Theory of Rectangular Plates-Part 1
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Lecture - 07
Tutorial: Reduced Stiffness & Plate Stiffness

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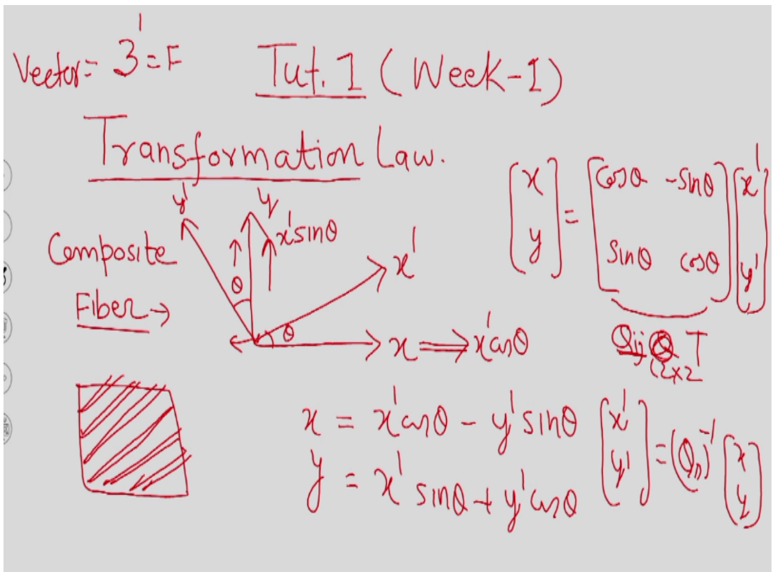

Welcome to our tutorial 1 for week 1. In this tutorial, I shall solve some basic questions on numerical, so that you can understand that based on the theory, how do you apply while solving the numbers. The very first topic is using the transformation law. I think in your undergraduate levels, 2 dimensional transformation is taught like if we have, this is my x-axis. This is my y-axis. If another coordinate system, which is making an angle theta with the x axis.

Let us say x dash and this is y dash, so it will make an angle theta. So if somebody is interested to find out the fourth order displacement, let us say we know that the components of. First of all, we will write a transformation matrix. So x dash will have 2 components, 1 is along x axis, 1 is along y axis. What will be that? that along x axis, it will be x dash cos theta. I am going to write like this x dash cos theta. Then along y axis, it will be x dash sine theta.

This y dash will have 2 component along this and along here. So it will be –y dash sine theta along x axis and y dash cos theta along y axis. Now you arrange the components. Let us say x, will be x dash of cos theta along x axis and –y dash of sine theta. Then, along y axis, the component x dash sine theta and +y dash cos theta. Now can we arrange in terms of matrix. Let us see, x, y cos theta-sine theta sine theta cos theta, x dash and y dash.

You will say that this is the mathematics that how do you transform or our aim is to find out in terms of x dash and y dash, let us see. This is your matrix Q or in general terms, we say that this is our matrix T, transformation matrix, but in general theory of elasticity or engineering mechanics, we define by Qij transformation matrix. So here we will say that Qij, which is a 2 x 2 matrix.

At some point of time, you may know that value along x and y and you are interested to find out the values alone root at x axis. In that case, x dash and y dash will be inverse of this and x1. This transformation is valid when you are having a vectorial quantity. Vectorial quantity means first order tenser. If you write that x and y interest, if you say that uvw, fx fi, so you can add and transform accordingly.

We are going to use this mathematics, let us say A composite. We know that in a composite, one is fiber, along that material properties highest. So let us say lamina. This is our A, rectangular lamina, in which graphite fiber or glass fiber are oriented like this. So we know only property of fibers that this is your direction.

