

Principles of Mechanical Measurement
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Module – 07
Force & Torque Measurement
Lecture - 19
Elastic & strain gage load cells

Morning friends so, we are into the week number 7, basically exactly at the halfway of this course, because it is a 12 week course that you know and we have so far completed 6 of the weeks, and 6 more to go including this one. Now, week-5 onwards, we are discussing about the measurement of one specific quantity every week. So, in week number 5, we talked about the measurement of displacement related parameters that is something having a dimension of length, something can be displacement or can be some distance traveled or can be strain or deflection whatever.

In the next week, that is in a previous week itself, where we just picked up one particular parameter, which we actually discussed in course of displacement only that was strain, and also stress in relation to that. And we have discussed more about the strain measurement of focusing mostly on the resistive strain gages with very briefly touching upon the inductive or capacitive devices.

So, first was displacement then was strain and stress. And then the one that in logical sequence should be the force only. And therefore, in this week we are going to talk about the measurement of force and torque. So, force measurement is probably one of the oldest possible way of measurement or oldest mean of measurement, which people have to think about or have to find a way to do, because this is something that we need in everyday life for this.

Now, where the most basic principle of force measurement is based upon something that you have learned in very early day physics in your school that is the Newton's law of motion or to be more specific Newton's second law of motion.

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Basic principle
Newton's Second law of motion
 When an object is being acted upon by a net force, the object accelerates in the direction of the net force. The magnitude of acceleration is directly proportional to the net force and inversely proportional to its own mass.

$$F \propto \frac{d(mv)}{dt}$$

$$F \propto m \frac{dv}{dt} = ma$$

$$\Rightarrow F = q_c(ma)$$

$$\frac{a}{F} = R(q_c n)$$

$$\frac{a_1}{F_1} = \frac{a_2}{F_2}$$

$$F = q_c(ma)$$

$m = 1 \text{ kg}$
 $a = 1 \text{ m/s}^2$
 $F = \frac{1 \text{ N}}{1}$
 $q_c = \frac{ma}{F} = 1$

Figure 1

Now, what Newton's first law of motion says, it says that if we have a body which is stationary or if we are a body which is moving, then if no forces are acting on this, then the body will remain its present position that is a stationary body will remain stationary or the moving body will keep on moving in the same direction with the same velocity.

Therefore, the first law of motion basically signifies the importance of a force. To cause a change in the state of a body, we need to have some kind of force acting upon it that is if the body stationary, we need to put a force on it to make it moving. Similarly, if the body is moving itself, then to make a change in this velocity either in terms of magnitude or in terms of direction, we need a force to act upon this one. So, the importance of force is being stressed upon during the first law of motion or in the first law of motion.

Now, comes the second law, which tells that when the object is being acted upon by force, then it experiences a kind of acceleration or some form of acceleration. And the acceleration will act upon the direction of the net force and its magnitude will be directly proportional to the net force itself, and also inversely proportional to its own mass. All of you know this particular statement, which actually more commonly said that the rate of change of momentum of the body will be proportional to the force.

And that is if we write in the mathematical form, force will be proportional to the rate of change of momentum of a body, where m is the mass, v is the velocity of the body and if mass remains constant, we can also write this one as $d t$ giving mass into acceleration.

And now converting this one to an equality relation, we generally get g_c as the constant of proportionality into mass into acceleration.

Now, this constant of proportionality some of you may be wondering from where it is coming in, because F equal to mass into acceleration is something that you have read from your high school level itself. But, that definition through, only when you are using the SI unit of dimensions or SI system of units.

Before, I come back to this, this is just a very common example of what we are talking about in conjunction with this Newton's second law of motion. Like when you are talking about one mass this is the mass is being dragged upon by two engines, which are putting identical force, then there is a large force acting upon your small mass. So, we have a large acceleration, the length of this arrow is proportional to the acceleration.

However, if we keeping the load constant that is keeping this mass constant, if we make the force half that is we have just one engine here, the acceleration also becomes half of that. That means, for a given mass, this acceleration by force that remains some sort of constant k , which is given as this g_c into m this we are talking about for a given mass or in other we can write that $a \propto F$ is equal to $a_1 \propto F_1$ is equal to $a_2 \propto F_2$.

For two different cases where the body is act upon by force F_1 , we have an acceleration A_1 , when is acting it is being acted upon by force F_2 , acceleration is A_2 like in this case, F_2 is half of F_1 , accordingly acceleration also becomes half.

Now, if we add come to the case number 4, this one to that same single engine. If we add another same amount of mass that is now mass has become doubled, then acceleration becomes half of that, whatever we got in this case, this acceleration is only half of that. And to increase this, of course you have to increase the force and here we have added one more engine. So, force becomes double, acceleration also becomes double to this particular value. This is Newton second law of motion, which you know from your school level itself.

