Theory of Rectangular Plates-Part 1
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Lecture - 08
Navier Solution + Levy Solution

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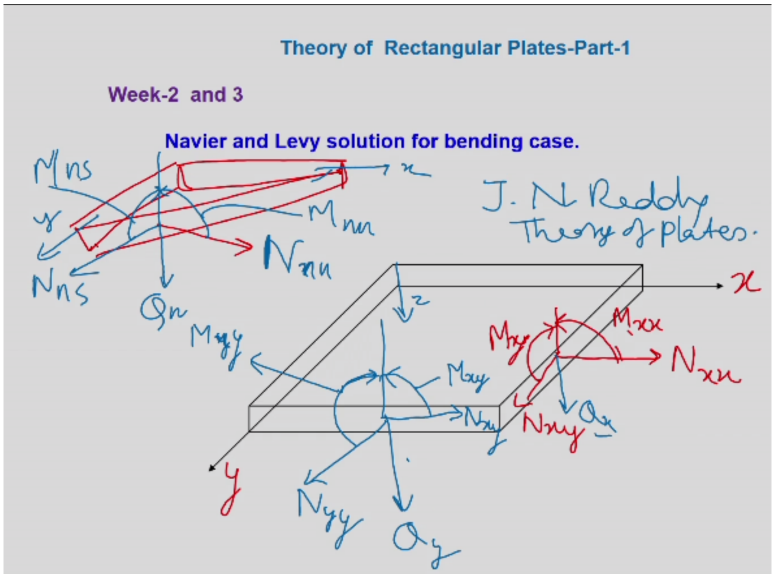

Welcome to our week 2 lectures, possibly this lecture will cover some part of the week 2 or some part of the week 3 lectures. So in this lecture, I will cover Navier and Levy solutions for under bending case. So before moving to that, I have forgot to explain that physical representation of in-plane stress resultants and movements, and how these are applied over the x axis. So basically if you say that, this is my x-axis, y-axis and downward is taken as z-axis.

This thing is given in Professor J. N. Reddy book, Theory of Plates. So I have taken that idea. You can see that when this x-axis is this phase, which is positive phase along this direction is x-positive, along this direction is y-positive, downward x, we can say that along the x and x-axis will be there, along y positive direction and y, y and along normal Qx will act. Now the movements, you see that the direction of movements Nx, x.

Try to bend the plate in this direction and then try to bend this plate along y-directions. So the sign conversion is given here, or you can also do in a vectorial form, because you know that, movement is basically force into perpendicular distance or distances along this direction is positive, k and this is you will say that I, so k cross i give you j. So from that also 1 can find it out. Similarly, this phase, this is a y-positive phase, so Ny, y, Nx, y, shear force and the Qi will act.

Similarly, Mx, y, My, y will act. Now in the boundary conditions, we have converted into a normal and shear resultant. So there I will say, let us see we cut a plate in between this phase, so we are interested to find out the stress resultant for this. So the normal direction to this phase will be Nm, then Nn, s and Qn, and corresponding Mn, m and Mn, s. We have already defined these variables. This is the representation on the mid plane or a reference plane, the stress resultant.

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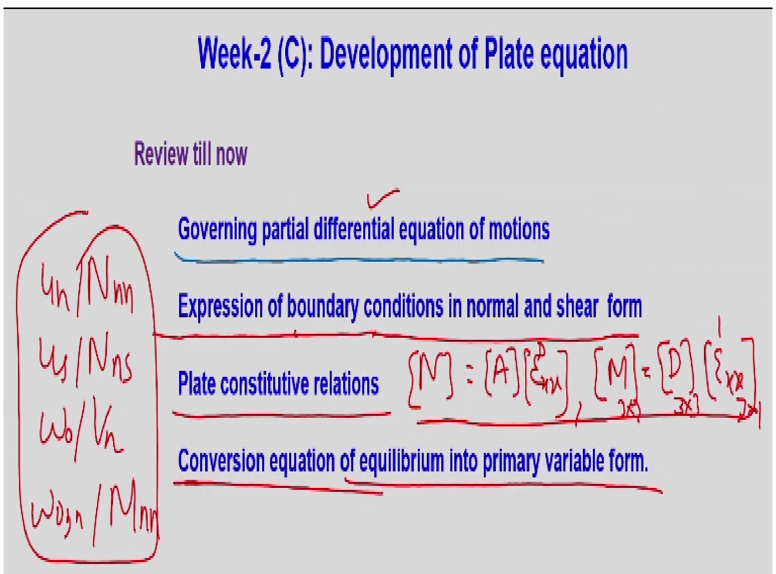

So we have covered, till now developed a governing partial differential equation and I expressed the boundary conditions in normal and shear force, if you remember that un/Nm, us/Nns, w/V and w not/Vn, we will say that form Mn, n. Then I have gone for plate constitutive relations. There we have represent and it will say in column vector A is a stiffness matrix and some we will say that epsilon x, x not.

Similarly, if you say that M is the column vector which contains Mx, x, My, y and Mx, y that can be represented as matrix B, 3 by 3 matrix, and epsilon x, x of 1. There are some thermal loading required, but for mechanical case, you can see where this is 3 x 3, 3 x 1, 3 x 3. Then I told you that whatever the partial differential equations, they are in the form of stress resultants, or the movement resultants like, Nx, x or Nx, x.

But our primary variable is u0, v0, w0 because we started with that assumption that we know, that u0, v0 if we can solve that u0, v0, then we can find out the stresses, strains everything. So we have to convert those resultant form of equations into a primary variable form. So now these are 3 equations of motion.

