**Theory of Rectangular Plates - Part 1**
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**Lecture – 14**
**Tutorial: Levy Solutions**

So welcome to tutorial 4. Here I would like to solve some problems based on Levy solutions.

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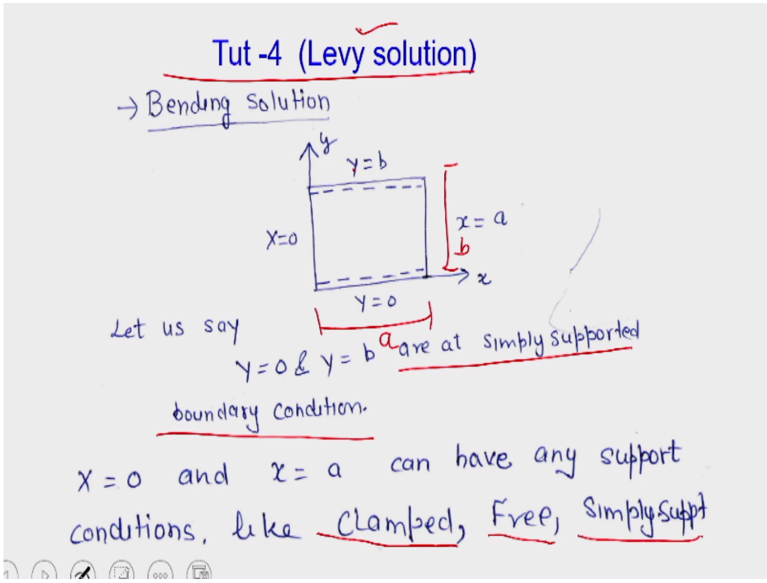


First of all, I would like to define a geometry let us say a rectangular plate whose length is a along x axis, width is b along y axis and y = 0 as y = b are at simply supported. What is the necessary condition for a Levy play that 2 opposite axis must be simply supported like this to opposite axis? Sometimes in some books you may find that x0 and xa are simply supported and the next that x0 and xa can have any support conditions.

What do you mean by telling any support condition? It means it may be clamped, free, or simply supported. x0 may be clamped, x = a may be free and so on.

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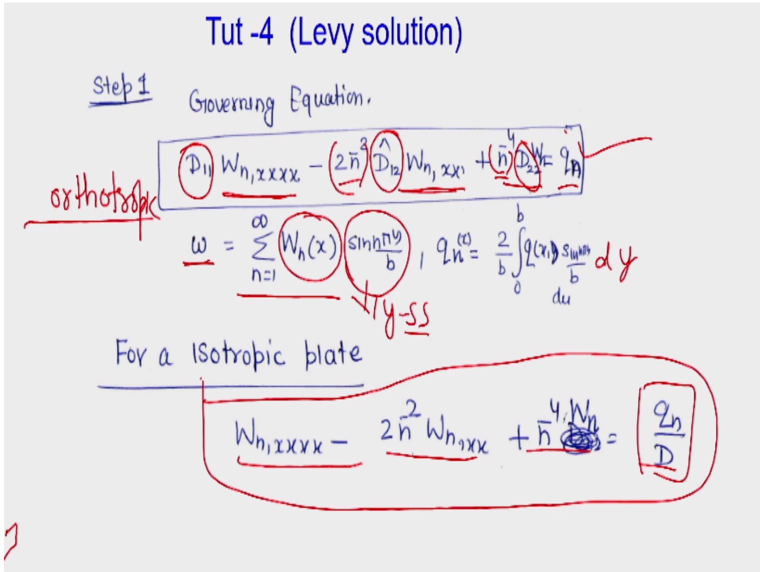


So very first step to analyze this problem is that suitable governing equation fourth order in x, see second order in x and wn and this is your qn. I am first giving you an equation for an orthotropic plate. So if somebody asks you that what will be deflection of a clamped free plate, Levy type and axis clamped and f is free and it is made of some orthotropic material. So, we will first come to this equation, substitute the value of D12, D22 and let us say that the loading is UDL or side loading.

