Theory of Rectangular Plates - Part 1

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Lecture - 03
Classification of Plate Theories and Some Basics

Welcome to third class of theory of plates.

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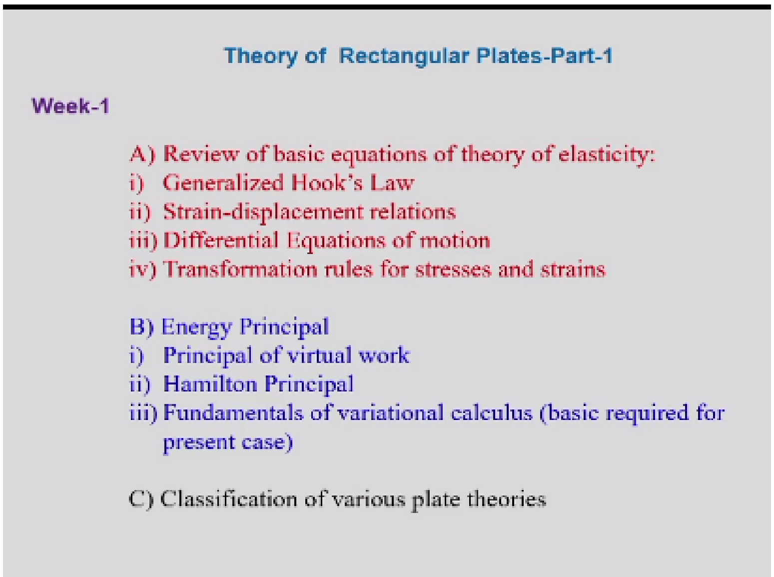

So we have already covered reviews of basic equation of theory of elasticity, energy principles and today we are going to cover that some basic assumptions and classification of various plate theories.

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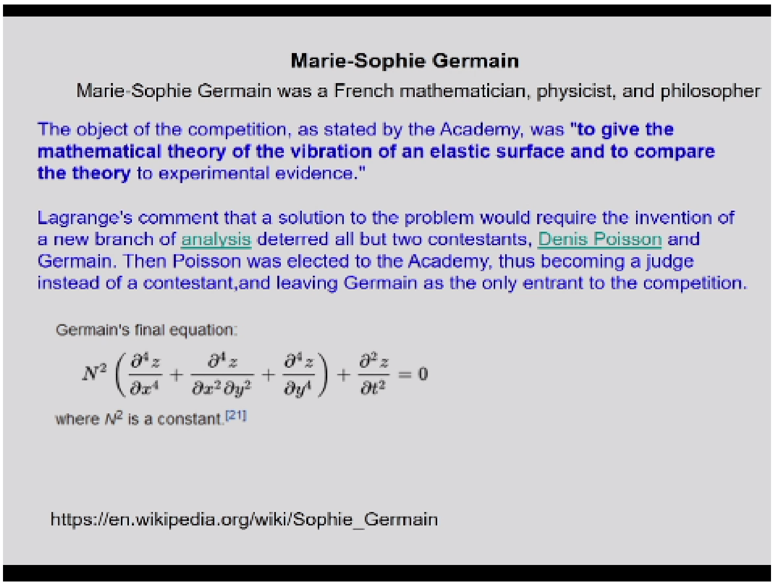

So before going to the main course I would like to tell you that Marie-Sophie Germain, she was the first mathematician or she developed plate theory and she does this is her equation of motion. So it is a very interesting story I have also written some part of it which is given on Wikipedia that they were a competition in the academy of Paris then they asked that to give the mathematical theory of vibration of an elastic surface and to compare the theory to the experimental evidence.

So during the competition this topic was assigned and asked that whosoever will give the equation of motion and later on that theory should predict the experimental results. So the importance of she is that you know that Euler Lagrange that famous mathematicians commented that this solution to the problem would require the invention of a new branch of analysis.

So then the Poisson's, Poisson's ratio which you call the scientist, the researcher Poisson was elected to the academy, thus becoming a judge instead of a contestant and leaving the Germany as the only entrant to the competitions and she developed that first theory of plate but there was the time calculation of variation was not evolved properly, it was in the initial state.

So there was some discrepancy so this theory could not give accurate results to the experiments, validations. So later on Kirchhoff did some correction and developed the plate theory. So these days we use to say that Kirchhoff's plate theory or Kirchhoff's assumptions that, sometimes we do not call classical plate theory, we say Kirchhoff's plate theory so the name that Kirchhoff did give the plate theory first time.

But before that these basic equations were given by Marie-Sophie Germain. This is very interesting story sometimes in area that it was area of mathematics like mathematicians or researches who are world level mathematicians they were in this field in the development of the theoretical analysis for a plate or a beam you see that Euler–Bernoulli beam, Euler was a mathematician.

So he developed a beam theory then the Kirchhoff's, similarly the Lagrange so both were of mathematicians of some times, they also did some good contribution to physics also.

