

Principles of Mechanical Measurement
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Module – 06
Stress & Strain Measurement
Lecture - 02
Strain gage rosettes & gage orientation

Hello everyone. Welcome back to the second lecture of our week number 6, where we are talking about different methods of Stress and Strain Measurement. In fact we are not talking about too many different methods here, we are focusing mostly on the resistive strain gages, and I shall be trying to touch upon one more device towards the end of today's lecture.

But we have restricted ourselves mostly to while discussion of resistive strain gages, because that is so prevalent in the industrial measurement of stress and strain that we hardly have to think about any other option. The principle of strain gage was introduced in the previous week itself, when you talked about the measurement of length scales. However, because strain itself is something like something can be related to the length.

Still here in this week, we have discussed about several different aspects of the strain gages. Like in the previous lecture, we have discussed about the material for the grid, and also the backing materials, different desirable characteristics of those materials, and also we have talked exclusively about corresponding circuitry. Like we have talked about the ballast circuit, which is similar to the potentiometer, and also we have talked about the bridge circuit. And we have roughly compared between both of them also.

Like in case of both of them were like in both ballast and bridge circuits for the change in or for a given amount of strain, the corresponding change in the output voltage more or less remains the same. However, in case of ballast circuit, you always have some kind of output voltage. And when you compare the strain corresponding, then that corresponding to the already existing output, the change in the output voltage may be extremely small. However, in case of a bridge circuit, the normal output is always 0. And therefore, whatever change may happen, we can directly distinguish that provided our voltage measuring instrument has sufficient resolution.

Now, there are three different kinds of circuits which are commonly used along with the strain gages. We have already discussed about two of them, the third one I was not able to discuss because of the paucity of time, I shall briefly we discussing that one today.

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A little exercise

The hoop & longitudinal strains on the outer surface of a cylindrical pressure vessel are measured to be 425 & 115 $\mu\text{m/m}$. If the modulus of elasticity of the material is 103 GPa and Poisson's ratio is 0.28, find the value of stresses in both directions.

$E_H = 425 \mu\text{m/m}$ $E_L = 115 \mu\text{m/m}$ $E = 103 \times 10^9 \text{ Pa}$
 $\nu = 0.28$

$$\sigma_H = \frac{E}{(1-\nu^2)} (\epsilon_H + \nu \epsilon_L) = \frac{103 \times 10^9}{(1-0.28^2)} [425 + 0.28 \times 115] \times 10^{-6}$$

$$= 51.1 \times 10^6 \text{ Pa} = 51.1 \text{ MPa}$$

$$\sigma_L = \frac{E}{(1-\nu^2)} (\epsilon_L + \nu \epsilon_H) = \frac{103 \times 10^9}{(1-0.28^2)} [115 + 0.28 \times 425] \times 10^{-6}$$

$$= 26.1 \text{ MPa}$$

A resistance strain gage with $R = 120 \Omega$ and $S_g = 2.0$ is placed in a bridge circuit, in which all resistances are equal to 120Ω . The power voltage is 4 V. Calculate the voltage indication for a strain of $1 \mu\text{m/m}$, when a high-impedance detector is used.

$R_1 = 120 \Omega$
 $S_g = 2.0$
 $R_2 = R_3 = R_4 = 120 \Omega$
 $E = 1 \times 10^{-6} \text{ m/m}$

$\frac{d\epsilon_0}{\epsilon_1} = \frac{\frac{\partial R}{R}}{1 + 2 \left(\frac{\partial R}{R} \right)}$

$\frac{R}{R_m} \approx 0$
 $S_g = \frac{\partial R/R}{\epsilon}$

$\Rightarrow d\epsilon = \frac{S_g \epsilon_1}{1 + 2(S_g \epsilon_1)} = \frac{2 \times 10^{-6} \times 4}{1 + 2 \times 2 \times 10^{-6}} = \frac{8 \times 10^{-6}}{1 + 4 \times 10^{-6}} \approx 2 \times 10^{-6} \text{ V}$

$\boxed{2 \mu\text{V}}$

And then moving on to discussion a few relevant parameters like the temperature compensation and also the effect of gage orientations etcetera, but before that let us do one small exercise. So, this is not relative strain gages, this is just related to the definition of stress and strain.

So, just read the question carefully, here we are talking about a cylindrical pressure vessel. The magnitude of the hoop and longitudinal strains on the outer surface of that cylindrical pressure vessels are given 425 and 115 micrometer per meter length. So, if we write, then so the strains are given, so epsilon H which is the hoop strain here is given as 425 micrometer per meter, sometime it is just also written as 425 mu just to indicate that this, we are talking about a micron change compared to some micron level change in each meter length.

And similarly, in the longitudinal dimension direction epsilon L is given as 115 micron per meter. So, the modulus of elasticity is given that the corresponding material, which is given as 103 Gpa that is we can write this as 103 into 10 to the power 9 Pascal, and the Poisson's ratio is given as 0.28.

So, we have to find the value of stresses in both directions. So, we can straight to use the relation of stress and strain which you have derived in previous lecture, if we go by that way, then how we can write say the stress in the hoop stress in the H direction. What will be the relation for this?

If you remember carefully, what we derive in the previous lecture, then you can directly make use of the relation, otherwise you can derive this one, it will be $E \nu \epsilon_H$ into ϵ_L that is what, similarly the stress in the longitudinal direction will be $E \nu \epsilon_L$ into ϵ_H . Because, what is the definition of ν , it is a ratio of just these two strains only.

So, in terms of ν , they are related to each other, and we have already derive this relations in the previous lecture. So, if we now put the magnitude for this one, our E is given as 103×10^9 divided by $1 - 0.28^2$ into what is ϵ_H that is 425×10^6 plus $0.28 \times 115 \times 10^6$.

So, if we continue with the calculation, you can just calculate the values, I have pre calculated the numbers, so it is coming 51.1×10^6 Pascal that is we can also write this as 51.1 mega Pascal. Similarly, if we put the values for the longitudinal one, then you will shall be having 103×10^9 divided by $1 - 0.28^2$ into ϵ_L , we have that is 115 plus 0.28×425 into 10^6 . So, if you calculate the value, even I have the number as 26.1 mega Pascal.