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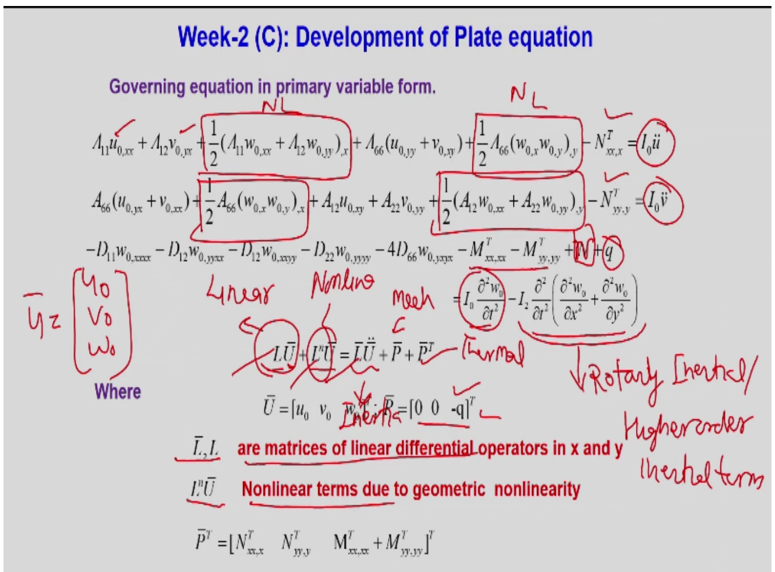

In the last lecture, I have derived and put it here. So these are the 3 equation of motion in the primary variable, u0, v0, so on. So this term is basically the non-linear terms contribution to that. Similarly, here, and here we have put like slightly italic of terms. Again, I would like to see that this term is known as rotary inertia, or higher order inertia terms. We can solve this equation, in this as it is form.

In this course, I will explain first the basic step, that how to solve A, governing partial differential equation, for the linear case. Then later on if you consider this terms, it makes further complications and then one can solve this type of problems. Now can we arrange this sort of equations in terms of some matrices, so that programming can be easy, reproduction can be easy, that one can remember the things very nicely, because remembering of this equation having all the terms is difficult.

If you go for a programming, then you have to write each and every terms. So, yes we can write in the terms of a matrices, let us say LU bar, Ln. Basically this contains, I would say that linear contribution. This contains non-linear contribution of the terms. This contains inertia. This contains mechanical loading, and this contains thermal loading. So we can write all these 3 equations in terms of a matrices LU bar, LnU bar and P bar and P top.

Where you can say that U bar is nothing but a column that contains u0, v0, and w0, where P bar is nothing but there is no mechanical load, the thermal so only the third term will contribute -Qt. Similarly, Pt will contain Mx, x, y, t, My, y, t and this movement resultant, thermal. Now what are L, Ln and L bar, so Ln, L bar are the matrix of linear differential operator in Lx, y and Ln bar is the non-linear terms.

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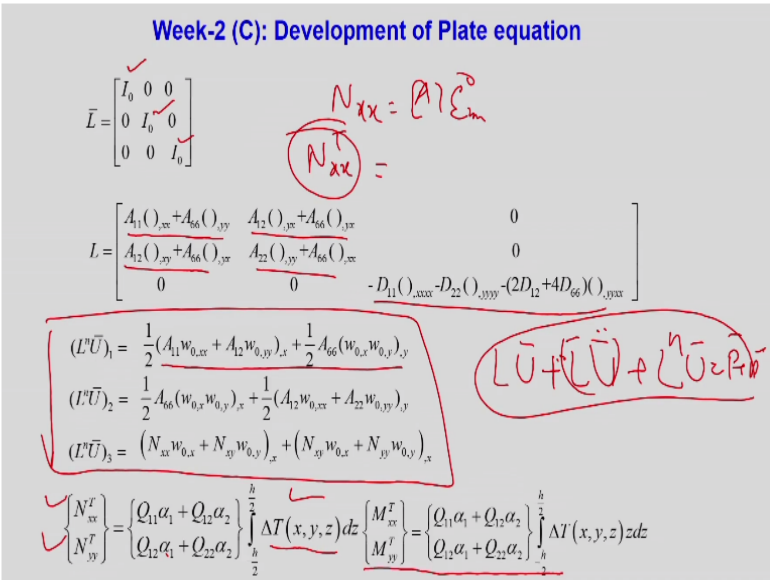

So let us say, so I will go back, slightly, so L bar is basically inertial term, how many terms? 1 this, second and third + this rotary inertia. So the subsequent solution purpose, I will not consider non-liner terms as well as rotary terms. So I will make up, L bar will contain I0, I0 and I0, then L the contribution due to the u0, the contribution due to the v0, again in the second equation and in the third equation contribution due to the w0.

So we have taken only the linear contribution of the terms, then the non-linear contribution in the first equation will be this, similarly the second equation and third equation can be written like this. So now you see that we have represented those sort of governing equations into a very beautiful form that, L of U bar+L bar of U bar double dot+Ln0 U bar = sometimes you put at -, then P bar+Pt like that and we know what are L, L bar, and Ln bar.

Again that we have discussed that what is Nx, x, like A1,1 of this some epsilon x, x, 0. Similarly, Nx, x of T, movement resultant due to the thermal load that can be represented that Q1,1 of alpha1, Q1,2 of alpha2, Q1,2 of alpha1 and alpha2 and variation of temperature. Similarly, the movement resultant due to the thermal load.

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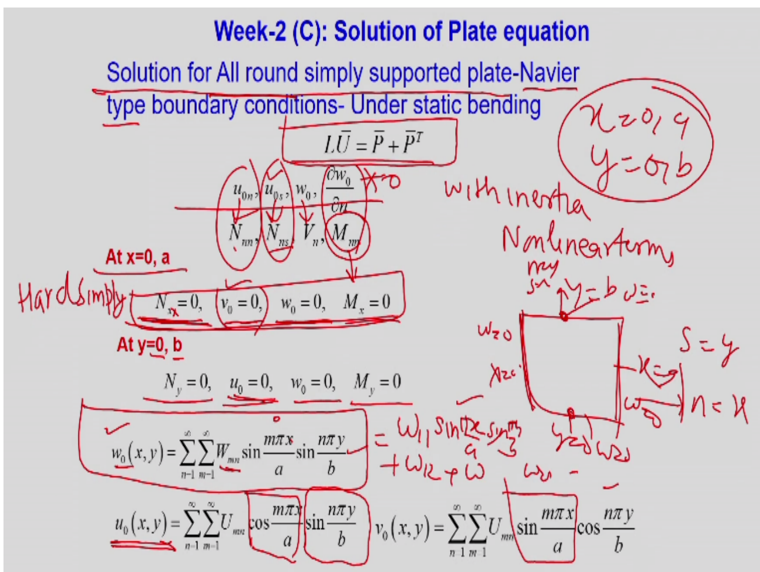

Now we are going to present a solution of all round simply supported plate. When a plate is subjected to a simply supported in all the directions, then we call it is a Navier type boundary condition. Wherever you come up with the Navier type solution, or a Navier type boundary condition it means a plate, or a body, whether it is a plate or a beam or a shell, solution becomes Navier type solution, means if we talk about a shell, it means all axes should be simply supported.