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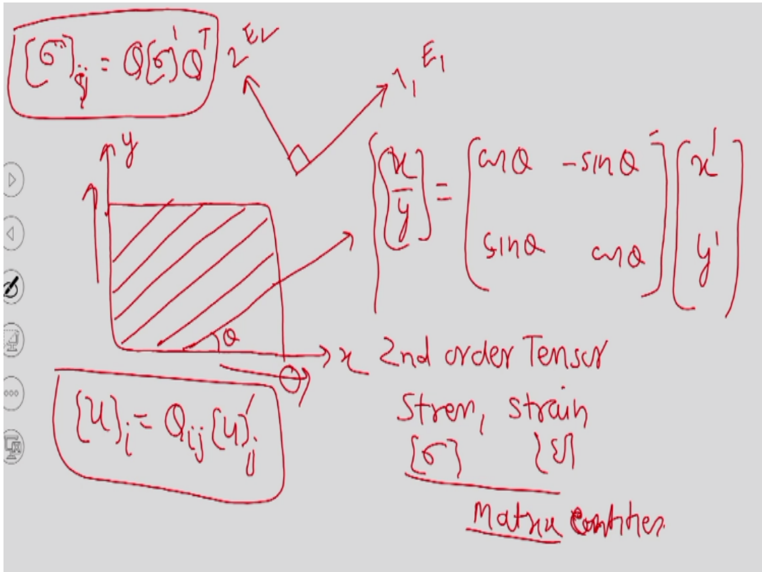

You take a lamina, in which direction of fibers is like this, at an angle, let us say theta degree with the x axis. So we know that material property of a fiber along this direction, so our fiber is in this direction, perpendicular to this 90 degree, 1 and 2. So if we know that E1, E2 and other properties, stresses, but we are interested to find out properties along x axis and properties along y axis. Then we have to use the concept of transformation.

But vector for, I say that, if it is x and y, then cos theta-sine theta, let us say I am going to write sine theta, cos theta and so on, x dash and y dash, but if you are talking about a second order tenser, like stress or a strain sigma. Sometimes we call it is a matrix entities. How do you transform this thing? For a vector, let us say any vector u can be transformed Q, you say i here, ij and uj or say it U dash.

For the case of a second order tenser, then it will be or in terms of a matrix, Q sigma dash and Q transpose like that. This I am going to explain you.

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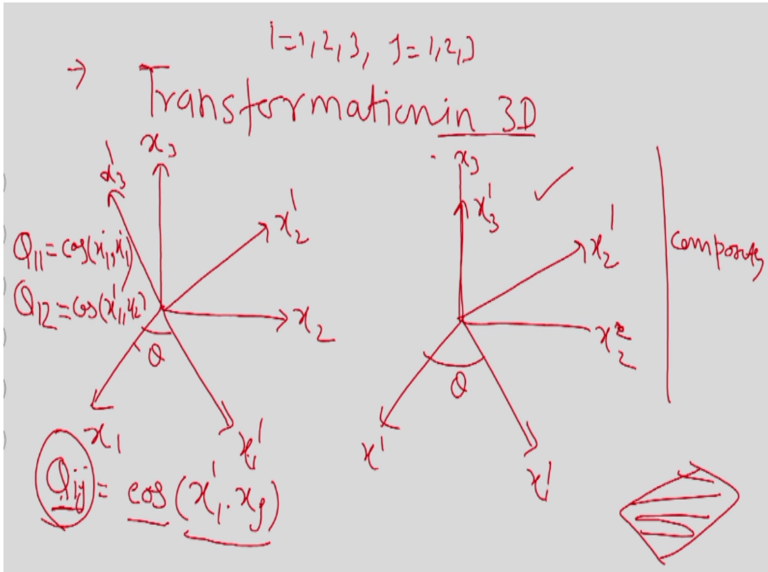

Now talk about transformation in 3D case where you have x1, x2, and x3. Then I have rotated this frame, let me say or slightly like this, let us say x1 dash, x2 dash and x3. This is your general transformation that in 3 dimensional how do you define accordingly, but generally for the case of our composite, we say that x3 is coinciding with, that rotating only x and y, no. All around the z, the rotation is there, because if you are talking about a composite material.

The fiber angles rotate about the z axis, so x and y. So our equation will become x1 dash, x2 dash and x3 dash is same. So mostly for composites, we use this kind of information that around the z axis is coinciding and in plane axis, the x1 is making an angle theta with x axis or y axis whatever you want to say, but in general sense it may also have a complete rotation about the z also. So how do you evaluate a transformation matrix Qij.

