

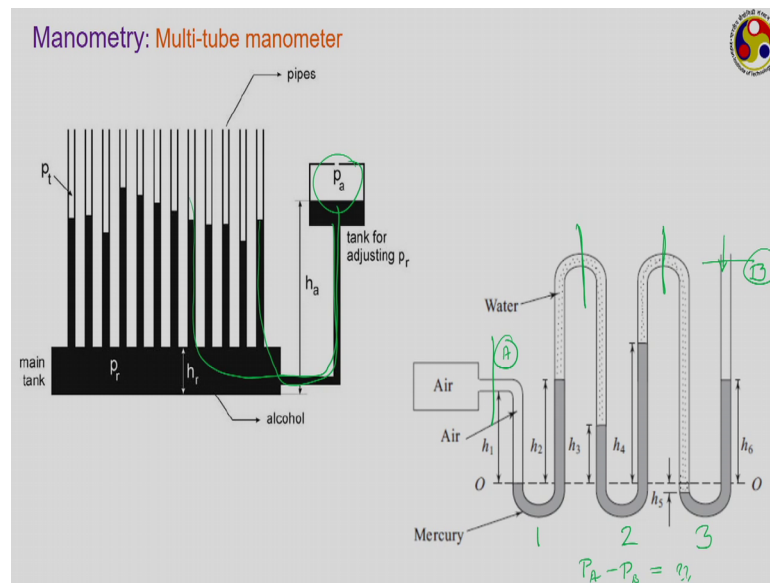
Principles of Mechanical Measurement
Dr. Dipankar N. Basu
Department of Mechanical Engineering
Indian Institute of Technology, Guwahati

Module - 08
Pressure Management
Lecture – 2
Piezometer & elastic pressure transducer

Hello friends. So, this is the second lecture of week number 8 on the topic of Pressure Measurement. I hope you have gone through the previous lecture carefully, and also try to read from the books to get the corresponding concepts properly. In fact, we have not discussed about something too new to you, because we have mostly focused on the use of manometry for pressure measurement which is always a part of undergraduate fluid mechanics courses.

And I am sure you are already aware about the terms like manometer or barometers, but we have tried to focus from a bit or we have tried to go into some more detail about the manometers discussing about topics like the inclined tube manometers, and also the interesting topics of increasing the sensitivity of the manometer in the form of a inclined tubes or micro manometers. We have also discussed about the inverted tube manometers, where we can use a manometric fluid lighter than the normal working fluid. So, today we shall be discussing slightly more about the manometers, solving some numerical examples, and subsequently we shall be moving to other kind of pressure transducers.

(Refer Slide Time: 01:43)



So, the manometers quite often in industrial applications, you will find we keep on using multiple manometers or also sometimes called the multi-tube manometers, where instead of one we need to have pressure measurement at different positions may of the same pipeline or same channel. And in that situation, we just have a common channel connect our common arm of the manometer coupled or connected to certain reference. Like in this case this is the standard reference arm which is open to the atmosphere that we maintaining the reference pressure as the atmospheric pressure. And then instead of having a single second arm, we are having multiple second terms each of them mounted a different location of the corresponding channel or tank in this particular case.

And as you can see each of them is giving a different reading, different reading about the correspondingly manometric fluid elevation, thereby allowing us to measure or know the variation in pressure in different parts of this particular tank. But we have to understand that here each of these manometers are actually coupled with the same reference side. Like we can visualize this particular one as the one U-tube manometer, similarly, we can also consider this particular one as another U-tube manometer. This way we are having several U-tube kind of manometers or maybe well type manometer coupled in same device.

These kinds of configurations are called multi-tube manometers quite frequently we will find the application of this one in industries and even in laboratories also. Like suppose

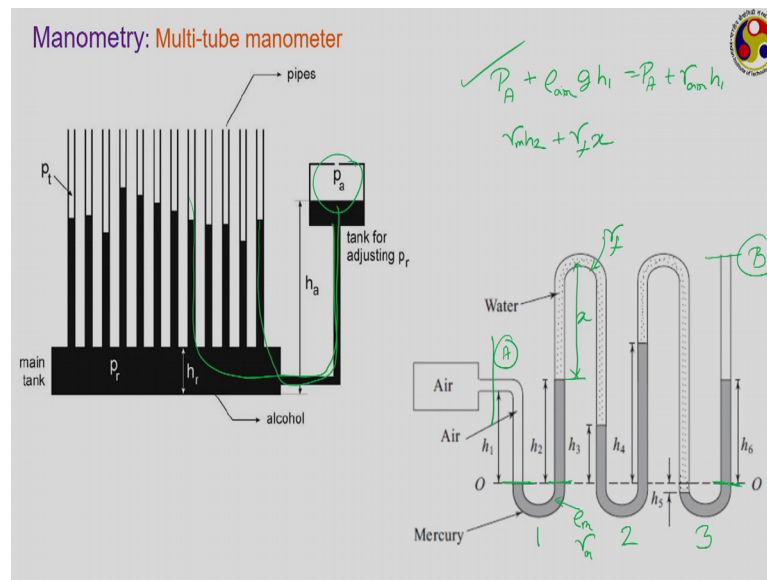
you want to may you are expecting rapid variation in pressure in a meaning some channel something like a shock tube, there we need to get the idea about the pressure value at very small distance apart and there we can easily mount such kind of multiple manometers or multi-tube manometers.

Another quite frequent variation you will find is this is also in a way multi-tube manometer or we can visualize this one as like a combination of three distinct U-tubes here. Each U-tubes are separate I should say each of the three manometers having two separate arms, but the thing is that there are two of they are connected to each other in other end of series formulation. Like we can see this is our manometer number 1, this is manometer number 2, and this is manometer number 3.

The left hand side of manometer 1 is connected to one position where you want to have some kind of pressure measurement, and but the other side of the manometer 1 is coupled to the left hand arm of manometer 2. And in the right hand arm of the manometer 2 is coupled to the left hand arm of manometer 3. And the right hand arm of manometer three is open to the atmosphere or may be connected to some other locations where we want to get the pressure measurement. Like say if this is your location number 1 and this is your location number 2, and our objective is to or instead of using 1 and 2, let us using some I will use some other symbol to differentiate with the manometer numbering.

So, this is our location A and this is our location B, and our objective is to get the pressure difference between these two locations that is we are trying to get a measure of P_A minus P_B , then we are coupling three different U-tube manometers in series like this, and we can easily get corresponding measurement. If you want to do a simple pressure balance for this manometer combination, then how we can do this, one dotted line is shown the section OO. If we do a pressure balance at this along this particular line let me just designate this one as B again.

(Refer Slide Time: 05:33)

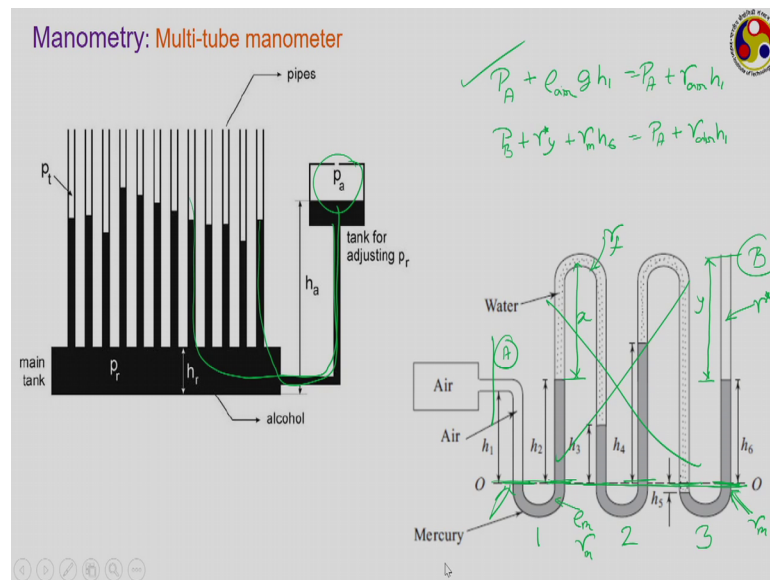


So, what is the pressure at this particular position this should be P_A plus ρ_{air} which refers to a density of air into g into h_1 or we can write say P_A plus γ_{air} into h_1 . So, this is the pressure that is acting at this particular position, then how we can say what about the pressure at this particular location how much will be that pressure of course, this one will be equal to this particular quantity that is P_A plus γ_{air} into h_1 . However, how much is the value of this, this refers to if we say this is your manometric fluid having a density of ρ_m or γ_m as ρ_m into g , then we are having a column of height h_2 of this manometric fluid say pressure corresponding to γ_m into h_2 .