If we talk about a beam, in the case of beam also in Navier solution means, whatever axes, or which you can prescribe the boundary condition, that 2 axis, x, 0 and x, a will be simply supported. For the case of plate, it will be x0 to a and y will be 0 to b. These are simply supported, first condition. Since I have already told you I will solve for first linear case, then 1 can go for a non-linear case also.

So the first equation becomes like this without inertia and non-linear terms. So since we are talking about a bending, we assume that this is not a function of time. Now these are the variables, either you have to choose this or this. When we talk about a single supported boundary condition, what are those variables, which has to be 0 or specified at the axis. So we talk about x=0 and a, if you say that x is 0 and x is a at this edge.

So first of x, Nx, x normal movement, either u has to be 0 or Nm has to be 0. So for this case n is x and s is y, so accordingly we will say that either you specify u or you specify Nm. So we will say that in-plane stress resultant is going to be 0. We are allowing to displace in that direction, then either v or Ns, if you have Nx/0 or u0x, so along shear direction is y, so v0 will be 0. We are talking about a simply supported, cannot go beyond, in the downward direction so w0 is 0.

So corresponding when we talk about a simply supported, it means slope can be anything. Deflection has to be 0, but slope can be anything, so it cannot be 0. So it is counter Mn, n will be 0. So this type of support conditions we can say that hard simply supported. Even in choosing the boundary condition v=0 is very, very important to discuss here. If you do not chose that, then we cannot solve the sort of this equations analytically, then we have to go for some numerical or approximate techniques.

So you have to choose for Navier case, we are talking about analytical solution hard simply supported Nx, v0, w0 and Mx. Similarly, along the y direction, when you talk about y=0 and y=b, what are those variables has to specify, for this case M is y, S is x, now you see Ny needs to specify, u0 need to specify, w0 and My. So we have 4 variables at x=0, 4 variables at x=a, similarly 4 variables at y=0 and 4 variables at y=b.

Now that the very first approximation, that how do you assume a solution. When we talk about Navier solution, generally for analytical case we assume a solution in a Fourier series, double

Fourier series. So you see that w is 0 here, w is 0 here, w is 0 here, w is 0 here. So we have to identify a function in which x=0, w is 0, x=a, w should be 0, y=0, and y=b. So in double Fourier series, if we can express sine m Pi x/a, sin n Pi y/b, Wmn is a constant, if you can express w0 is like this, then it satisfies the boundary conditions exactly.

When you put x=0, sine term is going to be 0. When you are going to put x=a, so basically sine m Pi, this is even number, so that again it will be going to be 0. Similarly, when you put y=0 and y=b, so this if you assume w like this sine along x direction, sine function along y direction, then it will satisfy the boundary condition exactly. Similarly, how do you assume, u0 and v0. First you see along y axis u0 is 0, when y is 0 ynb, but along x axis there is no information.

It can have any value, so that is why along y direction u is expressed as a sine function and along x direction it is assumed as a cos function. Similarly, v, along x axis it is 0, so along x axis it is expressed in terms of a sine function, and another direction you can assume as a cos function. So these are the series solutions where n goes from 1 to infinity, m goes from 1 to infinity, if somebody is interested to explicitly write down what does it mean.

It means you put n=1, m=1 and again you put sin Pi x/a sin Pi y/b, then you put m=1, 1 to and accordingly you change it, then 21, then 23 and so on. So basically we have to evaluate these constants w11, w12, w13, these are unknowns, if you know these constants then our solution is known. So we have assumed this thing.

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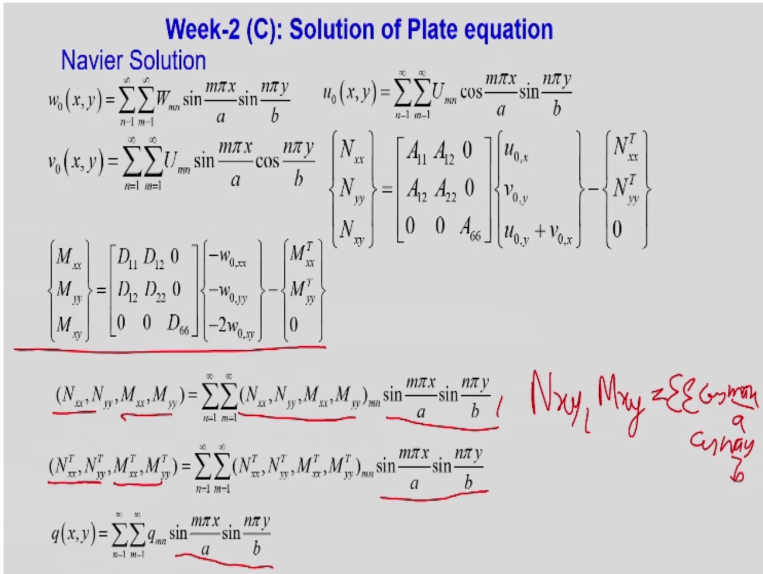

I think now it is clear that how to assume a Fourier series, the 8 satisfy the at least I like to say that essential boundary condition, u0, v0, and w0, wherever it is required they are satisfying exactly. Now what about the stress resultant, how do you find out that I should express Mxx as sin-sin or a cos-cos series or a cos-sin series, that you could find out by using plate constitutive relations. If you have u, x so cosine becomes sine, so it becomes sin-sin, I will write ss.

Then v, y, then again ss, this term is 0 so mx need to be expressed as a sin-sin series. Similarly, if you go through, Nyy sin-sin series, but if you want to known Mxy, so u, y cos-cos, cc, then v, x cos-cos. So Nxy you have to represent as a cosine series, similarly thermal resultant terms, if all has to be sin-sin, then only we can take some common out terms. So this has to be a double Fourier series, otherwise we cannot get the solution.

It will be very difficult to take out the common. Then the movement terms, so I have written, Nxx, Nyy, Mxx, Myy is a double sine series. Similarly, the thermal resultant of normal plane and movement can be represented as sin-sin, and the loading q, mechanical loading, if you all things in sin-sin, the mechanical loading is also has to be expressed in the double sin series. One I have forgot that Nxy and Mxy will be represented as a cosine series.