Then accordingly find out this value n bar and n bar 4. So, basically this W is expressed like Wn which is a function of x and sine n * phi /b because along y axis is it simply supported and what is qn? qn is nothing but 2/b * q(x, y) sine * n * phi /b * dy. Now reduces to its reverse I am here going to explain you for an orthotropic plate similarly one can go for an orthotropic plate. So for an isotropic plate this d becomes d only. So d may common out going there. Now this equation we are going to solve.

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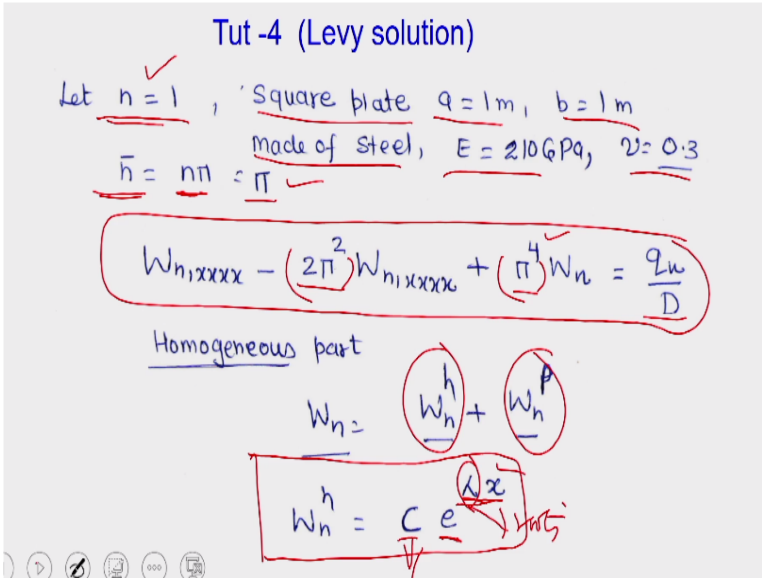


So very first case is that we say that let us say we are making some assumptions where n is 1, and a square plate a is 1, b is 1 and it is made of steel. E and these things are given. So n bar becomes only n * phi where n is 1 so it becomes 1. So this constant becomes 2 * phi square and these constants becomes phi * 4 square and qn/D. So this equation reduces to like this. Then again if you want to put the value of D now this is a differential equation fourth order.

So it will have a 2 solution, homogenous solutions and particular solution for the homogenous case this part will be = 0 and the homogenous solution is assumed like this. C is an arbitrary constant e raise to the power of lambda x. Lambda is unknown are the roots of the equation I would like to say that.

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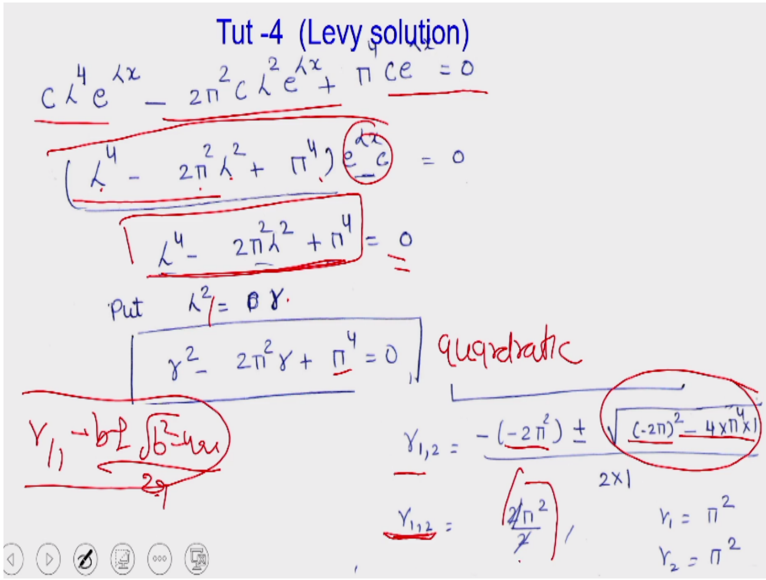


So you substitute it there. In that equation homogenous equation so it becomes a fourth order root, lambda 4 - 2 * phi square * lambda square + phi 4. So this is = 0. This cannot be so one has to be zero. So from here you substitute some terms. If you have a this kind of equation you can directly if you would have a program or something you can evaluate that a fourth order equation at 4 root or you come out lambda square = r and it becomes to a quadratic equation.