And then we are having another liquid column of this height, this liquid may be having say γ_f . So, we are having a fluid of density γ_f or specific weight γ_f multiplied by some height let us say this height is x . So, instead of h γ_f into x is the pressure that is acting at this position. But the issue is that do we at all need to know about this particular pressure, because how much is the pressure acting at this position our objective to get a measure about P_A minus P_B , or if P_B is atmospheric pressure only the value of pressure at a point A.

Then how much about the pressure at this last position; let us remove this one focus only at the last position is by going past all the intermediate pressure like this one, this one, this one, this one, we are just going back to this particular final plane.

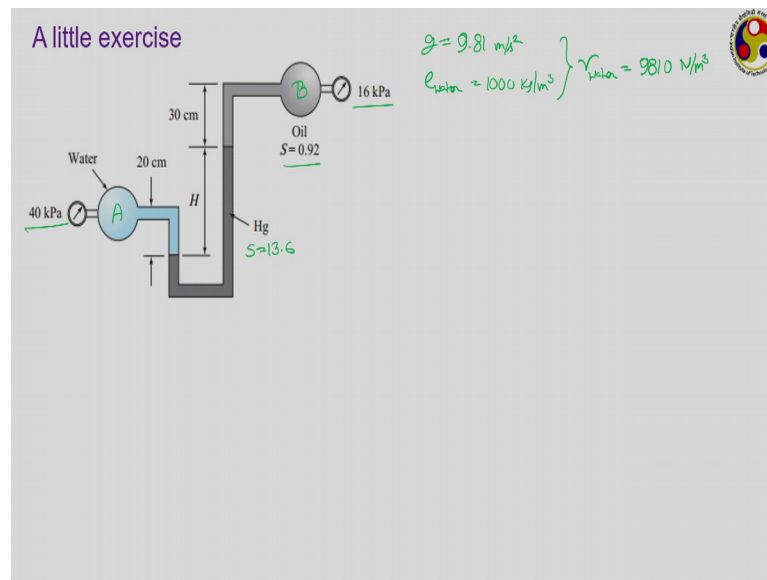
(Refer Slide Time: 07:30)



How much is the pressure acting there, this pressure has to be P_B plus let us say we are having a fluid heavy here of density γ^* . It may be air, it may be some gas and corresponding height of this particular thing is equal to y . So, $P_B + \gamma^* y + \gamma_m h_6 = p_A + \gamma_m h_1$, because this is the same line that we are talking about. So, pressure at all these points has to be equal to each other as this is a connected system of fluid.

And so we can just bypass this entire intermediate portion just focus on this starting point, and this finishing point or the first arm and the last arm, and get a value of this reading just by knowing the value of this h_1 and h_6 and the properties of all the involved fluids. So, this way despite visual visually a very complicated structure of manometer, our calculation may not be that much complicated at all. Now, let us try to use this concept to solve if you had a simple manometer based numerical problems.

(Refer Slide Time: 08:56)



The first one here is a very simple U-tube manometer. We have a simple U-tube manometer. One side of the manometer is connected to a tank containing water and having a pressure of 40 kilo Pascal. Other side is connected to a tank containing some oil having a specific gravity of 0.92 and pressure of 16 kilo Pascal. And in between we are having a manometric fluid of mercury something not given here, let us say the specific gravity of mercury is 13.6. And we are also going to use g that is gravitational acceleration is 9.81 meter per second square, and density of water as 1000 k g per meter cube in all these problems. Thereby giving gamma water value of 9810, what will be the unit of is Newton per meter cube. So, this is the situation that we are having.

We are having three fluids involved one is water the properties are known, and other is mercury which is a manometric fluid. And a third one is oil which is present in the third position that is in tank that in the second tank. Let us say this is your tank A having pressure 40 kilo Pascal. This is your position B having a pressure of 16 kilo Pascal. And now we have to identify a plane based with respect to which we are going to do the pressure balance.

So, what is the common choice of plane? This one looks like to be the most common choice because below this one you are having just single fluid. We generally always go by choosing a plane below which you have just single through the continuous span of one particular fluid. So, we can neglect the below portion from the from the calculation

like in this case below this particular line, this entire portion corresponds to only mercury as the fluid and so we can neglect this, because we know the density at this point and this point has to be equal and now that is given by equal height of mercury column on both sides.

(Refer Slide Time: 11:08)

A little exercise

Oil $S = 0.92$

Water

40 kPa

20 cm

30 cm

16 kPa

Hg

H

1

2

Handwritten notes:

$$g = 9.81 \text{ m/s}^2$$

$$\rho_{\text{water}} = 1000 \text{ kg/m}^3$$

$$\gamma_{\text{water}} = 9810 \text{ N/m}^3$$

$$P_A + \gamma_{\text{water}} (0.2) = P_B + \gamma_{\text{oil}} (0.3) + \gamma_{\text{Hg}} (H)$$

$$\Rightarrow P_A + 9810 \times 0.2 = P_B + 0.92 \times 9810 \times 0.3 + 13.6 \times 9810 \times (H)$$

$$\Rightarrow H = \frac{1}{13.6 \times 9810} [(40 - 16) \times 10^3 + 9810 \times 0.2 - 0.92 \times 9810 \times 0.3]$$

$$P_A = 40 \times 10^3 \text{ Pa}$$

$$P_B = 16 \times 10^3 \text{ Pa}$$

$$H = 17.43 \text{ cm}$$

So, how much is the pressure let us say this is position 1 and these are position 2. So, pressure at point 1, how much it will be, it will be equal to P_A plus γ_{water} into how much is the height that is 20 centimetre is the height, so 0.2 is the pressure that is acting at point 1. And similarly pressure acting at point 2, which will be equal to this is equal to P_B plus what we have a first a 30 centimetre column of oil, so which is γ_{oil} into 0.3 plus we have a column of mercury of height h . So, this is γ_{mercury} into height h .

Now, how much is γ_{oil} , the density of oil is not given, but what we have is a specific gravity. What is the definition of specific gravity? In this case, the specific gravity can be considered to be density of oil divided by density of water, that means, we can also write this one as g being a constant this is γ_{oil} by γ_{water} of truly speaking the definition of specific gravity involves the ratio of density of that particular fluid to the air or that particular material fluid or solid to the density of water at 4 degree celsius which is 1000 kg per meter cube.

However, as we have already taken 1000 kg per meter cube as the density of water, so we do not need to mention about the temperature here. Rather if we come back to this now we have P_A plus how much is gamma for water which is 9810×0.2 is equal to P_B plus how much is gamma for oil in this particular situation specific gravity is 0.92 into gamma for water which is 9810×0.3 . And that for mercury then how much is the density for mercury, the specific gravity is given as 13.6, so it is $13.6 \times 9810 \times h$ something which are looking to identify. So, you can take H on one side to do the rest part of the calculation.

However, one thing you have to be careful here, the unit of pressure here both P_A and P_B are given in kilo Pascal, whereas others you have already converted to the basic SI unit. Therefore, the value of P_A minus P_B that we are going to use in calculation that you should be putting as P_A equal to 40×10^3 Pascal and P_B equal to 16×10^3 Pascal. So, if you put it back there, what we are having is $1 \times 13.6 \times 9810 \times P_A$ minus P_B which is 40×10^3 minus 16×10^3 minus sorry 9810×0.2 minus $0.92 \times 9810 \times 0.3$.