Now what is the next step. Now you substitute this variation into that equation, what is that equation, LU bar = P bar+P bar t, whatever you want to say. So if you substitute this thing here, let us say, u, xx. so u, xx means derivative of u along x direction, of first derivative will give you -sine, second derivative will give you cosine, so basically if you see that, just I am going to explain u0=umn cosine of let us say m bar x, and sin of n bar y.

So first derivative u by x will be -m bar umn sin m bar x sin m bar y, then the second derivative again -m bar square, um cosine of m bar x and sine of m bar y, like this you can evaluate yy derivate, v0 derivative along y and x, and derivative along yy. So this matrix can be written like this co-efficient m bar, n bar, where m bar is nothing but m Pi/a, where n bar is nothing n Pi/y, just for a simplicity, so that every time we do not to say m Pi/a, n Pi/b. We can say that m bar square, n bar square, mnn.

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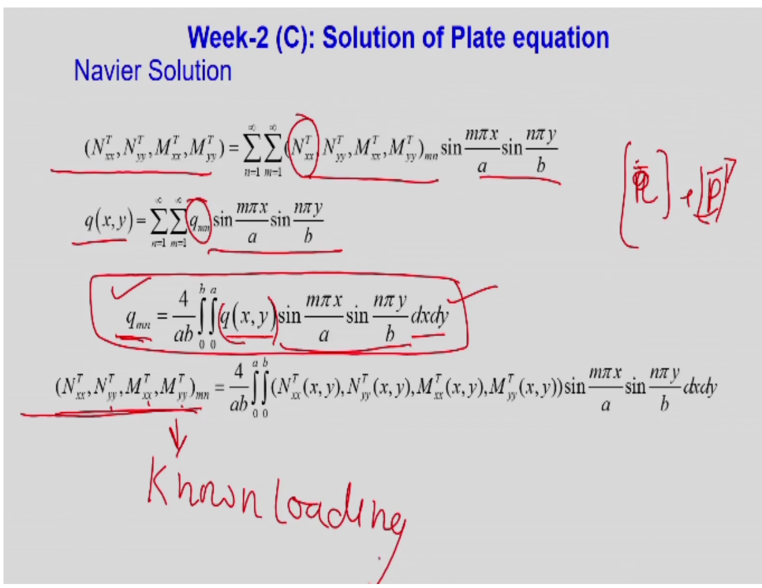

So you see that the matrix, now it is known as k, then you have to evaluate our P bar and P bar t matrix, what are those terms, the thermal terms is expressed, double sin series. Similarly, the loading is expressed, then what is this qmn, what is this Mxxt, these are not the unknown, these are the known things, and what are those, that qmn is nothing but, if you go through any mathematics books in Fourier series, so that function can be represented in one, 4, 0 to b whatever the limits, if you say that limit is 0 to Pi, so it will be pi/2, like that.

So 4 times of ab, 0 to b, 0 to a and q which is a function of x/n sin Mx/a and sin Ny/b. So we can say that this Qmn is known in this terms. So when you know the function of Qxy whether along x direction it is a uniformly distributed or sinusoidal loading or a hydrostatic, whatever load, you put that value here, may be in terms of a function or a constant and 1 can evaluate the value of qmn. Similarly, one can evaluate, the value of Nxt, Nyt, Mxxt, Myt.

These are the known parameters, known loading. Loading is known to you. So these terms you have to consider it.

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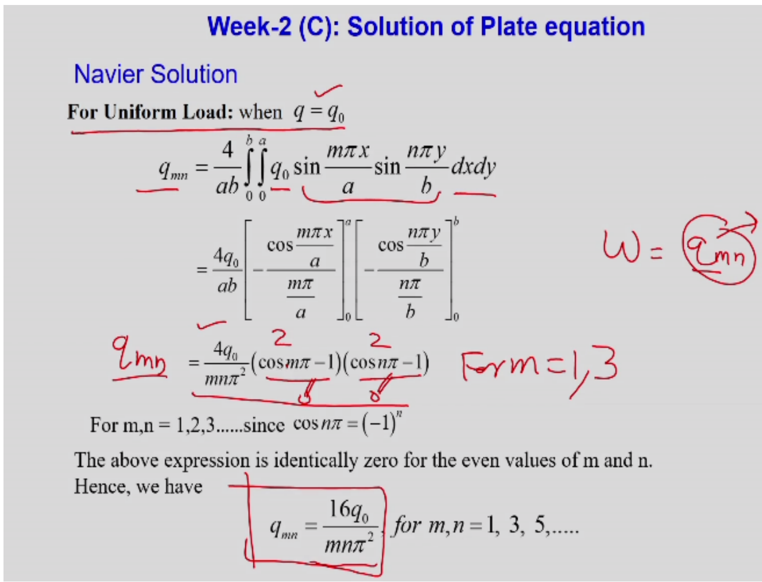

Now what is the next step just here I have few steps that that how to evaluate that double integral, if you say that we are talking a uniform load, it means it is not a function of x, nor a function of y, then Qmn can be written as q0 and this terms. So if you say that it becomes cosine, then again it become cosine 0 to a, 0 to b, if you evaluate, it gives a term like this. So basically this terms if you put m=1, cos Pi will be 1, -1 so it becomes -2, similarly -2.

So +4 contribution comes into the picture, 2 from here and 2 from here, for m=1. If you put m=2, m2 Pi, that =1, positive 1, it is becoming 0, this is becoming 0, no contribution. Then if you put m=3, then again you will get contribution to it. So basically Qmn contains when the series is odd number, 1,3,5,7 only for those cases, non-0 contribution comes and what is that non 0 contribution, 16q0 by mn Pi square. For the even terms there is no contribution.

So if you talk about a solution case, in that case if you write a w, I will tell you that some function of that. So basically this term will be non-0 for only odd terms, so again in this series, only odd terms will contribute, for we know k we can write k, now umn like this, double sin series, similarly the loading portion, so you see that these things are common, sometime say that orthogonal functions that you can take common these things, not always sin-sin.

Maybe for this case sin-cos umn then, I am talking about this is for w, w it will be sin-sin, but other case it may be sin or cos like that. So we can take here I can write say that sin and cos, I can say here cos and sin and again sin and sin. Let me just put it again, so basically for the u case it will be cos and sin along x axis is not there, so basically sin and cos and then v will be cos and sin, then this will be sin and sin.