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Stepan Prokopovych Timoshenko

He is considered to be the father of modern engineering mechanics

- *Theory of Elasticity* , McGraw-Hill Book Company, 1st Ed. 1934,
- 2nd Ed. 1951 (with J. N. Goodier)
- *Elements of Strength of Materials*, D. Van Nostrand Co., 1st Ed. 1935, 2nd Ed.
- 1940, 3rd Ed. 1949 (with G.H. MacCullough), 4th Ed. 1962 (with D.H. Young)
- *Theory of Elastic Stability*, McGraw-Hill Book Company, 1st Ed. 1936,
- 2nd Ed. 1961 (with J. M. Gere)
- *Engineering Mechanics*, with D.H. Young, McGraw-Hill Book Company,
- 1st Ed. 1937, 2nd Ed. 1940, 3rd. Ed. 1951, 4th Ed. 1956
- *Theory of Plates and Shells* , McGraw-Hill Book Company, 1st Ed. 1940,
- 2nd Ed. 1959 (with S. Woinowsky-Krieger)
- *Theory of Structures*, with D. H. Young, McGraw-Hill Book Company,
- 1st Ed. 1945, 2nd Ed. 1965

https://en.wikipedia.org/wiki/Stephen_Timoshenko

So but a person Timoshenko, Timoshenko name is very famous name I think everybody whether he is an undergraduate students or the master or the post graduate or PhD students have once or sometimes encountered that professor Timoshenko’s books, the books are that Theory of Elasticity or Elements of Strength of Materials, Engineering Mechanics. I would like to say that before I will comment that he is considered to be father of the modern engineering mechanics.

Though the theories were developed, but they were the engineering background persons because mathematicians developed the theory so they were difficult to understand, so the Timoshenko was the first person who tries to simplify or from the explaining physics point of view. So he contributed that for the engineering student or the engineering researches can understand their theories that theory of plates like theory of elasticity or in the book of engineering mechanics or element of strength of materials.

So they have written in a very simplified manner so that undergraduate students postgraduate students who has not very strong background in mathematics can understand this concept and afterwards so engineers like us can understand this theory and can contribute to the development of the theories of plates.

So I would like to also emphasis on that that one can develop the theory number of ways like using the complex analysis method or using the simple mathematics or using the differential equation of motions or using the variational calculus. So there are number of ways to present the equation of motion or to drive the governing equation of motion. So for engineers or the researches who works in that engineering mechanics field so they love to work in that simplified version of that theories so that it can be easily understood.

And then I talk about that in the last lecture also I talked about beam analysis. So this is you can treat as a beam. This is simple rule or you take the steel rule. You know the property of the steel, simple tool steel properties, what is the e, what is the mu and clamped at one bench and try to deflect it and measure that how much it is going or how much force you are applying there.

So whatever your analysis or in your undergraduate courses or standard formulas are given for that you can verify at your home, at your hostels. So these are not very difficult concepts. Similarly, I would like to tell that see this is a water bottle, if you make, this is thin so you can analyze this water bottle as a cylindrical cell with internal pressure whatever the water is having and outer is an atmospheric pressure.

So you put on the table on a rest so this acts on the simply supported other surfaces are free. So whatever the pressure it should not burst. So one more thing I would like to tell you, you take any water bottle and you feel that there is some roughness, jig jag portion is there. If it is made just plane what will happen, that this water bottle is very thin it cannot take that much of pressure.

If you make some wrinkles kind of thing or sometimes you see on a roof top in hill area or some where it is like rainy zones. So you will find corrugated sheets, this types of sheets, so these takes more strength compared to just a plain sheet. Similarly, in this bottles this pattern is created corrugated pattern so that the same thickness it can bear more pressure or it has more strength.

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Introduction-Week-1 (C)

Review Second Lecture:

Principle of Virtual Displacement:

$$\delta W = \int_V \sigma_{ij}.\delta\varepsilon_{ij}\, dV - \left(\int_V f_i.\delta u_i\, dV + \int_{\Gamma\sigma} T_i\, .\delta u_i\, ds \right) = 0$$

Hamilton Principle:

$$\int_T (\delta K - \delta W_I + \delta W_E) dt = 0$$

So now I am going to our main topic so in the last class we have formulated a principle of virtual displacement in which this portion is your work done internal work done and this portion is related to your external work done. Similarly, I have stated the Hamilton principle in which because the virtual displacement principle cannot be used for a dynamic case.

So if you are interested to analyse a body under the dynamic case under that time dependent analysis then we have to use the Hamilton principle where this portion is the contribution due to the kinetic energy, contribution due to the internal work done and contribution due to the external work done. So I would like to do slight correction here, it should be plus not the minus because already it is minus, if you put as minus so internal work done you will get minus sign external work done will get plus sign.

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Introduction-Week-1 (C)

Plane Stress :

For some engineering problems, state of stress for a thin body/structure is required in which dimensions along thickness direction (z) is very small compared to length (x) and width (y) of the body. For such problems, the stresses in the z-direction being very small, are neglected, and it is assumed that the body is two dimensional and in state of plane stress [K. Chandrashekar]

$$\sigma_z = \tau_{yz} = \tau_{zx} = 0$$

So most of the time during first course of solid mechanics we used to teach a concept of plane stress. This is very much required because it is simplified version most of the time thickness of the body is not very high compared to the length and width, then we can apply this assumption of the plane stress. So 90% engineering application this concept or this assumption holds good.

So before proceeding to the main theory I would like to explain what is the concept of plane strain that for some engineering problems state of stress for a thin body or a structure is required in which dimensions along the thickness direction is very small compared to the length and width of the body. If you encounter basic fundamental structural element in which length and width are comparable, but thickness is very small.

It means you can analyse or you can find out state of stress of that body using the plane stress assumption for such problems the stresses in the z direction become very small. There is also a physical concept behind that since thickness is very small it cannot resist, so it cannot take the too much stress. So thickness stresses in the thickness direction are neglected and it is assumed that body in 2-dimensional in a state of plane stress.

So for that case if you say this as statement about the plane stress is given in a book of theory of plates by K. Chandrasekhar. So you have a body so x and y and (()) (10:46) plate there is z thickness which is small, so for that case stress along z direction sigma zz, tau yz and tau zx will be 0. Most of the time students get confused here. Let us say a system like this, but I have assigned this axis is x and this axis is y and this axis is z.