If we do the calculation, then I have pre-calculated the final value, the result will be 17.43 centimetre in this case. So, I will urge you to do the calculation and check what it or result is matching with this. So, this is a simple manometer a U-tube manometer with both side connected to two different pressure locations or two different pressure vessels, and we are calculating the height of the manometric fluid column in this.

(Refer Slide Time: 15:07)

A little exercise

$g = 9.81 \text{ m/s}^2$
 $\rho_{\text{water}} = 1000 \text{ kg/m}^3$
 $\rho_{\text{mercury}} = 13600 \text{ kg/m}^3$
 $\rho_{\text{oil}} = 920 \text{ kg/m}^3$

$P_A + \rho_{\text{water}} (0.2) = P_B + \rho_{\text{oil}} (0.3) + \rho_{\text{Hg}} (H)$
 $\Rightarrow P_A + 9810 \times 0.2 = P_B + 0.92 \times 9810 \times 0.3 + 13.6 \times 9810 \times (H)$
 $\Rightarrow H = \frac{1}{13.6 \times 9810} [(40 - 16) \times 10^3 + 9810 \times 0.2 - 0.92 \times 9810 \times 0.3]$
 $\Rightarrow H = 17.43 \text{ cm}$

$P_A + \rho_{\text{water}} (0.02) + \rho_{\text{Hg}} (0.04) + \rho_{\text{water}} (0.02) = P_{\text{atm}} + \rho_{\text{Hg}} (0.16)$
 $\Rightarrow P_A - P_{\text{atm}} = 15.62 \text{ kPa}$
 P_{atm}

We have let us move to a second problem here again we are having a U-tube manometer one side is open to atmosphere. This is open to atmosphere. This is a tank full of water. And we want to calculate the pressure at this tank A. So, we want to calculate P_A , and for that purpose we are given with this manometer which is a quite peculiar situation, we are having a mercury column trapped in between water and on the other side we have a continuous column of mercury.

May not be very much feasible practically because mercury is much heavier than water, and so it is extremely difficult or near to impossible for a mercury column to be located on top of an water column still let us continue with this given situation. So, which plane should we select for our pressure balance, the we have to select a plane below which we just have a single fluid. So, this is the plane that we can go for below which we are having only water let us say this is a location 1, so location 2.

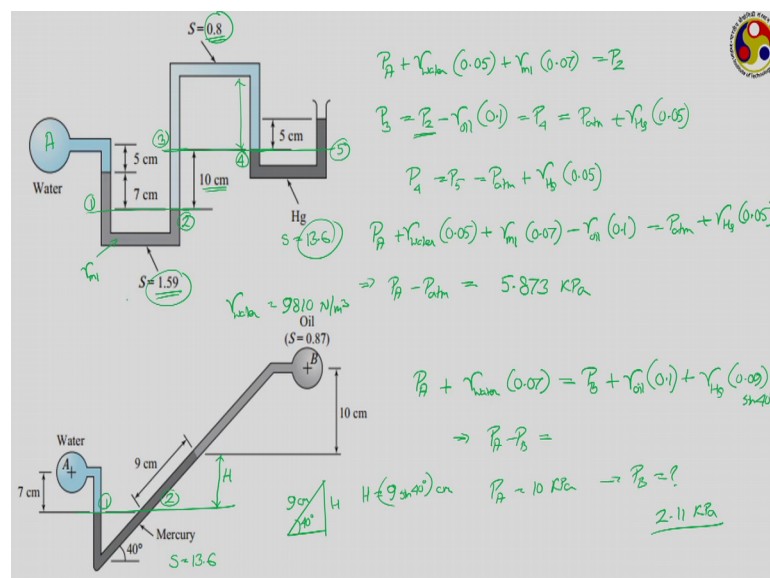
So, at location 1, pressure is P_A plus there are three columns on top of this. First we are having a column of height 2 centimetre, a column of water. So, this will be γ_{water} into 0.02 plus a 4 centimetre of mercury column, so γ_{Hg} into 0.04 plus another water column of high 2 centimetre, so γ_{water} into 0.02 that should be equal to pressure at 0.2, how much is pressure at 0.2.

Once I do have the atmospheric pressure plus you are having a column of mercury of this much of height it is given here. So, this is γ_{Hg} into how much is the height we

have 8 plus 2 - 10 plus 4 - 14 plus 2 - 16, so 0.16. And now if we separate that out, we can take P_A minus P_{atm} is equal to everything else going to the other side. And if we calculate this value is going to be 15.62 kilo Pascal that is the pressure value at this tank A is equal to 15.62 kilo Pascal.

But which pressure we are talking about, here we have evaluated P_A minus $p_{atmospheric}$, that means what we have got that is actually the p_{gauge} pressure at location A or tank A, which is 15.62 kilo Pascal. And if we know the actual value of the atmospheric pressure, then we can easily get the absolute pressure at tank A as well.

(Refer Slide Time: 17:58)



We have a multitude manometer now. Here again we are having a tank full of water let us say tank A. Our objective is to calculate the pressure at this particular tank, but it is coupled with multiple tubes, multiple manometers. The first one is having a manometric fluid of specific gravity 1.59, whereas second one is having mercury as a manometric fluid which is having specific gravity of 13.6. And in between means the portion connecting the first and second U-tube manometers, we are having a oil having a density of or i should say specific gravity of 0.8.

So, how many fluids involved in this problem? 4, one is water on one side, and then the oil in between portion, and then two different manometric fluids. And we may also have having air in this particular portion which is open to atmosphere, but density of air can be neglected compared to others. So, this one we are neglecting.

Now, how to proceed with this particular problem? There you have to be careful that here that elevations are also not the same. So, how should you go for or the first manometer we can take this as the level. So, let us say this is point 1, this is our point 2. And the pressure at this particular plane between point 1 and 2 has to be equal. So, at point 1, how much is your pressure that is equal to P_A plus γ_{water} into 0.05 plus $\gamma_{\text{manometer fluid}}$ for let us call this as γ_m into 0.07. This should be equal to or this should be equal to pressure at 0.2. And now if we do another balance along this particular plane let us say this is our point 3 and this is our point 4. So, point between point 3 and 4 what we can write, pressure at point 3, how much is that that will be equal to p_2 , p_2 is the pressure at this particular point. Pressure at p_3 will be higher than this or lesser than this? p_3 being at a higher elevation pressure will be lesser than this and the difference will be this 10 centimetre column of this oil. So, p_3 if we write this p_3 will be equal to p_2 minus γ_{oil} into 0.1, this is your p_3 .

So, this is γ_m into 0.07. This should be equal to or this should be equal to pressure at 0.2. And now if we do another balance along this particular plane let us say this is our point 3 and this is our point 4. So, point between point 3 and 4 what we can write, pressure at point 3, how much is that that will be equal to p_2 , p_2 is the pressure at this particular point. Pressure at p_3 will be higher than this or lesser than this? p_3 being at a higher elevation pressure will be lesser than this and the difference will be this 10 centimetre column of this oil. So, p_3 if we write this p_3 will be equal to p_2 minus γ_{oil} into 0.1, this is your p_3 .

This should be equal to p_4 . And how much is p_4 , p_4 we are having this particular column of oil, but no information is given about that. But what we know is that p_4 should be equal to p_5 , where 5 refers to this particular elevation. So, how much is p_5 , p_5 is atmospheric pressure plus we are having a mercury column of height 5 centimetre. So, this we can take back here as atmospheric pressure plus mercury column of this 5 centimetre.

So, if we combine all of this replace p_2 in this equation in terms of the first. So, we are having P_A plus γ_{water} into 0.05 plus γ_m of the first manometric fluid into 0.07 which becomes p_2 minus γ_{oil} to 0.1 oh sorry is 0.1 is equal to atmospheric pressure plus γ_{Hg} into 0.05, from there you can separate out the terms to have P_A minus $P_{\text{atmospheric}}$ on one side everything else goes to the other side.

