Principles of Mechanical Measurement
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Module – 06
Stress & Strain Measurement
Lecture – 17
Resistive strain gages & associated circuitry

Welcome back friends, to the week number 6 of our MOOCs course on the topic of Principles of Mechanical Measurement and this is the week where we are going to talk about the Measurement of Stress and Strain. Now, in the previous week we have discussed about the measurement of displacement or dimensional parameter where you are introduced to quite a few different kinds of techniques for measurement of length scale. We have primarily restricted ourselves to measurement of smaller dimensions and mostly to electromechanical transducers, because that is the most common kind of transducers that you will find in any kind of mechanical measurements.

That is some kind of mechanical input quantity being converted to an electrical output and then that electrical output being calibrated back to that mechanical quantity. Now, stress and strain or particularly strain is also something kind of a length related parameter because, it is just the change in length or I should say the ratio of the change in length to the original length. And, when I started our discussion on displacement measurement I mentioned that whatever you are discussing there they can be relevant to any parameters having a dimension of length including the strains or rather including strain.

Now, you definitely remember that we spend significant amount of time discussing about the resistive strain gauges and that will be coming back again in this week because that is the most common type of tool that we use for measurement of strain. But, while we restricted also mostly to the development of the mathematical expression for resistive strain gauges and also discussed about different types of gauges. Here we shall be discussing several other very important factors of the resistive strain gauges and also hopefully shall be able to discuss briefly about a few other kinds of strain gauges.

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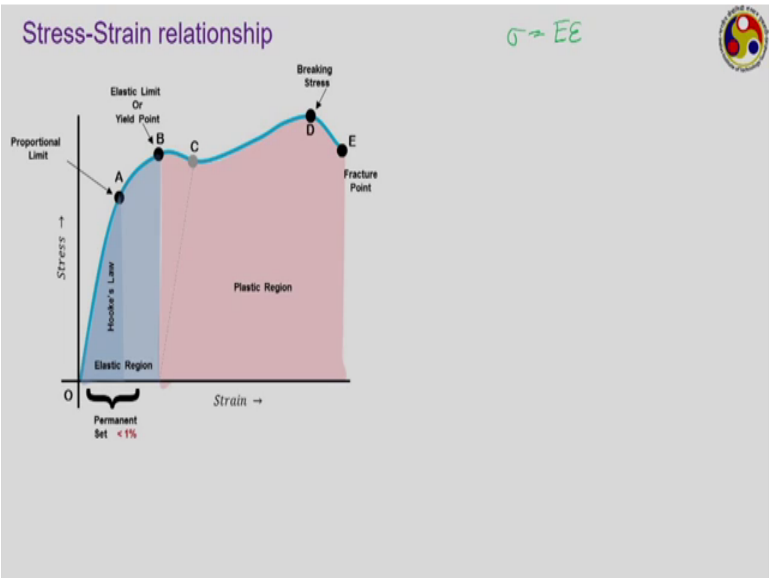

So, let us start our discussion the first thing that we have to talk about is of course, the stress-strain relationship where this curve I am sure all of you have must seen several times very early in your course. You know that within a particular limit which you generally call the proportional limit or the elastic region stress and strain are proportional to each other represent almost a straight-line relationship something like this kind of line where the Hooke's law is applicable that is stress and strain are proportional.

And, we get the simple relation that is the stress is some constant multiplied by the strain and this constant is known as what is the Young's modulus, all of you know, sure. And, once we go beyond this elastic region of course, the stress strain relationship does not remain linear, it is a highly non-linear relationship depending upon which zone of strain that we are looking for. However, our discussion will be limited only within this elastic zone or as long as this linear relationship is valid.

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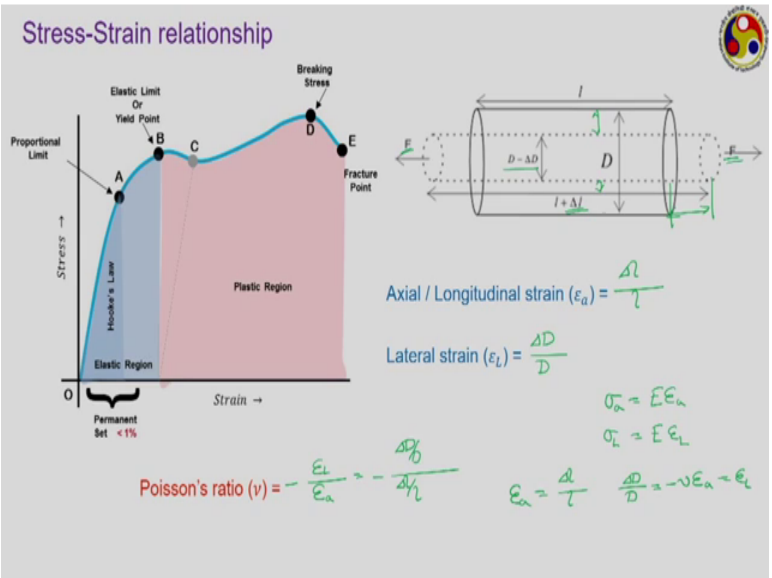

Now, if we talk about a very simple bar say a bar of initial length l and initial diameter capital D, it has been subjected to some kind of force F. These are subjective which is been subject to tensile force a from both direction thereby causing some kind of strain to this because, of this force is length changes by this amount like this particular amount change happens in one direction; similarly the other direction thereby causing a net change of length delta l. And so, this is what we referred as the axial or longitudinal strain epsilon a given by the ratio of the change in length which is delta l by the original length l.

However, if the volume of the material has to be maintained then this change in length must be a component by a corresponding change in the other dimension. So, the diameter D which was initially before putting a tensile strength which is the initial diameter that reduces to some value of D minus delta D because on this side we have some amount of reduction, this side also we are having some other amount of reduction or if I draw properly say this is the reduction on this side. So, this total leading to delta D amount of change in dimension.

And, that gives you the lateral strain that is the strain which is acting in a direction perpendicular to the direction of the force application. Epsilon L, again it can be defined the same that is a change in length in that particular direction that is delta D divided by the original length in that particular direction D. And, the ratio of these two is given by the very famous number which is a Poisson's ratio which is defined as minus of lateral strength to the axial or longitudinal strain that is minus of delta D upon D by delta l upon l.

This is a Poisson's ratio how the minus sign comes is because while generally, if for the kind of situation that you are showing here while delta l is positive delta D is negative. So, to avoid a negative sign the Poisson's ratio is given this minus symbol. Now, if we want to find the relation of these two strains the longitudinal or axial strain and the lateral strain with the corresponding stresses we can all as long as we are within the elastic limit we can always mix with the Hooke's relationship or Hooke's law.

So, the stress in the axial direction epsilon a will be the Young's modulus trying times the corresponding strain, similarly in the stress in the longitudinal direction will be the Young's modulus time the corresponding L. However, this particular situation is valid when we are dealing with only an uni-axial kind of load that is the load is being applied only on one direction.

If you are dealing with the compressive load then what will happen compressive means on both this force F is acting in the inward direction in both cases in that case delta will be negative and delta D will be positive. So, here is the Poisson's ratio will always remain to be a positive quantity. However, the magnitude of the longitudinal and lateral strain while the magnitudes may change the direction of course, definitely will get reversed. And, also once we know one of the strain we can always calculate the other one in terms of the Poisson's ratio.

Like, suppose we know here the epsilon a is given as delta l upon l; now, how we can define the ratio delta t upon D that can always be represented as minus of nu into epsilon a which is our epsilon l therefore, using the idea of Poisson's ratio just one of the strain sufficient to represent the other one as well. Now, let us move to a more general situation where not one, but the forces applied in two different directions. So, for that purpose let us initially consider a small rectangular block.