Similarly, you have this contribution like sin-cos, then cos-sin and sin-sin, so 1 can take common those things and make a sort of equations like this, k and I will say umn and I will like to say qmn. So we can solve it, this equation KU=P, so U can be equal to K inverse of P, or you see that the matrix K is not coupling between the third equation. So we can solve this third equation independently, sometimes we called a plate under bending case.

So we can solve those sort of equations independently. So if you substitute in this equation =Qmn so Wmn will be qmn/D11m4, D22 and so on. This is represented for an orthotropic material. Anytime you can convert into an isotropic case for example, D11 and D22 is nothing but D, D12 will be u times of D, twice of D66 is 1-u times of D, and we have D can be written as. So if you replace all Ds in terms of like this, so this Wmn will be valid for an isotropic case.

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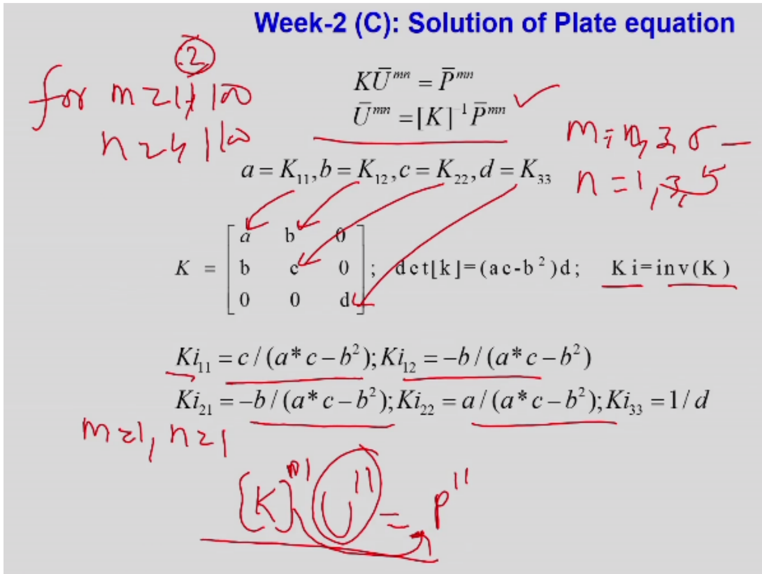

So how do you write a program, let us say you know that K matrices, P matrices, and let us say the first element we say a, second element b, then 21 is same, 22 is c and k3 is d. So you have to just make the inversion of that, you can make a symbolic programming in the MATLAB, or Mathematica, there you can obtain those variables, so K inverse matrix, inverse of K, Ki11 is this one, Ki2. So these things you can code.

If you know that elements for each mn1, so you apply a loop like if you talk about a MATLAB case for m=1 to say 100, n=1 to say 100, 1 increment or you already know that for the loading is odd increments, so you say that about 2 increment, 1, 3, 5 and then you can find out the things. So this sort of equation, which I would like to say, let us say m=1 and n=1, so you will have a matrix k1,1 and u1,1 and p1,1. So you will get the solution of this according to m=1, n=1. Similarly, 21, 22, 23, so you are going to solve in a series form.

Most of the times, students get confused here. When we are taking a Fourier series, this is valid for a series of m, n, m goes from 1, 3, 5 and so on and goes from 1, 3, 5. You have to evaluate for each terms. Not just solving one equation and you say that okay, my solution is there, no. You have to go for each m and n and then solve it. So this is the way of solving a 3 x 3 equation together.

But in rest of the books, in theory of plates, generally we talk about bending case, that we assume that there is no stretching. There is only bending there. So we will solve only the third equation, which is this above. We will solve it like this.

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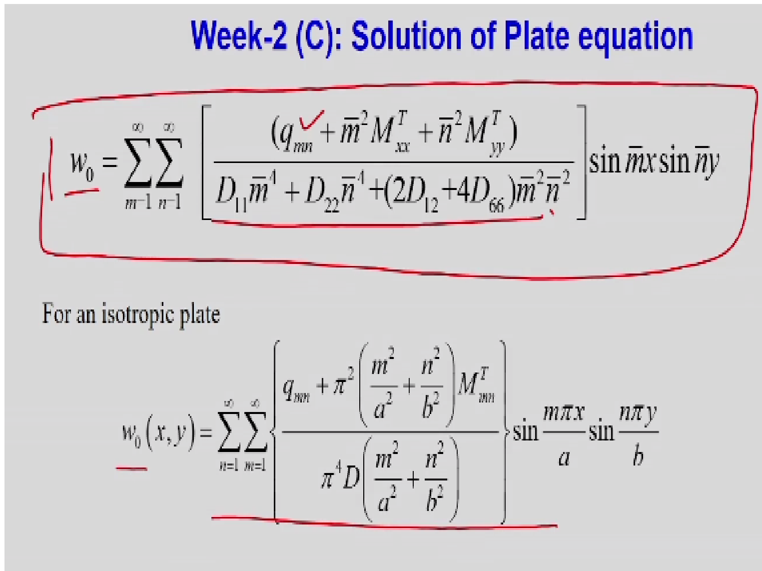

Single equation, where 2mn can be calculated for each Mnm, which is 16 times of q not upon a and pi 4 square and then you substitute value of Mnm, so these terms can be put together. Again, for isotropic case, this w not can be represented like this. So now you know u not, v not and w not and you substitute those wm, n all these things together or in a series form. How to evaluate the bending movements or the stress resultants nx, ny.

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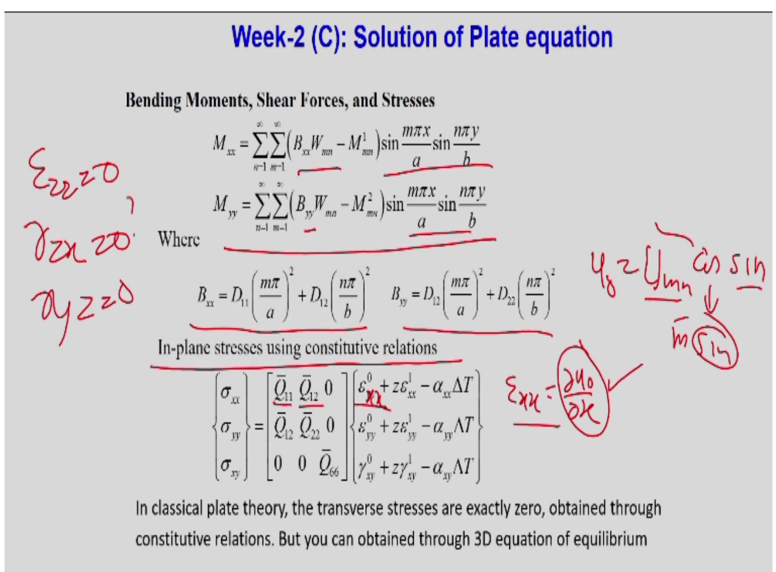

So we can calculate the bending movements using the plate constitute relation. That is why in the starting phase, you used the plate constitute relations, converted partial differential equation into a primary form, then from the primary form you get u not, v not, w not and again using the constitute relations, you obtain the bending movements. So here b axis again, some constants b1, b2, byy in a series form.