Now, what information is unknown to you know all the heights which I have already mentioned the equation, γ_{water} can be taken to be 9810 Newton per meter cube like in the previous problem. So, γ_{water} as per the previous probability is 9810 Newton per meter cube γ_m will be equal to 1.59 times this γ_{water} γ_{oil} will be 0.8 times for γ_{water} and γ_{mercury} will be 13.6 times of γ_{water} . If we combine all of them, again I have pre calculated

the number, your answer will be coming as 5.873 kilo Pascal. Be careful about the unit because we have put everything in SI units and so the output that you are going to get actually is coming in terms of Pascal. So, you can present a that in terms of you can continue to be in terms of Pascal, or you can represent this one in kilo Pascal or mega Pascal as per your convenience or as per the need of the situation.

And let us close this out with another simple problem, this time involving an inclined tube manometer. We are having an inclined tube manometer two tanks A and B. The first one contains water; second one contains an oil of specific gravity 0.87. However, they are coupled by an inclined tube manometer containing mercury as the manometric fluid. So, mercury as usual we are taking S equal to 13.6. And this inclined tube is making an angle 40 degree with the horizontal.

So, how can we calculate the pressures or how can we perform pressure balance? Let us take this particular plane because, below this you are having only mercury. We can continue this. Let us say this is our position 1; this is our position 2. And let us consider this particular height to be equal to capital H . So, pressure at point one is equal to P for water or P in tank A plus and water column of height 7 centimetres. So, γ_{water} into 0.07 that is equal to P at point 2, now how much is P at 0.2. If we look at carefully at the diagram pressure at 0.2 will be equal to P_B plus there is a 10s this portion is inclined, but the height is given, (Refer Time: 24:38) the inclination length, the height is given to be 10 centimetre which is filled with this particular oil.

So, here the pressure is γ_{oil} into 0.1, because we always have to go by the vertical distance not by the distance along the inclined plane as we are talking about the gravitational here only. And now along with this we are having this γ_{Hg} into H which is the height of the corresponding mercury column. Now, how much is this mercury column height, how can you relate this to this 9 centimetre inclination length. Here you can talk about a triangle, this triangle is having a height of H , this length is nine centimetre and this intruder angle is 40 degree. So, how H is related to this 40 degree, we can easily say that H is equal to 9 into $\sin 40^\circ$, this much of centimetre.

So, if we replace this, then this becomes 0.09 $\sin$ of 40 degree putting this into equation, you can get an expression of P_A minus P_B . And the final result will give you the pressure difference between these two tanks. Actually I have not noted the complete

definition of this problem, actual problem was if P_A is equal to 10 kilo Pascal, how much is the value of P_B , I am giving you the answer which will be coming as 2.11 kilo Pascal.

Please try to solve and see whether it is matching with your calculation, yes. So, this way we can easily solve any problem involving simple manometers or even also inclined tube manometers. We have also discussed about the micro manometers in the previous lecture, but I have not included any problem related to that you can refer to any fluid mechanics textbook to get relevant problems, and to find any more elaborate discussion on the relevant topics.

(Refer Slide Time: 26:56)

Dynamic response of manometer

Leg I Leg II $P_1 > P_2$

Force due to P_1 - Force due to P_2
 - Force due to gravitational head in leg II
 - Frictional forces = mass of liquid $\times$ acceleration

$$\Rightarrow P_1 A - P_2 A - \rho(A2h)g - \left(\frac{L}{D}\right)\left(\frac{1}{2}\rho v^2\right) = m \frac{dv}{dt}$$

$$\Rightarrow (P_1 - P_2)A - (2\rho g A)h - \left(\frac{8\mu L}{\rho D}\right)\left(\frac{1}{2}\rho v\right) = m \frac{dv}{dt}$$

$$\Rightarrow (P_1 - P_2)A - (2\rho g A)h - \left(\frac{4\mu L}{R^2}\right)v = (\rho AL) \frac{dv}{dt}$$

$$\Rightarrow (P_1 - P_2)A - (2\rho g A)h - \left(\frac{8\mu L}{R^2}\right)\frac{dh}{dt} = (\rho AL) \frac{dh}{dt}$$

$$\Rightarrow \left(\frac{L}{2g}\right)\frac{d^2 h}{dt^2} + \left(\frac{4\mu L}{g R^2}\right)\frac{dh}{dt} + h = \left(\frac{1}{2g}\right)(P_1 - P_2)$$

$f = \frac{64}{Re}$
 $\sim \frac{64}{\frac{\rho v D}{\mu}}$
 $R = \frac{D}{2}$
 $A = \frac{\pi D^2}{4} = \pi R^2$
 $v = \frac{dh}{dt}$

Let us move on to some other pressure measuring device. And the one that we have now I am sorry because before I move on to any other device I quickly want to talk about the dynamic response of a manometer. Like we have seen that we can categorize the measuring instruments into different categories zeroth order, first order, second order, etcetera depending upon the relationship between their output and input in mathematical form. Very simple instruments like thermometers etcetera we have already analyzed to identify thermometer as a first order system, and like the nozzle flavour transducers etcetera also we have analyzed in earlier lectures.

What about manometer it is such a simple instrument, but what is the mathematical nature of this. So, to understand that I have got a simple U-tube manometer, this

particular line is the corresponding steady state line or the initial level, now suddenly both the legs have been subjected to certain pressure difference. The leg 1 is subjected to higher pressure p_1 , leg 2 is subjected to a lower pressure p_2 . Here p_1 is greater than p_2 accordingly there is a reduction in the water sorry the manometric fluid elevation column in leg 1.

And we have seen there is a reduction of height h in this instantaneous instantaneously the liquid column is going down by height h in the first leg, and increasing by the same amount in the second leg because the liquid is incompressible nature. So, how can we perform a mathematical analysis of this or develop a corresponding governor conservation equation or I should say governing equation.

We have to do a force balance on this. So, what are the forces that is acting on this particular system of fluid, the manometric fluid here, we can write that on one side we are having the force by or force due to p_1 minus the force due to p_2 , these two are definitely acting p_1 is acting on leg 1, p_2 is acting on leg 2 minus there is a because this $2h$ amount of increase in liquid column is happening in leg 2 that is happening against gravity.

So, we may be having a force due to gravitational head in the second leg minus the frictional resistances friction as the liquid column tries to move, the viscosity of the liquid resists that particular motion. So, it also has to go against this particular frictional force. This summation of force these are the forces that we can realize on this. This should be equal to mass of this manometric liquid into its acceleration.

So, how can you write this? Force due to P pressure p_1 , P_1 into A , where A refers to the cross section area of this manometer we have assumed it to be uniformly cross having an uniform cross section. Similarly, it is P_2 minus A . Now, how much is the force due to the gravitational head, how much it should be. So, how much is the corresponding volume of this fluid, the volume density is ρ volume is A into $2h$ this much is the volume, and this against the gravity functional force g this is the corresponding gravitational head minus the frictional forces.

How much the frictional forces, assuming a one-dimensional approach the concept of friction factor we can write this one to be fL by D into $\frac{1}{2} \rho v^2$ where small v is the velocity and L is the total length of this fluid column that is starting from this

particular point going up to this. Whatever is the total length of this mercury column that we are representing with this capital L. D refers to the diameter of this particular tube, and small v is the velocity with which the movement is happening. This should be equal to the mass into acceleration, acceleration can be written as $\frac{dv}{dt}$, and plus P_1 minus P_2 into the area minus ρg or I should say twice $\rho g A$ into h minus what can be f, now f we know can generally be related to the Reynolds number of the flow. And here we cannot we generally do not expect very large more velocity.