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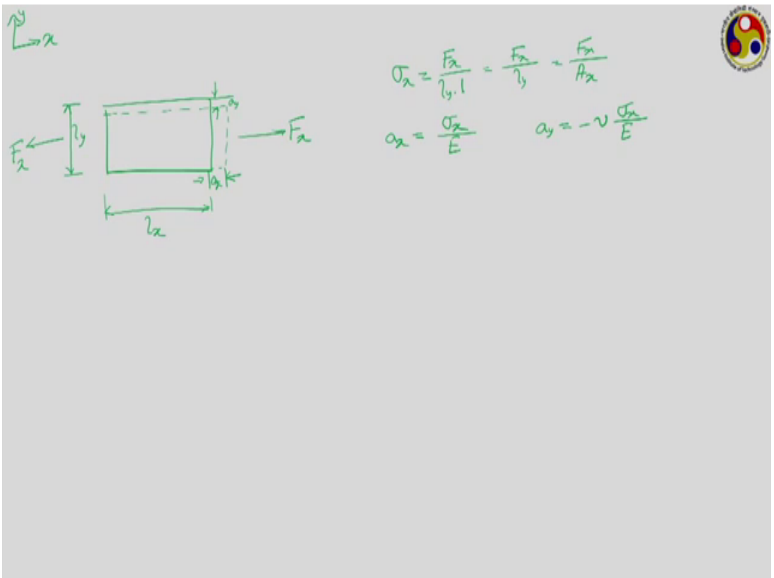

Here we are restricting ourselves to two-dimension initially. So, let us say this direction is x, this particular direction is y. Let me draw the rectangle properly. So, this is our initial rectangular block. This block is having a length of say, l in this direction and or say l x in this direction and the length of l y in this direction. Now, we are subjecting this block to an axial strain or axial force acting in the x direction. So, let us say some force F x we are putting in this then what is the corresponding stress that we are subjecting this to? F x will sorry sigma x will be equal to F x divided by the area.

So, the area over which it is acting the cross section area is l y into 1, where 1 is the dimension in the third direction which we are not considering that is F x divided by l y or we can also add this one as F x by A x where A x refers to the area perpendicular to the x direction. So, that is your stress that is acting in the x direction. Now, what will be the effect of the stress this being a tensile strength. So, there will be a increase in length in the x-direction and to compensate for that there has to be a decrease in the dimension in the y-direction.

So, because of this your system may take a shape somewhat like this to clarify the image I am erasing this force F x for now just to make some. So, maybe something like this let us say this is your F x. So, this is the chain increase in length in the x direction whereas, this represents the decrease in length in the y direction. Now, how much will be the increase in length let us say this is small a in x direction, this is small in y direction referring to the change in length because of the force acting in x direction.

Now, how much will be your a x? a x definitely will be equal to once we know the value of the stress we can write this one as sigma x by E this will be the strain that is acting in the x direction because of this force F x. Similarly, how much will be a y? Here we can make use of this Poisson's ratio to be the strain as again sigma x upon E because Poisson's ratio represents the corresponding change in dimension. So, this is the deformation in the x and y direction because of this force F x. Now, let me remove this l x to make it get some more spores face space.

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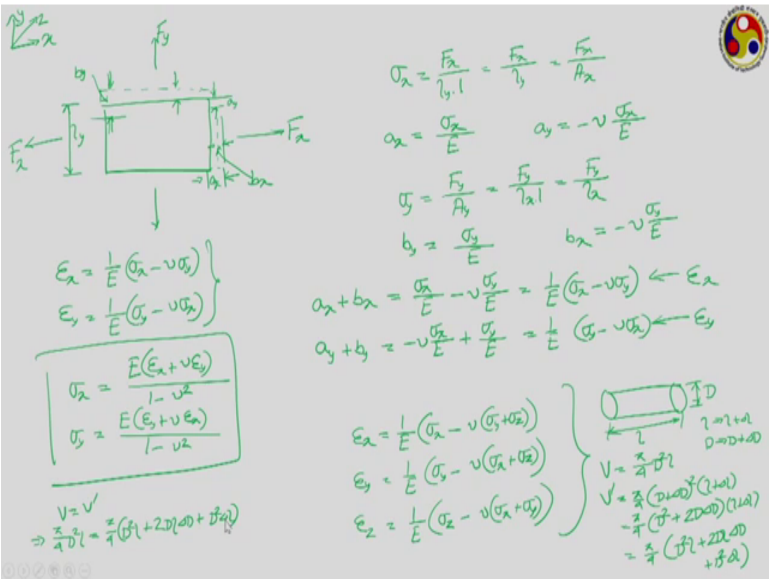

Now, we are putting another tensile force in the y direction let us say it is magnitude is F y because of the presence of this F y there will be further change in the dimension of this body or in this rectangle in both directions. So, there will be elongation in the y direction whereas, there will be a reduction in length in the x direction. That means, if I remove the previous dotted line now, in the y direction initial length or somewhere here. Now, that will get elongated; let us say this because the final dimension in the y direction corresponding this becomes the final dimension in the x direction which was initially somewhere here.

So, in the y direction how much is the change in that length scale? The change in the length scale is this to this, where actual change or the final change is only this much, but this much change takes place because of the forcing F y direction. Let us say sigma y is the stress that is acting in the y direction which is again F y divided by A y; A y being the area which is having normal to the y direction or we can write this on to be equal to F y by l x into 1 or F y by l x. So, let us say this particular thing is represented by b y; b y referring to the change in length because in the y direction because of the presence of this F y.

Similarly, this particular dimension is being referred as b x which refers to the change in length in the x direction corresponding this force F y. So, how much will be your b y? b y will be equal to very straightforward we know it can be written as sigma y upon E. Similarly, how much will be b x? We can make use of the idea of the Poisson’s ratio. So, it will be minus nu the represents ratio into sigma y by e. So, how much is the net change of dimension in the x direction then? There was initial and a x increase because of the presence of force in the x direction or opposes force F x plus there is a further b x change because of the F y. So, net change corresponding to E x our net change was sigma x upon E and corresponding to b x near change is nu into sigma y upon E that is 1 upon E into sigma x minus nu into sigma y.

So, this is the net change in the x direction or net change in length in the x direction. Similarly how much is the net change in length in the y direction? That can be a y plus net b y. So, your a y was minus nu into sigma x upon E that is not change because of the F x plus the change corresponding to F y or sigma y upon E that is this much into sigma y minus mu into sigma x. So, what is this then? This is the net change in length in the x direction or this one can then be related to the sigma x, the net strain in the x direction because of both the forces, similarly this is sigma y the net change in length or nets train I should say in the y direction because of both the forces.

So, this way we can write then sigma x is equal to 1 upon E or epsilon x I should write the strain in the x direction net strain sigma x minus nu into sigma y, similarly epsilon y is equal to 1 upon E into sigma y minus nu into sigma x. If our interest is to know only the strain then this is sufficient. However, if our interest is to know the stresses then these are two equations which can be solved to get the two unknown sigma x and sigma y and I am leaving the solution to you, but if you solve this you will get that sigma x and sigma y will be coming something like E into epsilon x plus nu into epsilon y by 1 minus nu square whereas, epsilon y will be coming as E into epsilon y plus nu into epsilon x by 1 minus nu square. These two are the expressions for the corresponding stresses.

And, we can easily extend this one to 3 dimensional analysis where we shall also have this dimension z, the third direction and by extending this if we want to write a generalized expression then your epsilon x will be equal to 1 upon E into sigma x minus nu into sigma y plus sigma z epsilon y will be equal to 1 by E into sigma y minus nu into sigma x plus sigma z. Similarly, the strain in the z direction, the new dimension, that is being added to sigma z minus nu into sigma x plus sigma y.

So, this way we can generalize the Hooke's relationship to get a more generalized relation between the stress and strain in all possible direction. One important factor to consider here is the value of the Poisson's ratio. Poisson's ratio of course, can have any value typically most of the materials having a Poisson's ratio in the range of 0.3 to 0.4, but what can be the ideal value of Poisson's ratio if there is no change in the volume of the material.

Let us say we are having a cylindrical element of a diameter D and length l. So, how much is the volume of this the volume of this material is pi by 4 D square l and because of some kind of stress suppose this l changes to l plus delta l and D changes to D plus delta D then the modified length is pi by 4 into D plus delta D whole square into l plus delta l. If we assume that there is no change in volume this is the V prime that is volume after modification, ok. Let us simplify this delta D and delta l both are being very small quantities we can neglect any product involving them.