So you see that along y direction thickness is small compared to along x and z direction. If you assign the coordinate system like this then along the y direction stresses will be 0, what will be that sigma yy, or I will say that tau yz and tau yx will be 0. So at the time of interview or when you are going to develop a theory and just you did not take care and you have model this plate like that y vertical x and z longitudinally in that case following components has to be 0 if you are applying a plane stress assumption, not this.

So this is when, when z is the thickness direction. So plane stress application how much I said that length and width are comfortable, but thickness is small. What do you mean by the small, this is a qualitative term, what is in terms of quantity, what is small, which thing you connect small 0.1 mm is small or 100 mm is small as per your dimension, sometimes in civil structural application the slab width may be 100 mm = 10 cm.

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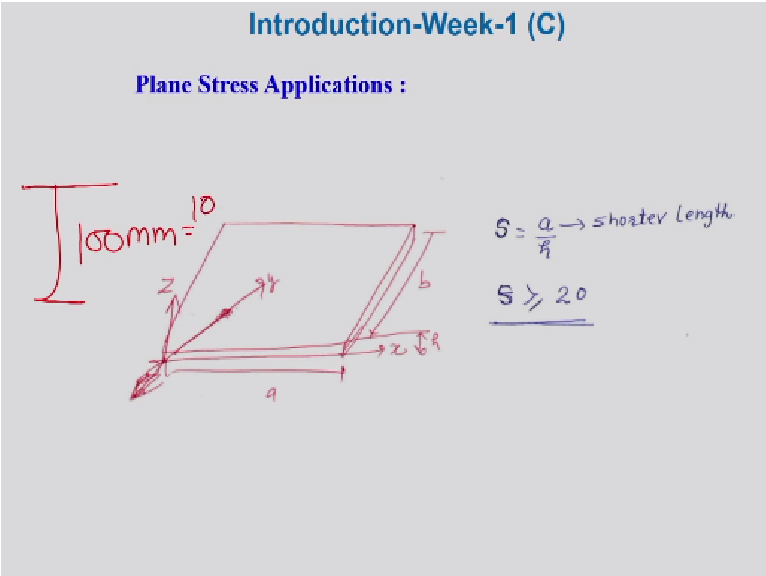

You say no it is very big, but if you see there this may be 10-meter-long like the roof of this room or 5-meter long. So compared to that if it is 500 centimeter and it is 10 centimeter then ratio is coming 50. So that slab can be analysed as a plane stress analyse. So you see s, length to thickness ratio, if this is greater than or equal to 20 definitely we can apply the plane stress assumption or the theory will predict the more accurate result and near to the experimental results.

But if you have a S = let us say 4 then it is not like 4 is a mechanical components you have some according to maybe 1 mm thickness and this maybe 10 mm and this maybe 2 mm thickness. You say that I would like to analyse or I would like to analyse this as a thin plate or under that assumption that it cannot be possible under plane stress because thickness 10 mm and 2 mm.

So it is giving 5 length to thickness ratio, so it is becoming thicker. So this thickness can resist. So most of the time these basic concepts are not well taken care. So you have gone through that course of advanced elasticity, theory of elasticity and theory of plate and cell and so on, but if you do not take care of this and you are going to develop without taking concentration of this aspect ratio that whether my theory will able to predict these things or not.

So when you are going to analyse a structural element you must first find out these things and then accordingly choose the theory, I will explain number of theories are available for different kind of thickness and ratios.

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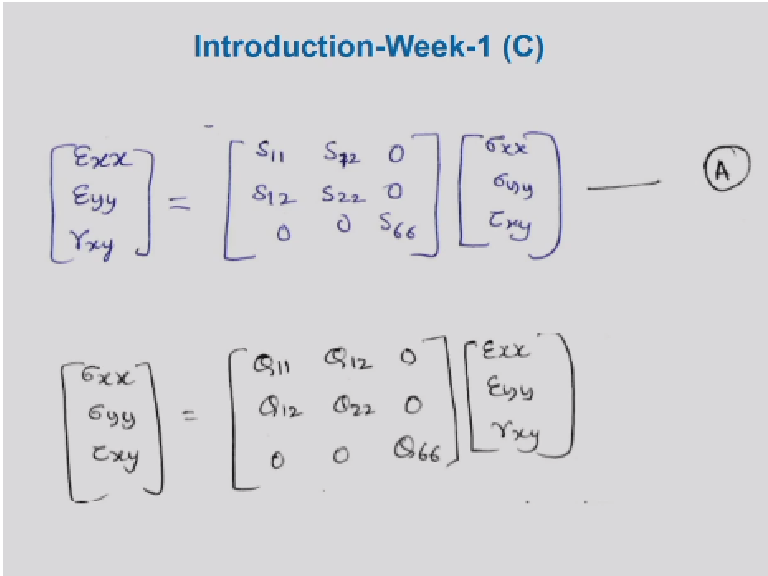

$$\begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{bmatrix} \quad \text{— A}$$

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{bmatrix}$$

So under that plane stress assumption the constitutive relation or I would like to say strain and stress relations we have assumed that sigma zz is 0, tau yz is 0, tau zx is 0. So you will get only nonzero components sigma xx, sigma yy, tau xy and their compliance coefficients. So this is the, I would like to say that reduced strain stress relations for the plane stress case. Now if you want to invert it, inverting it gives stress strain relations.