And so we can assume to be laminar condition, so that f can be represented as $64 \mu R e$ that is $64 \mu \rho v D$. So, if we take it back here we are having f as $64 \mu \rho v D$ into we are having this L by D into half ρv^2 is equal to $m \frac{dv}{dt}$ or P_1 minus P_2 which is the imposed pressure difference between the two legs minus $2 \rho g A$ into h minus we just have to simplify this.

So, what we can do here is, we are having a $32 \mu L$ and v let us keep that v separately. So, you are having $32 \mu L \rho$ cancels out and v cancels out. So, we are having a D^2 this into v. This should be equal to, now how much is the mass of fluid inside this, total mass can be related to this ρ into total area that into the length of this which A into L refers to a total volume of liquid in this column. And if say capital R is the radius, so it will be equal to $D/2$. And area will be equal to πR^2 that is $\pi (D/2)^2$. So, converting this D^2 in terms of R^2 what we are having then D^2 square can be written as $4 R^2$. So, we can separate it out as P_1 minus P_2 into A minus $2 \rho g A$ into h minus $8 \mu L$ by R^2 into v is equal to $\rho A L \frac{dv}{dt}$.

Now, what is the relation between v and this h, h is the height or the difference the change in the liquid column height from its initial level to the final level. And v is the velocity with which it is happening. Therefore, we can always write v is equal to $\frac{dh}{dt}$. So, if we replace that, then here this v can be written as $\frac{dh}{dt}$, similarly this one can be written as $\frac{d^2 h}{dt^2}$.

And now if we divide this entire equation by this particular term $2 \rho g A$, you divide everything and if we reorganize, then what we are having, so it becomes L by $2 g$ into $\frac{d^2 h}{dt^2}$ plus then dividing it by this we have $4 \mu L$ by $\rho g R^2$ $\frac{dh}{dt}$ plus h is equal to $\frac{1}{2} \frac{P_1 - P_2}{\rho g}$ which is the pressure difference.

(Refer Slide Time: 35:56)

Dynamic response of manometer

Force due to P_1 - Force due to P_2
 - Force due to gravitational head in leg II
 - Frictional forces = mass of liquid $\times$ acceleration

$$\Rightarrow P_1 A - P_2 A - \rho(A 2h)g - \left(\frac{L}{D}\right)\left(\frac{1}{2}\rho v^2\right) = m \frac{dv}{dt}$$

$$\Rightarrow (P_1 - P_2)A - (2\rho g A)h - \left(\frac{64\mu L}{D^3}\right)\left(\frac{1}{2}\rho v^2\right) = m \frac{dv}{dt}$$

$$\Rightarrow (P_1 - P_2)A - (2\rho g A)h - \left(\frac{32\mu L}{D^3}\right)v = (\rho AL) \frac{dv}{dt}$$

$$\Rightarrow (P_1 - P_2)A - (2\rho g A)h - \left(\frac{32\mu L}{D^3}\right)\frac{dh}{dt} = (\rho AL) \frac{d^2 h}{dt^2}$$

$$\Rightarrow \left(\frac{L}{2g}\right)\frac{d^2 h}{dt^2} + \left(\frac{4\mu L}{g D^3}\right)\frac{dh}{dt} + h = \left(\frac{1}{2g}\right)(P_1 - P_2)$$

$$\left(\frac{D^2}{g L} + \frac{25D}{g \mu} + 1\right)h = K(P_1 - P_2)$$

Now, how this is looking like, can you find similarities any of the equations that you have we have seen earlier? Here $P_1 - P_2$ which can also be written as Δp is the imposed pressure difference that is the input to your system, h is the output to your system, then is not it looking like a second order system. If we compare this one to the standard form of second order that we have derived earlier, then what was the form, do you remember? We had a form of D^2 by ω_n^2 plus $2\zeta D$ by ω_n plus 1 into h is equal to K into $P_1 - P_2$, was not it the form?

(Refer Slide Time: 36:42)

Dynamic response of manometer

Force due to P_1 - Force due to P_2
 - Force due to gravitational head in leg II
 - Frictional forces = mass of liquid $\times$ acceleration

$$\Rightarrow P_1 A - P_2 A - \rho(A 2h)g - \left(\frac{L}{D}\right)\left(\frac{1}{2}\rho v^2\right) = m \frac{dv}{dt}$$

$$\Rightarrow (P_1 - P_2)A - (2\rho g A)h - \left(\frac{64\mu L}{D^3}\right)\left(\frac{1}{2}\rho v^2\right) = m \frac{dv}{dt}$$

$$\Rightarrow (P_1 - P_2)A - (2\rho g A)h - \left(\frac{32\mu L}{D^3}\right)v = (\rho AL) \frac{dv}{dt}$$

$$\Rightarrow (P_1 - P_2)A - (2\rho g A)h - \left(\frac{32\mu L}{D^3}\right)\frac{dh}{dt} = (\rho AL) \frac{d^2 h}{dt^2}$$

$$\Rightarrow \left(\frac{L}{2g}\right)\frac{d^2 h}{dt^2} + \left(\frac{4\mu L}{g D^3}\right)\frac{dh}{dt} + h = \left(\frac{1}{2g}\right)(P_1 - P_2)$$

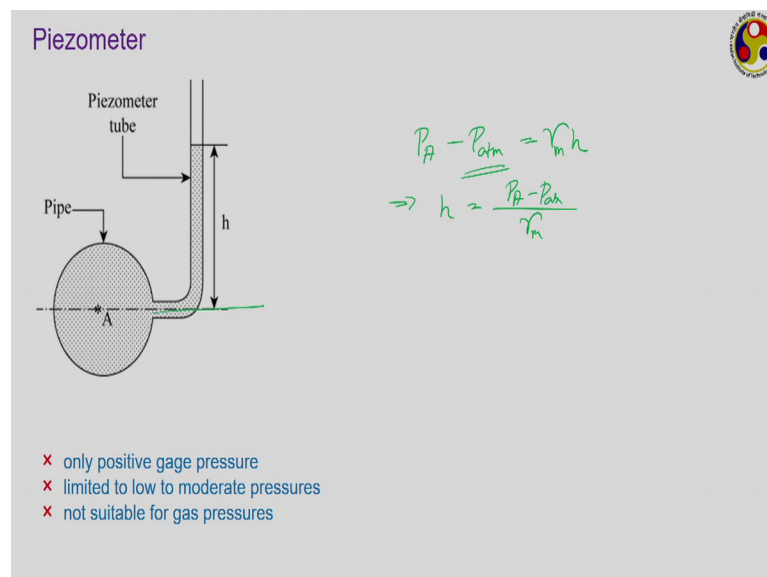
$$\rightarrow \left(\frac{D^2}{g L} + \frac{25D}{g \mu} + 1\right)h = K(P_1 - P_2)$$

$K = \frac{1}{2g}$
 $\omega_n = \sqrt{\frac{2g}{L}}$
 $\zeta = \frac{2\mu \sqrt{2L}}{D^2 \sqrt{g}}$

So, if we compare here, your K is most straightforward, K will be equal to $1/2 \rho g$. ω_n by visual observation, it becomes $\sqrt{2g/L}$. How much will be the ζ then so we have $2\mu \sqrt{2L} / \rho R^2$ into $\sqrt{g}$. So, we are having the natural frequency as this. This is the corresponding damping coefficient, and we are having the static gain as well. So, say a simple U-tube manometer can be viewed to be a second order instrument with this particular form, yes. And this way we can perform any kind of dynamic analysis for a second order or a second just the way you perform dynamic analysis for any second order system we can do the same on manometer.

Like, a manometer which is initially under steady condition may be having equal height of fluid column in both fluid in both the T columns. If it is suddenly subjected to some kind of ramp input or maybe just a step input that is suddenly you are imposing a different pressure on both the arms, then how that is going to be f . You can easily do this by comparing with this corresponding reference equation. So, that is it for the manometer let us quickly move to a few other devices.