So, it becomes D square plus 2D delta D; delta D square can be neglected into l plus delta l into pi by 4 into D square l plus 2D l delta D plus D square delta l the product of delta D delta l can be neglected. If there is no loss of material then V will be equal to V prime. So, if we equate these two, then we have pi by 4 D square l on one side is equal to pi by 4 into D square l plus 2D l delta D plus D square delta l. Let me erase this now because this part is done to make some more space for a writing.

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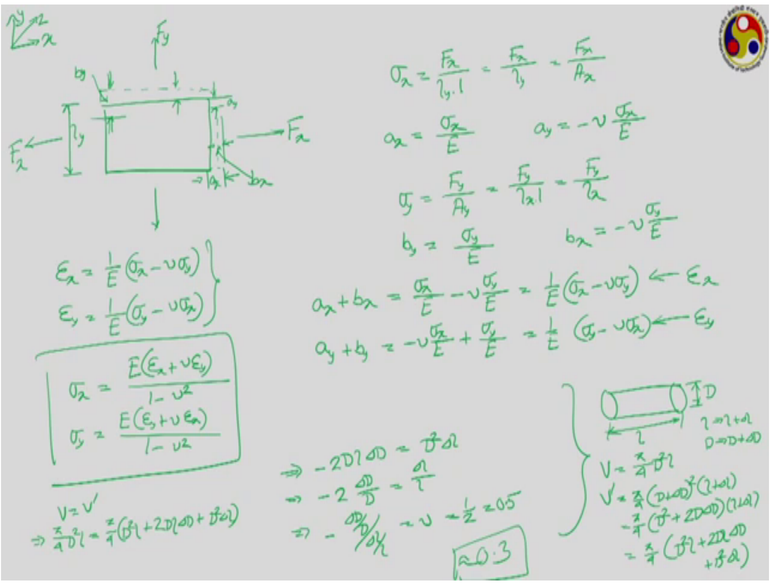

So, if we simplify this now then we have minus 2D l delta D is equal to D square delta l and if we write this way divide everything by D square l, then we have on this side and this side we have delta l upon l that is minus delta D upon D divided by delta l upon l which as per our definition is a Poisson's ratio is half or 0.5. This is the standard value of Poisson's ratio when there is no loss of volume this or volume remains conserved, but practically their volume may not remain conserved even within the elastic limit and Poisson's ratio typically takes a range value around 0.3 only which we shall be using in subsequent discussions.

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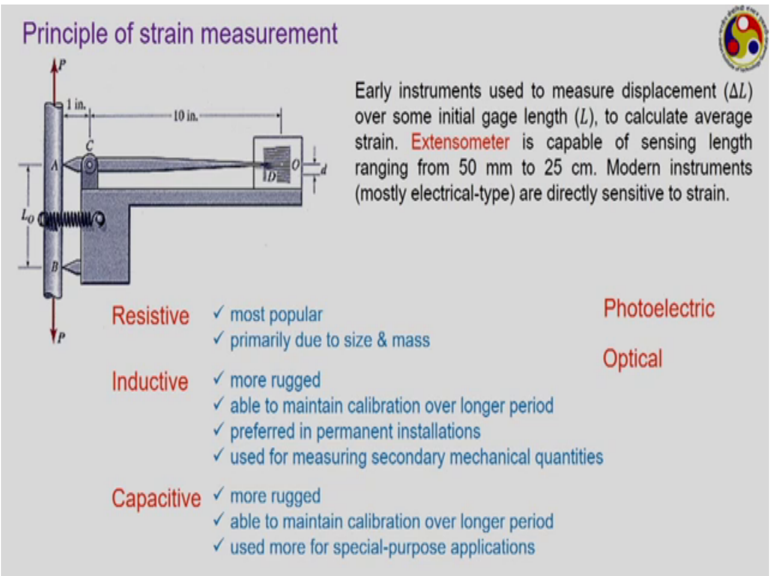

Now, the principle of strain measurement; the there are several ways we can measure strain. In early day instruments generally they used to measure the displacement delta l over a certain length and then the displacements were used to be divided by the actual length thereby getting the value of the strain. Extensometer is one of the earlier instruments where this displacement delta l used to be measured actually modified form of this one is still in use in certain applications.

But, the idea in all such cases is to measure the displacement, then divide that by the initial length to get the strain it was capable of measuring length changes over quite a large scale something like in the 50 mm to 25 centimeter like mentioned here. So, it is a quite decent range. However, the division was there it was not sensing the strain it was actually sensing the displacement. So, it actually falls in the domain of the displacement measurement like we have discussed in the previous week, but there are modern instruments they are actually directly sensitive to the strain. So, there is no need of just measuring the displacement and then divide that by al in length to get the strain rather we can directly get the value of the strain from the modern instruments.

And, this is how an extensometer used to be looking like, but there is no point discussing more about this. The modern instruments for strain measurement can primarily be classified into three categories resistive, inductive and capacitive all these are so called electromechanical transducers, that is the electrical quantity or I should say mechanical input the strain is being converted to some kind of electrical output. In resistive instruments the output change is caused by a change; in resistance of the corresponding element in inductive cases it is caused by a change in inductance of the corresponding element; whereas, in capacitive instruments it is definitely a change in capacitance.

Resistive instruments are the most common on the resistive strain gauges they are mostly popular because they are generally a value in very small size, so that they can directly we fixed on the specimen where we are looking for the measurement and their mass is also negligible therefore, it hard once we attach them to the actual specimen it hardly causes any change in the characteristic of the system itself. However, inductive and capacitive ones are generally more rugged and they are able to maintain their calibrated value over significantly long time period. So, there is no need to have repetitive calibration and also resistive ones generally once they are attached to a particular instrument we cannot detach it and use it anymore.

So, there something like a permanent fixation to the specimen that may not be always true for inductive and capacitive instruments means they can be used for different kinds of systems. Inductive ones are generally preferred in permanent installations and also they are very commonly used when our objective is not to measure the strain, but some other secondary parameter like pressure, temperature etcetera. We shall be discussing in corresponding module there are several strain gauge kind of instruments where actually measure the strain and then we calibrate that to something like pressure or temperature. In those situations the inductive instruments are preferred whereas, capacitive ones are more preferred in some special purpose applications.

Their applications are much more limited compared to the resistive and inductive one. We can also have photoelectric or optical kind of instruments where again the final output comes in the form of some electrical quantities, but they are again generally special purpose uses.

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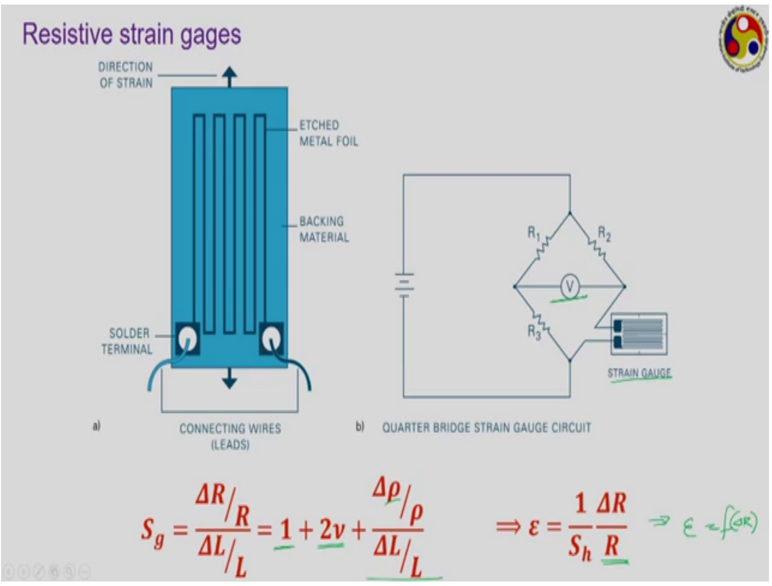

This is how a resistive strain gauge may look like. Some resistive element generally etched to a metal foil with a proper backing material and depend and they are resistive strain gauges are generally fixed with the specimen. So, once the specimen goes to some kind of deformation they are able to directly sense that and corresponding change in resistivity or resistance I should say is generally leads to a change in some kind of electrical quantity. The change in resistance can be sensed by a Wheatstone bridge kind of arrangement or maybe some other arrangement we shall be seeing one shortly.