So, this way just using the stress strain relationship and using the knowledge of Poisson's ratio and the Young's modulus, we can calculate strains stresses in both directions. This is something that we shall be using very shortly regarding the stress measurement because, in the strain gage, we are talking about the measurement of strain. And once we know the strain, then just from the knowledge of this two parameter, we can easily calculate the corresponding stresses as well. This is principally the way we go for the measurement of stress. ahLet us see one more parameter of a problem, which involves resistance strain gages now. Here you are talking about a strain gage, which is having a gage resistance of 120 ohm, and a gage factor of 2. We are placing this one in a bridge

circuit, in this bridge circuit all resistances are also of the same value that is 120 ohm. The power voltage is 4 volt, so the excitation voltage is 4 volt in this case.

And we have to calculate the voltage indication means, when the bridge is balanced initially all resistance are of equal value, and therefore the output voltage should be equal to 0. But, whenever there is some kind of strain, that the strain gages able to sense, its resistance will change, corresponding there will be some kind of voltage indication.

So, we have to calculate the voltage indication, corresponding to this much of strain. When we are using a high impedance detector means, here the high impedance detector term refers that we are using some kind of voltage measuring instrument, whose impedance is extremely high almost infinitely high, when we compare that with the resistance of the strain gages, so that it does not impose any kind of loading effect that means, we are talking about this R divided by R for the voltage measuring instrument, this virtual need to be equal to 0, therefore no loading effect. This is being imposed by this particular term high impedance detector.

So, what are the given information? Here for the gage we have R equal to 120 ohm, and the gage factor is 2. For the bridge circuit, all the resistances are of the same value that is if we take all this one as say, if we call this gage resistance as R_1 , let us draw one bridge circuit first.

So, let us say, this is the bridge circuit that we are dealing with, and here this is the excitation voltage. So, this is your excitation voltage and say 1 is the strain gage, and others are 2, 3, and 4. Here R_1 refers to a gage resistance, and as for the given situation R_2 , R_3 , and R_4 all are equal to be your 120 for ohm which is the gage resistance itself. And this e_i is given as 4 volt, and we are looking to sense a strain of 1 into 10 to the power minus 6, this is a magnitude that is 1 micrometer per meter or 10 to the power minus 6 meter per meter. This is how we are going to sense it.

Those frame is a non-dimensional parameter, because it is just a ratio of 2 length. The change in length divided by original length, but just to indicate that quite often we put this kind of unit. Now, how to do it? For a bridge circuit, we have already done the mathematical derivation in the previous lecture, I hope you remember.

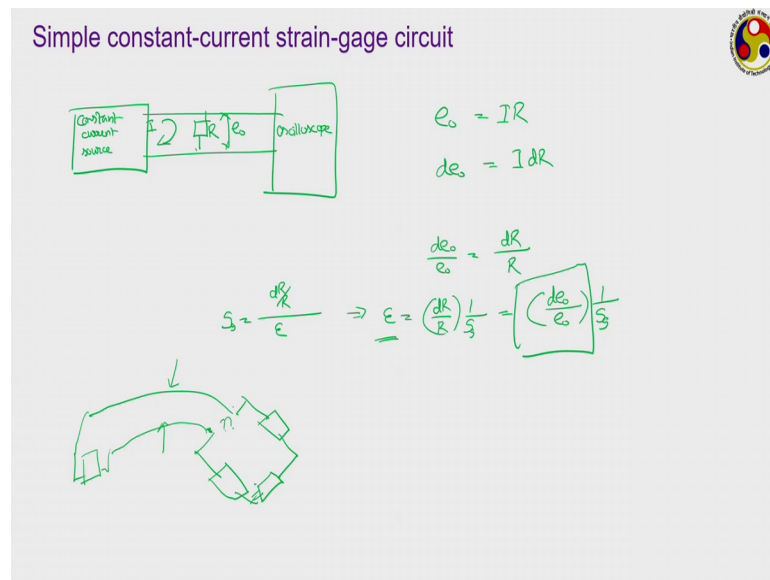
And what we got there, there through our derivation, we finally ended up somewhere here is not it, it was R_1 by R_1 by $4 + 2$ into ΔR_1 by R_1 . And as per the definition of the gage factor ϵ is ΔR_1 by R_1 divided by ϵ . So, if you replace this one, then we have ϵ a numerator divided by $4 + 2$ into again ϵ , which means ΔR_1 is equal to ϵ into R_1 divided by $4 + 2$ into ϵ .

So, if we put the numbers, our gage factor is 2, ϵ is given as 10 to the power minus 6 and R_1 is equal to 4 volt, so that is in SI unit only, so do not need to convert in the denominator $4 + 2$ into gage factor is 2, resistances 10 to the power minus 6 . So, if we carry on in the calculation, we have so 8 into 8 into 10 to the power minus 6 , and in the denominator we have $4 + 4$ into 10 to the power minus 6 , an extremely small number.

So, if we calculate this one, then we are again I have pre calculated this. So, we are going to get roughly 2 into 10 to the power minus 6 volt or we can write this one to be equal to 2 micro volt. So, this is the final answer that we are looking for that is this 1 micron amount of strain will be indicated by 2 micro volt.

So, your voltage indicator should have this much of resolution, so that it is able to sense a change of 2 micro volt amount so that we can able to sense 10 to the power minus 6 level of strain or 1 micro in micro meter per meter in amount of strength. So, this is a way, we can solve any problem associate with a bridge circuit or even ballast circuits also in conjunction with this resistive strain gages.

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Now, let us move forward to the third kind of electrical circuit that we quite often use in conjunction with the strain gages, it is the simple constant-current strain-gage circuit. So, what we have here? Here we have a constant current source which is connected to some kind of voltage measuring instrument, which can be an oscilloscope.

So, this is a if we write properly, then this is called the constant current source, so which is able to supply constant amount of current, and this is a voltage indicator. Now, so this oscilloscope is going to indicate the voltage, what we are going to get in the output.

So, this oscilloscope is going to indicate the voltage that we have in the output, but where is a strain gage? The gage is mounted in parallel to this constant current source, this is the gage of resistance R . So, if this oscilloscope is having near infinite impedance, then no voltage drop will be in or no current will be flowing through the oscilloscope, rather we shall be having current flowing only through this particular circuit. So, this current is say I .

If this current is I , then the voltage that we are that gets induced across this resistance, say this voltage is e naught. Then how much is your e naught? Your e naught is equal to I into R . Similarly, if there is a change in this resistance by the R amount, then corresponding change also will be to this much.