So where it is represented by Q11, Q12, Q12, Q22, Q66. So stress strain relations for the case of plane strain case is reduced to like this where Q11, Q12, Q22 are nothing but sometimes I would like to say ij where i goes from 1, 2 and 6 and j also goes from 1, 2 and 6 they are known as reduced stiffness.

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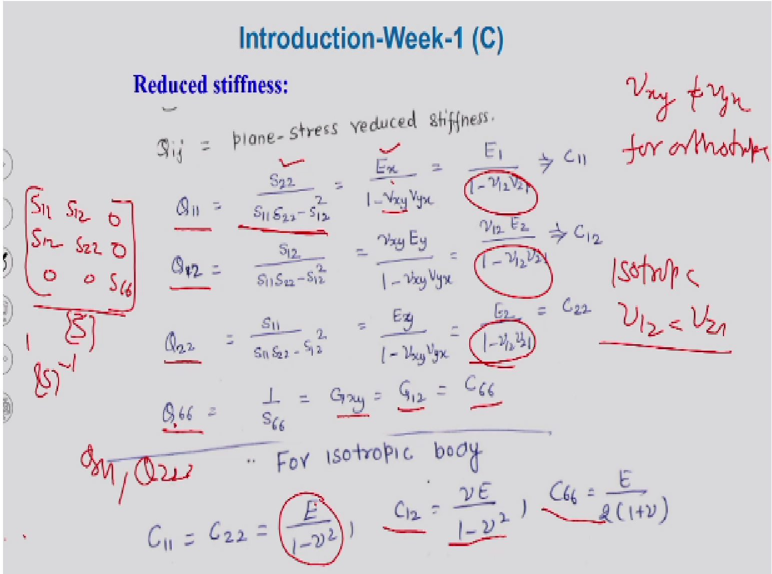

Then what are these in terms of compliances you can see if you are inviting a 3 by 3 matrix S11, S12, 0; S12, S22, 0; 0, 0, S66. If you are inverting that matrix let us say any matrix S have, so invert, so in that it will help those components S22/S11 S22 - and (()) (17:48) these are in terms of compliances. As already I have told you that engineering constants.

So we are more familiar with that like Young's modulus, Poisson's ratio generally we are able to calculate the mechanical property in terms of engineering constants because we have a UTM that we can directly find out the Young's modulus E1 and similarly we can find out the Poisson's ratio or Shear modulus.

So in terms of that one can say that if it is a orthotropic material it will be Ex Young's modulus along x direction, 1 – mu xy and mu yx. So the Poisson's ratio mu xy not equal to mu yx for orthotropic material. This is very important thing and if you want to write in terms of number 1, number 2, number 3 index form, so E1, 1- mu 12, mu 21, which is equivalent you can say that C11.

Similarly, Q12, Q22, Q66, Q12 is the C12 there E2 1- this denominator remains constant, do you remember that thing see Q12 means mu12 and E2, say Q22 means E2, Q66 is nothing but Gxy or G12 C66. For the case of isotropic body that mu12 = mu21 then this relation reduced to E/1-mu square. So most of the time when you going to develop a plate governing equation for isotropic plate then you need to just E1 – mu square.

So you can obtain that Q11, similarly Q22 will also same and Q12 will be mu times of E 1 – mu square and this standard set of relations are given in the book. You can go through K. Chandrasekhar book or K. Bhaskara book, both things are given. You have to just be aware okay these kinds of things are available, you need not remember that what is Q11, what is expression of this just you must know okay they are something like Q11 and Q22 and they are the reduced stiffness matrix.

And their elements look like that, so for when we are going to apply a plane stress assumption then we have to use Q11, Q22 instead of normal stiffness matrix.

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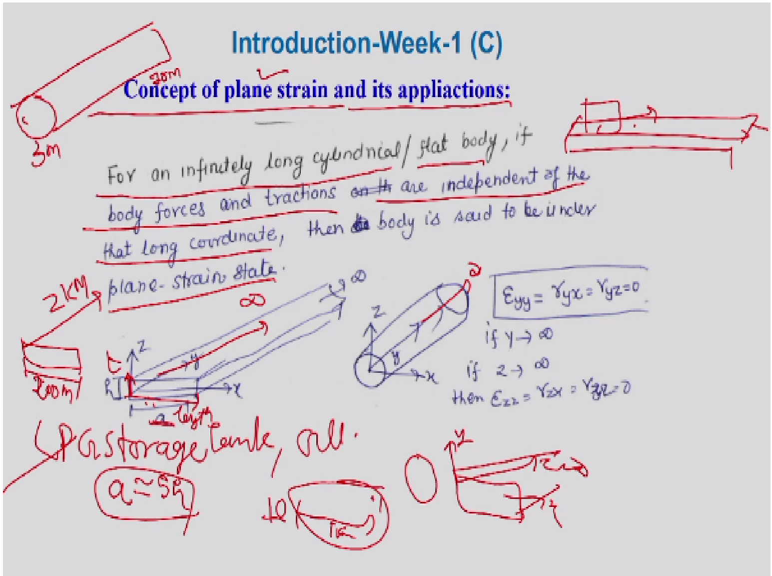

Now a concept of plane strain and its applications. Plane stress I have told you that thickness is very small, what is the concept of plane strain when you are going to analyse. Let us say there is a bridge or there is LPG storage tanker, I would like do it here, LPG storage tank or you say that oil storage tank, you will feel that whatever the radius and this length may be up to 70 meter.