But, now we come to this constant g_c . How you get the value of this g_c that is how we define the unit of mass or I should say unit of force, because mass is a fundamental quantity. So, if we say that our we are talking about a mass of 1 kg and so a body of mass

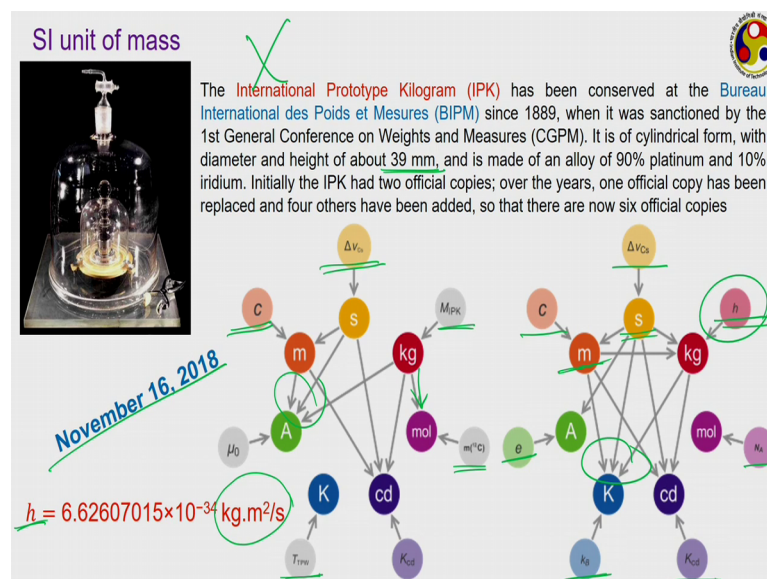
1 kg is being acted upon by a force, which leads to an acceleration of one meter per second square which is unit acceleration in SI unit.

So, we are having a body of 1 kg mass and that after being acted upon by force is having an acceleration of 1 meter per second square, then if we call that force as 1 Newton then what we have? As for the relation $g = c$ is equal to force divided by mass into acceleration, and as we are taking this force to be as 1 Newton, then this is equal to 1 into 1 that is $g = c$ is 1.

The now the of course we can also identify the unit of $g = c$ by combining units of this, it is not that $g = c$ will always be equal to 1, it is only because the way we are defining this force of 1 Newton. 1 Newton, we are defining as a force which corresponds to 1 meter per second square acceleration of a body of mass 1 kg or I should say when a body of 1 kg is being subjected upon by a force, which leads to a acceleration of 1 meter per second square. Then that magnitude of that particular force is 1 Newton, thereby giving $g = c$ to be equal to 1.

So, your choice of unit is different or if we are defining the force like 1 Newton this case in a different way then $g = c$ will be having some other value also. So, it is important to remember that the relation that we are talking about that is not just force equal to mass into acceleration, but there is a constant of proportionality also which only in SI unit, when you are using Newton as the unit for force $g = c$ becomes equal to 1.

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Now, the measurement of force always comes down to the measurement of these two quantities. Once we know the value of g and as we are restricting our self to the SI unit, therefore g will be equal to 1. And hence measurement of any force reduces to a measurement of mass and acceleration.

Now, mass is a fundamental quantity all of you know, because out of the seven fundamental units in according to the SI unit, mass is one of them. And in the very first lecture, we have discussed about the fundamental dimensions and there we have talked about this particular thing the unit of mass is kg, which is being defined by this particular mass.

Like as per the convention of 1889, one particular cylinder which is having a mass of or I should say the diameter of about 3.9 centimeter and height of about the same value. And it is made of platinum iridium that is 90 percent platinum, 10 percent iridium.

The mass of this particular cylinder was identified to be 1 kg, during this first general conference on weights and measures. And that was preserved in the Bureau of International weights and measures, since 1889 at Paris. This is often called the IPK or International Prototype Kilogram.

So, this particular mass was identified to be 1 kg and since then everything was calibrated with respect to this. This is served as the most fundamental unit general, initially it has only two official copies; over the years, one of the copy was replaced, and four others were added. So, now we have 6 official copies of this IPK. Different countries have their own prototype, which each of which being calibrated in terms of this IPK. And then industries use tertiary level of standards, again this mass of 1 kg is being considered based upon this IPK.

Now, this is something that I have mentioned, during the first week of this particular course. And if you remember, there I when we talked about all the seven fundamental units I mentioned that, while the other fundamental quantities like length and time is generally being calibrated or measured in terms of some natural phenomenon. Like both the measurement of length and time or I should say the definition of meter and second are based upon the velocity of light in vacuum, which is a very much natural phenomena.

Similar effort is also on to define kg in terms of some naturally occurring event or some natural phenomenon, so that we do not have to depend upon a particular body of mass or any particular physical quantity. And that change actually has taken place. Initially, when I record with the first lecture, it was before the particular date that I am going to talk about now which I am sure all of you already have some idea being science students.

But, there I definitely have mentioned that effort is on to replace the definition of kg is to in terms of some natural phenomena instead of using IPK within next 5 to 10 years. Before, I go to that the problem with IPK is that, it was definitely preserved in the at most possible way of preservation. Like it was stored in a triple log vault, 1 meter below the particular laboratory and there are two bell jar that you can find with temperature controlled environment, and on this particular platinum iridium alloys are also chosen, because it is extremely hard and corrosion resistance etcetera.

But, still over all these years, its masses change by a few microgram. And that change this being the primary standard of measurement of mass that change is going to affect any kind of change mass measurement, where we are going to take this one as the standard.

