common orifice meters that we have here this beta which is just the ratio of d c or to the total pipe diameter capital D that is generally in the range of 0.25 to points 0.75, whereas the value of this C d lies something in the range of 0.6 to 0.9, which is reasonably good meant the actual theorem flow rate may will be 80 to 90 percent of the one that is created by theoretical calculations, so much less a loss involved.

Orifice meter is a very simple device as you can see, here we have do not have to do anything, just you have to incorporate or introduced on orifice plate and this particular flow section. And just by measurement of this pressure drop, and fluid density, we can get the reading of the flow rate. The biggest advantage of this one is of course it is very cheap, and very simple in construction, we can measure a good range of flow rate, low to quite moderate level of fluid can be measured, and relatively high pressure recovery. Pressure recovery I shall be coming slightly later on, it actually is related to how much of the initial pressure the fluid is able to regain back its 65 around 65 to 70 percent for orifice meter which is quite decent.

However, it has because of its simplicity, it has advantages and it also has disadvantages. Like it can support only homogeneous fluid, if the fluid is having some kind of particulate suspension or fluid density is varying and the flow direction, then this one fails quite miserably. It works only for a limited range of viscosity, accuracy is strongly dependent on with the fluid properties like density, viscosity etcetera.

Only horizontal orientation is possible, we can not use orifice meter in inclined or vertical position. And also it is prone to clogging. Generally, the orifice diameter is quite small like the beta value that I have shown here. If the beta can be only one-third to one-fourth of the actual pipe diameter, so if there is some kind of dirt or soil elements present in the fluid, then that can easily block their orifice diameter.

The, but the while everything looks quite good as long as they are sticking to a general fluid and with moderate level of velocities, but the biggest challenge is to know the position of this vena contracta, because that is strongly dependent upon the channel configuration, dependent upon the fluid itself. And even for a given fluid on the flow velocities, larger the flow velocity, larger the Reynolds number the vena contracta may appear for the downstream will orifice location. And therefore, it is not a fixed location for which you can mount, your manometer to get the pressure reading. So, held p 1 or other one can be any given location in a fixed location p c, we have to be a bit experienced to locate it properly or locate is appeared in appearing location properly.

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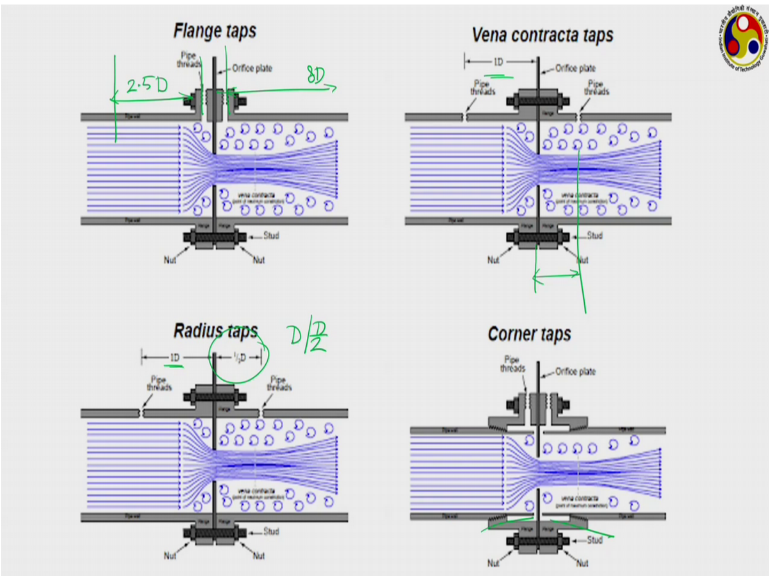

Accordingly, there are different kinds of tapping position that has been tried. Like the flange dropping tapping is the probably the easiest one, where you do not have to make any hole on the channel rather just the location, where you are putting the orifice on the same flange, you are putting the tapping's. You are having one tap on this side, and another tap on this side, this is the you do not have to their pre drilled coming with the orifice for any other kind of tapping, we actually have to make holes on the walls which technically may not be difficult, but merely to pipe failure or leakage problems later on.

In from that point of view, lunch flange tapping is quite convenient and that is why, it is quite popular in US in particular. However, it is not suggested if the flow or pipe diameter is less than 2 inch. The other one, which is not shown here that is called pipe tapping that is not shown in this particular slide. Pipe tapping in case of pipe tapping, we are putting the one location at a distance of 2.5D downstream, and other about 6 to 8 D downstream, these are the two locations where you are putting the pipes which are far away from the orifice itself.