And this may be up to 2 meter or 3 metre or you are talking about a bridge civil engineering structures, let us say in that bridge width and this length goes up to 2 kilometre. This may be around 200 meter or more than that even 100 metre maybe and this length maybe 200 km or if in the big cities you see these metro flyovers like specifically in Delhi or in Bombay are in Calcutta.

The flyovers over which metros is running, they can also be analysed under the case of plane strain. So one major thing you have observed that if one direction length is infinity okay and these 2, this is your thickness direction and this is your length direction, then we treat as a plane strain case.

You may question that it can be treated as a beam, but the condition of a beam is that this length a nearly or equal to the maximum 5 times of h, not more than that, but if is a is bigger than that for the case of thickness let us say it maybe 10 times you go for that at least 1 metre or it may be 0.1 so 10 ratio, ratio of 10 then you cannot treat this structural element as a beam element, then you have to say it is as if plane strain element.

Now you have got the concept, when plane strain element the width is considerable more than the thickness by at least 10 times the thickness, then you have to treat as a plane strain element. If your thickness and width is equal to 2 or maximum up to 5 then you can treat as a beam and that comes under plane stress case, but it is a plane strain case.

Similarly, a cylinder which is very long here also the co-ordinate system plays a major role in which you took the direction. So in some of the olden book when we talk about a plane strain case they said that or you say that it is a flat panel that this axis is x, this axis is y and z is infinity along the z direction, then we say that under the case of plane strain epsilon zz, gamma zx and gamma yz will be 0.

It is saying that for an infinitely long cylinder or a flat body if body forces and tractions are independent of that long coordinate then the body is said to be under plane strain state. So if I took this is as my y axis in the case of plate if you just want resemblance then z is my thickness, x is my longitudinal and let us say y may be length and it is very long.

So in that case for that if you want to apply the plane strain assumption so components of the strain will be 0, epsilon yy, gamma yx, and gamma yz they have to be 0 if y tends to infinity which I have written here.

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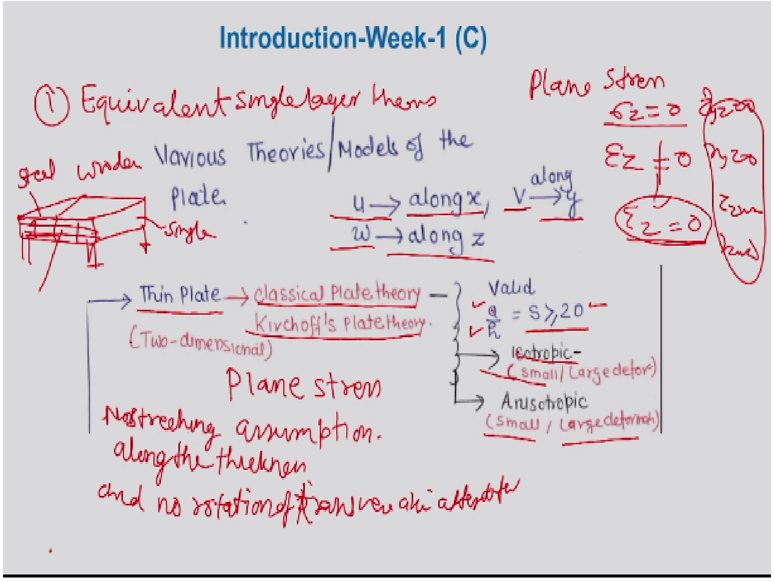

Now I am coming to I have told you that what is the plate, what is the beam, what is the shell, what is the plane stress assumption, what is the plane strain assumption. Now we have a number of models to analyse those plate problems. So we can categorise those theories into some basic classification code that let us say you displacement along x-axis v displacement along y-axis, w displacement along z axis.

So very first way is that equivalent single layer theories, under that head it means that we are analysing let us say a plate and is a single layer, just as your isotropic plate made of steel or a tabletop made of wooden, width maybe steel just as if one layer, but if you talk about a composite it may be multilayer case of thing. So here we can also apply these theories to analyse the composite plates or laminated plates.

But assumptions remain same that it treated as a plate as a single layer okay. So under that classification first is the thin plate theory or sometimes we call classical plate theory or Kirchoff's plate theory and these are the 2-dimensional plate theories and based on plane stress assumption. Their results valid holds when length to thickness ratio is greater than 20.

Suppose you develop a plate theory and get the result solving by equation of motion then you want to compare with the experimental results or the real analysis then it will hold good when a to h is greater than or equal to 20. Under this head there are theories applied for isotropic case, isotropic plates they are both small deformation considering as well as the large deformation consideration.

Similarly, thin plate theories also develop for the anisotropic material which I am saying that for let us say my plate is made up of composite or my plate is made of some different advanced materials which is anisotropic or orthotropic or transversely isotropic, we can develop the thin plate theory under that assumption. If the thickness is small compared to the length and width.

So there may be small or large deformation both cases theories were developed, in this theories basically that assumptions hold that no stretching along the thickness and no rotation of the plane, transverse axis, after deformation. So basically in these theories the shear stress or the normal, I must talk about the shear strain not the shear stresses, shear strain and normal shear strengths are neglected.

If you go to the under plane stress case sigma z is 0 okay, but it does not mean epsilon z is 0 right, it is not, but when we are going for a thin plate theory, specifically classical plate theory, there we are saying epsilon z is also zero. If tau yz is 0, it means gamma yz is 0, tau zx is 0, gamma zx is 0, they come from there, but this is an extra assumption.