And therefore, it was important to define either correct the mass of this one or define a new unit of mass without changing the actual magnitude of this thing. And therefore, the change took place in November 16, 2018 just about 3-4 months back. Truly speaking, that units that I am going to mention here that is yet to come in practice, it will be active in just 3 months time that is from May onwards; May 2018 onwards, these definitions will be active. But, the consensus has already taken place, during the convention held on November 16, 2018.

Now, this was the earlier convention, these are the seven different units of mass. And out of this, the actually this particular date is important, because this can be considered to be the most important day in the history of science since 1960. What happened in 1960? In 1960 the unit of length that is meter was define in terms of the velocity of light in vacuum and same as for same for time.

But, the definition of mass was based upon the mass of IPK and others like the temperature was independent, it was define in terms of a triple point of water you know

that the triple point of water is given the value of 0.01 degree Celsius or 273.16 Kelvin, accordingly the value of Kelvin was defined like the mole.

Mole is the fundamental unit of quantity of matter and it was defined in terms of ^{12}C atom; that means, the mass of ^{12}C atom, I should say the quantity of ^{12}C atom was defined as 12 mole and or 12, I should say 1 kg of ^{12}C mass was defined in terms as 12 kilo mole. So, the mass or the definition of mole was dependent upon the definition of kg.

Now, during 16 November, 2018, this is the change that has come into play. Here the mass or I should say the definition for meter and second remains the same. And the definition of this luminous intensity also remains the same, but all others are change.

Like for mole the definition now includes Avogadro's constant; the definition of mole is based upon Avogadro's constant. We know that one kilo mole of any matter will contain Avogadro number of molecules and from there we can define the value of mole. The definition of Kelvin has been defined in terms of the k_B , what is k_B do you think what or can you guess, I am sure some of you may be able to guess this, this is a Boltzmann constant.

But, actually the value of Boltzmann constant of the measurement of Boltzmann constant depends upon the measurement of all the three fundamental ones that is length, time and mass, so it becomes dependent on all three of them whereas, it was independent in the previous case. Definition of ampere that now is given in terms of e that is electrical charge or I should say electronic charge. So it is again based upon the definition of second now whereas, in earlier case it was based upon all the three fundamental ones.

So, this we have changed, but the biggest change has taken place here in terms of the definition of mass or 1 kg. 1 kg is now been defined in terms of Planck's constant, which is given a value of this. Actually, scientist always thought about possible replacement in terms of the Planck constant or if you other relevant parameters, but they took so much time to define this, because the exact value of Planck constant itself was debatable.

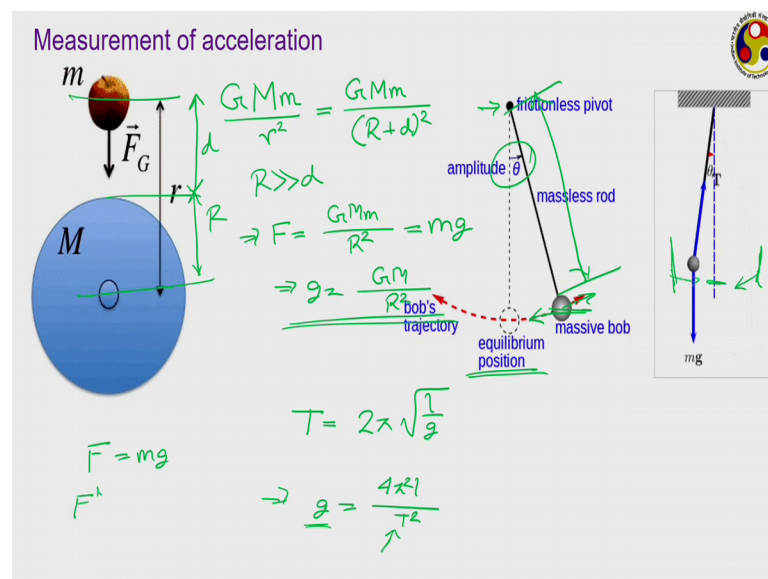
And now they have converted to this particular value of Planck constant and accordingly the definition of kg is given in terms of this, but issue is that the Planck constant unit for this is $\text{kg meter square per second}$. So, this definition of kg is dependent on accurate

measurement of meter and second yes, so it is depend upon both of them. And accordingly it effects, if you others as well.

So, from now onwards this concept of IPK becomes invalid still actually I should not say that, because it will become invalid from May 2018 onwards I have forgotten the date, but from that onwards the definition of IPK becomes invalid. And you always have to define kg in terms of this Planck constant. So, all such textbooks has to change to incorporate this particular change in the definition of kg.

However, of the magnitude or definitely has not change that is something which we are used to call 1 kg that still remains 1 kg or a mass of something that used to be 5 kg earlier that still remains 5 kg. But, what we mean by that kg that has changed as per the newest convention.

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Next comes acceleration. Acceleration is always related to the gravitational acceleration or a means to we do not talk about general acceleration, rather we always talk about gravitational acceleration in relation with the measurement of force. Now, what we have in case of gravitational acceleration, we always know that anybody always experiences some kind of attraction towards the earth.