Similarly, 1 can solve the stresses using the linear constitute relations or non-linear whatever you want have. So you know Q11, Q12, u not, epsilon not xx. So epsilon not xx is del u not, x. You know what is u not, umn and cos and sine. So along x axis it will become sine. Some contribution will come m bar and umn if you multiply that and you can find it out that epsilon xx. Similarly, you can solve sigma yy, sigma xy.

Now the very important step. Though we have assumed in the classical plate theory that epsilon zz is 0, gamma zx is 0, gamma yz is 0. So based on that tau yz, tau zx and sigma z is become 0. If you use the constitutive relations, can we find it out these stresses. Definitely plate whatever the thing, there may be some stresses there. How to calculate? If you go for using the constitutive relations, we cannot get.

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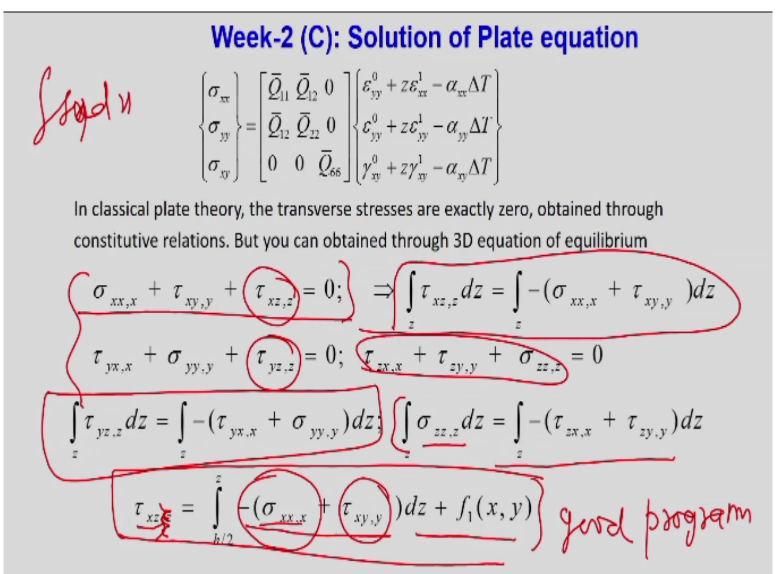

To evaluate this, we need to use 3D equation of motion. You see that first equation of motion, sigma xx, x tau xy, y + tau xz, z. So if you say that tau xz, z- of that and integrate both side along z direction, that will give you this thing, tau xz will be = -x/2z and this equation. Now you know these things sigma x, x tau x, y in terms of primary variables. Definitely you can take the derivative of that and substitute it here and take the integration.

Similarly, tau y, z so the equation for the tau y, z and the third equation can be used for to find out the stresses sigma z, z inside the plate. Though it looks very simple integrating like this, but when you are going for actual programming, these are not so simple. Then, you have to think how to take integration or each point, the stresses where you have to take. So these require good programming skills. Sometimes, writing just an integration of something looks very easy.

But to evaluate or to make a program of this, is a difficult thing. So this is not very easy to evaluate, so you need to have some background of mathematics or how to evaluate numerical integrations. So generally when we go for this, we used to numerically integrate not functionally. Sometimes if you write in terms of function, you can do the functional integration also.

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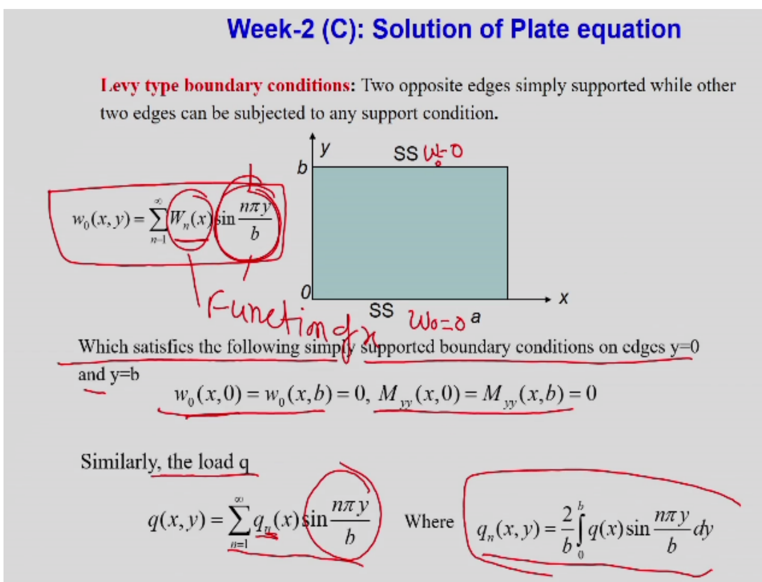

Now the next step. Before going to the Levy solution, now you have obtained sigma xx, sigma yy, sigma xy, as well as tau yz, tau zx and sigma zz. So all the stresses are known. All the displacements are known. The ending movements are known and Xmy are known, now you can plot it, you can program. Now the next analytical solution is Levy type boundary conditions. When we say Levy type, it means 2 opposite axis simply supported.

The very first condition that whether it is a plate or a shell, so 2 opposite axis must be simply supported, other 2 axis can have any boundary conditions or any support conditions. For the present case, let us say this is my x direction, this my y direction, along this is my x0, xa and y0 and yb. We are assuming that along y direction at y=0 and y=b the plate is simply supported. So x=0 and x=a can have any boundary conditions.