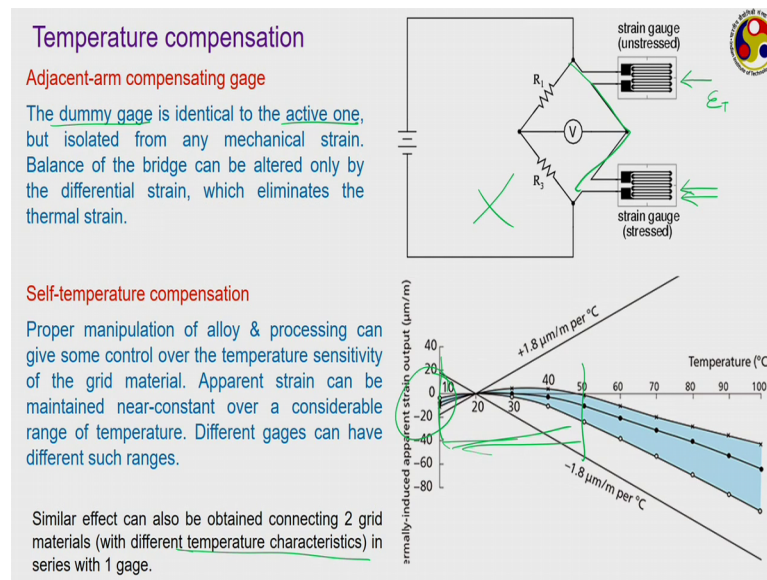
So, if we take the ratio of this to $d e$ naught upon e naught should be equal to $d R$ by R that is the change in the resistance can directly related to the change in the voltage that has been sensed by the oscilloscope. And now, if we relate this one to our gage factor, we know our gage factor can be written as nearby R by ϵ which gives ϵ is equal to $d R$ by R into 1 upon S_g or divided by the strain gage the gage factor. So, this way we can again use this constant current sources to measure the resistance.

And if we relate this one to the voltage output e divided by 1 by S_g , so by measure in this particular factor using this oscilloscope or any other voltage measuring instrument, we can get the value of this strain. So, this is a third way of strainless circuit. Though the bridge circuit is the most common one, but there may be several other situation several situations, where you are not able to use a bridge circuit, we have to go for a either a ballast circuit or maybe a constant current kind of circuit.

Here just one final thing, before I move on to another topic, one final thing I would like to mention about the bridge circuit. We know that commonly in bridge circuit, where we have four resistances like this. And in the quarter bridge configuration, only one of these resistances are the strain gage like this one. Say we generally keep this part open, and this part is connected to the strain gage, which may be mounted somewhere here.

Now, the length of these wires, they are very important. If this lengths are quite large the magnitude of the length of this wires are quite large, then there can be significant amount of ohmic losses or because of (Refer Time: 16:13) losses as the current passes through this wires, so that can also affect the final value that we are going to get in this position. So, this kind of error that needs to be avoided, these are often referred to as the lead wire errors. However, we can avoid them by using wires are very low resistances, but or the more logical situation, we always have to try to keep the length of this wires short, so that the corresponding losses are minimum.

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Now, we move on to the next topic, which is the temperature compensation. Now, you always know that the temperature of any kind of resistance is strongly affected or I should say the value of the resistance of any register is strongly affected by the temperature. Generally, the nominal value that are specified with a register corresponds to any particular reference temperature. And whenever the temperature varies from that position, there is a change in the resistance.

So, accordingly the performance of the strain gages are also going to get affected. There are primarily two different mechanisms by which we can eliminate that kind of effect because of the pressure variation in temperature. The first one is the adjacent arm compensating gage is something that we have already discussed, in conjunction with the resistive strain gages or bridge circuits.

Here along here instead of one, we use another second strain gage which is identical to the active one like here we have one active gage which is mounted on the specimen, where you want to measure the strain. But, you also have another dummy gage, which is very much identical to the active one.

However, it is not in contact with the specimen itself, rather it is isolated from the specimen, but still kept at a position which is very close to the specimen so that its surrounding temperature, he remains also very similar to what the active gage itself is

sensing. Then what will happen, this dummy gage that is in this case this is the dummy gage, and this is the active gage.

Now, the dummy gage is sensing some stress, but that is only because of the temperature, whereas the active gage is sensing stress, because of the presence of both of them that is because of the mechanical stress plus the thermal stress. Accordingly, we can eliminate the thermal stress by comparing these two that is what we are trying to say is that when the strain gage active sorry the this active gage, whatever stress it is sensing say one is coming from the mechanical part, the physical strain which you want to measure. And other is coming from the thermal part, because of the temperature variation.

But, this one is sensing only the thermal part, so by proper or positioning or proper connection of these two gages, it is very much possible that the net strain sensed by this particular arm is this minus ϵ_T giving or leaving out only the mechanical component, so that depends upon the orientation of these two gages which you have will be discussing shortly. This is one way of temperature compensation.

However, if it is there are several situations, when it may not be possible to place this dummy gage at an environment having same temperature. Like if your strain gage is mounted on a specimen, which is go through some kind of welding process or maybe some kind of very some kind of hot metal treatment, then it may be exposed to very high temperatures, and it is not possible to put the dummy gage in any other situation, which can guess which can sense similar level of temperatures.

There may be several other situations also, where it is not at all possible to go for a bridge circuit, we have to go for a ballast circuit. In those kind of situation, this adjacent arm compensating gage option is invalid and we have to depend on the second option, which is called the self-temperature compensation. Self-temperature compensation refers to an inherent property of the gage itself.

By proper manipulation of the alloy which we are using to make the grid and also the proper processing of that one we can have some control over the temperature sensitivity of the grid material. Here like what we are trying to mean is that as the temperature senses, there will be two different effects that will becoming on the gage. One is the change in the value of the resistance because of the temperature, second is the mechanical deformation of the gage material, and also is backing material because of the

change in temperature like if the temperature increases, there will be an elongation of the physical length of the resistance, and also it is backing material. So, if we can treat the alloy properly and process and fabricate the gage properly, then it is possible that these two opposing factors may eliminate each other over some range of temperatures.