So, if you are looking for a Wheatstone bridge kind of arrangement then this is the arrangement of where one of the resistance actually can be the gauge itself. Under normal situation this voltmeter senses zero voltage, the null situation. However, whenever there is a change in the resistance of the strain gage, there will be some voltage flowing through this and that voltage will be a direct measure of the change in resistance and which can directly be calibrated to the change in the strain or I should say the strain.

The mathematical part of this one has already done last week. We know that the gauge factor can be related to the gauge factor defined as the per unit change in resistance to the corresponding strain can be expressed by relation like this where the first part refers to the change in resistance corresponding to the change in length. This is the change in resistance the Poisson's ratio appears there. So, this is the change in resistance corresponding to the change in the lateral dimension and this is the change in resistance because of a change in resistivity; rho is the resistivity here.

So, we can rewrite this as a relation like this epsilon is a strain S h is the gauge factor which comes from the manufacturer commonly and R is also the initial resistance of the resistor which again is given by the manufacturer. Therefore, epsilon is a direct function of this delta R that is any change in delta R can always be directly related to the strain.

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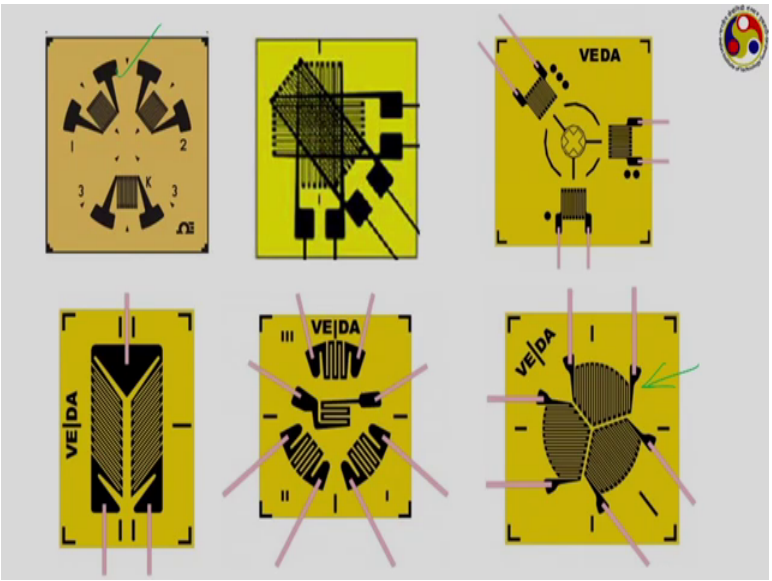

But, strain gauges can sense only the resistance or I should say the strain in one direction like here this particular strain gauge will always sense their strain in this particular direction tensile or compressive whatever, but direction remains unaltered. So, if we are looking for the measurement of multi-dimensional strain scenario or we are not sure about the direction from which the force is getting applied we should have to go for such kind of strain gauge rosettes, but different gauges are placed in different directions.

Like in this case, you can see three gauges placed over one eighty degree apart in this scenario whereas, we can have much more complicated structure something like this also, right. Each of the gauge will sense over a particular direction or over a range of directions and then they can be combined suitably to get the final value strain. In the next lecture we shall be discussing much more about the strain gauge rosettes and how to do the corresponding calculations.

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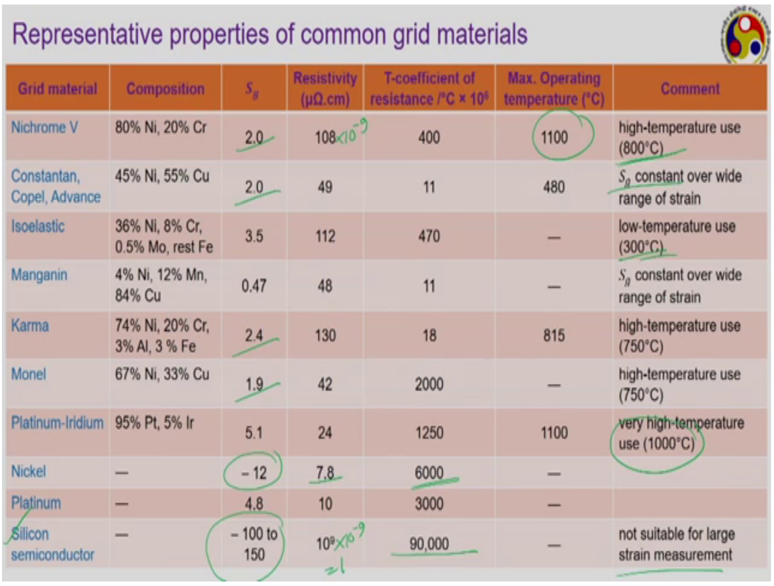

Representative properties of common grid materials

Grid material	Composition	S_g	Resistivity (μΩ.cm)	T-coefficient of resistance /°C × 10⁶	Max. Operating temperature (°C)	Comment
Nichrome V	80% Ni, 20% Cr	2.0	108	400	1100	high-temperature use (800°C)
Constantan, Copel, Advance	45% Ni, 55% Cu	2.0	49	11	480	S_g constant over wide range of strain
Isoelastic	36% Ni, 8% Cr, 0.5% Mo, rest Fe	3.5	112	470	—	low-temperature use (300°C)
Manganin	4% Ni, 12% Mn, 84% Cu	0.47	48	11	—	S_g constant over wide range of strain
Karma	74% Ni, 20% Cr, 3% Al, 3 % Fe	2.4	130	18	815	high-temperature use (750°C)
Monel	67% Ni, 33% Cu	1.9	42	2000	—	high-temperature use (750°C)
Platinum-Iridium	95% Pt, 5% Ir	5.1	24	1250	1100	very high-temperature use (1000°C)
Nickel	—	– 12	7.8	6000	—	
Platinum	—	4.8	10	3000	—	
Silicon semiconductor	—	– 100 to 150	10^9	90,000	—	not suitable for large strain measurement

These are some of the common grid materials that we use for making the resistivity resistance of the strain gauges. Their common values are given generally different kinds of alloys are preferred nichrome, constantan and these are very common alloys. You can see most of them having a gauge factor of 2 something or in and around 2. But, some of them are not. So, I am coming their operating temperature is something very important like that nichrome is able to operate to very high temperatures and that is why they are generally preferred for high temperature use when you are looking for a measurement something in the range of 700, 800 degree Celsius nichrome is the preferred as a material. Whereas material a isoelastic or etcetera preferred generally low temperature.

There are a few other high temperature material as well like letting a medium can go to very high temperature in the range of 1000 degree Celsius. The there are other materials like this constantan and strain here the value of S g, that is shown that is in only the nominal value it may change a bit depending upon the range of temperature. Why it can change it is temperature? The logic is very simple as that emperor changes resistance can also change and that can lead to a change in the corresponding gauge factor. The constantan is able to maintain a constant value of gauge factor over a sufficiently wide range of strain and wide range of temperature as well.

Now, look at nickel, what it is giving? Its gauge factor is negative. It is a very high value 12 and also it is negative; that means, the change in resistance is opposite to the direction compared to the others. It has also a very high temperature coefficient of resistance and therefore, it has quite encouraging prospect and even higher gauge factors actually extremely high gauge factors you can find for semiconductor strain gauges. They have also very high temperature coefficient and they are able to sense extremely small change in resistance as well.

Because the corresponding resistivity of the material is 10 to the power 9 micro ohm or I should say ohm centimeter whereas, for other common materials something if you see nickel has only 7.8 whereas, nichrome has 108 micro home that is it is 108. If we compare their magnitudes it is 108 into 10 to the power minus 9 ohm centimeter whereas, this is 10 to the power 9 into 10 to the minus 9 that is one ohm centimeter it is much much higher 10 to the power 7 times higher than nichrome.

So, it can give you extremely high resolution can sense very very small change in strain or very small amount of strain as well. However, they are not suitable for large strain measurement because a large strain would lead to a very large change in the corresponding resistance which is not preferred. So, semiconductor gauges are primarily reserved where we looking for very precision instrument or we are looking for measurement of extremely small values of strain.

However, the materials like nichrome or platinum iridium or constant ins are the ones that are very commonly used depending on the corresponding temperature ranges.