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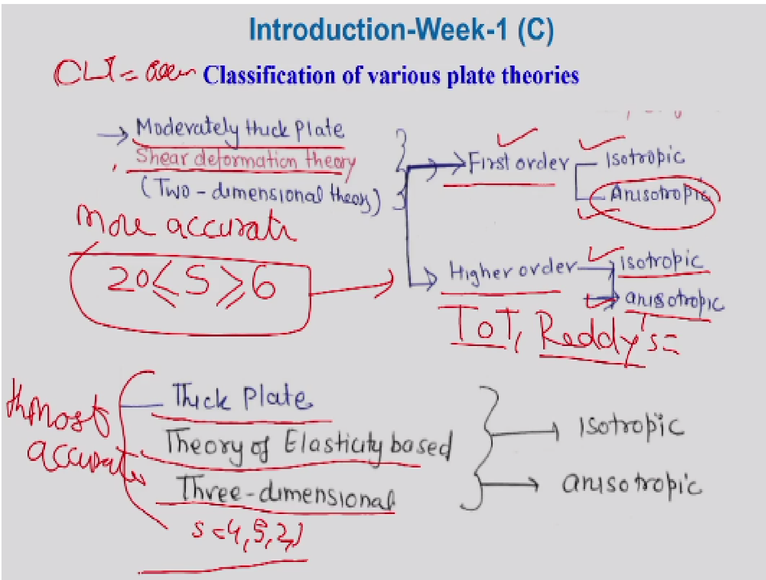

Next, moderately thick plate theories, here we assume that let us say thickness is playing a role and there is a deformation that it can resist the shear stresses or the transverse stresses. So that is why moderately think plate theories are also known as shear deformation theories and they are also 2-dimensional theory, they may be categorised like first order shear deformation theories and higher order shear deformation theories.

In higher order generally we take or terms like third order, fifth order, seventh order. I will explain later on why we are considering only third order, fifth order, not second order and fourth order. So first order theories for the isotropic and anisotropic, higher order isotropic and anisotropic. So I would like to emphasis here that let us say first order theory was developed for a isotropic plate later on in 1960s or 70s.

Now you have material is changed, your material is piezoelectric or your material is functionally graded which is an isotropic material or some other material advanced ceramic or you have included something else layer wise composite, then the same theory can be used to analyse anisotropic plate, people have done, developed their results for using the first order assumptions.

It has been found that first order theory gives slightly less accurate results and they require some shear correction factor for properly or accurately describing the shear stresses, but when we go for the higher order theories like the very famous theory TOT or Reddy's third order theory. So they do not require the shear correction factor and even these theories can predict the thick blood behaviour very accurately.

They are basically good for isotropic, when you go for anisotropic material one can develop third order theory for anisotropic plate but their behaviour may not be predicted very well because when you have a layer concept. So something extra element you require then thick plate theories, when your thickness is not negligible then you have to go for thick plate theory.

So that theories are generally based on theory of elasticity and these are 3-dimensional in nature. I would like to say that most accurate, here I would put that more accurate, not most and classical maybe just accurate. If you want to understand the mechanics or really very accurate estimation of the stresses or the strains or the different boundary effects, then 3-dimensional theories is required.

Whether the thickness is small or big they can capture both the effects, but for these theories you have to be in that range. So for this case S must be greater than I would like to say 6 or 5 and less than 20. So in that range higher order theories accurate more good results and this basically when you have S4, 5, 2, 3 whatever they can give you exact solutions.

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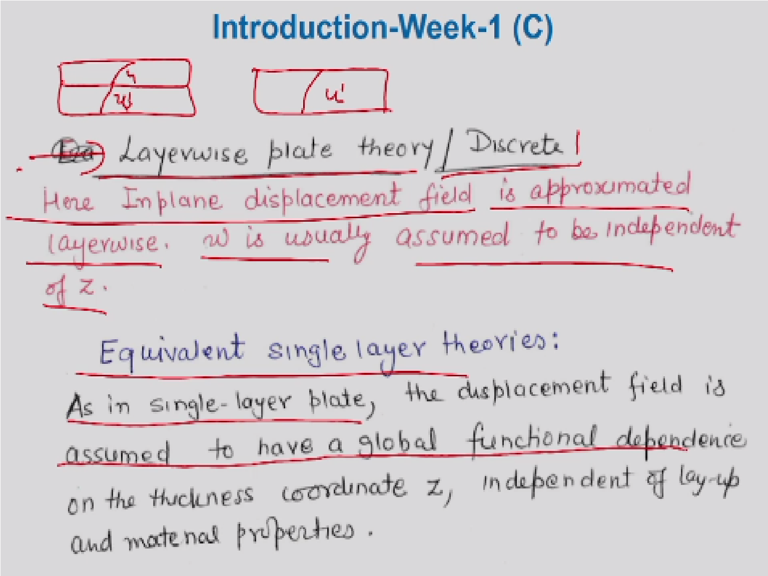

Now a concept when the plate is made of an isotropic material there is no layer wise concept than you do the analysis by using weather third order or higher order or classical if it is in that range accurately predict the behaviour, but when you have a concept of anisotropic material then those equivalent single layer theories produces inaccurate results not completely for but slightly inaccurate results.

So to overcome those limitations researchers developed that layer wise plate theories or discrete plate theories. Here in plane displacement fields is approximately layer wise whereas w is usually assumed to be independent of z, like if you have these layers that is a 2 layer so here to here you will have some variation that is U1 and here to here you will have U2. So each layer will have displacement field.

Previously it was having same thickness only one variation. So you see that as the number of layer increases, number of variable increases. So computational cost increases, the disadvantage of this theory, though it had predict the results very accurately and practically the composite laminate or you call a composite plate are made of at least 100 layers or 90 layers, very thin layers.