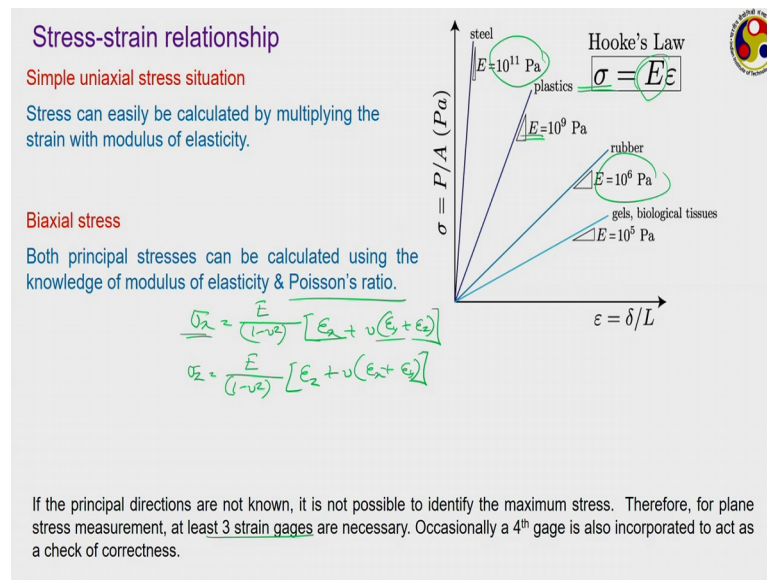
Like the figure that is shown for a situation, there you can see over this particular range, the value the corresponding the apparent strain R is extremely small. In fact, we can extend this one to some level somewhat like this also means, over this range of 10 to 15 degree Celsius. We can say that we do not need any other kind of compensation, the device itself is able to go for some kind of self-temperature compensation, and thereby corresponding temperature induced apparent strain remains extremely small within this range only.

However, if we the temperature range that we are dealing with that is quite large, this one may not be able to sustain. But, we can by proper metalworking, we can develop gages which can maintain a near constant value of the gage factor over a sufficiently long range of long interval of temperatures.

So, long means, I some can say it can be in the range of 200 to 300 degree Celsius. And also we can develop gages which shows this particular property in different ranges of temperatures. Therefore, depending on which range of temperature we are dealing with, we can choose the corresponding gage. And we hardly have to think about any kind of temperature compensation then.

The similar effect can also be obtained, if we use two different grid materials having different temperature characteristics, and connect in series in a single gage. But, here importantly we have to control the temperature characteristics of both materials properly, and we have to connect them suitably as well. Then the combined effect of these two gages will also be more or less independent of the temperature effect. But, still temperature composition is a very important option important tool that we always have to be mindful of whenever working with the resistive strain gages.

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Next we discussed very briefly about the stress strain relationship. One option can be simple uniaxial stress situation, where we know that the stress and strain are linearly proportional to the elastic limit. And so once we know the value of this E , then just the knowledge of epsilon gives you the value of this corresponding stress.

Like the values for the modulus of elasticity shown for different kinds of material, whereas material like rubber can have only in this range steel, they have much higher level much higher modulus of elasticity and so the knowledge of this E therefore, can very easily give you the value of corresponding stress only from the strain measurement.

However, this is what we are talking about uniaxial stress, there is a force applied only in one direction. But, practical cases we may have to go for biaxial stress measurement, where the stress is acting in both directions. And they are along with the modulus of elasticity, you also need to know the value of this Poisson's ratio and how we are going to use it, just like the problem that we have solved, today to start with like the stress which is acting in a particular direction say σ_x , we can always write this as E upon $1 - \nu^2$ into $\epsilon_x + \nu$ into $\epsilon_y + \epsilon_z$. So, all three strains are contributing for the final calculation of ϵ_x .

Similarly, we can calculate say the ϵ_z is our interest, then this will become $1 - \nu^2$ into ϵ_z , which is the primary direction plus the Poisson's ratio

into the other two or summation of the other two. So, this way we can calculate the biaxial stresses.

If the principal direction is not known, then however it is not possible to identify the maximum stresses. Like x, y, z, it is coordinates are only theoretical idea in a practical stress measurement situation, you may not be completely knowing the corresponding coordinate directions or the direction of application of the forces or the application of force the direction of force may keep on changing during a dynamic measurement scenario.

So, in that case, we cannot just use one gage, and get the idea about all three different kinds of all three components or the stresses or rather we just cannot place the gages, which you can directly give you the value of this epsilon x, y, and z, rather we have to go for some kind of combination of strain gages.

And then we have to calculate the maximum and minimum value of the corresponding strains by suitable mathematical manipulation. At least three such strain gages are required to obtain any kind of combination, when operating with unknown principal directions. But, sometimes a four gage is also incorporated, which can act as a check for the correctness and such kind of combination of gages are known as rosettes or gage rosettes.

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Strain gage rosettes

$\epsilon_a = 72 \mu\text{m/m}$
 $\epsilon_b = 120 \mu\text{m/m}$
 $\epsilon_c = 248 \mu\text{m/m}$

$\nu = 0.3$
 $E = 207 \text{ GPa}$

$$\epsilon_1, \epsilon_2 = \frac{1}{2} \left[\epsilon_a + \epsilon_c \pm \sqrt{2(\epsilon_b - \epsilon_a)^2 + 2(\epsilon_b - \epsilon_c)^2} \right]$$

$$\sigma_1, \sigma_2 = \frac{E}{2} \left[\frac{\epsilon_a + \epsilon_c}{1-\nu} \pm \frac{1}{1+\nu} \sqrt{2(\epsilon_b - \epsilon_a)^2 + 2(\epsilon_b - \epsilon_c)^2} \right]$$

$$\tan 2\theta = \frac{2\epsilon_b - (\epsilon_a + \epsilon_c)}{\epsilon_a - \epsilon_c} \rightarrow 2\epsilon_b > (\epsilon_a + \epsilon_c)$$

$$0^\circ < \theta < 90^\circ$$

$$\sqrt{2(\epsilon_b - \epsilon_a)^2 + 2(\epsilon_b - \epsilon_c)^2} = \sqrt{2(120 - 72)^2 + 2(120 - 248)^2} = 193 \mu\text{m}$$

$$\epsilon_{\max} = \frac{1}{2} [72 + 248 + 193] \times 10^{-6} = 256 \mu\text{m/m}$$

$$\epsilon_{\min} = \frac{1}{2} [72 + 248 - 193] \times 10^{-6} = 63 \mu\text{m/m}$$

$$\sigma_{\max} = \frac{207 \times 10^9}{2} \left[\frac{72 + 248}{1 - 0.3} + \frac{1}{1 + 0.3} \times 193 \right] \times 10^{-6} = 62.68 \text{ MPa}$$

$$\sigma_{\min} = 31.94 \text{ MPa}$$

$$\tan 2\theta = \frac{2 \times 120 - (72 + 248)}{72 - 248} \Rightarrow 2\theta = 25^\circ \text{ or } 205^\circ$$

$$\Rightarrow \theta = 12.5^\circ \text{ or } 102.5^\circ$$

$$\rightarrow (\epsilon_a + \epsilon_c) = 72 + 248 = 320 \mu$$

$$\rightarrow 2\epsilon_b = 2 \times 120 = 240 \mu$$

Rosettes primary combine of primary is a combination of three different parameters like shown here, this is just one particular rosette, which is very commonly called the rectangular rosette. Now, in this particular rosette, you can see there are three gages and they are placed 90 degree apart from each other.