So everyone knows that roots of a quadratic equation is - b + - b square - 4 * ac/2a where a is the coefficient of r square, b is the coefficient of r, c is the coefficient of constant just c is a constant so in this way one can evaluate so I am writing that r 1, 2 is nothing but - b + - b square - 4. So you see that for isotropic plate this term becomes 0. You take any nn1 combination. You say n = 1, n = 2, n = 3 and 4 and so on always this will be in same order. So it is becoming gamma 1 and 2 phi square.

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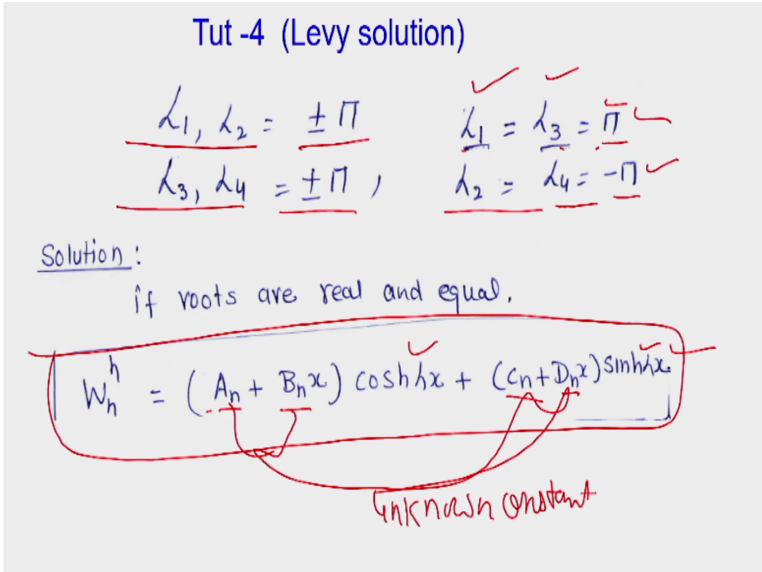


You have now roots lambda1, lambda 2 is + - phi . Lambda 3 and lambda 4 is + - phi . So now you have 4 root; 2 roots are same, phi and 2 roots are - phi . So already these things I have discussed in the course during the course that if my roots are real and same then the solution wnn would be look like this. We can write a solution like this that W and homogenous can be 2 roots that if roots are equal because we are writing for just for 4 roots if you have multiple then this shape will be slightly more different.

So (An + Bn of x) * cosh * lambda x + (Cn + Dnx) sin hyperbolic lambda x. So this is the form of my homogenous solution where lambda is phi again and An, Bn, Cn, Dn are the unknown constant. Why are saying An and Bn basically if you say that my n is 1 then it will be A1, B1, C1, D1. If you see n is 2, A2, B2, C2 for case of a plate uniformly distributed loading then you will at least require no number of constant. If you say that is sine loading, then first term is sufficient enough.

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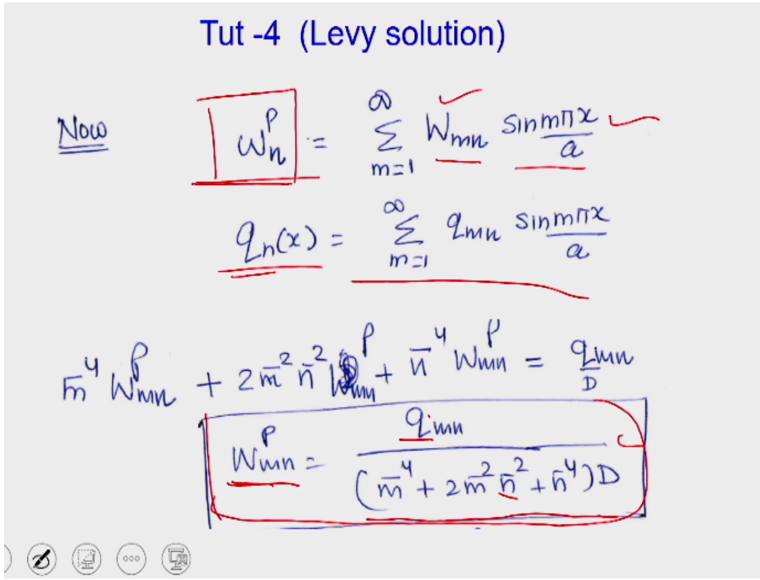