Sometimes it may be 50, but if that theory is applied you see that number of unknowns will increase too much then the equivalent single layer theories as I told it in the single layer plate displacement field assumed to have a global function dependence.

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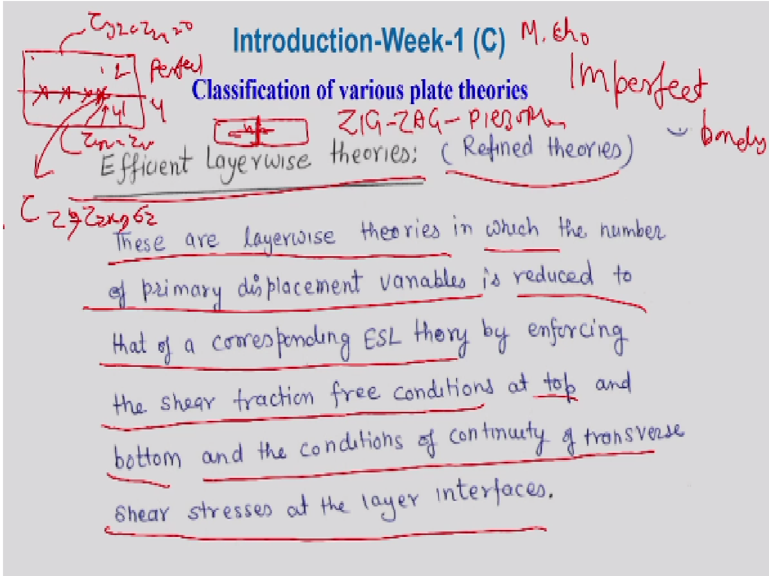

Then the researchers have come up okay, equivalent single layer theories are not giving good results and layer wise theories having that if you increase the layers number of variables increase, so number of differential equations you have to solve. So these days now if you talk about now, okay, we have a very good storage space, big computers, we can even go for layer wise, no problem, good results.

But it is in 1970s or 1980s or 1990s when we do not have that much of high capacity of computers. Then we talk about okay, some theories which can give lesser, which have some lesser unknowns. So a concept come that efficient layer wise theories or sometimes we called as refined theories.

So in this layer wise theories, these are layer wise theories in which number of primary displacement variables is reduced to that of a corresponding ESL theories is a major concept by enforcing the shear traction free conditions at the top and the bottom and the condition of continuity of transverse shear stresses at the layer interfaces. What is the meaning of this let us say if you have a plate and just I am giving an example of 2 layer.

It is saying that at the top and the bottom there are no shear stress, only top and bottom, and at this interface. For a perfect bonding case because materials are perfect, if it is not perfect (()) (38:06) So it will not provide the required function. So for the perfect bonding whatever the u thought it is a layer wise variation whatever u you will get from let us say at this u at this interface from the bottom approach.

And then you go for u2 from the top approach. So at this interface they should match. If you have gone through a finite element analysis or you have sometimes developed a solution in any abacus or and see, then there you will also in the elements that at the junction of that this should match if you get from there or there. Their variation is away from there but at this junction for a perfect bonding case.

So that is the height is known as continuity of transverse shear stress, so there anyway are saying that tau yz and tau zx basically transverse and sigma z has to be continuous over the surface as well as displacements latter on otherwise there will be slipping of the plate if this continuity is not maintained. So people have done the analysis perfect bonding. There you can assume that whatever you are getting from u there is a certain jump or whatever shear stress you are getting from the bottom and the top plate it may have different.

So in real scenario yes, but theory becomes more complicated that this theory will not be able then we have to consider imperfect bonding so it will have some more complex cases, but it will be near to the real picture and actually there is slight motion may take place, you cannot like if I talk a concept of engineering mechanism the perfectly rigid body say it is just a hypothesis that a body cannot deform but if you apply a pressure it may deform into slight that pinch.

Similarly, here there may be some imperfection there, so that can be taken care, but that theory will be slightly basic this concept will change. So I am talking about this layer wise theory where we say that condition of continuity of shear stresses and top and bottom free. So one of the theory which is developed by professor Kapuria for the piezoelectric plate, zigzag theory. For elastic plate it was developed by M. Cho, so these theories are known as efficient layer wise theories.

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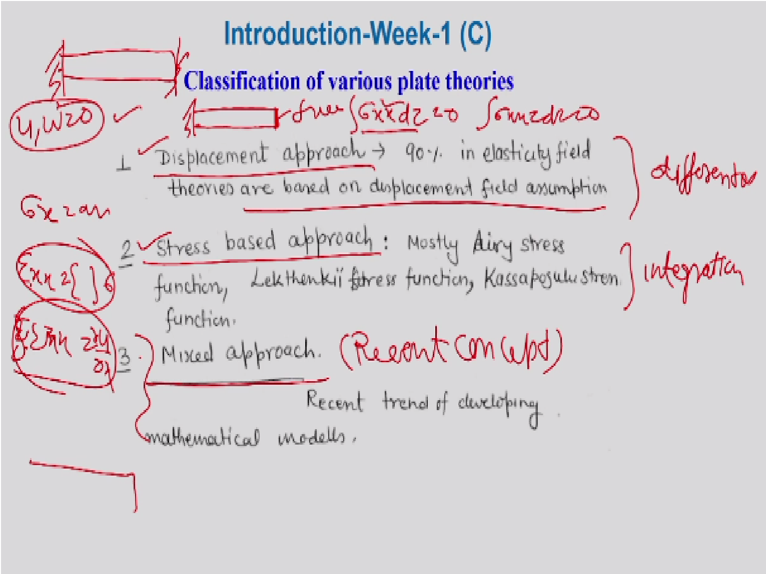

Now I would like to explain that you have thin, thick okay, you can define based on some geometric parameters, but there are classification based on the approach. So basically 3 approaches and there. One is the displacement based approach, then second is the stress based approach and third is the mixed approach which is the I would like to say recent concept.