So, 8 D selected, because it is believed that this is sufficient distance for the fleet to cover to regain back to its max original pressure level or at least whatever vortices are generated, they should it should get rid of all those vortices. However, it is a large distance that we have to consider; therefore the pipe itself needs to be quite long. And also whatever pressure losses that it is it the fluid may be encounter because of the pipe roughness, and some other in homogeneity in the channel itself, they will also get reflected providing lots of noise in the measurement, so that mean is not the pipe tapping is not a very popular one.

Then we have the vena contracta tapping as it is shown here, it is 1 D on this side, but the location of vena contracta being unknown. It is generally given a particular a fixed location that is generally decided. If we can properly identify the location of vena contracta that is perfect because, you can get the perfect value of the delta p; but quite difficult to locate.

So, people keep on going for this radius tapping, where we have on the upstream side we have at 1 D which is very similar to the vena contracta, but instead of in case of vena conductor, while we may have to keep on shifting the downstream tapping. Here the downstream trapping is fixed at D by 2 location. So, this tapping is also sometimes called D slash D by 2 kind of tapping. So, here D by 2 as a thumb rule week of quite often found that the vena contracta appears around that D by 2 location. So, this radius tapping is a quite popular, particularly when you are talking about a large diameter pipe.

And final one is the corner tapping, as it is shown the corner tapping's are located at the corners something like this kind of orientation. This particular one is quite popular in Europe. And in US for small diameter pipes, they keep on going for this corner tapping. So, radius and corner topping probably, and the flange tapping of course are quite popular, but depending upon what kind of system you are working with, what level of accuracy you want, we keep on going for one each of them or any one of them.

So, this is about orifice meter. Orifice meter is a very easy configuration as we can see, he just have to introduce someone orifice into the flow channel. And then pressure drop across the orifice, you will is going to give you the measurement of the flow rate. However, in case of orifice as there is a drastic change in the flow dimension or pipe dimension, the fluid has to add just quite a bit, thereby creating lots of disturbances and vortices formation.

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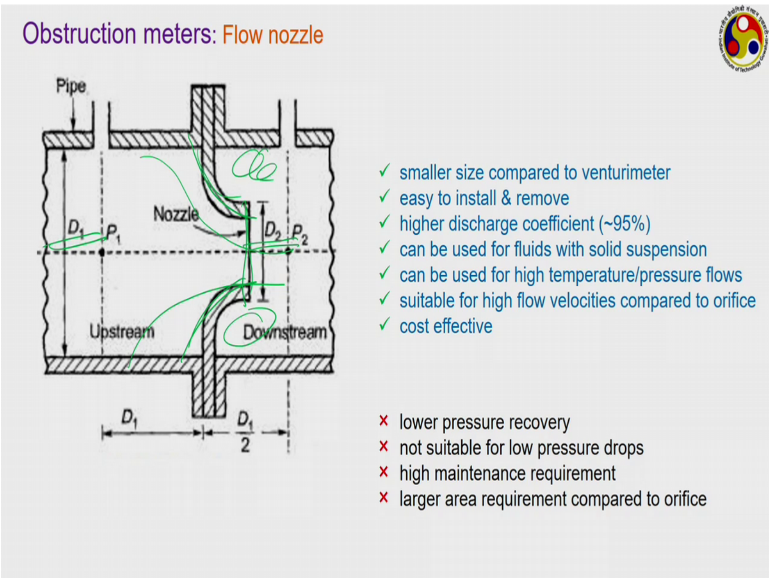

So, to eliminate that next step, we go to the flow nozzle. In case of flow nozzle, on the upstream side we provide a quite smooth change in the cross section area. From the initial pipe diameter D 1, it smoothly moves to the orifice diameter D 2 through this nozzle kind of configuration. However, at the exit it again is suddenly gets exposed to the entire cross section, so leaving for vortices formation in this particular zones.

So, this flow nozzle you can say, you can say it is some kind of improvement over the orifice meter, but still leaves a big scope of improvement in form of downstream design, its size is smaller compared to the venturi meter which we will be talking about next. It is easy to install and also remove, if required. And the discharge coefficient is quite high, actually after around of 95 percent which is excellent. Can be used for fluids with solid suspensions as there is a is smooth transition for the upstream fluid. On the upstream side at least, the solid particles can easily follow that without leading to the clogging of the orifice.

This orifice portion often called the throat of the channel also in a way, because that is a portion with the smallest diameter. We can also use flow nozzles, when there is we are dealing with a high temperature or high pressure flow situation. High flow velocity is compared to orifice, we can go for it is also cost effective.