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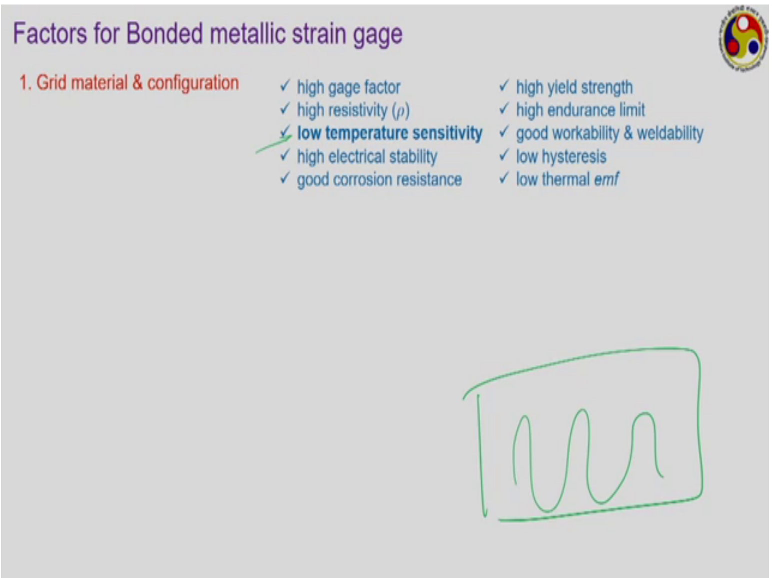

Now, you know that the strain gauges can be of several kinds, but the bonded metallic strain gauges are the most commonly used ones and so, we shall be discussing a bit more about them the performance of bonded metallic strain gauges can depend on five different factors.

The first factor is the grid material and configuration. Grid material there are these are the desirable properties of the grid material like we always want to high gauge factor we want the resistivity to be high. For most of the common materials the resistivity does not change that much with the strain. Thereby the gauge factor S g comes out to be 1 plus 2 nu and as ideally nu is 0.5. So, this comes 1 plus 2 into 0.5 giving you 2, that is the most common reason for having a gauge factor to be around to, but if the new changes plus if we are having the resistivity change with strain then it is value can be something else. But, that is something that happens to the semiconductor strain gauges in particular.

But, in general you want the resistivity to be high. We want high electrical stability and good mechanical properties, low hysteresis any kind of device you generally do not want hysteresis because hysteresis is induced some error, but something of particular interest is low temperature sensitivity. We know that the resistance of register changes with temperature and as if the register change your instance changes then of course, the calibration has to be done freshly and the output also will be erroneous if we do not do corresponding correction or compensation.

There are two ways the change in temperature can affect the performance. One you know that the strain gauge materials these are connected on some kind of backing. They are generally fixed by adhesive or some other kind of mechanism. Now, if the temperature increases then the bonding between the wires and the corresponding backing material that also keeps on changing that becomes weaker thereby producing an additional strain on the register element.

Now, your sensor generally is not able to distinguish this thermal strain with the impose strain and thereby putting an additional factor in the final output and the other way it can change is a change in the resistivity with the correspond with temperature. So, these are the two ways the temperature sensitivity can affect the output and that is why we always prefer the temperature sensitivity be low or you want the material grid material to retain its resistance over a wide range of temperature.

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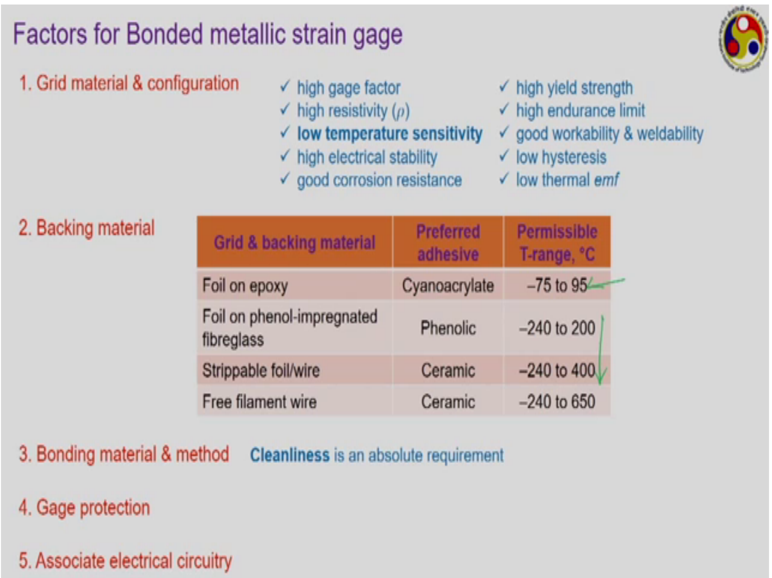

The second factor is the backing material the choice of backing material is important particularly from mechanical stability point of view. These are some of the common backing materials that we can use you can see their ranges. Epoxy is generally quite well preferred because this is the normal range for several industrial applications. However, if we are looking go for higher temperatures then we have to keep on going in this direction and ceramic bricks advanced adhesives will be more preferable.

Bonding materials and methods; now, bonding materials definitely matters means how you are connecting the grid material to the backing. There are several kinds of common bonding materials are available and that is why several kinds and generally all of them provides similar kind of performance. However, one parameter that is of utmost importance is the cleanliness of the measuring site. The measurement site has to be absolutely clean, perfectly free from any kind of dirt or oil or moisture even paint also that is we always want the first the specimen to be cleaned to the as much clean we can have, thereby removing any kind of dirt and moisture from them, removing any traces of paint from that surface and then fix the strain gauge on the bare surface with utmost cleanliness.

Just related to that is the gauge protection the gauge must also be protected from the surroundings. Surrounding can be coming from the dirt or dust particles flowing in; it can be related to the moisture, it can be related to some in particularly in when you are working a workshop you needs to be protected from some kind of mechanical. In fact, generally the gauges the manufacturers supply the gauges along with some kind of protective material in the form of some regimes or maybe works or some kind of epoxy kind of protective gears so that the gauge can be protected from the surrounding.

And, the factor number 5 a very very important one where we have to go for much more discussion is the associated electrical circuitry. Now, we know that the change in resistance has to be measured by some kind of electrical circuit may be a Wheatstone bridge kind of arrangement or some other kind of circuit, but actually the change in resistance is finally, going to cause a change in some kind of output voltage or output current like in a bridge circuit normally the output voltage is zero and whenever the change in there is a change in resistance in any of the arms of the bridge then there will be a change in the output voltage or some output voltage we are going to get.

Now, we have to change that output voltage using some kind of suitable voltage measuring instrument and the depending upon the nature of you knowing that you are using the effectiveness of the strain gauge that keeps on changing quite a lot. So, we need to discuss a bit more about possible kinds of electrical circuits.

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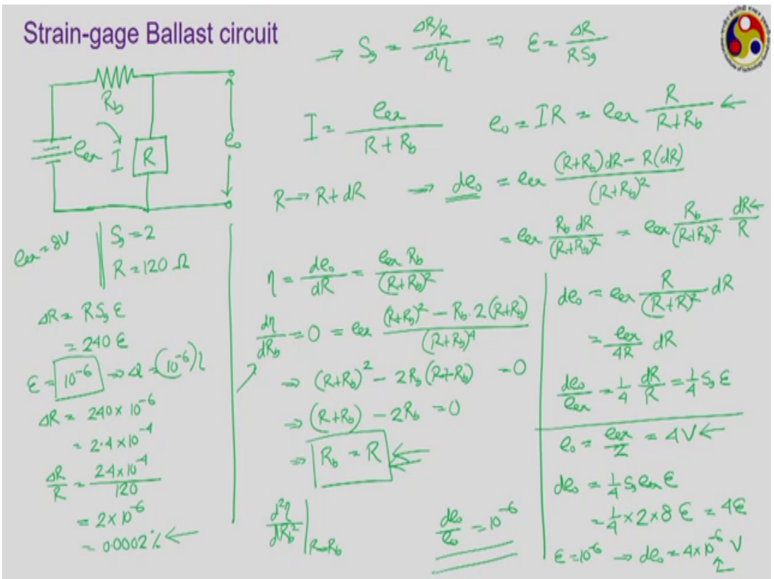

The first kind of electrical circuit that we use in connection with strain gauge is something known as a Ballast circuit a ballast circuit is somewhat similar to a potentiometer application very simple we have a voltage source something like this we have a resistance connected with this and this is your strain gauge. Let us say this voltage is e e x, R is the gauge resistance and R b is this ballast resistor and then across this you will be measuring this output voltage e naught.