And therefore, a four earth imposes a force on this and hence depending on its mass it experiences some kind of acceleration, and that acceleration is a gravitational

acceleration. Like applying a Newton's law of gravitation, we know that if the mass of earth is capital M and the mass of the small body is small m , then the corresponding gravitational acceleration acting on them will be equal to $G M m$ by r square, where r is the distance from the center of earth to the centroid of the body. Or we can also write this one as $G M m$ by capital R plus d whole square, where capital R is the average radius of earth and d refers to the distance of that body from the surface of the earth or I should say distance of the centroid of the body from the surface of the earth.

In most situation this; in the general situation always this capital R is extremely large compared to density, this leading to the force that we are talking about as $G M m$ by capital R square. But, this force as; now this is coming from Newton's law of gravitation.

And now if we go back to the Newton's second law of motion, this will be equal to the mass of the body into the gravitational acceleration that is acting on this, thereby giving g equal to $G M$ by R square that is a constant, because M refers to the mass of earth, R refers to the average radius of earth, and G is a gravitational constant. So, we get some value of g uniform value of g at the surface of the earth.

Of course, we know that earth is not a perfect sphere, so accordingly the value of G keeps on changing along the surface of the earth itself. Now, though capital G and capital M remains constant, but capital R is not constant, accordingly we make a different value of G , if we are at the equator or if we are the polar regions or as the distance keep on changing along with the landscape. So, whatever value of G , we experience at the seabed or maybe at the seashore the value will be different, when you go to the top of any mountain or maybe top of the Everest, but still that difference may not be very significant.

Now, the way we measure the gravitation acceleration is with the most commonly is used; with the use of this pendulum. Now, pendulum you know the principle here we have a massive bob, which is almost which can be assumed to be an unit mass that is being connected by a mass less wire or mass less rod and fixed to this particular frictionless pivot.

So, when it; this is the equilibrium position for the bob and when we put some force on this, it follows a trajectory to move something like this. That is as you can see, the

gravitational force always is acting downwards, but the direction of the tension in the wire that keeps on changing depending upon the position of this.

At also the acceleration that is acting on the body that also gives some changing like, when it is at the topmost position. Like if it is the topmost position of the body here, the force is acting in this particular relation towards the equilibrium, but its velocity may be acting in this particular direction.

So, the force is highest, when it is at the topmost position somewhere here and is here that magnitude of force is largest when we; but the force is always acting towards the equilibrium position, and us when the body reaches towards this extreme position is velocity is 0. And in the equilibrium position, where the force is 0, it is velocity is a maximum.

So, if there is no resistance, no friction acting on this, it will keep on oscillating following a simple harmonic motion like this. I am sure most of you have done this experiment of pendulum at your school level physics laboratory, to identify the value of G using the equation of pendulum.

And we know that if capital T represent the time period of oscillation of this pendulum, then and the simple relation is $2\pi\sqrt{l/g}$, where l is the length of this particular wire that is I am talking about this particular length, the straight line distance from the pivot to the centroid of the bob and T is the time period of oscillation, of course this equation is valid on this wire, θ is very small, generally restricted to 1 radian plus minus 1 radian I should say.


So, from there we get that g is equal to $4\pi^2 l / T^2$. So, generally at school level laboratory or any other situation, we measure the length of this particular rod or particular thread and we do experiment to get the value of this T . Generally, we repeat the experiment for several runs to get an average value of T from where we can get an average value of this g .

And any force measuring instrument, generally there or ratings and its specified in terms of this g that is how much force it can measured that is always scaled in terms of g at that particular location or I should say as per the Newton's second law of motion, while know

that this F equal to m upon g . The ratings are commonly specified for some F star, which is F upon g .

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Common methods of force measurement



- ✓ balancing against gravity either directly or through a system of levers
- ✓ measuring the acceleration of a known mass
- ✓ balancing against a magnetic force
- ✓ transducing force to fluid pressure
- ✓ applying force to an elastic member & measuring the resultant deflection
- ✓ measuring precision change in a Gyroscope
- ✓ measuring change in natural frequency of a taut wire

Now, the common methods of force measurement, there are several ways we can measure force. And you have to remember that always we are actually trying to play around with a mass and corresponding acceleration to get the magnitude of force. And now what kind of acceleration we are talking about accordingly, the measurement principle may vary.


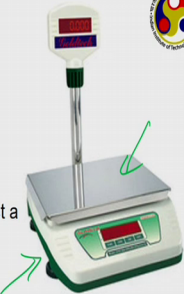
The most common method is balancing is a gravity either directly or through a system of levers, then measuring the acceleration of a known mass. So, that mass into acceleration becomes a force, then balancing against a magnetic force. So, here the force source of the force is different, but we can calculate mass or acceleration or their product using this principle.

Then transducing the force in to a fluid pressure like in earlier models, we have talked about the sorry the pneumatic mode of measurement, so this is what we are talking about here. Then applying the force to an elastic member and measuring the resultant deflection, something very similar to what we have done in the displacement measurement or deflection measurement. Just their objective was to measure the deflection, here objective is to measure the force.