Then the next step, evaluation of a particular solution. So we are assuming that Wn p in this form Wmn * sin * m * phi * x/a and similarly qn can be represented like this and finally I would like to say that particular solution will be qmn/D mn for a case of isotropic plate like this m bar and m bar square and so on. So this qmn will depend upon the type of flow if it is UDL or side loading and so on.

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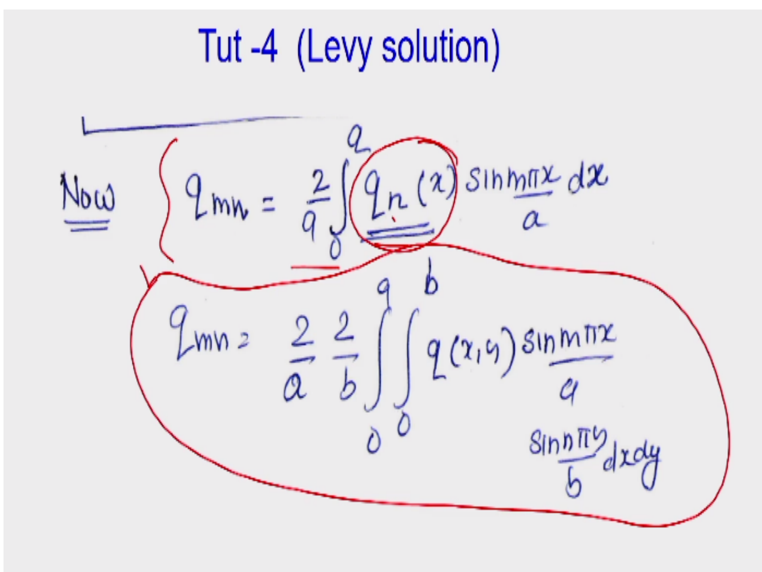


So qmn can be evaluated like this. Already, I have told you that qn is 0 to b so ultimately it can be represented like this.

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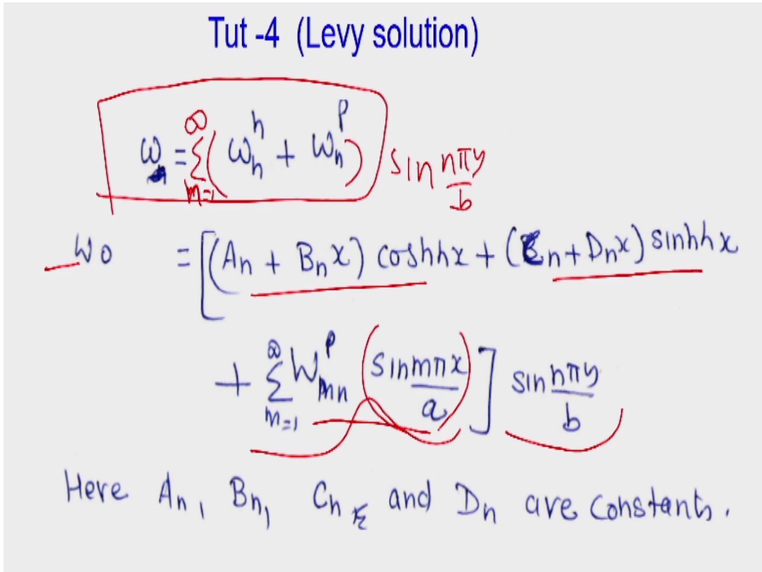


Now you write the final solution. Wnh + Wnp and sum n = 1 to infinity * sine * n * phi * y/b so W0 is finally written in this form. This was homogenous and particular solution sine m* phi * x/a and sin * n * phi * y/b. So here An, Bn, Cn, Dn are the constants which we need to evaluate and further this also C here is a function of sine. If you assume UDL then it will be a constant. It is a function of hydrostaticular or some kind of variation then it will be a function like this.