So displacement based approach you will find 90% of the plate theories are based on the displacement field assumption. Under this approach we assume the displacement to be field first and then find out the stress and strain and then using the principle of virtual work develop a set of governing equation and solve it and the another case is the stress based approach.

If that stress is known or stress are assumed first and then by integrating you can find out that from the stresses you find out the strains and the strains, you find out the displacements. So reverse approach. So in this first approach you do differentiation, here you do integration. So you are going back, so stress you assumed something then you find out the strengths using the constitutive relations and then find out the integrating of u.

So generally in theory of elasticity we go that Airy's stress function, Lekthenkii stress functions or Kassapoglou stress functions, so you assume those functions and try to satisfy the equation of motion and solve those stresses. Once you know the stresses then you can find out the strengths using the consecutive relations, once you know the strengths then you can go for the displacements.

Then we have a missed approach concept under this mixed approach concept we take field variable displacement as well as the stresses. So generally if you see my research papers there you will find we have used the mixed approach for developing a 3-dimensional solutions. The concept why we are going to this, what is the discrepancy with that.

Generally, the boundary conditions, displacement based approach can give very accurate result if the boundaries are in terms of displacement. Let us say if we have a fixed beam, it means where u is 0 and w is 0 in terms of boundary conditions or in terms of displacement and let us say again you maybe have fixed. There may be cases so that displacement based approach that displacement boundaries are satisfied exactly.

But if here is free, it means displacement can be anything, but stress must have to be 0, stress resultant if I talk about. So this around the thickness direction we are satisfying the stress resultant conditions, not the stresses at every point, similarly the moment resultant has to be 0. So when we are interested in terms of some local effects then these theories may not provide accurate results.

Now come to the stress based approach, the boundary conditions in terms of stresses, they satisfied very nicely, but displacement boundary conditions again they have to be integrated form or some other form has to satisfy. So mixed approach is required because if you see the cantilever beam, it is a mixed kind of boundary conditions, at the one edge all the stress components are 0 at the other S all the displacements are 0.

So this is the mixed kind of boundary condition. So if we have a mixed approach in which you have a displacement as well as the stresses so you can satisfy the boundary condition exactly or I would like to say in a more accurate sense, if you talk about 3D then we say exactly, if you talk about 2D then we say that in more accurate sense.

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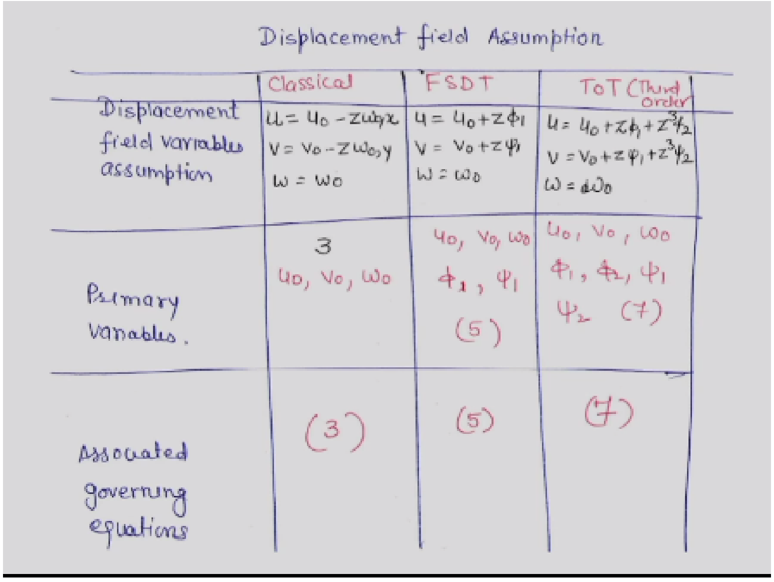

Then displacement field assumptions, this is the very important slide I would like to say that nobody gives you like this in a books that how much primary variables you have for a classical, FSDT on TOT. So I am going to tell you let us say for a classical theory so u is assumed as like this, v is assumed like this and w is assumed like this, so u have a 3 primary variables u0, v0, w0.

So just by looking your displacement field assumption I can tell you how many number of governing equation or equation of motion you will get, you will get 3 equation of motion. If you have FSDT theory, then you have 5 (()) (46:39) and 3 this so 3 + 2 there will be 5 unknowns. Then if you go to the third order, so you may take, sometimes they are not taking 5 as an unknown, they just replacing with this then it will also reduce to 5.

But if you go that let us say this is also unknown and this then it will be 7 and 7 equation of motion. So basically you have to solve 3 simultaneous differential equations, 5 simultaneous differential equations and 7 simultaneous differential equations. So solving simultaneous differential equations or developing a program for that is really huge task.

It is not that you are solving just algebraic equation like a matrix this thing. So later on for some cases it becomes like just a linear algebra if you talk about all are simply supported, but if you have different conditions or a Levy type support conditions then you have to solve OD, simultaneous OD, so their algorithms or some difficulty level are there.

So here our week 1 ends so I have completed the basic principles then the basic relations and the classification of cell theory. So next in the week 2 we will develop classical plate theory considering the (()) (48:13) linearity. Thank you.