Measuring precision change in a gyroscope then measuring the change in natural frequency of a taut wire. These two are a bit more specialized situations. Let me see depending on time, I shall be discussing about them, but all others we shall be touching upon briefly.

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Mechanical weighing system

- null-type of device
- simple to use; uses symmetry
- requires careful design
- operator skill
- for heavier loads, balance against a system of levers

In case of precision measurement,

- ✓ buoyant force
- ✓ temperature-controlled chambers
- ✓ operator body heat & convection current

Description	Range (g)	Resolution (g)
Macro analytical	200- 1,000	10^{-4}
Semimicro analytical	50-100	10^{-5}
Micro analytical	10- 20	10^{-6}
Micro balance	less than 1	10^{-6}
Ultra micro balance	less than 0.01	10^{-7}

Now, the first one that we have in line is the mechanical weighing system. I am sure each of you have seen this particular thing, this is what we commonly use for measurement of mass. But, actually does it provide the measurement of mass, actually what it measures is the force. Because, whenever you are putting, something say on this particular arm, we are trying to balance that by putting a counterweight on this other arm.

If suppose the right hand side arm that is my right that I am talking about is empty and we have put some quantity here, some item here for which we want to get an idea about the mass. Then initially both the plates were empty and as we are putting something here, initially this particular indicator was at zero position, as we are putting something, it will get deflected towards this direction.

And now our objective is to get it back to the zero position by placing some counter weights here. We keep on adding more on more weights here, duly at in that zero position. So, this is a perfectly null kind of measurement, where there is a deflection, but we are trying to negate the deflection or get it back to the null position.

So, it is very simple to use, we generally use symmetry, because one common principle is that the distance of this pivot here from should be equal on or from the should be equal from both of sides, so that we can get a proper force balance. Generally, the design needs to be very much careful, so that the lengths of both the arms are equal and if particularly, if you are looking for a precision measurement, we need high degree of accuracy in the fabrication step itself and also skillful operation.

But, if you are talking about a very small value of mass, this is fine. But, if you are talking about heavier loads, then this is something we commonly identify at common shops or markets, which are also known as a platform balance. While this one is commonly called the analytical balance or precision balance, this one is called the platform balance, where actually do not have two arms. We just have one arm or one platform, where we put all the stuffs that you want to know about. And inside the body, there is a system of levers, which tries to get it back to that null position; get the indicator back to the null-position.

The analytical balance and also such kind of measuring system can be classified into different categories depending on their depending on the mass that they can measure. Like the macro analytical device or macro devices can measure quite large values from 200 to 1000 and their resolution is of the order of 10 to the power minus 4g. Whereas, semi-micro analytical devices have a lower range and the micro analytical differences, I have only 10 to 20 and you can see that the resolution, keeps on increasing in this direction.

So, when you are talking about the micro balance, actually we are trying to measure a force less than g value and for ultra micro balance it is even slower or I should say smaller and extremely high resolution that is required. Depending upon what kind of application you are looking for, you have to pick up one among this five primarily based upon the value of mass in terms of this g.

In case of precision measurement, however there are several factor that you have to be careful of. Like the first important factor is the presence of buoyancy force, which we often neglect. There can be also factors like the temperature effect on this, because we change in temperature the there can be physical expansion or volumetric expansion of both the body and also corresponding counterweights. So, temperature control chambers

may be essential, operator body heat and convection currents also can significantly affect the performance.

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Diagram showing two pans of a balance. Pan 1 contains a plastic material with density ρ_u . Pan 2 contains brass weights with density ρ_s . The ambient air has density ρ_a .

$$W_u = \rho_u V_u \quad W_1 = (\rho_u - \rho_a) V_u$$

$$W_s = \rho_s V_s \quad W_2 = (\rho_s - \rho_a) V_s$$

$$W_1 = W_2 \Rightarrow (\rho_u - \rho_a) \frac{W_u}{\rho_u} = (\rho_s - \rho_a) \frac{W_s}{\rho_s}$$

$$\Rightarrow W_u = W_s \frac{\rho_s}{\rho_u} \left(\frac{\rho_u - \rho_a}{\rho_s - \rho_a} \right)$$

A quantity of a plastic material having a density of about 1280 kg/m^3 is weighed on a standard equal-arm balance. Balance conditions are achieved with brass weights totalling 152 g in a room, where the ambient air is at 20°C and 1 atm . The specific gravity of brass may be taken as 8.49 . Calculate the true weight of the plastic material and percentage error that would result if the balance reading was taken without correction.

$$\rho_u = 1280 \text{ kg/m}^3 \quad W_s = 152 \text{ gm} \quad \rho_a = \frac{p}{RT} = \frac{1.023 \times 10^5}{287 \times 293.15} = 1.2049 \text{ kg/m}^3$$

$$\rho_s = 8.49 \times 1000 = 8490 \text{ kg/m}^3$$

$$W_u = W_s \frac{\rho_s}{\rho_u} \left(\frac{\rho_u - \rho_a}{\rho_s - \rho_a} \right) = 152 \cdot \frac{8490}{1280} \left(\frac{1280 - 1.2049}{8490 - 1.2049} \right) = 153.839 \text{ g}$$

$$\rho_u = 100 \text{ kg/m}^3 \rightarrow W_u = 153.839 \text{ g} \rightarrow 1.2\%$$

$$\text{Error} = 0.122 \text{ g} \quad \frac{0.122}{152.122} \times 100\% = 0.08\%$$

Let us quickly check, how the presence of buoyancy can affect. While it is very difficult to element the convection currents and the effect of temperature, but the with simple calculation we can eliminate that buoyancy part. Like suppose, we are trying to measure the mass of a body, say this is our body for which you want to measure the mass, this is 1 and this is a one, this is the counter weight that is 2.