So how do you assume the solution. You know only that w is 0, I am solving only third equation, so we have only 1 variable, special case, for that case, w is 0 here and w is 0 here. So you will use a single Fourier series, that along y direction yw is 0, so the boundary condition is satisfied exactly. When we see here, wn, when we are talking about a Navier solution, then wny is a constant, but for the present case, it is not a constant. It is a function of x.

This we can solve, so now again, which satisfy the boundaries in the supported boundary condition at yn0 what are the variables. Deflection and movements, so these has to be satisfied. So similarly the loading has to be expressed in a single sine series along the y direction as the same w. These things you have to remember. Whenever we are going to solve an analytical solution, the function we assume along x or y, same assumption should be taken in the loading case.

Otherwise, the left hand side and right hand side will not be same and we cannot take the common. Again Qms can be found out like this. Now only one integration 2/b 02b qx sine n pi/b.

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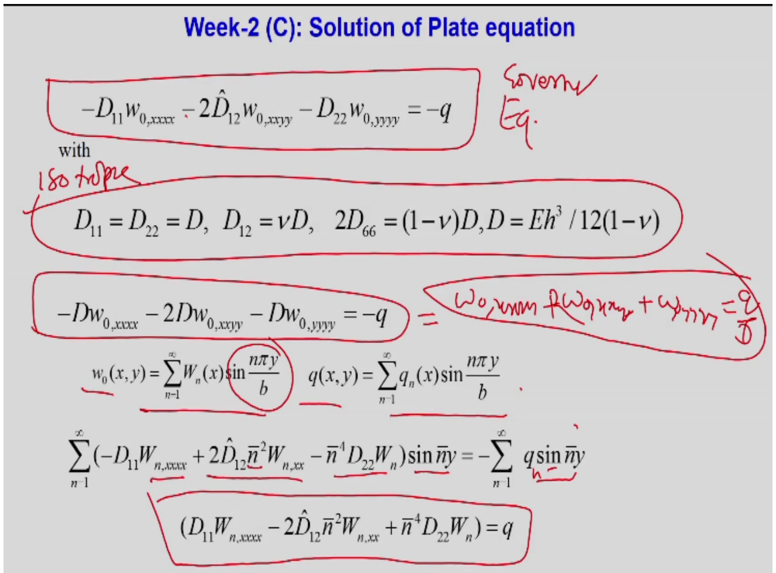

This is our equation of motion or I will say that governing equation, partial differential equation. Again if you use this for isotropic case, this becomes like this, all D's you can take common and then q/D you can write that w, xx + double derivative of double Oxx, Xyy of twice + Y derivative = q/D and one can solve that problem. We are going to solve for the orthotropic case. We have assumed that w is along sine y direction and similarly loading is as well.

Now we substitute this w not and q not here. What does it give, x derivative? So w and derivative 4. Double derivative of y and double derivative of x, it gives stable, no effects, but along y direction it is arranged in sine series, so it will be m bar score, the – sign will be there. So it becomes +, then 4 derivative along y direction, you see. In single equation, you can see clearly, sine and y, sine and y, write qL is here.

So ultimately these equations become like this, which is an ordinary differential equation of 4th order. Now we have to solve this equation. So this equation can be solved analytically or can be solved by approximate method, Ritz method, finite difference method, finite element method.

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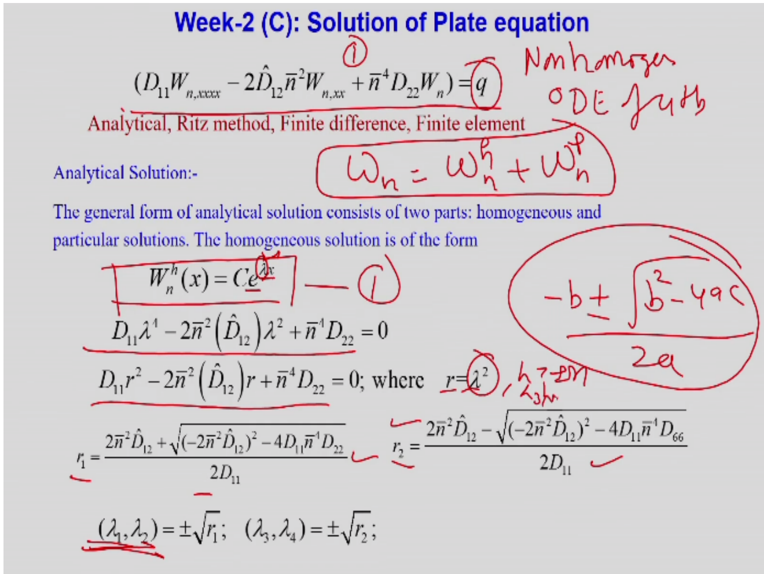

So the first solution, this is the ordinary differential equation of 4th order+ non-homogeneous, that k is here. So it's solution will have 2 part. Let us say q1 will have 2 part, homogeneous + particular. When we have a non-homogeneous ordinary differential equation, we can divide it into 2 parts, homogeneous and particular part. So homogeneous solutions can be written as, we can assume a solution of a differential equation like this. E lambda*x.

Where lambda are the roots of the equations. So if you substitute this equation into here for a homogeneous case, it leads to a 4th order equation. Again, if you say that r is my lambda square, so it becomes a quadratic equation. Then you know the roots of the quadratic equation. Roots of the quadratic equation can be b square – 4ac/b2a. So you can write that r1 can be nothing but v square+first 1 b square-4ac/b2a, then similarly r2-of that 2a.

So lambda1, lambda2 will be +/-of r1 and lambda 3 and lambda 4 will be +/- of r2. So now we have a four roots. The very important case that when you are going to program this or you are going to solve this type of equation, you need to know the type of roots whether they are same, complex conjugate, real, different, then the homogeneous solution type will be different. You cannot say for everything same type of solution.

Depending upon the roots, these are given in a mathematics books, 1 can go through that, high engineering mathematics, any books. Basically, we use Erwin Kreyszig that how to solve ordinary differential equation, that 2 roots, now first case.