That is the standard rule. If I talk about a single element, it will be cosine of xi dash.xj. So we are interested to find out a Qij matrix done, x dash is making with x1 will be a cos component. So according to if you say i=1, j=1, then Q1,1 will be nothing but cos of x1 dash xj dash, again 1 dash making in with angle, here is comma. Now Q1,2 x1 dash will make x2 cos that angle, similarly we can calculate all the elements if you say that i is taking 1, 2, 3 and j is taking 1, 2, 3.

You can evaluate all the elements by following this law. Now I am going to implement a numerical.

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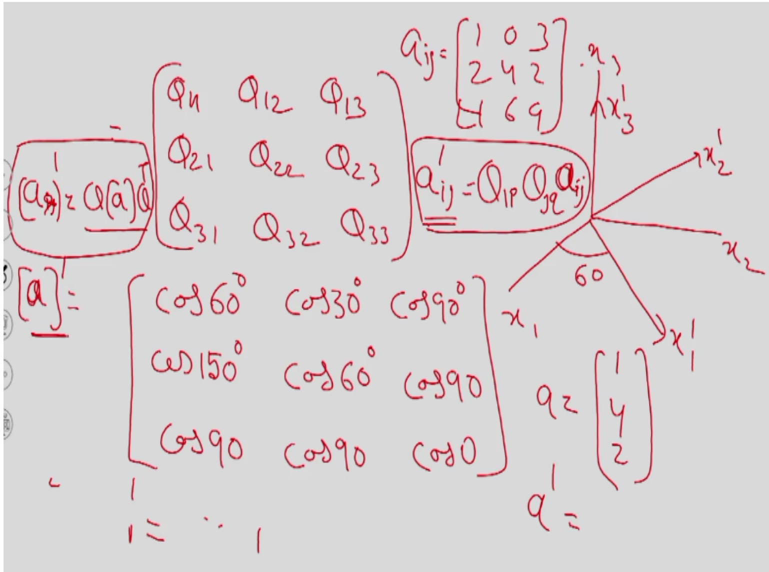

Let us say your x1, x2, x3 and another x1 dash is making an angle of 60 degree with the x axis and this is x2 dash and x3 dash is same with x3,3. Now you have to evaluate Q matrix. First of all, we will write all the elements Q1,1, (1,2), (1,3) Q2,1, (2,2), (2,3), Q3,1, (3,2), (3,3). So the first element x1 dash is making with x1, what will be that cos of 60 degree. Then x1 dash is making with x2 dash, which will be cos of 30 degrees, then x1 dash is making with x3.

It will be 90 degree. Then x2 dash is making with x1, which is coming 150 degree. Then x2 dash is making with x2 cos 60 degree and cos 90 degree. Then x3 dash is making with x1 cos 90, x3 dash is making with x2 cos 90 and x3 is making with x3 cos 0. This is your 3 dimensional transformation matrix when x3 dash = x3. So now you just put the value. It is becoming 0, this one. Basically you have to evaluate, if I tell you that evaluate any f or evaluate any u, you have to just multiply this matrix with that.

Let us say, we are interested to find out any vector a whose values are given, let us say 1, 4, and 2. Now we are interested to find out a dash. So a dash will be nothing but Qij and a1. If you multiply this matrix with this a1, you will get a1 dash. If you know the components along x1, x2, and x3, then you can rotate along the rotated component. The vice versa you take inverse of this, let us talk about a matrix component aij.

Let us say (1, 0, 3) (2, 4, 2) (1, 6, 9). We have this. Now we are interested to find out aij dash. What will be that? It will be qip, qjq and a(ij). This is an index form, but if you are interested to know in matrix form, then it will be aij or I will say that a dash will be equal to qa and q of transpose, like that. So you put Q matrix, then a matrix, then transpose of this matrix multiplied together, you will get a, a matrix, along the transform coordinates.