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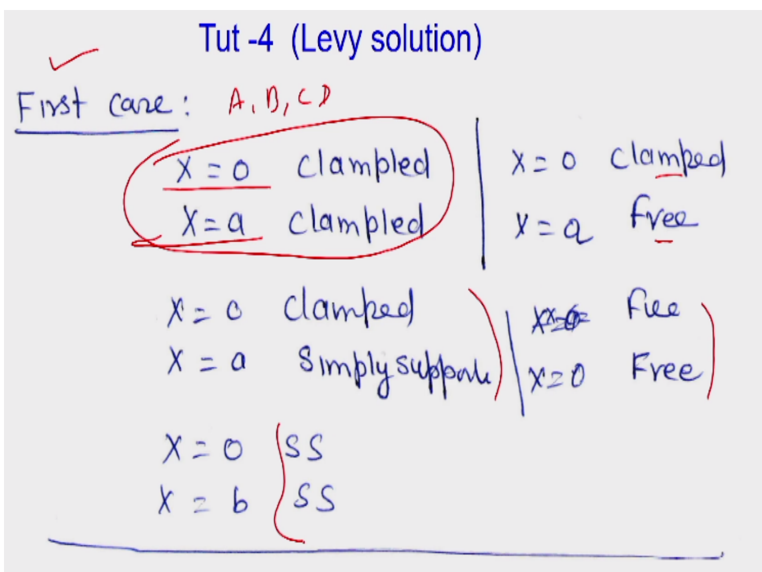


So now I am going to a special case that first what type of boundary conditions. So this is your the main form of deflection where An, Bn, Cn and Dn are the unknowns which you have to evaluate. So there these are depend upon the boundary conditions. If I say that my plate x = 0, x = a both are clamped so you will get some An, Bn, Dn. If I say that clamped and free different kind of that clamped and simply supported free and free. So any kind of boundary conditions one may assume and can solve for that even though simply supported one.

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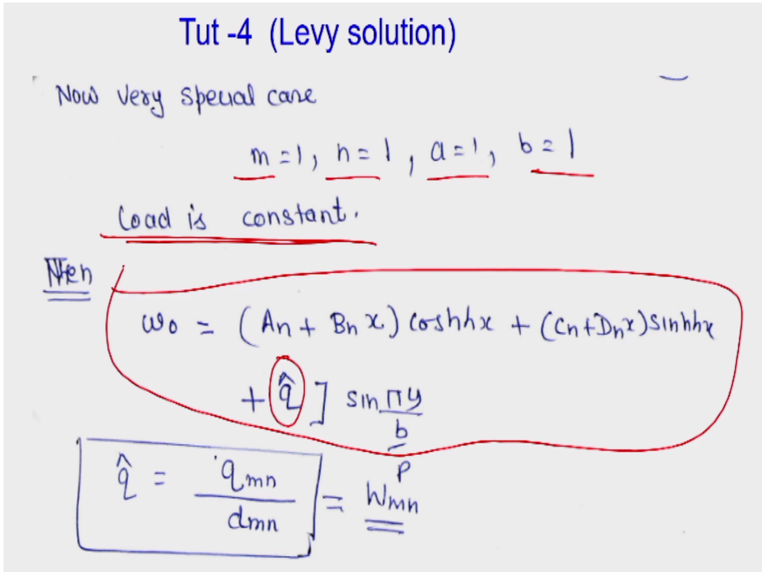


So now I am talking to a very special case where m is 1, n is 1, a is 1, b is 1 and load is constant. So in that case let us say q is q hat not sine, cos nothing. The solution can be represented like this. So again qmn is written as wmn of p.