Mind you here we are not at all talking about the loading effect which will be coming in by the when we mount some kind of voltage measuring instrument here we want to check the sensitivity of the instrument or sensitivity of this particular circuit. Before that let us write the general relations which you are going to use as per the definition of the gauge factor S g is equal to delta R upon R by delta l upon l the denominator being strain we have epsilon is equal to delta R by R into S g. Let us take some rough values let us say we are taking S g is equal to 2, which is the most common value of gauge factor and R equal to 120 ohm. This is also quite a common value of this gauge resistance in several industrial applications.

Now, look at this gauge circuit, ok. Before doing that let us see how much will be the value of delta R then in this scenario your delta R is equal to R S g epsilon that is if we use these two values. So, we have 240 into epsilon; now, suppose you are looking to measure is strain of 10 to the power minus 6, a strain value of 10 to the power minus 6 that is we are talking about delta l to be 10 to the power minus 6 times of the initial length. Then how much is the corresponding change in the register delta R that is 240 into 10 to the power minus 6 that is 2.4 into 10 to the power minus 4 if we compare with the original value of the resistance that is delta R upon R is 2.4 into 10 to the power minus 4 divided by 120.

So, how much is it? It is so we can write this on to be something like 2 into 10 to the power minus 6 or if we convert this to percentage 0.0002 percent that is you are talking about an extremely small change in the value of this delta R when you are looking for measurement of strain like this. So, your circuit should be capable of sensing such small amount of strain or small amount of change in resistance I should say. So, let us check the performance of the ballast circuit in this point of view.

So, in this ballast circuit say I is the current that is flowing through this. So, how much will be the current I will be equal to the corresponding voltage divided by the net circuit resistance upon R b. So, your e naught will be equal to I into R that is ee x into R upon R plus R b. So, this is the initial situation. Now, the because of some action of strain this R changes to say R plus dR. So, how much it will be? If we differentiate this particular equation on both side then we have de naught is equal to e x is and R b both are constant because they are unaffected by the strain only thing that is changing in this R by this amount dR.

So, it will be equal to e e x into we can perform the differentiation just a normal way the denominator into differentiation of the numerator minus numerator into the differentiator form the denominator R b being constant here R b whole square in the numerator R dR cancels out. So, we have e e x into R b dR by R plus R b whole square or we can also write this one as e e x into R b by R plus R b whole square into dR upon R.

So, this is the change in output e naught initially having some value this is the corresponding input dR visit and your ballast circuit we have to check the sensitivity of this one. Now, how can we define sensitivity? Do you remember what is the definition of sensitivity? Sensitivity is the change in output corresponding to per unit change in input. What is your output here? e naught is the output. So, this is the change in output corresponding the change in input dR. If we take the help from the previous expression this one can be written as e e x R b by R plus R b whole square.

Now, let us try to check what is the ideal value of this R be the ballast resistance so that we can get the maximum sensitivity of this instrument how can we check that that is possible if we differentiate this with respect to R b and set to 0, I repeat we are trying to identify the value of R b or the relation of this R b with the initial gauge resistance R to get the maximum sensitivity of this instrument. So, we are differentiating the sensitive with respect to R b and setting that to 0, so, what we are going to get? You are going to get e e x was there. So, we have R plus R b whole square dR b oh sorry we are just differentiating with respect to R b. So, minus R b into R b into 2 into R plus R b divided by R plus R be whole to the power 4.

So, if we simplify this then we have R plus R b whole square minus 2 R b into R plus R b is equal to 0 or R plus R b minus 2 R be equal to 0. So, if we simplify this what we have R b equal to R that is when R b and R that is when the value of this ballast resistor is equal to the initial value of the gauge resistance then the system will be having the maximum sensitivity. We can it can be proved like if we get the second derivative and put R equal to R b you can check what values you are getting that can prove that this particular value corresponds to a maximum in the sensitivity. And, this I am leaving to you can check the second derivative and put R equal to R b there to confirm whether it is a maximum or minimum this should give you a maxima in the sensitivity.

Now, with this we are going back to our previous discussion. So, if we are having R equal to R b then your de naught is equal to e e x into R b becomes equal to R by R plus R whole square into dR that is e e x by 4R dR or something just like or if we want to write in a different way it can be a de e naught by e e x is equal to 1 by 4 dR by r. So, this is the output of the instrument. So, corresponding any change in resistance dR correspondingly you will be getting de naught. If we want to compare this with the initial input that we have set in that is this set of input we are talking about. We know the gauge factor we have said to be 2, R equal to 120 m and now know you know that ballast resistance also should be equal to 120 m 120 ohm to get the maximum output.

Then how much will be your initial e naught? Your initial e naught should be equal to this much that is e e x by 2. Let us fix up a value of say e e x is equal to 8 volt which is a common battery voltage. So, this comes to be equal to 4 volt. Now, how much is then de naught? Let us relate this dR to corresponding gauge factor that is using this particular relation or the corresponding strain. So, it is 1 by 4 into dR by R will be equal to S g into epsilon. So, corresponding d e naught will be equal to 1 4 into S g into e e x into epsilon. So, 1 by 4 into S g is 2 into 8 epsilon that is finally, 4 epsilon.

So, when your epsilon is equal to 10 to the power minus 6 your corresponding change in voltage we are talking about is just 4 into 10 to the power minus 6 volt that is an extremely small change in voltage that you are talking about. Look at the initial voltage was 4 volt and now we are talking about a change 4 into 10 to the power minus 6 volt. So, de naught by e naught is equal just 10 to the power minus 6 which is an extremely small resolution that we are looking for from our voltage measuring instrument.

So, despite putting the optimal condition like this the ballast resistor circuit requires very high voltage measurement resolution to get a proper observation that is the biggest problem with the ballast resistor circuit while it is a very simple arrangement quite similar to the potentiometer. But, using this we generally demand a very high voltage requirement and therefore, is not always the preferred option in industrial application. However, if a we are looking for some kind of dynamic arrangement this can be preferred because there we are not at all looking for the absolute change in strain, but we are trying to get a profile of the change in strain and sometimes this one can give a good readings, but that is beyond the scope of present discussion.

So, the second kind of circuit that we can have is the strain gauge bridge circuit what we have a bridge circuit a standard Wheatstone which kind of arrangement? I am sure all of you have some idea about the Wheatstone bridge.

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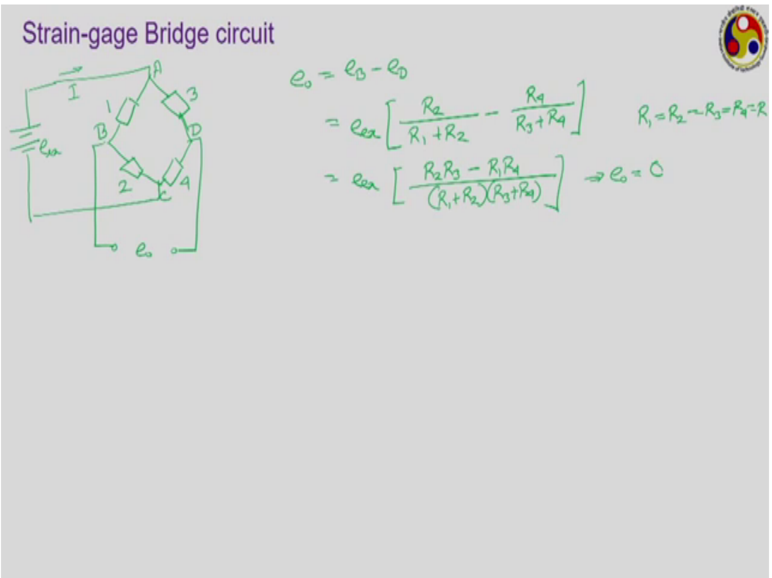

But, still I am just on drawing a simple Wheatstone bridge to refresh their knowledge. Let us say this is 1, this is 3, 2 and 4. This two ends will be connected to some kind of voltage source let us say e e x. Let us give some name say this is a this particular point is B, this particular point is C and this particular point is D. So, e naught is the output voltage. So, in a normal situation or if we drop write the equation then e naught should be equal to e B minus e D.