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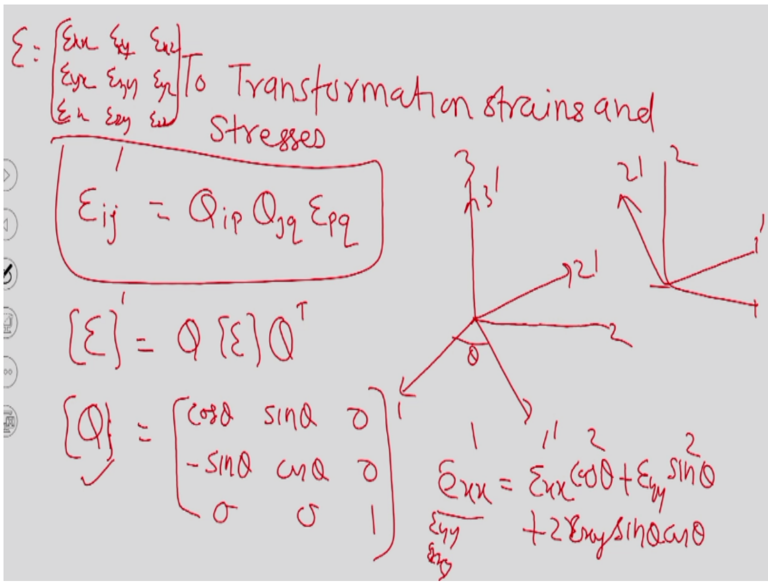

Now we can implement to transformation of strains and stresses. So here you see. Generally, in lamina case, we ask about 2D or around 3D, it is a constant. So the same way, you say 1, 2, and 3 and then some rotation, 1 dash, 2 dash and 3 dash is same, making an angle theta. Basically epsilon i, j dash can be written as qip, qjq, epsilon pq or in matrix form, epsilon dash can be written q epsilon qt. So q for this case already if you are talking about even for 2D, I have explained 1 and 2, 1 dash and 2 dash.

If you are talking about 3D, so third element about that axis will become 1, so we can say that cos theta, sine theta, 0. Then –sine theta, cos theta, 0,0,1. You say that cos 90, cos 90 and if you take the transformation, this becomes like this. So if you put it here, so cos square, sine square and some sin-cos terms will be there. So if I write epsilon x, x dash will be epsilon x, x cos square theta+epsilon y, y sine square theta.

Then +2 of epsilon x, y sine theta and cos theta. If somebody is interested to find out epsilon x, x dash, 1 can find out like this. Similarly, epsilon y, y. So basically you substitute those values. What is your epsilon, epsilon is like epsilon x, x epsilon x, y epsilon x, z epsilon y, x, epsilon y, y epsilon y, z epsilon z, x epsilon z, y and epsilon z, z. This is your matrix. So if you multiply with this, you can get all the components. What will be my epsilon 1,1? what will be my epsilon x, y?

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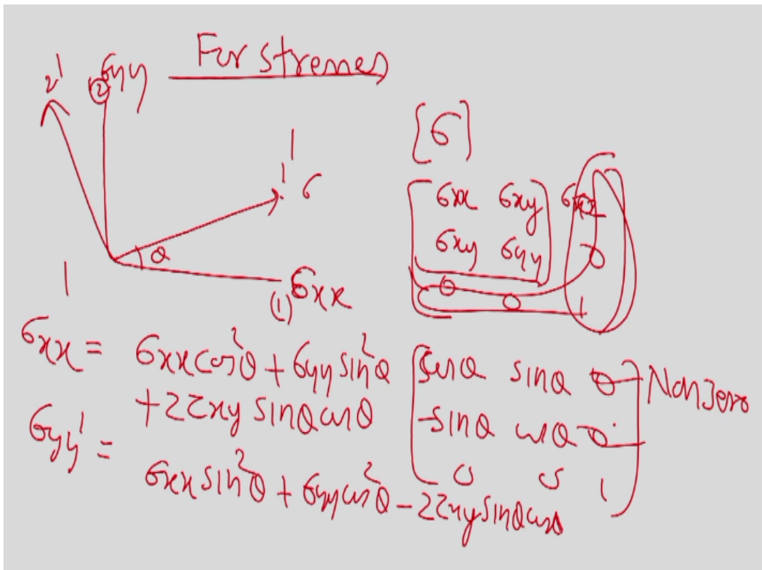

Now for stresses, generally in theory of elasticity, we used to ask that let us this angle is this one. So we know that stress is along x axis sigma x, x, sigma y, y, so what will be or I would like to say that not 1 axis, 2 axis 1 dash and 2 dash. Our aim is to find out sigma x, x dash sigma y, y dash. So here also, again you have same way, sigma, you put it. If you talk about it 2 dimensional, then it will be sigma x, x, sigma x, y, 0.