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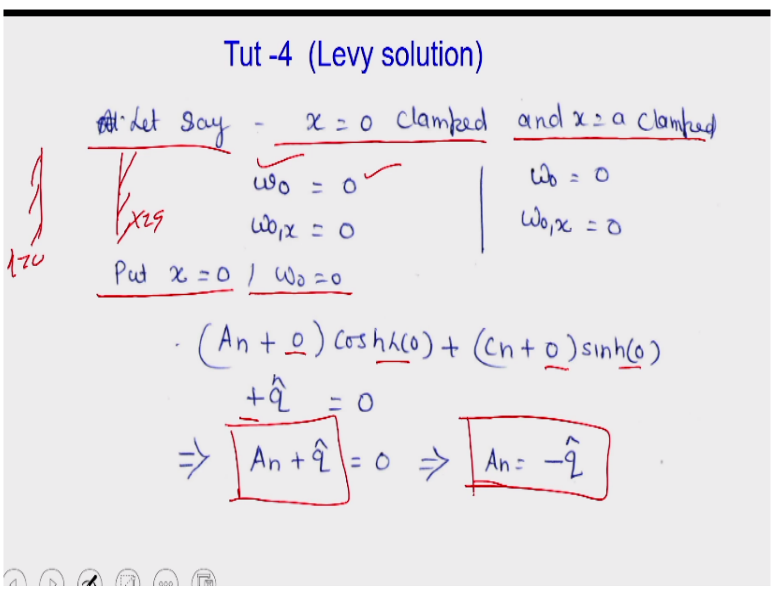


So let us say x = 0 clamped and x = a = is also clamped that both of the edges x0 as well as x = a are clamped. So you put x = 0 for that case, deflection must vanish. So put x = 0 in that form.

Then from there you come to know that An + q hat = An - q hat. So your first constant will be the negative of your load vector.

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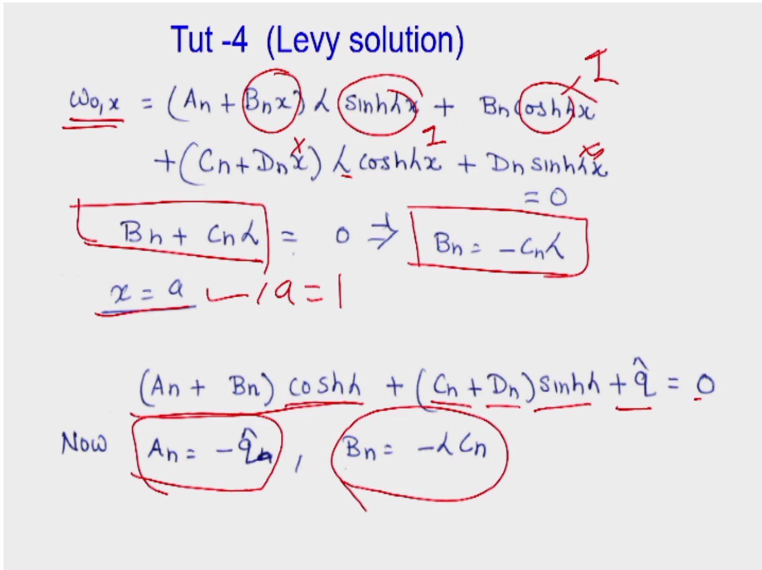


Then slope w, x at x = 0. So if you put here x = 0, 0 it becomes 1, it becomes 1, it becomes 0, it becomes 0. So from this equation Bn + Cn * lambda tells you that the coefficient of Bn = - lambda * of coefficient of Cn. Then x = a. Put x is a and we know that a = 1. So I have just simplified this expression that (An + Bn) * cos hyperbolic lambda + (Cn + Dn) sine hyperbolic lambda + q hat = 0 and substitute this. An = - q hat and Bn = - lambda Cn. If you substitute these values.