(Refer Slide Time: 38:08)



The next one in line actually is a manometer itself, but still it is given a separate name that is called piezometer; historically speaking the it is probably came earlier than a manometer. This idea is very simple. Here we are just having a tube, a small diameter

tube which is being connected to either a pool of fluid or maybe just a free stream of fluid flow or maybe fluid flowing through a channel.

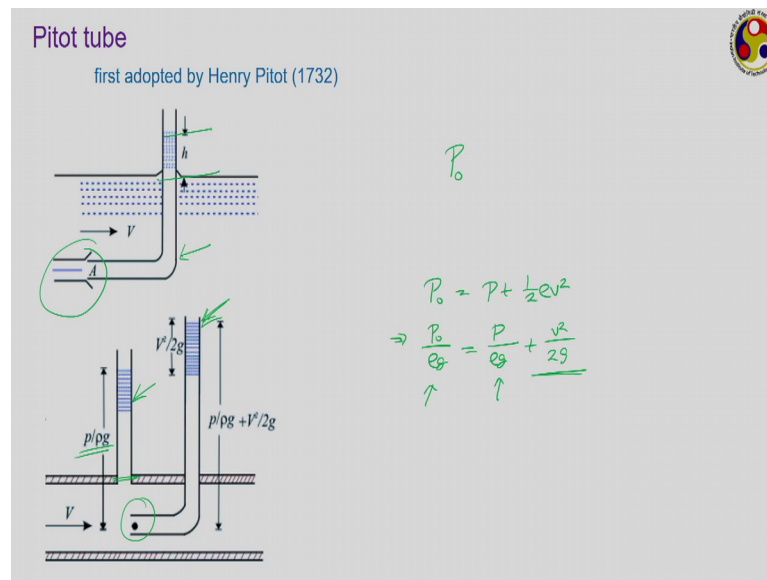
Now, as the one side of this tube is open to atmosphere, so in the location where you are connecting the other end if that is having a pressure higher than atmosphere, then that will cause an increase in the elevation of the fluid or that will force the fluid to form a column inside this particular channel, and accordingly we can get a measurement of pressure there at this location.

Like we can easily draw a simple pressure balance along this particular line which will be P_A minus $P_{\text{atmospheric}}$ will be equal to γ of this manometric or piezometric fluid into h . This h is often called which can also be written as P_A minus $P_{\text{atmosphere}}$ by γ of this fluid is often called the piezometric height as well, but it is of course, refer can refer to a particular a fluid.

So, a piezometer is probably the earliest way of measuring pressure inside big pool of liquid maybe an open channel flow situation, where we can just deep a small diameter tube open to atmosphere into that particular pool and get an idea about the pressure just by measuring the height of the liquid column there. Despite being is simplicity there are a few things that we have to consider about this, one problem it can only give you gauge pressure and because we are doing everything with respect to this atmospheric pressure and that to positive gauge pressure. Because in that location a is having a pressure lower than atmosphere instead of having a liquid column formation, we shall be having air increasing into the fluid. So, we can only measure positive gauge pressure using a piezometer.

It is an a limiter to low to moderate pressure levels because if you are looking for a very high pressure measurement, then correspondingly you are going to get a much longer or much taller column of liquid, and we need a very tall tube to deal with which is practical not practicable. So, we are limited with low to moderate pressure levels something quite close to atmospheric pressure. Then it is not suitable for gaseous at all, because in case of gaseous we are not going to get any kind of free surface and so it is not going to form any kind of column like this. It is limited only to liquid columns or liquid pressure measurement; generally the liquids which are not corrosive and not at very high temperatures.

(Refer Slide Time: 41:01)



But one important application of this piezometer can be in case of a pitot tube. Pitot tubes are truly speaking are more commonly used for measurement of velocities or flow rates for fluids. And in next week itself we shall be going to talk more about pitot tube, but still it is working being on the measurement of pressure I should mention about the pitot of tube it here. The idea of pitot tube is just a piezometer, but it is immersed into a flowing stream of liquid, and it is then bend to given L kind of shape such that this particular end of this pitot tube is facing the flow.

Now, as the fluid comes in contact with this, because this particular tube is stationary. So, fluid will immediately try to adhere to the wall thereby attaining a stationary condition. In the previous lecture, we have discussed about the concept of static and stagnation pressure. So, from there I assure you I am sure you are able to relate this, which pressure it can measure it will definitely going to be giving you the stagnation pressure that is P_0 naught, because liquid coming in contact with this tube will become stationary. Thereby converting entire of its dynamic pressure or entire of its kinetic head to pressure and so the liquid column height we are going to get here that will correspond to the stagnation pressure.

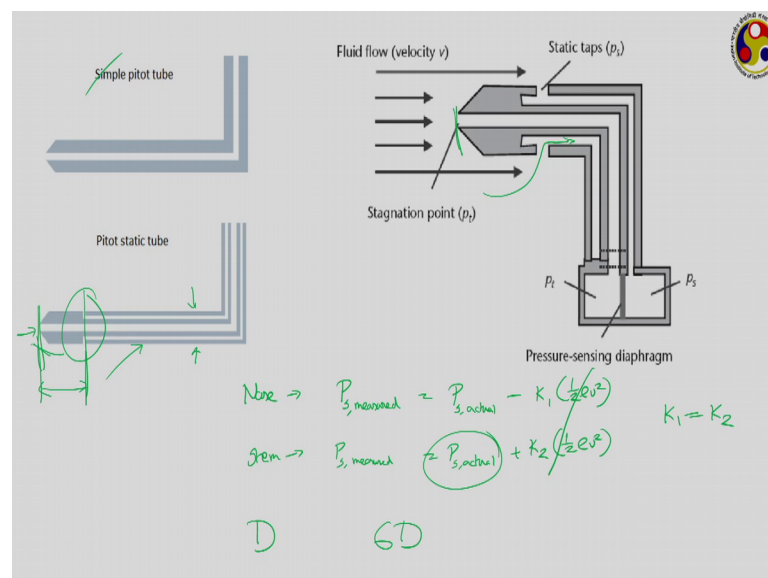
But if your objective is to know the static pressure or maybe the velocity of the fluid, then we have we need not one, but two tubes. This particular tube shown here is called the stagnation tube, and then we connect a static tube with this. A static tube is nothing

but an open tube mounted on the wall which is not adhering or which is not blocking the flow at all. Fluid is able to flow tangentially to this. And therefore, depending on the pressure at this particular level, there will be a liquid column formation in this particular piezometer thereby giving you an idea about the corresponding static pressure.

So, if P is the static pressure in this particular location, then P upon ρg or P upon γ will be the corresponding liquid column height and that will be a measure of the static pressure. And now at this point, as the kinetic energy is getting converted to pressure energy, here we are going to get a measure of the stagnation pressure, and that stagnation accordingly the liquid column height will be more here.

So, we know that P_{naught} will be equal to P plus half ρv^2 or if we divide this by γ or ρg , then P_{naught} by ρg is equal to P upon ρg plus v^2 by $2g$. This is the stagnation pressure head, this is the static pressure head. And they are different, this is a static, the first one is a stagnation pressure head which you will be getting from here. The static pressure head will be coming from here, and their difference is their dynamic pressure head. So, just from the knowledge just by measuring the difference between the stagnation static pressure head and from the knowledge of the local gravitational acceleration, you can measure the velocity of this location.

(Refer Slide Time: 44:19)



But quite often instead of using a separate static and stagnation probes or tubes, we combine them to something known as the pitot static tube. This is the simple pitot tube.

In case of pitot static tube, instead of having one, we actually have two tubes. The first one remains the standard simple pitot tube. And around that one we are having another tube. However, the other tube starts a bit upstream from the first one that like this is the nose of the pitot static tube, and then a distance after a distance apart, after some distance apart from this you are having the second tube starting. Here at this particular location, you will be having a few holes mounted on the periphery of this first tube. We can depending on the configuration, we can have 4, 6 or 8 number of holes made on the circumference through this the fluid can enter the annulus form between the two channels.