So, each of them say this one a, this is b, and this is gage number c. So, each of them is showing a reading of corresponding strain in that particular direction. Now, we have to combine them, to identify the maximum or minimum strain that is acting in this situation. Let us assume that the maximum or minimum strain is acting in some theta direction, you have to identify the magnitude of this particular thing, and also the magnitude of the theta.

So, I am not going from the mathematical calculation that can be obtained quite straightforward way. But, what we can write is the final result, where epsilon 1 and epsilon 2 which can refer to maximum and minimum stresses. It is not that the epsilon 1 is always larger, epsilon 2 may also be larger, but these two are the two extreme values of the stresses.

So, if we do the calculation properly, their values will be something like this, epsilon a plus epsilon c plus or rather I should say again the problem with writing in the pen, so plus minus root of 2 into epsilon b minus epsilon a whole square plus 2 into epsilon b minus epsilon c whole square.

So, once we know the value of epsilon a, b, and c, then we can always combine these three to get when you are using this plus, we are talking about the maxima; when you are using the minus, we are talking about the minimum. And what will be the corresponding principal stresses? Say if sigma 1 and sigma 2 refers to the two direction of the principal stresses, then that will be equal to E by 2 into epsilon a plus epsilon c. Here I have written epsilon 2 in the previous case, there should have been epsilon c, epsilon c by 1 minus nu, it should have been minus 1 minus nu plus minus 1 by 1 plus nu into with your square bracket the same term that is 2 into epsilon b minus epsilon c whole square plus 2 into epsilon b minus epsilon a whole square.

So, and this allows us to calculate the value of the strains in the corresponding epsilon 1 and 2 directions, there are two principal directions. And how can you identify the value of this theta? The same way we can calculate the value of this tan of twice of this theta.

Generally, is given us twice of epsilon b minus epsilon a plus epsilon c divided by epsilon a minus epsilon c, which will give you the value of theta. Generally you can get two different values of theta, working two different coordinate directions or in two different quadrants.

Now, if the value of this twice of epsilon b is greater than epsilon a plus epsilon c, then we can say that the theta lies between 0 degree and 90 degree. However, otherwise the theta lies in the other coordinate that we are going to get by solving this particular situation. I say it is plus 90 degree, where you are talking about plus in the counter-clockwise direction like shown here.

Let us do with a small problem here. We are not deriving this equation, because this have quite standard equations, we hardly have to remember them also. But, here we are dealing with the situation, where the material properties are given as either a modulus of elasticity to is 207 GPa and 0.3 as the Poisson's ratio, and the values of the strains in three directions a, b, and c are shown here.

So, how can we calculate the values? Let us first calculate the square root. So, the value of the square root that is 2 into or let me start writing from top, so that we can get some more space. So, 2 into epsilon b minus epsilon c whole square plus 2 into epsilon b minus epsilon a whole square that is if we put the values 2 into epsilon b is 120 minus 72 whole square plus 2 into 120 minus 248 whole square.

And the calculation of this one is giving us 193 mu or 193 micron per meter. So, how we can calculate? Now epsilon max then will be equal to half into epsilon a that is 72 plus epsilon c that is 248 plus the particular one that we have just calculated that is 193. So, if we put the numbers, it will becoming 256 micron or micrometer per meter. Similarly, epsilon min will be equal to half into 72 plus 248 minus 193, so the value is going to be coming to be 63 micrometer per meter.

And if you want to calculate the stresses, so sigma max will be equal to we can put the numbers here the modulus of elasticity is given as 207 into 10 to the power 9 divided by 2 into epsilon a plus epsilon c epsilon a 72 plus epsilon c is 248 divided by 1 minus 0.3 plus minus or sorry we have to put the plus sign here, as we are trying to get the maximum here.

So, 1 plus 0.3 into the square rooted value which we have already calculated here which is 193 into 10 to the power minus 6 we are taking out, because remember the 72, 242, and 193 all are having 10 to the power minus 6 incorporated. In fact, in this calculations we have all should have also written 10 to the power minus 6, because here all the values are given us micrometer per meter, so this is the epsilon max. And if we put the number, you are going to get 62.68 MPa.

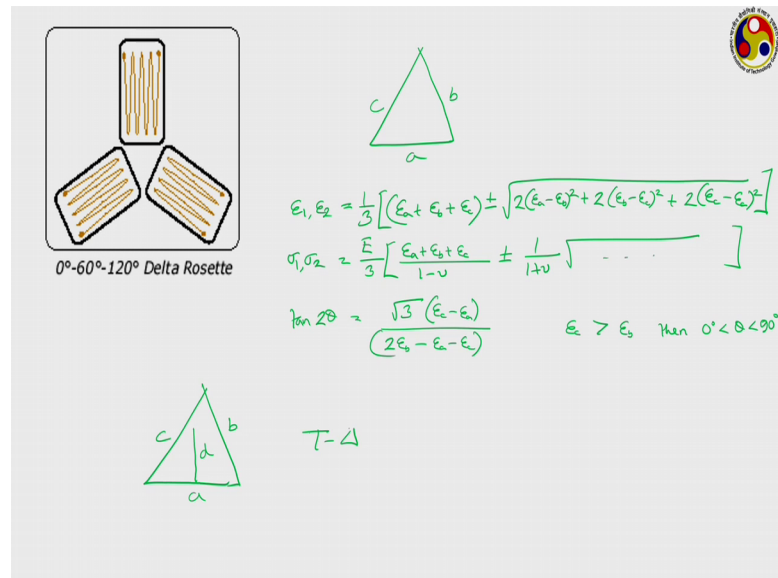
Similarly, sigma mean how much it will be? Here this positive sign will become negative or because here we have this plus minus sign. So, once you put the minus sign, then you will be getting this one as 31.94 Mpa. I am just giving you the final numbers, you can calculate, and get the final values of whatever you have.