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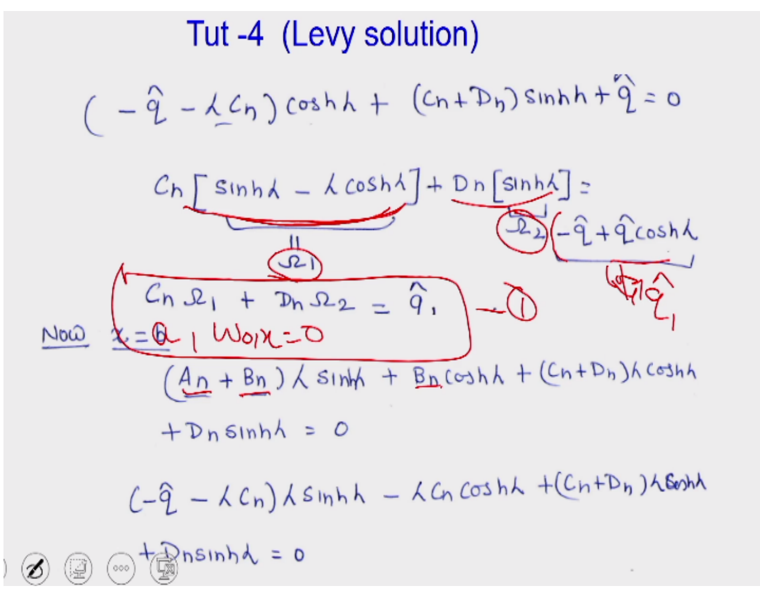

So it becomes Cn * some constants or some high sine Dn something and some loading constant. So I am saying so let us say these coefficients are noted as capital omega 1 and these coefficients are known as capital omega 2. So this is a load term and this loading is denoted as q hat 1. So you got this equation Cn and Dn 1 equation. Then at x = a, w, x = 0 slopes so put that A value in the slope equation and from there substitute the value of An and Bn wherever is Bn so it reduces to.

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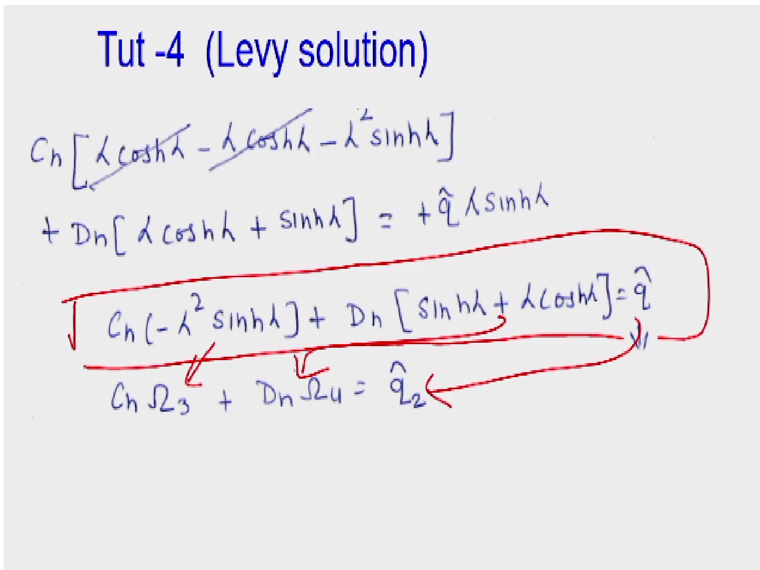


A equation like this where we say this is omega 3 and this is omega 4 and this is also q2 hat.

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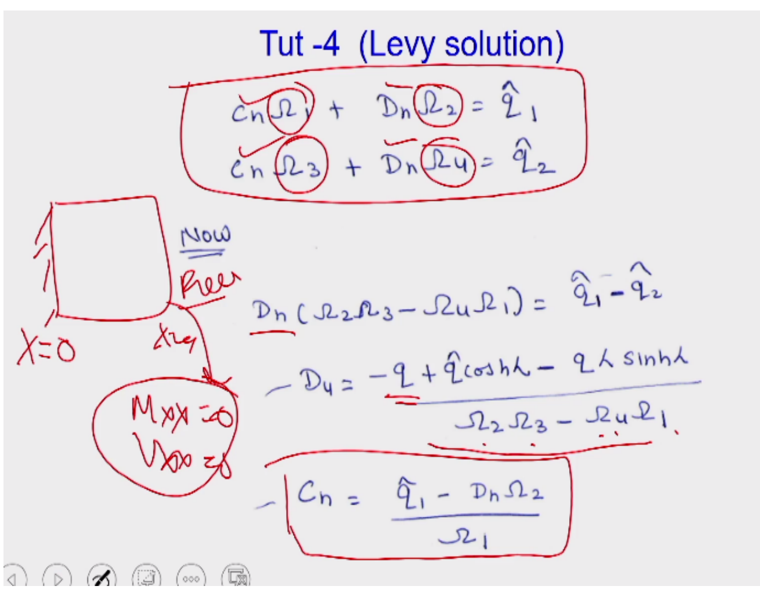

