Theory of Rectangular Plates-Part 1
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Lecture – 13
Matlab Coding + ABAQUS

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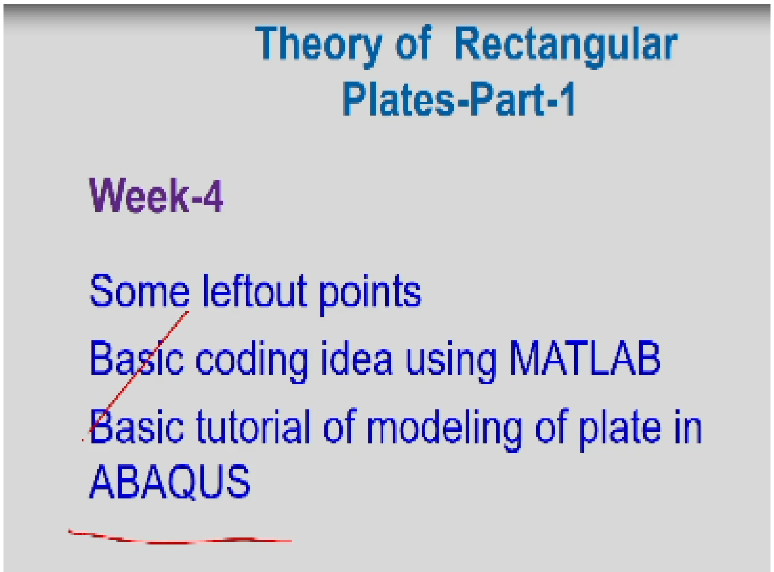

Welcome to our last lecture in this series Theory of Rectangular Plates Part 1. So, I have covered from the basic equation of motion or the constitutive relations. Then I have developed classical plate theory then gave the solution Navier and Levy solution and the 4th I have given that basic structure to develop a solution for a case of Ritz or Kantorovich. Then I have given the some information about how to develop a 3D solution for a rectangular plate.

Now while going through all this lectures you have come to know that how to develop a governing differential equation for a plate or how to analyze for that its bending effect or its frequency or its buckling critical buckling. So, all these things you have done and now I am going to test some of the very basic requirement and what is the outcome and why we are all going or studying these things?

Then we have to prepare or one to analyze a plate can we just do by a calculator. Okay for a single layer if you talk about navier solution you may try for it or you can try that just using the calculator. But as you are going to use for multilayer plate, composite plate, sandwich plate later on plate made of some advanced material like functionally graded or piezoelectric material. Then you have to write your own program or a code so it is my; I would like to share experience that some of the master's students who joined us when I ask okay.

Do you know some idea about programming? very first they said no we do not have any idea or if I ask to write a small code of a some like a quadratic equation or the Navier solution. They are not able to do that. So, now I feel that if I am going to give you some basic hints that how to write a simple code. If you have analyses in navier solution or levy solution based on classical so later on you can modify the codes or you can include more effects so this, I am going to tell.

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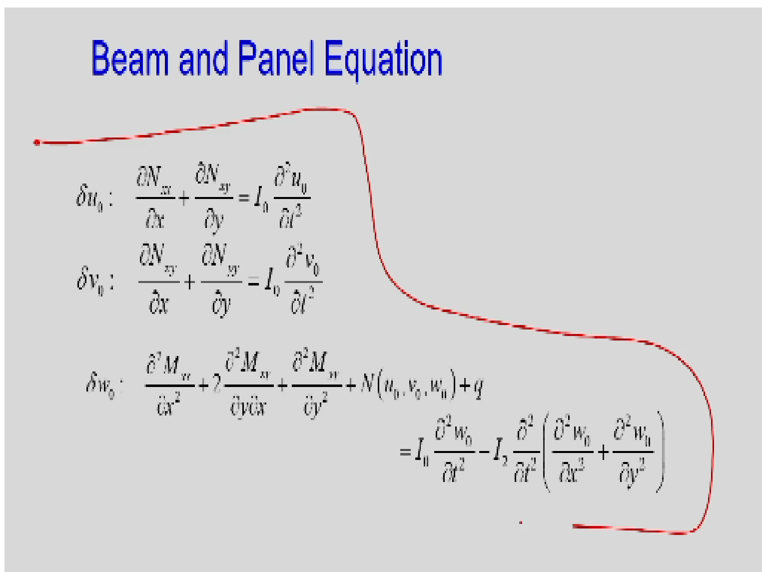

Next here you see that we have a 3 primary variables u0, v0 and w0. So, we get 3 I would like to say that partial differential equations that derivative along x, derivative along y and partial differential equations in x and y. If we have a more number of primary variables like in FSDT or in third order or in higher order maybe 7 primary variables, then we will have 7 corresponding partial differential equations.