So, the density of this body is say unknown mass is ρ_u , density of our counter weights which you generally know that is say ρ_s . And this entire operation is being done in air, which is having a density of ρ_a . Then while actual mass that we are trying to measure is equal to ρ_u into V_u , but because of the presence of buoyancy the force that your balance is sensing is ρ_u minus ρ_a into V_u , because following the Archimedes principle, we know that any body immersed in a fluid. We will replace the concern fluid of its equal to its own volume and therefore the buoyancy force acting upon this will be equal to the weight of the corresponding volume of fluid.

Similarly, W_2 that is the mass of the counterweight, where it should be ρ_s into V_s , but because that is immersed in this fluid of density ρ_a , corresponding force that your balance is sensing is ρ_s minus ρ_a into V_s .

And now by performing the balance, we should have W_1 equal to W_2 , so we have ρ_u minus ρ_a into V_u is equal to ρ_s minus ρ_a into V_s . Now, let us suppose you replace this V_u with W_1 by ρ_u or let me do not write W_1 . Let me change the symbols, let me say W_u is the actual mass that we are trying to measure, W_s is the actual mass of the counterweights, but actual reading that we are getting is W_1 .

So, V_u becomes W_u by ρ_u , similarly V_s can be written as W_s by ρ_s . From there the true mass becomes W_s by ρ_u upon ρ_s into ρ_s minus ρ_a divided by ρ_u minus ρ_a . You can simplify in whatever you want, this particular principle or this particular. Commonly we prefer writing this one as W_s into dividing the numerator and denominator by ρ_s , we have 1 minus ρ_a upon ρ_s similarly, dividing both by ρ_u , we have 1 minus ρ_a upon ρ_u . This one common way we can we prefer to replace write this, but the initial expression itself is sufficient.

Let us take a test case. So, you have a problem here, where actually the sorry for the lines are getting overlapped a bit, but please read the problem statement carefully. Here we are talking about a quantity of plastic material having a density of about 1280 kg per meter cube. So, this is the one that we are trying to measure that means, your value of ρ_u is 1280 kg per meter cube.

We are waiting this we are measuring this using a standard equal arm balance, this terms equal arm balance is important, because this always we consider the arms to be equal, but and this information is generally never mentioned, but practically we are always talking about an equal arm balance.

So, balance conditions are achieved with the brass weights totaling 152 gram in a room that is what we are when you are doing a measurement, we have to provide 152 gram of weights, so which weight we are talking about this one what is this 150 gram W_u , W_s , W_1 or W_2 , this is actually this W_2 , I should say this is actually equal to W_s , this is the 152 gram.

So, 152 gram of mass that you have provided into the other arm; so, as per the reading that you are taking you may feel that your W_u is equal to 152 gram, but there may be the effect of buoyancy, because you are doing this entire measurement in ambient air which is having at a temperature of 20 degree Celsius and pressure 1 atmosphere.

Now, the specific gravity of brass may be taken as 8.49, how we can get the density ρ_s from there, it is 8.49, so its density will be 8.49 into 1000 that is 8490 kg per meter cube. So, specific gravity is generally a common way, we see industrial density specifications.

Now, we have to calculate the true weight of the plastic material and also the percentage error that would result if the balance reading was taken without correction. So, we need to know the value of ρ_a . If we assume air is an ideal gas, then we can apply the principle P equal to RT . Here P is equal to 1 atmosphere that is 1.023 into 10 to the power 5 Pascal's, for air R is 287 joule per kg Kelvin into temperature T is here 20 degree Celsius that is 293.15. So, I have pre calculated the value, it comes to be 1.2049 kg per meter cube.

So, now we can calculate W_u directly, W_u is equal to W_s into the principle that we used earlier that is $1 - \rho_a$ upon ρ_s divided by $1 - \rho_a$ upon ρ_u , you have all the values available. So, if you put it, you are going to get it as 152.122 gram. So, this is actually a true mass of that plastic body though your balance is showing only 152, but it is actual mass of that is 152.122 gram and this 0.122 gram is the error that is coming in because of the buoyancy effect.

So, if we want to check the percentage error, then here error is 0.122 gram. If we want to calculate the percentage, this much error is present in 152.122 into 100 percent, so that will lead to something like 0.08 percent, so it is extremely small for this case.