So, now we have to get the direction in which direction is a epsilon max and min are working or the principal direction for this. So, $\tan 2\theta$ is equal to twice of epsilon b that is 120 minus epsilon a is 72 plus epsilon c is 248 divided by epsilon a minus epsilon c that is 72 minus 248.

So, if you put the values here, then two theta will become it to be 25 degree or 205 degree approximately, which gives theta equal to 12.5 degree or that is 102.5 degrees. Now, which one will be the correct answer that will be coming from this criteria. Let us check the value of epsilon a plus epsilon c, so epsilon a plus epsilon c in this case is 72 plus 248 that is 320. And twice of epsilon b is 2 into 120 that is 240 or I should write 240 mu and 320 mu.

Now, this one is greater in this case. So, what does that indicate? Theta hat this twice of epsilon b greater than this quantity, they would have it should have lied in the first coordinate. But, as this is smaller, so it is not lying in the first quadrant, then your value of theta is 102.5 degree. This direction on which the maximum or minimum strain is stresses working. This way you can do the calculation for any strain gage rosettes. There are several other strain gage rosettes also, this is the most standard one the rectangular strain gage rosette.

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But, another one I shall would like to talk about, and that is a delta rosette or also called the equi triangular rosette. Because instead of presenting this way, we could have also presented this as three triangles like this, where say a, b, and c are the three strain gauges.

Again here I am directly writing you the expressions epsilon 1 and epsilon 2, if we do the calculations properly, then you are going to get as epsilon a plus epsilon b plus epsilon c plus minus, and this plus minus will decide whether you are talking about the maxima or minima plus for maxima minus for minima. So, again we have a root 2 into epsilon a minus epsilon b whole square plus 2 into epsilon b minus epsilon c whole square plus 2 into epsilon c minus epsilon a whole square. So, this is a quite straightforward formula, we can easily calculate this.

And similarly, if you put say sigma 1 and sigma 2, then that will be equal to just definitely E will be coming in E by 3, the rest part remains quite same. Like if we compare with the previous case, whatever we had before the square bracket that was divided by 1 minus nu, here also during divided by 1 minus nu plus minus what we are going to have 1 by 1 plus nu, and this entire square term. The same square term the or square rooted term that we have in the previous line.

And the criteria for tan theta in this particular case or tan 2 theta, it is slightly different, it is root 3 into epsilon c minus epsilon a divided by twice of epsilon b minus epsilon a minus epsilon c. And the criteria is if epsilon c is greater than epsilon a, then theta will

lie in the first quadrant. So, using this we can calculate how would that look in the calculation for this delta rosette. So, we have talked about the triangular and the equi triangular or you can or I should say the rectangular and the equi triangular or commonly called the delta rosettes. And some rough mathematical relations that are given about.

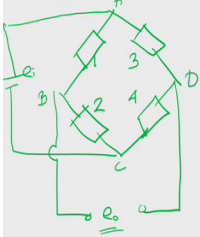
So, once you know the values of all these three resistances that is a , b , and c or ϵ_a , ϵ_b , and ϵ_c , we can easily calculate the maxima and minima stresses, and also the direction of principle stress. Thereby we can combine three resistances, these are just two examples. But, we can combine resistances in different other combinations or I should say resistive strain gages in different other orientations, just to get an idea about the maxima and minima about minimum strain gages, maximum stress that is acting on the system, and the direction of its application.

There is another variation of this delta rosette that is quite often is also used, where we have the same three resistive strain gages a , b , and c . However, there is a fourth one, which is also put in from the center point of this a that is d . And this d is used like we have mentioned earlier, d is generally used to check the correctness of the solution or correctness of the measurement for this. This particular one is called a delta star orientation or sometimes also called more popularly a T delta orientation, because the bottom part looks like an inverted T, there so it is often called T delta orientation.

These are just too common or two three common examples of strain gage rosettes, but we can have several other orientations also which may be very very difficult to deal with, at least through simple analytical cases such situation.

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Gage orientation



$$e_o = e_b - e_d$$

$$= e_i \left[\frac{R_2}{R_1 + R_2} - \frac{R_4}{R_3 + R_4} \right] = e_i \left[\frac{R_2 R_3 - R_1 R_4}{(R_1 + R_2)(R_3 + R_4)} \right]$$

$$\frac{de_o}{e_i} = \frac{\partial e_o}{\partial R_1} dR_1 + \frac{\partial e_o}{\partial R_2} dR_2 + \frac{\partial e_o}{\partial R_3} dR_3 + \frac{\partial e_o}{\partial R_4} dR_4$$

$$\Rightarrow \frac{de_o}{e_i} = -\frac{R_2 dR_1}{(R_1 + R_2)^2} - \frac{R_1 dR_2}{(R_1 + R_2)^2} - \frac{R_4 dR_3}{(R_3 + R_4)^2} - \frac{R_3 dR_4}{(R_3 + R_4)^2}$$

$$R_1 = R_2 = R_3 = R_4 = R$$

$$\frac{de_o}{e_i} = -\frac{1}{4R} (dR_1 + dR_2 + dR_3 + dR_4)$$

$$= -\frac{1}{4} (S_1 \epsilon_1 + S_2 \epsilon_2 + S_3 \epsilon_3 + S_4 \epsilon_4)$$

$$= -\frac{S}{4} (\epsilon_1 + \epsilon_2 + \epsilon_3 + \epsilon_4)$$

$$= e_i \left[\frac{R_1(R_3 + R_4) - (R_2 R_3 - R_1 R_4)}{(R_1 + R_2)^2 (R_3 + R_4)} \right]$$

$$= e_i \left[\frac{-(R_2 R_4 + R_2 R_3)}{(R_1 + R_2)^2 (R_3 + R_4)} \right]$$

$$= -\frac{R_2 e_i}{(R_1 + R_2)^2}$$

Finally, we have to talk a bit about the orientation of the gages. When we are talking about a bridge circuit, we like in a quarter gage configuration, we connect the strain gage with or one of the arms that is one of the resistance in the bridge is being replaced by the strain gage.

However, if you are going for a half gage, we replace two of them two of the resistances, and in a full bridge on configuration all the four resistive strain gages or all the four resistives of the bridge are replaced by corresponding strain gages. Now, which of the configuration, we should go for or if we are combining 2 or 3 or 4 strain gages into the same configuration, how we can calculate the final value that we would like to check very briefly here, through some kind of mathematical orientation.