So, we can make use of the corresponding flow of current like if I is the current that is flowing through this circuit then I will get divided into two parts; one flowing through one and another length one referring to 3 and that way we can form the circuit, but we can always get their corresponding relation. So, that e B minus e D can be represented as something like this. e B should be equal to R 2 divided by R 1 plus R 2 that is the voltage deviation voltage dividing relation. Similarly, e D should be equal to R 4 divided by R 3 plus R 4 or if we simplify this one then what we are going to get? R 1 plus R 2 R 3 plus R 4; so, if we simplify this then we have R 2 R 3 minus R 1 R 4.

And, if all these resistances are of equal value then definitely we are going to if all these R's are of equal value that is R 1, R 2, R 3, R 4 all are equal to say some R, then what we are going to get from here then you will get e 0 equal to 0 under normal situation. Now, if one of these resistances are connected to a strain gauge or rather instead of connecting one of the resistances what we do we connect two of them to two different strain gauges, but those two strain gauges are used in different ways I am erasing this so that I can draw the situation.

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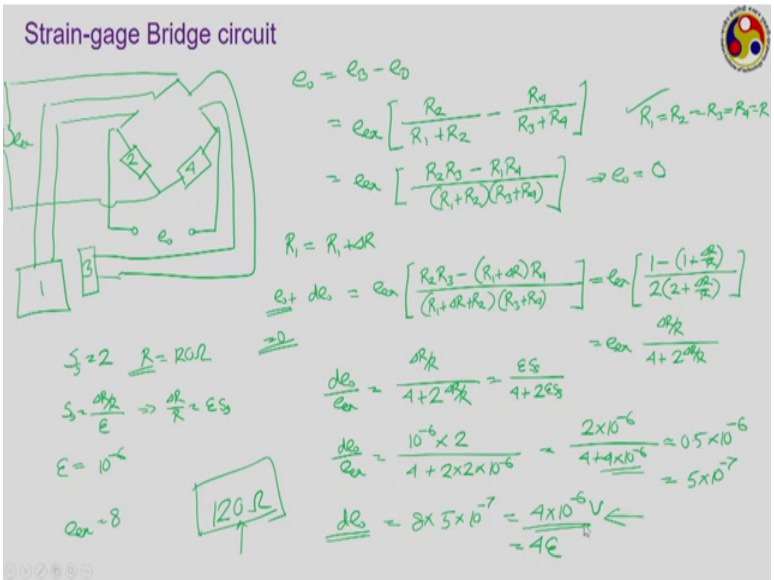

So, here situation is this 1 and 3 are cave vacant, 2 and 4 will be the normal bridge resistances. So, this is your 2, this is your 4, the normal bridge resistances they will be connected to the output voltage measuring instrument which is giving e naught. And, you also have the supply voltage at these two points which I am finding some difficulty to draw this say, ok. So, over here you have this e e x.

Now, what about 1? The 1 is connected to the strain gauge using which you are doing some using doing the measurement. This is your strain gauge or mounted on the specimen where you are getting a measurement and now what about your 3, this particular one? This one will also be connected to another strain gauge which will be not mounted on the stranger this is more free, but that is kept that is identical to the one means they are in the same strain gauge and they located very close to each other as well only difference is that one is mounted on the instrument and this 3 is not at all mounted on the instrument rather 3 is kept free that is it is not at all not sensing any kind of strain. When one is undergoing some kind of strain, this 3 is not sensing any kind of strain.

So, we have to now go for an analysis of this particular circuit. Let us just change our terminology a bit there is no need say 1 and 3. So, 2 and 4 are the normal bridge resistances; 3 is a strain gauge which is not at all sensing any kind of strain, but one is the actual strain gauge which is sensing the strain. Then 2, 3 and 4 their resistances will remain unaltered, but R 1 will change by some delta R amount. So, corresponding change in voltage say e naught plus de naught will be equal to this R 2 R 3 main unaltered R 1 will become R 1 plus delta R into R 4 divided by R 1 plus delta R plus R 2 into R 3 plus R 4.

And, if we put all the R 1, R 2, R 3, R 4 to be equal to R like this relation that is the initial values all are equal then what we are going to get, ok. We should simplify it a bit before going for that particular step is it at all required no maybe we can divide the entire the both the numerator and denominator by R square then what we are having we are having say 1 minus we are dividing by R square. So, we are having 1 plus delta R by R similarly here we have R 3 plus R 4 giving us giving us a value of 2 and another goes here. So, you will be having 2 plus delta R by R e e x. So, you have a delta R upon R divided by 4 plus 2 into delta R by R.

Or you have to remember this initially this one was equal to 0. So, from there we get de naught is or I should write de naught by e x should be equal to delta R by R by 4 plus 2 into delta R by R. So, if we use the previous scenario that is use the concept of the gauge factor we take the same values S g equal to 2, R equal to 120 ohm and as for our definition is S g is equal to delta R by R by epsilon giving us delta R by R is equal to epsilon into S g. So, if we put it back here we have epsilon S g here divided by 4 plus 2 into epsilon S g here.

Now, if you are looking for an extremely small resistance measurement like epsilon is epsilon equal to 10 to the power minus 6 like in the previous scenario, then what we are going to get? In this case this will be equal to 10 to the power minus 6 into 2 divided by 4 plus 2 into 2 into 10 to the power minus 6, that is 2 into 10 to the power minus 6 divided by 4 plus 4 into 10 to the power minus 6. So, which can approximately 0.5 into 10 to the power minus 6 because this particular part can be cancelled compared to 4 or we can write this to be as 5 into 10 to the power minus 7.

So, again looking for an extremely small sensitivity, but the advantage here is that here the value of R is not coming into picture that is here we are not at all looking for the we are not at all looking to measure this one with respect to the initial value of e naught; initial value of e naught was 0. So, whenever there is a small change in R, let us if we take say the same value e x equal to 8 then your de naught will be equal to 8 into 5 into 10 to the power minus 7, that is what we have is 4 into 10 to the power minus 6 volt that is 4 into epsilon whatever I epsilon. So, the voltage that we are getting at is same the change in output voltage is the same what we got in case of ballast resistor.

But, the advantage here is there the ballast resistor value we are trying to compare in terms of a potentiometer where initially already 120 ohm almost there and with respect to 120 ohm we are trying to get a voltage value of this. But, here we are not at all comparing this particular value with respect to this one rather we are only sensing this amount of voltage. Sorry, I am making mistake it is not 120 ohm. 120 ohm was the initial value of I forgot the initial value.

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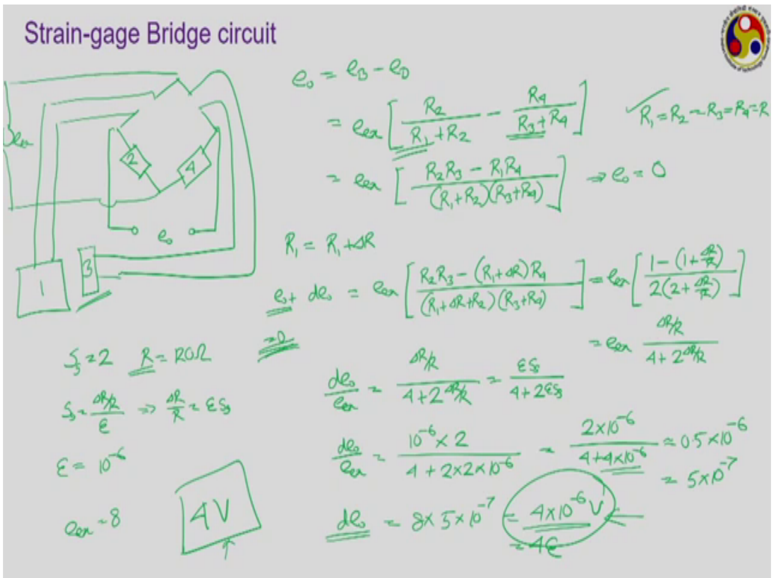

I think the initial value we calculated as a something like 4 volt. So, with respect to 4 volt output voltage initially we already have a 4 volt value at the output. Assuming a voltage measuring instrument which is having infinite resistance you are already measuring a voltage of 4 volt and in that 4 volt now you are trying to measure a change of 4 into 10 to the power minus 6 volt which is extremely difficult.