And this outer one is likely to give you the measurement of the static pressure. Just like shown here this is the one that is going to give you the stagnation pressure and now the fluid enters through this out through the circumferential holds, the annular which is going to give you the measure of the static pressure. And now the difference between these two can be measured using a manometer or any other kind of pressure measuring device to give you an idea about the fluid velocity. So, pitot static tube is a very standard method, quite accurate method and also a very simple and cheap one to measure the fluid velocities, and correspondingly the flow rate of the fluid.

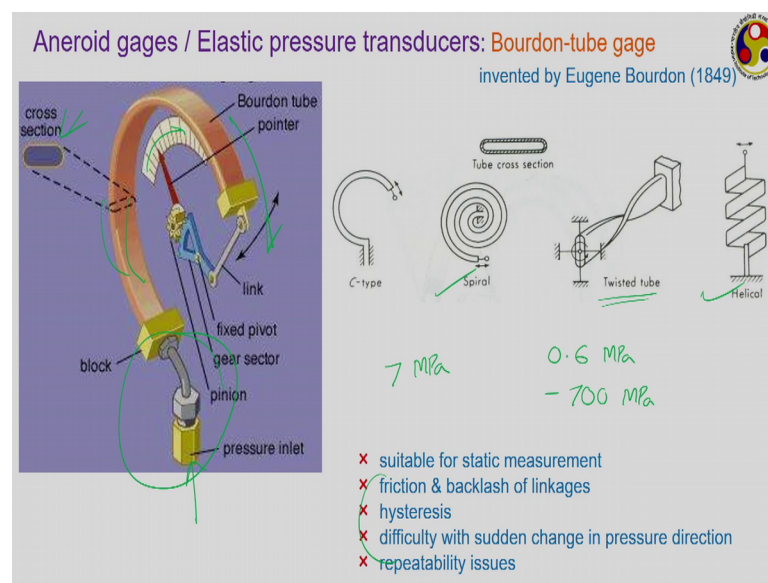
However, one important thing which definitely affects the accuracy of this one is a position of these holes. And for that the position of the nose and the stream is very important. This is what we call the nose and out this particular portion or rather this portion is the stem of this. If the holes are located too close to the nose because at the nose position the velocity becomes 0. So, immediately after the nose that is in this position the fluid accelerates this velocity increases pressure decreases. Whereas, when again the stem comes into effect the pressure will increase and velocity will start to decrease. Thereby there are two opposing effects.

And well the position of the hole depends on how we can counterbalance these two opposing effects. Like the effect because of the effect of nose, if we the location where you are measuring the static pressure, their measured value will be lower than the actual one, where K_1 being some kind of corresponding constant. Whereas, because of the effect of steam this is coming because of the effect of the nose K_1 is the corresponding constant or coefficient. Because of the effect of the steam, because of the steam there is a

reduction in velocity there is an increase in pressure according in the measured static pressure will be higher by some related by some coefficient $K \frac{1}{2} \rho v^2$.

And we have to design such that if we can properly design such that K_1 and K_2 are equal to each other then these two cancels out and what we get that is the actual static pressure only. So, we have to locate the holes properly that generally comes from experience, comes from repeated experimentation. Like a common design you will find that if the outer diameter of this tube is D , then generally the static holes are placed at a distance of something like $6D$ this, but this is only a thumb rule kind of thing depending on the design depending on a scenario you may place your holes earlier or later than this $6D$ position. It also depends on the velocity levels. We shall be talking a bit more about the pitot tubes maybe in the next week.

(Refer Slide Time: 48:33)



Let us move on quickly to another kind of pressure measuring devices which are elastic pressure transducers. In case of elastic pressure transducer, here our operation is quite similar to the elastic transducer that you have you are introduced to in conjunction with the displacement measurement. And the most common type of velocity pressure transducer is a Bourdon-tube gauge invented by Eugene Bourdon in 1849. Here the idea is you are having a sea kind of structure a sea kind of tube which is having a non circular cross-section generally an overall kind of cross-section something shown here.

One end like this particular end is free and this particular end is where we are sensing the pressure. As the pressure is imposed on this the inside the pressure inside that u increases and because of the difference in pressure between inside and outside, the tube tries to adjust to this one by converting its non-circular cross-section towards a more circular one.

But as the length of the outer and inner surface that is I should say this particular surface and on the one on the backside of this that remains more or less the same, the only way that is possible is by a circumferential movement of this particular portion the free end. And as the free end starts to move because of this linkage and the gear mechanism that is shown here, this dial or this indicator starts to move over the dial, thereby giving a measure of how much deflection that has taken place. And this deflection can directly be related to the pressure that has been imposed at this position. And the scale can directly be calibrated in terms of pressure thereby getting a measure of the pressure. It is one of the classical instruments very, very well used instruments, and there are several designs possible.

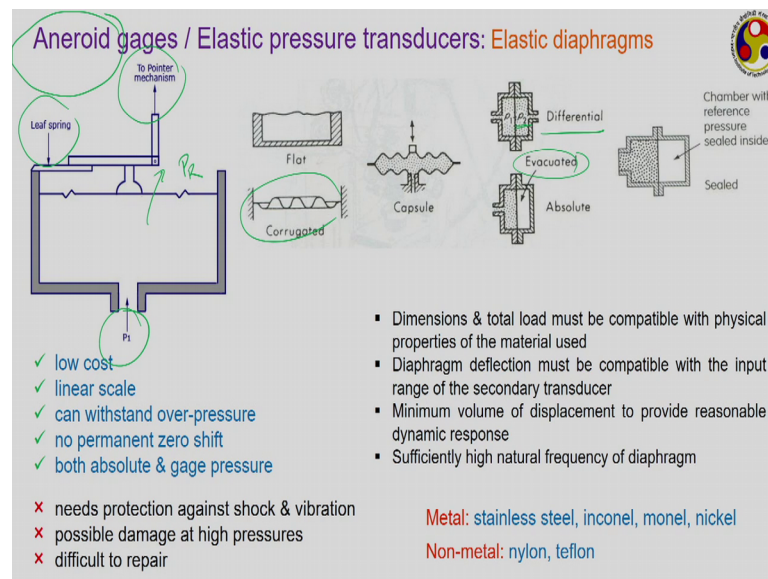
Like these are some of the designs C-type designs, this spiral design, the helical design. If we go for if we require more sensitivity, we go for the spiral or helical designs. All of them lead to the movement of the free end. Whereas, we can have a twisted kind of design twisted tape kind of design where instead of our circumferential movement we generally have an angular deflection. Twisted tape or twisted tubes are more suitable when we want to sense motion only one direction and we want it to be very steep to motion in any other direction, then we go for the twisted tubes. It generally gives you a very high sensitivity as well

The C-type probes can operate over a very very wide range like they have been practically applied to pressure very low as something like 0.6 mega Pascal to as high as 700 mega Pascal, whereas spiral or helical ones they are much more sensitive and they generally have been used a pressures up to 7 mega Pascal which is also not a quite small pressure levels.

But one problem with this these gauges are suitable only for static measurement, because they need some time for the indicator movement etcetera and. So, their response is a bit slow. The friction and backlash of the linkages are important particularly in the load

change is quite fast. Hysteresis can come into play, difficulty with sudden change in pressure direction, all these are actually limited application during the dynamic pressure admission measurement situation. And they are can be repeatability issue as well particularly if it is subjected to suddenly very high pressure load, it may call there are as there are mechanical linkages, so it may cause certain kind of a repeatability issues

(Refer Slide Time: 51:59)



Another variation of the elastic pressure transducer is a diaphragm where we just have a plain diaphragm, one side it maintained at certain kind of reference pressure, like this is a diaphragm then this side is maintained at some reference pressure P_R , other side is exposed to the location where you want to measure the pressure. And because of the difference in pressure there will be a deflection in the diaphragm, which will be sensed by this spring and corresponding mechanism giving a pointer deflection.