Why I am coming to this, I would like to tell you that let us say you have developed a solution for a plate any time you can convert it for getting equation of Beams or Panels. So, in the Beams there is no y coordinates and no dependent on the that so this equation will not contribute and this will not be there and similarly here this will not be and this will not be and some of the components will not be there and here this will not be there.

So, this equation reduces to a beam equation or a panel equation suppose in future you develop a sort of certain partial differential equation for a shell. So, you can get a special case for a plate and for a beam and later on you may presently we have developed a solution for by considering classical laminate theory and later we may go for a higher order theory. Then also from higher order theory by taking some special assumptions you can get back to CLT theory.

So, my suggestion is that when you are going for a theoretical formulation try to develop a general formulation So, for the result point of view you can go for specific let us say you develop a solution for theoretical formulation for younger plate then you can go for a solution for cross by plate Navier solution because this is valid for cross by plate. Then similarly if you have you can sometimes very famous this unified theories.

That using some index is delta 1, delta 2 and delta 3 where is single formulation based on using this Index will give you results classical as well as 1st order and 3rd order or whatever your special case may be refined or layer wise so you may go like that. So, you need not do again and again make those formulations so for your PSD program or your master programs or some general program point of view try to develop a general formulation and come to specific one.

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Programming for Navier Solution

$$w_0 = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left[\frac{(q_{mn})}{D_{11}m^4 + D_{22}n^4 + (2D_{12}+4D_{66})m^2n^2}\right]\sin\bar{m}x\sin\bar{n}y$$

$$q_{mn} = \frac{4}{ab}\int_0^b\int_0^a q(x,y)\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\,dx\,dy$$

$$M_{xx} = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left(B_{x}W_{mn} - M_{mn}^1\right)\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$$

$$M_{yy} = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left(B_{y}W_{mn} - M_{mn}^2\right)\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$$

$$B_{xx} = D_{11}\left(\frac{m\pi}{a}\right)^2 + D_{12}\left(\frac{n\pi}{b}\right)^2$$

$$B_{yy} = D_{12}\left(\frac{m\pi}{a}\right)^2 + D_{22}\left(\frac{n\pi}{b}\right)^2$$

Now see for a Navier solution this is the form of deflection where qmn can be find out using this form. Okay if you substitute this qmn there and taking the summation of this will give you the deflection off a plate under bending case. Very first question that a student asked that this series if you say that udl load uniformly distributed load for that case qmn comes I would say non-zero for m =1,3 and 5 and this also = 1,3 and 5 and so on.

Okay but how will you evaluate this that the student asked madam it will be just 11 33 or 55 will only be this as contribution or we will have 11 13 15 and if we talk about again instead of 11 13 15 then 31 33 35 all combination of that are odd combinations will contribute so while writing theoretically it looks very simple but when you go for a programming or when you go interested to actually calculate then these questions come to your mind.

That what should I take 11 or 13 or 31 or 0 or just the diagonal terms or how to make the loop that every m and n is going to be evaluated and how to store the data. Okay this was the question and now similarly we have a question it depends on sign mn and bar x bar y and here m and n. So, again you have a 4 different m n value you will get some values summation of that and like that. I am going to expose you some very small.

I would like to say that beginner steps let us say m=1, n=1 and evaluate all these things for first that case then manually go for m=1 and n=3 then try to evaluate and similarly m=3 and n=1. So, you will find then m=3 and n=3. In this way you are going to move so how to store and how to write those kind of loops. Now similarly you have a moment here.