So now you have 2 equations and 2 unknowns one can solve class X problem that basically if you substitute the value of lambda hyperbolic of lambda can be evaluated so this becomes a number basically. These are your numbers. So Dn can be evaluated if I multiply with this and q hat - q2/omega 2 * omega 3 - omega 4 * omega 1.

Similarly, Cn can be evaluated. Now we have evaluated all the constants. For a particular boundary condition that x = clamped and xa = clamped. Suppose if I ask x = 0 is clamped, but x = a is free or simply supported free so for this case here moment and Vxx needs to be specified. So there are boundary conditions so there are some different kind of variation may come, but the procedure remains same.

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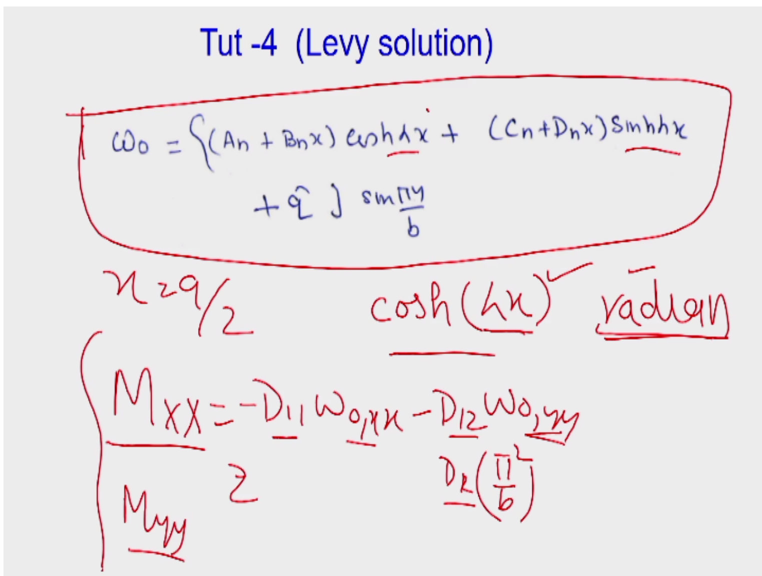


So finally deflection can be written like this. An, Bn and for particular if you say that central deflection that x = a/2. One can evaluate cos hyperbolic, sine hyperbolic. One generally students makes a mistake that when they evaluate using the calculator so these things are in meter or something so basically you have to say that radiant so in calculator radiant function should be 1 if you evaluated in terms of a degree then you will get a wrong number.

So wrong deflection so all these things are in radiant. So accordingly one has to evaluate. So first I had given this thing to one of my M. Tech student and they said okay everything evaluated by just assuming that degree, degree and that deflection and moment all things were coming along, so then I realized this you must remember this thing whenever we do things in the plate so cos hyperbolic or sine x * sine phi all these if things has to reevaluate in terms of a radian.

So, now if somebody is interested in terms of a moment or this plate it will be - D11 of W, xx - D12 of W, yy. So w, yy (FL) (16:23-16:25) only phi square/b D12 and the same term similarly you take double derivative of this substitute those values and evaluate that similarly evaluate Myy and so on. So now by going through this tutorial 4 you have learned that how to solve a Levy plate. How to solve the different constants?

So one may write a program very simple program in Matlab that lambda 1, lambda 2, lambda 3, all are given evaluated Bn, evaluate qn for a different material may be isotropic, may be for a steel and then you change the material to aluminum and if you are very much interested go for a orthotropic one may try for that. So with this our tutorial 4 ends here.