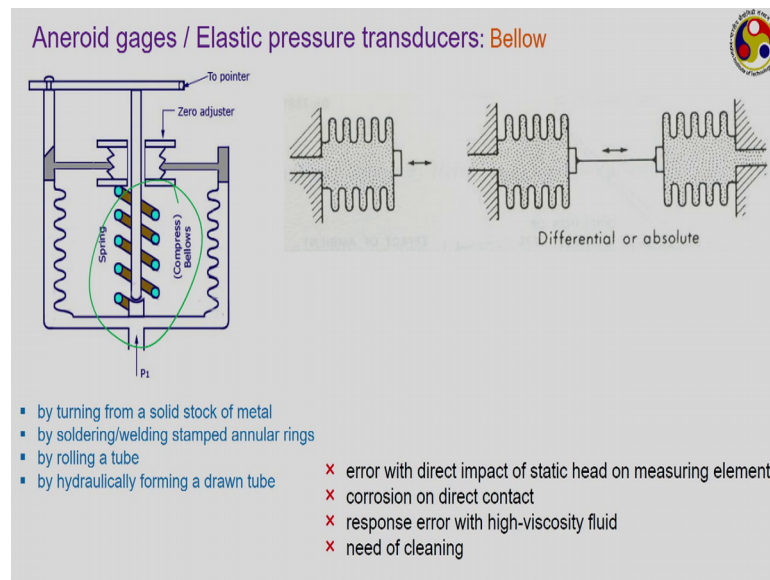
So, again or not only for this one in any kind of elastic pressure transducers, we are actually converting the pressure reading to some kind of deflection based or displacement based reading, and that which can be sensed by any kind of device like an LVDT or simple strain gauge and we can convert that to pressure reading. One important thing here, here we are using this term aneroid gauges aneroid means no fluid. Like the previous gauges manometers etcetera they were hydraulic gauges uses some kind of fluid these are no fluid kind of devices.

There are some several designs for these elastic diaphragms like the diaphragm can be flat, can be corrugated also like in this, we can have a capsule kind of configuration. We can get both differential and absolute pressure measurement like here differential, where we are maintaining a reference pressure p_2 on one side, whereas in case of absolute it is perfectly evacuated. So, these are very low-cost devices can provide a linear scale, can withstand over pressure because of the stiffness of the diaphragm itself. There is no permanent zero shift and we can get both absolute and gauge pressure. However, issues that it needs protection against shock and vibration there can be possible damage if it is subject to very high pressure and also high temperatures. And they are generally not difficult to they are difficult to repair, they are just one of use or maybe just permanently connected to certain kind of installations

So, while designing such kind of elastic diaphragm based pressure transducers, we have to be careful about these factors. The dimensions and total load must be compatible in the physical properties of the material that you are using material means I refer to the diaphragm material. Diaphragm deflection must also be compatible with the input to the secondary transducer which you are using to sense the deflection. The minimum volume of displacement or I should say the volume or displacement should be minimum, so that we can have reasonable dynamic response and sufficiently high natural frequency of the diaphragm again to give a good dynamic response.

These are the common materials that are used for forming diaphragms metals like stainless steels, or inconel, monels or nickels. And we can also have certain non-metal things certain grades of nylons or teflons can also be used.

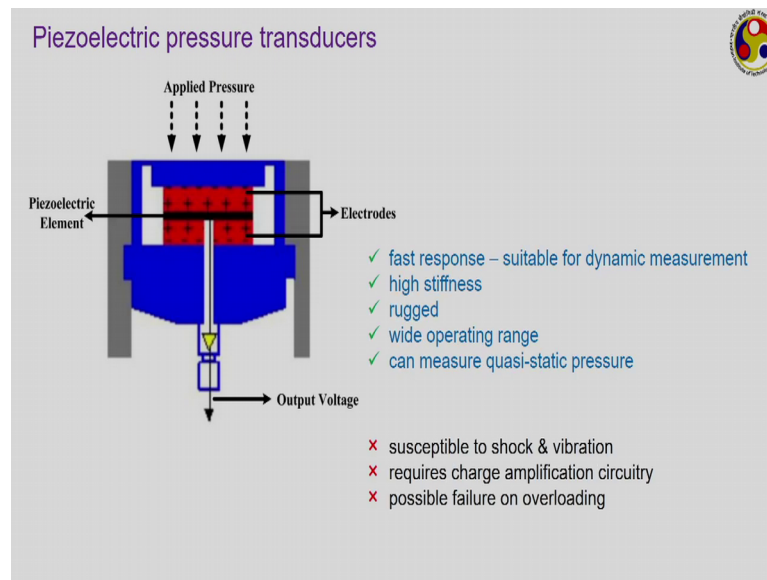
(Refer Slide Time: 54:44)



Another transducer quite similar to the diaphragms or bellows; where we just have one material which is folded to a structure like this. And like shown here, as they are subjected to pressure there is a change in their configuration, and accordingly some deflection can be sensed which can be converted to pressure. This bellows can be formed by turning from a solid stock of metal by soldering or welding annular rings or by hydraulically forming a drawn to tube. But they are all there have advantages quite similar to the diaphragms, but problem is they can show some error with direct impact of static head on measuring elements, there can be corrosion on direct contact and response over high viscous the presence of fluid actually these bellows are generally immersed in certain kind of fluid here.

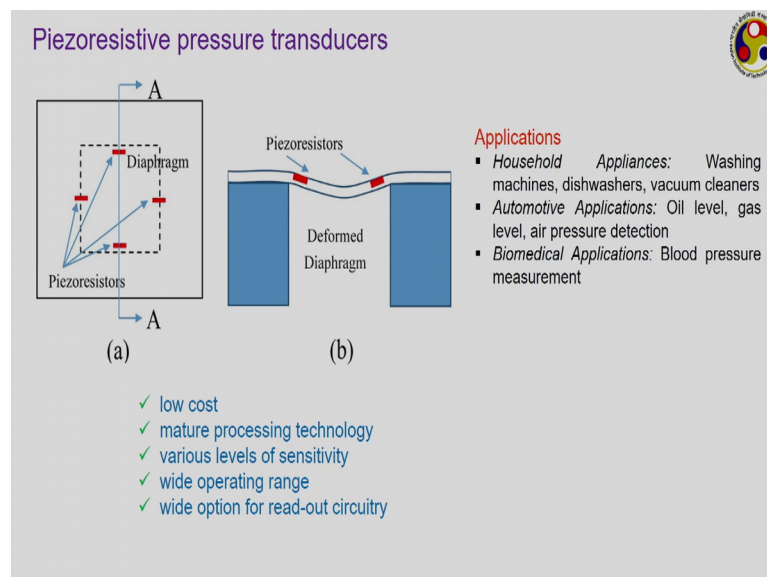
The viscosity of the fluid may dampen the deflection they are very causing some problem in response. And if we are using in some complicated or sticky environment, it needs frequent cleaning. Still they are also quite well used devices like the other elastic transducers.

(Refer Slide Time: 56:00)



I would like to keep it up to this today. I had plan of talking about a few other devices, but I shall be coming back to this in the next lecture.

(Refer Slide Time: 56:03)



(Refer Slide Time: 56:04)

Summary of the day 

- Dynamic response of manometers
- Piezometer & Pitot tube
- Elastic pressure transducers
- Piezoelectric & piezoresistive sensors
- Ultrasonic displacement sensors

.....to be continued

So, to summarize today's context, we have talked about the dynamic response of the manometers, then we talked about piezometers and pitot tubes, how to measure the static and dynamic pressures or the stagnation pressure separately, and also providing a way of measuring the velocities. Then we have discussed about the elastic pressure transducers. The other that other I have mentioned here those I shall be discussing in the next lecture.

So, thank you very much for your attention for the day. In the next class, I am hoping to finish the discussion on this pressure measurement, where we shall be discussing about the capacity transducers, piezoelectric and piezo resistive transducers, and also the options of measuring very high or very low pressure measure pressure situations.

So, thanks a lot.