Then again sigma x. y. Your matrix will look like this. When you are talking about purely a 2 dimensional matrix, then cos theta, sine theta, 0, sine theta, cos theta -0, 001, so ultimately this is your matrix basically. You do not even require all these things. They are going to be 0 affecting that or if you can put, you can put z, but for our present case, we have chosen that allow z axis, it is not rotating.

If it is rotating with the z axis, this component will also become non-0 components. So this is an important point that whenever we say that around z axis rotating, these components are going to be zero. But in some special questions, there are some problems like robotic hands or some others, where even around z axis, it is also rotating. Then this cannot be 0. For our present structural mechanic's theory of plates point of view, we generally take 0, 0 and 1, like this.

So it will be cos square theta+sigma y, y (sine square theta) +2 tau x, y (sine theta and cos theta). Then sigma y, y will be sigma x, x of sine square theta+sigma y, y (cos square theta)-2xy sine theta and cos theta. Now you have understood that how transform a second order tenser, but you have a fourth order tenser also Cijkl, which is a fourth order tenser.

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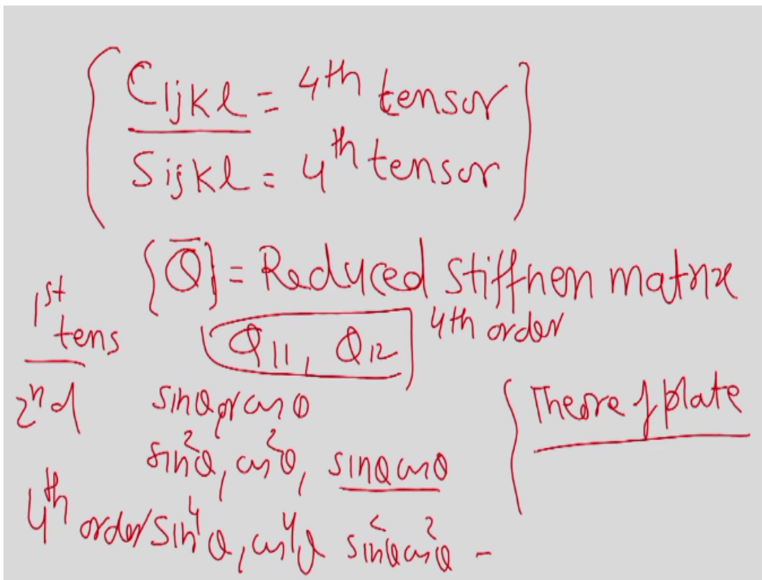

Then Sijkl and what about how to transform these things. Then again if I talk about Qij. Again I am saying Q reduced stiffness, can say it Q bar, reduced stiffness matrix, standard notation, do not confuse with that transformation. That is why I would like to say that the transformation you can denote by T and for in between we shall say that reduced stiffness matrix, the components are Q1,1 and Q1,2 and so on. Do not confuse with that transformation.

So all these are fourth order. Now you see that when you are talking about a first order tenser in a reactor, you have either sine theta or cos theta, but when you have a second order tenser, then you have sine square theta, cos square theta and combination of these things. When you have a fourth order tenser, it means you will have a 4 times of this and different combination of this, sine square theta, cos square theta and so on.

If I say that that is the third order, it may be sine cube, cos cube and so on. One can understand that how to start a transformation of this. These are the standard rules given. You go through any theory of plate. In the very first chapter, the basic even I have shown you in my presentation also that how Sijk can be represented or Q1 or Q2 can be represented in a transform coordinate system. These are variables required, if you talk about a composite.

If you are talking about only isotropic plate, then this transformation no meaning, but when you have a composite plate in which fiber make any angle with geometrical axis. For that case, transformations play a major role.