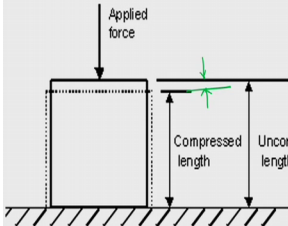
However, if we talk about a material having a different density, let us consider this body is made of some arbitrary material, which is having a density of your ρ_u is just 100 kg per meter cube. If you do the calculation now, then your W_u will becoming as 153.839 gram, so much higher level of error. So, your error is here 1.839 gram and it can be significant in several cases which actually amounts to about 1.2 percent. So, such amount of error can be quite significant in different applications.

Like if you are talking about some chemical experiment, measurement of the mass of some chemical reagent to perform every intricate chemical experiment 1.2 percent error can be very very significant. So, this buoyancy effect is something that we have to be very careful of during the measurement. Of course, the use of temperature control chambers, avoidance of the convection currents, those are also important, but those we

cannot eliminate by using mathematical means which you can do for in case of buoyancy.

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Elastic transducer / load cell



- elastic load-bearing member or combination of members
- application of load leads to deflection
- deflection measured either directly or indirectly
- use of secondary transducers is more common
- electrical output through resistive strain gage or LVDT

$$F = Ky$$
$$\Rightarrow K = \frac{F}{y}$$

- ✓ high sensitivity
- ✓ small response time
- ✓ stress maintained within elastic limit
- ✓ temperature sensitivity
- ✓ manufacturing tolerances

Next type of device that we have elastic transducers or load cells. Load cells are devices quite similar to the one that we have seen in case of displacing measurement. Here we have an elastic member, when subject to some kind of force, it suffers some kind of deformation. So, there is a change in length, and also change in the cross section area.

Now, accordingly we get a measurement that is the difference between this final length and initial length that has to be proportional to the applied force. Here we having elastic load bearing member or may be a combination of several members, because as we are applying the force there is a deflection, and the deflection can be deflection is proportional to the application of the force.

Now, the deflection can be measured either directly, but very rarely done or more commonly do; we go for indirect measurement. Indirect means, this deflection is measured using some kind of deflection measuring instrument, quite often leading to an electrical output that is the elastic transducer or load cell is connected to a secondary transducers. So, to the elastic transducer your input is the force, output is a deflection.

And in the secondary transducer, your input is a deflection an output can be electrical or whatever we would like to have very commonly we go for some kind of resistive strain

gauges or LVDT, so that we get electrical output. Now, most commonly this deflection or rather always this deflection is directly proportional a force that has been applied to this.

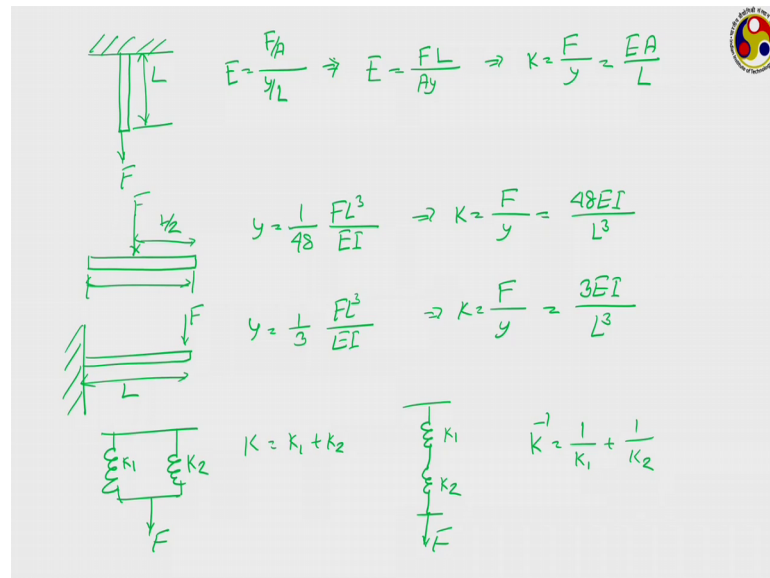
So, we can always write F is equal to K into y , where F is the force, and y is the corresponding deflection or K is equal to F upon y . Here K is known as the deflection constant. Depending on the nature of the membrane that we or I should say nature of the member that you are dealing with the expression for K can keep on changing.

In such kind of elastic transducers, you have to be very very careful of its configuration, because you always want very high sensitivity, and your sensitivity has to be high, then K should need to be very high. But, there can be practical difficulties depending on which level of application, you are going for so we may have to go for some kind of some kind of trial and error to identify the value of K .

Another; we also want a very small response time, so that we can get the reading quickly but generally, if we want to have a small response time, we need to have a smaller value of K . So, these two factors are actually counteracting to each other and we have to find some in between values. Stress has to be maintained within the elastic limit, because if you are crossing the elastic limit, then there will be permanent deformation for the member which will lead to error in all subsequent measurements.

There can be temperature sensitivity issues, because generally the value of the deflection coefficient decreases that is the sensitivity decreases with increase in temperature and manufacturing tolerances has also be to need to be considered with.

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Now, the value of K that depends on what kind of member that we are using, like if we talk about some of the common members, the most common situation that you can have say you have just a bar like this a bar of length L and say cross section area A and you are applying the force F here.