So, the let us considered first a bridge circuit, so this is the standard Wheatstone bridge kind of circuit that we have. Let us say this is resistance 1, this is 2, this is 3, and this is 4, let us give some names at this point is a, this b, this is c, and this is d. Here we have the source, so our source is some B C source having a voltage of e i, and our final measurement is taken across these two points between b and d, so e o is the measurement voltage. So, when 1, 2, 3, and 4, all resistances have their nominal values, then e 0 should ideally be 0, this one will show a 0 voltage value.

However, whenever there is a deflection in any one of the arms or may be at least in one of the arms, we are going to get some value of this. Let us first try to relate this e naught

with the resistances with an access which you have already done in the previous class for (Refer Time: 42:10) circuits.

So, what will be your e_{out} that definitely is e_b minus e_d . Now how much is your e_b ? We can make use of the current that is flowing through this circuit. And from there we can say that e_b should be equal to e_i into R_2 divided by R_1 plus R_2 the resistance of that particular side. And e_d should be equal to e_i into R_4 divided by R_3 plus R_4 that is if we simplify this, then we have R_1 plus R_2 into R_3 plus R_4 in the denominator. And in the numerator what we have? We have $R_2 R_3$ plus $R_2 R_4$ which cancels out minus $R_1 R_4$, so this is what we are having in the numerator.

Now, let us assume that 1, 2, 3, and 4, all can act as strain gages that is in any situation all of them can change or may be any one of them can change or any arm means any of the resistances can change their values that is R_1 , R_2 , R_3 or R_4 . Any one of them or several of them can change simultaneously they are causing a change in this e_{naught} .

So, correspondingly $d e_{naught}$ can be calculated as not $d e_{naught}$ R_1 into ΔR_1 or just to be consistent with our notation should write as say $d R_1$ plus R_2 $d R_2$ plus $d e_{naught}$ R_3 $d R_3$ $d R_4$. So, make use of the previous line e_i being a constant, I am taking it on the left hand side.

So, if we make use of the previous line, what is going to be; what is going to be the differentiation first we have to differentiate this expression with respect to R_1 . So if you differentiate with respect to R_1 , then what we are going to get?

Here if we differentiate with respect to R_1 , say let us calculate the first part $d e_{naught}$ $d R_1$, so what we have? We have e_i as outside in the denominator R_1 plus R_2 whole square R_3 plus R_4 , it is a constant, so that whole square the in the numerator differentiate with respect to R_1 each resistance are independent of each other. So, we are having minus of R_1 plus R_2 into R_3 plus R_4 , the inter denominator.

And now what we are having here $R_2 R_3 R_1 R_4$ that is a numerator into the differentiated form of the denominator with respect to R_1 , which is just R_3 plus R_4 . So, $R_3 R_4$ one cancels out from there, and that is e_i . So, what we are left with is minus R_1 plus R_2 minus $R_2 R_3$ is $R_1 R_4$, so $R_2 R_3 R_1 R_4$ divided by R_1 plus R_2 whole square by R_3 plus R_4 .

Now, I miss something, I feel I have missed something. When doing the first step numerator, and the numerator there should have been an R_4 in the numerator, which I have missed. So, it is minus R_4 in the numerator that means, we have an R_4 here this.

So, if we simplify this, that is minus $R_1 R_4$ cancels out. So, we have ϵ_i into in the numerator R_1 plus minus $R_1 R_4$ cancels out, and we have minus of $R_2 R_4$ plus $R_2 R_3$ plus $R_1 R_4$ cancels out by R_1 plus R_2 whole square R_2 plus R_3 . And if we simplified even further taking R_3 plus R_4 common, this is R_3 plus R_4 .

So, if we simplify it even further, then it becomes if you take R_3 and R_2 common from the numerator, we have just R_2 left in this ϵ_i by R_1 plus R_2 whole square. So, this is what we have, so if we take it back there, so this will become minus R_2 by R_1 plus R_2 whole square plus the same way, we can continue with the others. So, it becomes minus R_1 by R_1 plus R_2 whole square $d R_1$, this is $d R_2$ minus R_4 $d R_3$ by R_3 plus R_4 whole square plus R_3 $d R_4$ by R_3 plus R_4 whole square.

This is the entire expression that we are dealing with now. And I just want to ensure, if I made any mistake with the symbols anywhere, because that will lead to a wrong expression at the end. When you are differentiating with this first with respect to R_1 , so this is what we have now. Let me erase this entire portion to make some more space for the next part of our calculation. And this plus sign, then we will also become negative in that case.

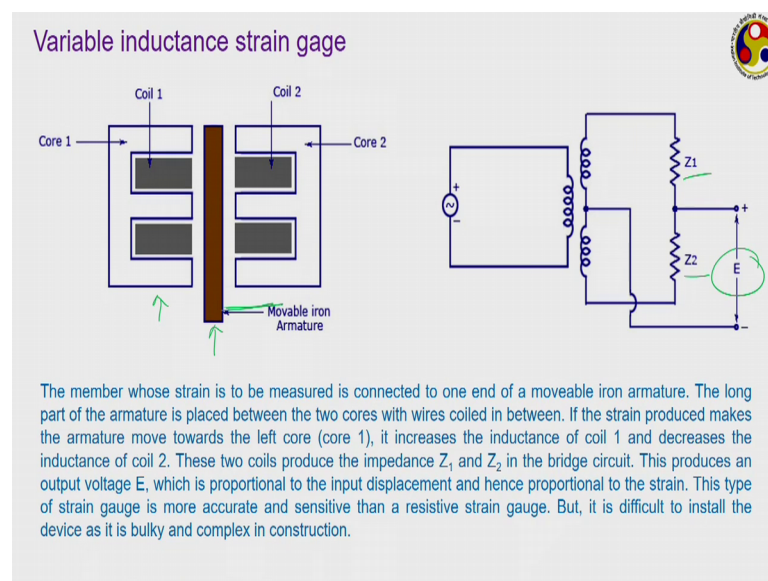
Now, in a special situation actually not a special in most of the common gages, you will find this R_1, R_2 all are equal in magnitude means, all of them are all the resistances in a bridge are given the same value equal to R . In that case, what we have? $d \epsilon$ naught by ϵ_i being the values as R . So, we have minus of 1 by $4 R$ into $d R_1$ plus $d R_2$ plus $d R_3$ plus $d R_4$.