But, the problem is the pressure recovery is a bit lower, we shall be again seeing this one shortly. It is not suitable for low pressure drops, because we need to have a sufficiently or significant pressure drop across the nozzle to get a suitable measurement. Maintenance requirement is quite high, larger area requirement compared to the orifice. So, to eliminate all the problems with flow nozzles, but continue with all its advantages.

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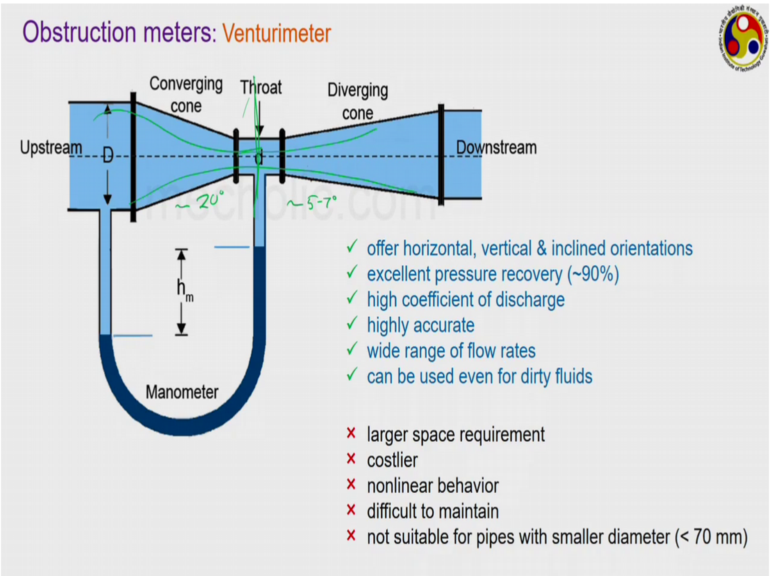

Next we move to the venturi meter, which can be visualized to be a combination of two channels. One with a converging section, other with a diverging section such that there is a common area in that throat at the throat. So, here both on upstream and downstream side, there is a smooth transition of the flow. The fluid is able to change the area quite smoothly on both upstream and downstream side.

On the upstream side, generally that angle is a bit steep in the range of 20 degrees, whereas on the downstream side is much more moderate about 5 to 7 degree. So, fluid is able to with without incurring too much loss the fluid is able to recover the pressure quite smoothly. Generally, we have one arm of the through one arm of the manometer cam mounted on the throat, and other on the upstream side.

So, this is an x very common device for flow measurement very very effective one, because it offers horizontal vertical or inclined kind of orientation which is not possible with orifice meter, and also not very easy to do with flow nozzles, but venturi meter you can mount in any kind of situation. Excellent pressure recovery, you can go up to 90 percent recovery in the initial pressure or the downstream side. High coefficient of discharge, it is highly accurate as well, and it can give wide range of flow rates. We can use this one even for dirty flows like flow nozzles, here there is no chance for or very less chance for solid particles to get clogged. So, thereby allowing us to use for situations with particle suspension or maybe some dirt or mud present in the flow.

Disadvantages definitely, it requires much more space compared to an orifice or a nozzle. It is definitely costlier, in the installation is also a bit costly. The behavior is non-linear, which is actually true for any of the obstruction meters. It is difficult to maintain, because it is not that you can just open the two parts, and clean the dart whatever gets accumulated inside, it is a it generally comes as a single set. So, maintain is sometimes may be difficult.

And not suitable for pipe is smaller diameters that is less than 70 to 75 mm mostly, because you need to have sufficient change in area from the extreme side to the throat which may not be possible for smaller dimension channels. Still orifice meter or compared to orifice meter flow nozzle, venturi meter is a best possible option among all the obstruction meters. No mathematical derivation shown there here, because I have already shown the derivation in the earlier slide. Once we have information about the coefficient of discharge, and the dimensions of this channel, then you can easily do the calculation for any kind of venturi meter. In the next lecture, we shall be solving numerical examples based on venturi meters.