So, as per the definition of the Young's modulus force A is equal to stress by strain stress is force by A, where F refers to A cross section area. And strain is Y upon L that is we can write, so E is equal to F L by A y or K is equal to F upon y that is equal to E A upon L. So, by changing the dimensions of the system in terms of length or area or by using a different value of this Young's modulus by increasing using material having higher value of Young's modulus you can get a higher value of this K.

But, if your member is something like this, suppose it is in a position like this, this total length is L and the force is acting upon at the midpoint, this is the force and just this length is L by 2. In this case, if you go for the measurement, then by using I am directly giving the expression for this, but by drawing a free body diagram. We can always calculate this; this will be equal to F L cube sorry in the previous case I have written small L which should have been capital L as per the drawing that I am drawn.

So, it is F L cube by E I, I being the moment of inertia, correspondingly K is equal to F upon y that is 48 E I by L cube. Whereas, if we are using the same membrane or rather

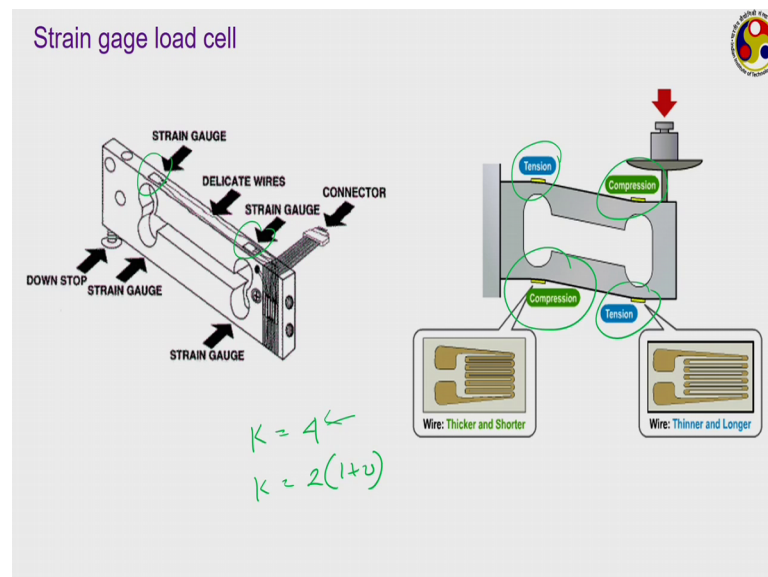
same member, this side is block and forces acting at this particular position, this total length is L.

So, in this case your y will become $\frac{1}{3} \frac{F L^3}{E I}$ or K is equal to F by y that is $\frac{3 E I}{L^3}$. So, depending upon the nature of the membrane, depending upon the position of the force application, the expression for K can keep on changing. Several cases, we may have to go for a combination of several members.

If your members are supposed combined like this, two members are combined in parallel and the force is being acted upon at this particular position. Then we know that here your K will be equal to K_1 plus K_2 , where K_1 refers to the deflection coefficient for the member one and K_2 for the second one.

Whereas if they are connected in series like this. This force F acting here, this is having a deflection coefficient of K_1 , this is having a coefficient of K_2 , then net coefficient K or K inverse I should say will be equal to $\frac{1}{K_1} + \frac{1}{K_2}$. So, this way for different members using the simple principles of mechanics, we can calculate the corresponding deflection coefficient.

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I shall briefly be touching upon on another one that is on strain gauge load cells. We have already had extensive discussion on the strain gauges; strain gauge load cell just uses a

principle of that. But, generally we can have four different strain gauge like, the diagram shown here this is number 1, this is 2, and there can be two on the other side also.

All these four strain gauges can be connected, they are connected or they are placed on the four arms of Wheatstone bridge kind of configuration. And depending on what kind of situation, you are going for either one or two or maybe all of them can change the load. Like on the other diagram that is shown here, two of the gauges are experiencing the tension, two of them are experiencing compression.



Now, accordingly we can get a measurement about the change in the resistance of the gauges or we can magnify the resistance. Like depending upon what kind of configuration we can go for, we can have a H factor of K equal to 4 that is we can magnify the factor to be 4 or we can sometimes also have it 2 into 1 plus ν which refers to two of them are sensing longitudinal strain, and two others are sensing lateral strain, whereas in this case all are sensing the same direction or strain. So, strain gauge load cell are also quite commonly used to measure very small amount of deflection and by converting into a change in resistance and subsequently electrical signals.

For force measurement however, while we definitely use a elastically members or elastic load cells and strain gauge load cells, the use of hydraulic and pneumatic load cells are very very common. So, in the next lecture, we shall be talking about the hydraulic and pneumatic load cells. And also we shall be talking about the torque measurement.

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Summary of the day

- SI unit of mass
- Common methods of force measurement
- Analytical balance
- Elastic load cell
- Strain gage load cell



So, I would like to keep it here itself for the day. So, today, we have talked about the SI unit of mass, particularly about the new convention that is going to be in effect in just two month time then the common methods of force measurement that we have summarized. We have discussed about the mechanical weighing system, particularly the analytical and the platform balances. Then on elastic load cells and a very brief mention about the strain gage load cell. So, I would like to wrap it up here today. In the next lecture, we shall be starting with the hydraulic and pneumatic load cells.

So, thanks for your attention for the day, thank you.