And if we replace this, now we know that say for the first gage say s_g will be $d R_1$ by R divided by ϵ_1 . So, if all of them generally are having same gage factor, then we can always replace this one as minus 1 by 4 into s_g into ϵ_1 plus s_g into ϵ_2 plus s_g , so I should write this as s_g for the gage 1, this is s_g for 2, s_g for 3 ϵ_3 plus s_g for 4 ϵ_4 .

And if all gage factors are equal, then this will become minus s_g by 4 into ϵ_1 plus ϵ_2 plus ϵ_3 plus ϵ_4 . So, depending upon what is our objective, we have to connect the gages properly. This gives us a way of connecting multiple gages, whatever may be the orientation that we prefer. Like if we are talking about only a quarter bridge say where the strain gages connected only in the arm 1. Then $\epsilon_2, 3, 4$ has does not have any value, because they are all 0, because they are fixed resistances, however ϵ_1 will be there.

Whereas, if suppose 2 and 4 are the arms, where you have strain gage connected, then only ϵ_2 and ϵ_4 will be having will be there in the picture. This way we can combine several strain gages to obtain a proper circuitry of gages, and we can most commonly go for this bridge orientation to get the final reading. So, this is all for resistive strain gages, which is a primary instrument for measurement of strain and stress where I briefly would like to talk about another kind of gage which is the variable inductance strain gages.

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Similar to the displacement measurement like resistive instrument, you can also have inductance and capacitance base instrument. And that is why this is a very brief introduction about the variable inductance strain gage. Here we have a moving an iron armature, and the instrument for which we want to measure the strain of the specimen that is connected with this armature.

So, in the initial position armature is at the neutral position between coil 1 and coil 2, so that the voltage induced in the both the coils are equal. Now, if there is any strain because of which, the armature moves closer to the left core that is closer to this particular one, then the inductance same coil on will increase, and inductance coil to that will decrease.

So, correspondingly the two the impedance produced in the corresponding z circuit z_1 corresponding to coil 1, and z_2 corresponding to z_2 , they will not become they will not remain same. And an output voltage you will be produced, which should be proportional to the displacement of this moving iron armature.

Similarly, when the coil moves towards right that is towards core to then the inductance in core to will be higher compared to the core 1, and so the z_1 and z_2 values again will change, according it will produce again a voltage e . If the movement is same, movement towards right and movement towards left are same, then the magnitude of e will be same, but his symbol will be different that is the way, we can get a quite accurate measurement of the strain and also the direction of the same as well.

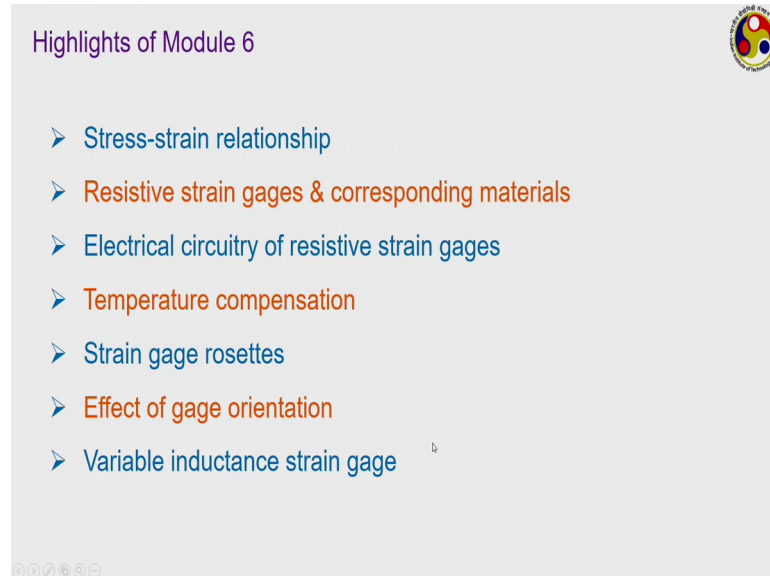
However, this instrument generally is quite bulky and complex in construction, then therefore they are mostly used either in a permanent installation or more commonly both inductance and also similar sort of capacitive base strain gage. They are used more for measurement of secondary parameter, that is instead of strain we are measuring strain, but our final object is not to measure strain, rather something like say pressure or flow.

In those kind of situation, generally go for inductance and capacitance base instruments. So, if such situation appears, when you talk about those parameters, we may be discussing a bit more about inductance or other kind of strain gages. But just to finish on I would repeat again that the resistive strain gages are the most common type of measurement tool that we use for practical measurement of stress and strain. And that is why this entire lecture and also the previous one that is this entire module we have discussed only about the resistive strain gages and different aspects of the same.

So that takes us towards the end of this particular week. We have kept it quite short, because if the idea of strain gages are already discussed in the previous week, and here we have just tried to see some of the practical aspects like the material, like electrical

circuitry, and like the combination of different gages either in the bridge circuit or in the form of rosettes that is what we have tried to see.

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So, what we have learned just a very very brief summary, we have talked about the stress-strain relationship you start with. Then we move to the resistive strain gages. If first we discussed about the materials, like I just mentioned. Then electrical circuit circuitry, we have seen that there are three kinds of circuits, which are commonly used in conjunction with the strain gages, the ballast circuit which is similar to potentiometer, the bridge circuit very very popular. And some situations, we can also go for a constant current source we have circuitry.

Then we have discussed about the option for temperature compensation, when you are using a bridge circuit, we generally go for the adjacent arm based compensation. However, if the arm we it is not possible to put a bridge circuit or if it is not possible to have multiple strain gages or is dummy strain gage located in proper environment, then we have to go for or we have to depend on the self-temperature compensation, which can be achieved by proper material working.

We have discussed how the strain gage rosettes, primarily we have discussed about the rectangular and equilateral or the delta rosettes. And effect of gage orientation was briefly touched upon. And finally, a very brief introduction about the variable inductance strain gage.

So that is it for module number 6. Please go through the lecture, try to study the books and solve the assignments. If you have any query, please write back to me, I shall be very happy to answer and several mathematical derivation that we have done in this particular module. And also in the previous week, if you have any difficulty, I would suggest that first you try to do that on your own without listening to the lectures or without looking into the books. And if you find any difficulty, then first look at the books otherwise, please write back to me, I would try to again clarify. So that is it for this week, I am signing off from here. And we shall be seeing again in the next week, where we shall be talking about the measurement of force and torque.

So, thank you very much.