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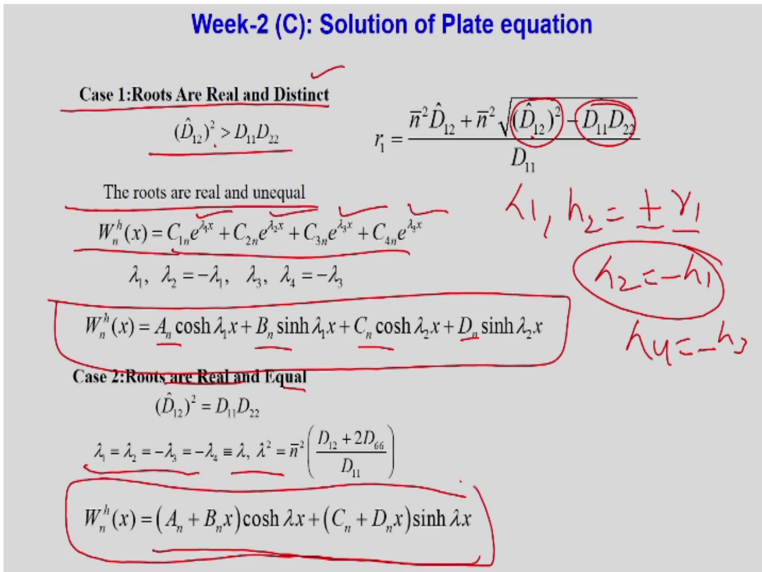

Roots are real and distinct. When this case will be there, when this is greater than this. Then it will be a positive under root and definitely will have difference. So we will have a 4 roots, lambda 1, lambda 2, lambda 3, lambda 4. We can direct a solution like this. Now again, you see that lambda 1 and lambda 2, they are just same with – sign of r1. So lambda 2 is nothing but – lambda 1, similarly lambda 4 is nothing but –lambda 3.

If you take consideration of this, we can write a homogeneous solution, like this in cosine hyperbolic, sine hyperbolic and these are the unknown constants. Then, if roots are real and equal, then both are equal, it becomes 0. So divide it by D1, so the solutions can be written like this. We have standard cases, but the aim to show this solution is here that you must be aware we can use these type of analysis for later stages.

When you are going for a complex material or advanced material, then their roots may be complex, then you can proceed. You must be at least aware, okay these kind of solutions exist. If you know the roots, you can write the solutions.

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Week-2 (C): Solution of Plate equation

Particular Solution:

$$W_n^P(x) = \sum_{m=1}^{\infty} W_{mn} \sin\frac{m\pi x}{a} \quad q_n(x) = \sum_{m=1}^{\infty} q_{mn} \sin\frac{m\pi x}{a}$$

$$(D_{11}W^P_{n,xxxx} - 2\hat{D}_{12}\bar{n}^2 W^P_{n,xx} + \bar{n}^4 D_{22} W_n^P) = q_{mn}$$

$$(\bar{m}^4 D_{11} W_{mn} + 2\hat{D}_{12}\bar{n}^2\bar{m}^2 W_{mn} + \bar{n}^4 D_{22} W_{mn}) = q_{mn}$$

$$W_n^P(x) = \sum_{m=1}^{\infty} \frac{1}{d_{mn}}[q_{mn}]\sin\frac{m\pi x}{a}$$

$$d_{mn} = (\bar{m}^4 D_{11} + 2\hat{D}_{12}\bar{n}^2\bar{m}^2 + \bar{n}^4 D_{22})$$

$$w_0(x,y) = \sum_{n=1}^{\infty}\left(W_n^h(x) + W_n^P(x)\right)\sin\beta_n y$$

Next, if roots are complex, D1,2 is greater than –i, so root may be in the form of complex conjugate. Frankly speaking when we solve a plate problem, let us say for my case in 3 dimensional equation, then we may have both cases. Sometimes real and distinct combination of that and complex conjugate. So in this case, let us say 8 Eigen values or 8 roots, so 4 may be real and different and 4 may in terms of a complex conjugate.

There may be both cases. Here you have only 4 solution, you have 3 cases, so we can write a homogeneous solution, like this. This is standard homogeneous solution given in any mathematics book. Now we have to evaluate the particular solution. So for this case, we can assume a particular solution that along x axis function was unknown, so we are assuming in a sine series. This is the important part. Most of the students makes mistake here.

They forget how to solve a particular equation. So you have to assume a solution in the sine series and substitute it in here in terms of particular solution. Then you will get Wm, m. So again, like when I get this kind of thing, it becomes W particular solution, just like this. So final solution will be homogeneous solution + particular solution. Now beta is nothing by N pi/b.

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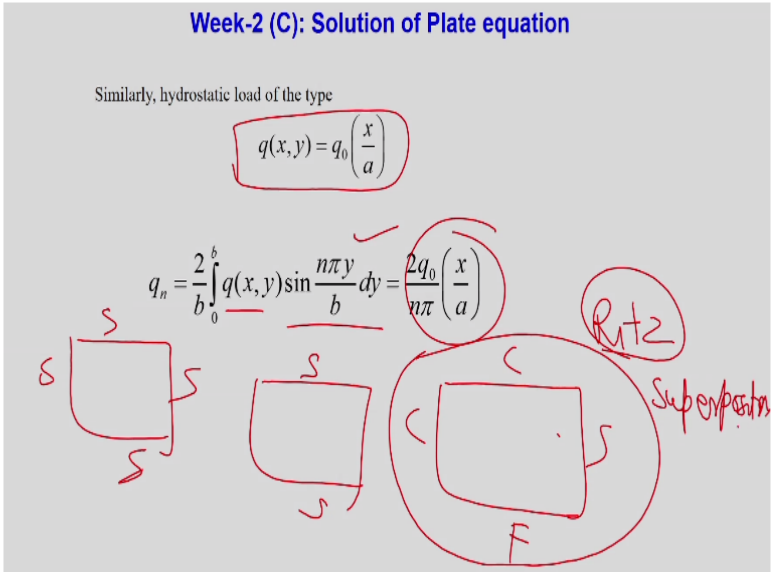

If we have a load of hydrostatic type, that along x direction, it is wearing like this, so you put it here, evaluate Qn. So I have explained how to solve a Navier solution, how to develop a Navier solution for a plate and a Levy solution for a plate. In the next section, we will steady that approximate solutions since, I have showed on a plate, which is having a boundary condition of all adjust simply supported or two opposite as simply supported.

A plate in which any boundary conditions you can say. So that solution analytically solutions are not available, but we have given a try in my research. So I will explain a little bit of that. Before that, I will explain how to solve a 2 dimensional plate if a plate having boundary conditions like this. So we will have a Ritz solution and next 1 is a principle of superposition. Thank you.