However, here initial value was 0, and there you are trying to measure a change like this and that is possible to measure that definitely can be measured therefore, very small even very small change in resistance can be sensed by a bit circuit like this and once you are able to sense a small change in resistance, then you can definitely change that small amount of strain as well which is of the order of 10 to the power minus 6 in this particular scenario.

Now, the question is why we are putting this second one as well? You are having one resistance number 1 in the in the form of the strain gauge that is fine. But, where you are having this 3 as well; because, this 3 will be acting as some kind of temperature compensator. Like here whatever we are doing that is corresponding to the reference temperature at the temperature at which this gauge factor and others were calculated. But, if there is a change in the surrounding condition then the value of register R we will change in one similarly the resistance in 3 will also change because these two are identical and so, whenever their means their values we will respond or their resistance will respond identically to the change in temperature there.

So, it is very important to keep this 3 to a location very close to 1, so that it is able to x b able to sense the same environmental condition. And, now whenever there is a change in the resistance 1 because of the change in temperature, 3 will also suffer an equal change and that will get reflected equally in this particular relation, like this. Thereby without altering your final output voltage and giving your final output as a temperature independent away. There are several others we can define the bit circuit also like the one that we have shown earlier that is known as a quarter bridge arrangement, where you have the strain gauge mounted only on one of the bridge.

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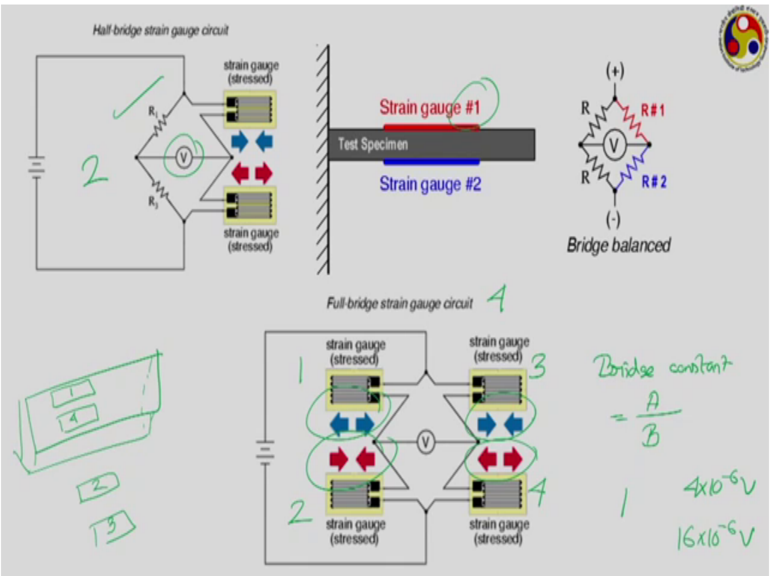

But, we can also have a half-bridge arrangement where on the specimen on two sides of the specimen we are putting two strain gauges. Now, if this specimen is suffering some kind of bending kind of arrangement let us save this specimen bends like this, then what will happen? Strain gauge number 1 will sense a tensile load, where the number 2 will sense a compressive load and that will get reflected on this gate circuit properly and that will give that will double your output voltage actually. This output voltage we will get doubled. This is called the half-bridge tension in this circuit.

If we want further sensitivity we can even use a full bridge where all four are mounted on the on in the form of strain gauges where both these two are like here all the gauges are stressed, but they are stressed and in different way. You can see this one and this one are sensing the compressive strains whereas, this one and this one are sensing the tensile stress means it is something like that you have this specimen this is a 3-dimensional trying the way I am trying to draw it.

Now, two of the gauges are loaded here. Let us say let us give some name to this a 1, 2, 3 and 4. So, your 1 and 4 are mounted on the same side and on the opposite side you have 2 and 4 mounted on the opposite faces so, sorry not 2 and 4, 2 and 3. So, 1 and 3 not rather 1 and 4 will says the same kind of loading whereas, 2 and 3 we will sense the same sense the opposite kind of stress and now, when we connect them in a circuit shown here then each of them we will modify the voltage value in the same direction.

Thereby, quadrupling or I should say making the output voltage four times then what we should have got with a quarter be judgment. We call this particular term a bridge constant. Bridge constant is defined as A upon B, where this A is the actual output that we are getting and B is the output that we should have got if we are using just a single strain gauge in a quarter which kind bridge kind of arrangement. So, when you are using a quarter which kind of arrangement the bridge constant is 1.

However, in this kind of situation your bridge constant is 2 because your output will be twice than what you have got with the quarter bridge and what about in this scenario with full bridge? Here the output will be 4 or I should say the bridge constant will be 4. So, like in case of quarter bridge if your output like in the previous case we have got 4 into 10 to the power minus 6 volt in if you go for a full bridge arrangement it will be 4 times of this that is 16 into 10 to the power minus 6 volt, thereby increasing the sensitivity of the output circuit.

So, these two are the two most important kind of electrical circuitry that are used in conjunction with resistive strain gauges the ballast resistor and this bridge circuit. We have a third one which is called the simple constant current strain gauge circuits, but today I do not have time. So, I am finishing the discussion here itself for the day the simple constant strain gates circuit we shall be discussing in the in the next discussion.

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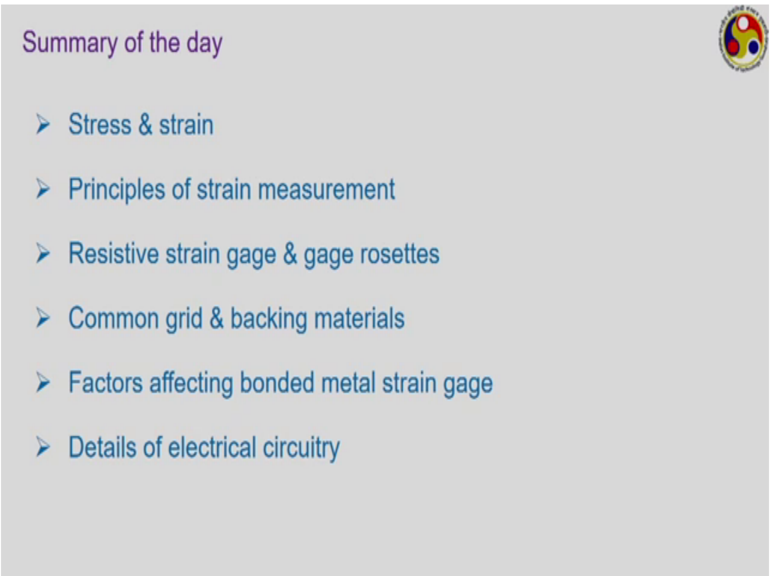

So, the summary of the day we have discussed about the stress-strain relationship, starting from there I discussed over the principle strain measurement and then we focused only on the resistive strain gauges then we discussed about different kind of eleven materials, the grid and backing materials; we discussed different factors which can affect the performance of bond and metal strain gauge. And, then we have discussed the details of two electrical circuitry, you are supposed to discuss three, but I think we should stop here because it is already late for the day.

So, in the next lecture I shall first be discussing about the third kind of electrical circuit that we can use in resistive strain gauges; then we shall be discussing about the temperature compensations the way temperature compensation effect can be done. We have already seen that effect here particularly in conjunction with the bridge circuits, but we shall be discussing a bit more on that and then, we shall be discussing about the strain gauge rosettes. There are several kinds of rosettes that we can have we shall be discussing about a few sample rosettes and finally, we shall be trying to see if you other ways of stress and strain measurement in mechanical systems.

So, that is it for the day. I am signing off for the day. Thanks for your attention and hopefully we shall be back very soon for the next lecture.

Thank you.