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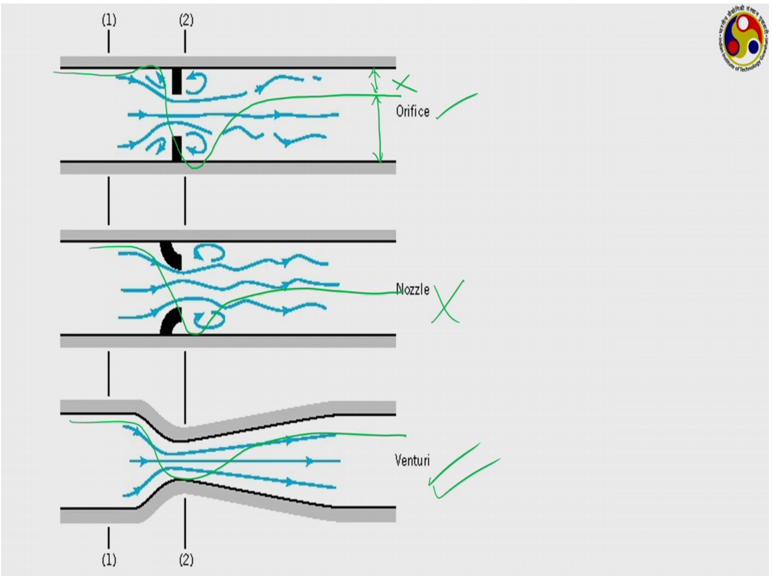

And finally as before I close in, I need to talk about the pressure recovery. In case of an orifice meter, there is a sudden change in flow direction. So, accordingly, there is the sudden drop in fluid pressure. If we follow the fluid pressure, you may find there is a slight increase the fluid pressure, then there is a drastic drop across orifice. And then it tries to recover to fall gate regain something like this.

So, while it is able to regain, this much of percentage of the initial pressure, this portion is lost. And this pressure recovery coefficient for orifice meter as I have shown, it can be in the range of 60 to 65 percent. In case of a flow nozzle, the loss is a bit less drastic, but it is not able to recover that much. Pressure recovered is quite poor see mostly, because while on the upstream side there is a smooth transition in the flow, on the downstream side, it is suddenly exposed to the entire cross section leading to significant amount of pressure loss.

However, in case of venturi it is smooth on both upstream and downstream side, accordingly the pressure is also able to change quite smoothly. And we can recover if I plot properly, we can able to recover almost 90 percent of the initial pressure. Thereby, very less loss leading to quite high value of discharge coefficient and also the velocity coefficient.

So, from pressure recovery point of view, venturi meter is the best one, and orifice meter comes again, flow nozzle is not that good. However, discharge coefficient for north flow nozzle is much better than orifice meter, and can be quite close to that of the venturi meter. Venturi meter is the best option from both discharge coefficient and pressure recovery point of view, but it is costlier and requires more space as we have seen.

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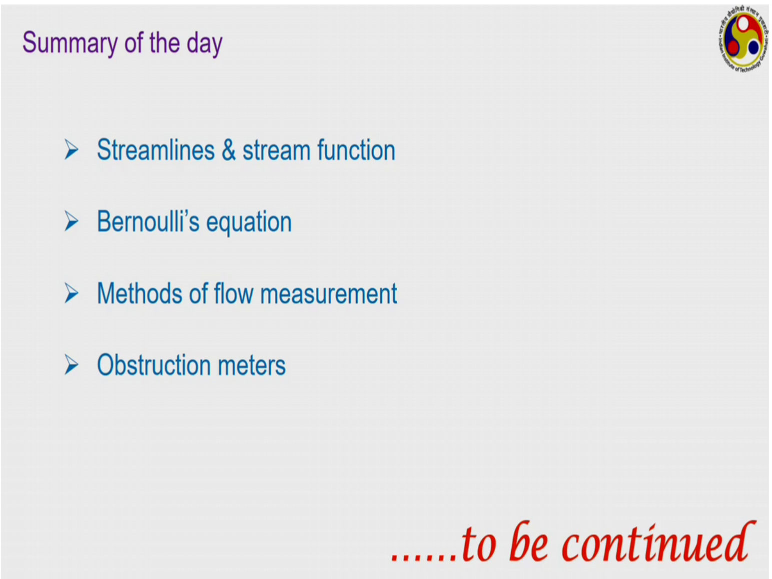

So, that is where I would like to conclude for the day. Today we have discussed about some initial concepts of fluid mechanics, particularly the streamlines and stream function, because those are required. Actually, the discussion of these orifice meters are all based upon the concept of stream lines and stream functions, because here we are applying the Bernoulli's equation and Bernoulli's equation which we have derived today that is based upon integrating along a stream line. So, the concept of stream line is important. We have also talked about the path line and street line, which are more important for flow visualization.

Methods of flow measurements that we have categorized into flow for different categories, and we have discussed about one of them which is obstruction meter under which we have talked about three tools or orifice meter, flow nozzle, and venturi meter.

So, in the next lecture, I shall be talking about the second category of flow measurement, where we shall be having different kinds of flow meters like ultrasonic or electromagnetic kind of flow meters to be talked about, so that is it for the day, just go through the lectures properly. And we shall be shown back very soon in the second lecture on this particular week.

Thank you very much.