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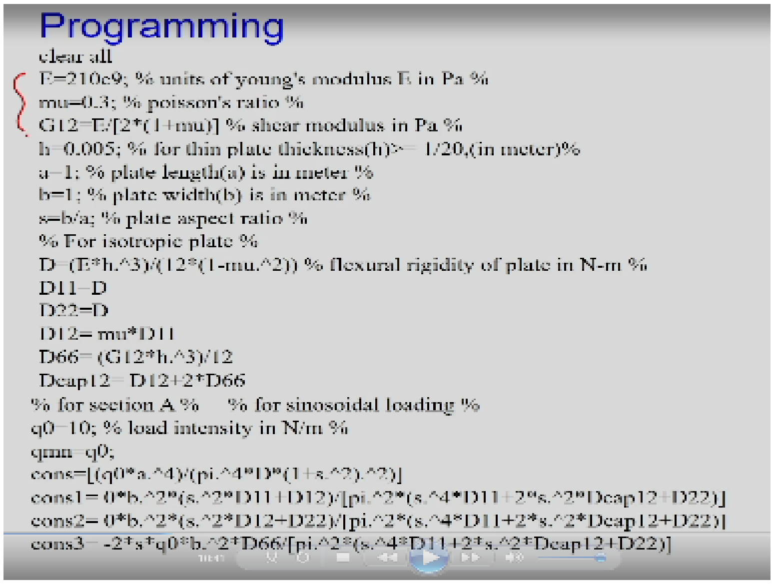

So, there is the thing whenever you write a program or a code first of all you must write in a general form try to give in so that through the inputs. Let us say you write say only for isotropic you give D ultimately and later on somebody say my E is changed or I have changed the mu and how will the deflection look like or what is the plot of that your guide will say that. So, you write in terms of that and you give input.

Like here we have written in and this is also basically I would say it is a hard form and it is not soft form and it should input through outside and it should ask enter the value of E or value of mu or the material is isotropic or orthotropic you may ask. For isotropic you ask only 2 constant if you are interested to evaluate for an orthotropic plate you may ask for some more constants so basically this is for isotropic plate.

So, under the value of E then mu and evaluate G12 and given the value of thickness of the plate so generally it should be >= or they have taken as 100 so sometime you may just take h as if you may use 20 and it will be 0.05 then length of the plate, width of the plate and aspect ratio b/a sometimes you may use that. Now you say that isotropic plate D is the value and bending stiffness.

Now D11 D12 and D 22 D66 and D cap basically like this you can evaluate. So, first of all like we have different sections that a plate is subjected to a sinusoidal loading, plate is subjected to a udl loading and plate is subjected to a linear vary loading. So, you will make comment that for section A for sinusoidal and for q0 you have taken 10 and whatever you want to take intensity of the pressure so for sin loading it will be = q0.

So for different constant so what are this con1, con2 and con3 go back these are nothing but this is con 1 and for this, this is con 2 and for this, this is con 3 like this you are evaluating that.

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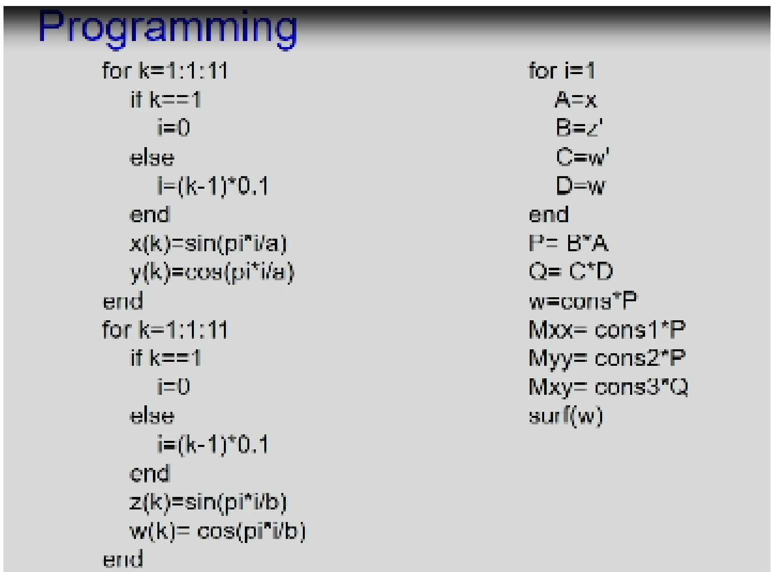

Then we have to say that if you are interested to for a plot along by taking 10 points along x axis and 10 point along y axis. So, how do you evaluate that value and going to store in a array form. So, basically this loop is written that k going from 1 to 1 and if k=1 and i=0 else as this. You are going to evaluate sin pi x/a and sin pi y/b. Okay then similarly some other constants they may be required for calculating the deflection.

So, basically A B C Z and so they have say that x z dash w dash and Dw and then again some constants is required cons * P. This will give you the value of deflection.

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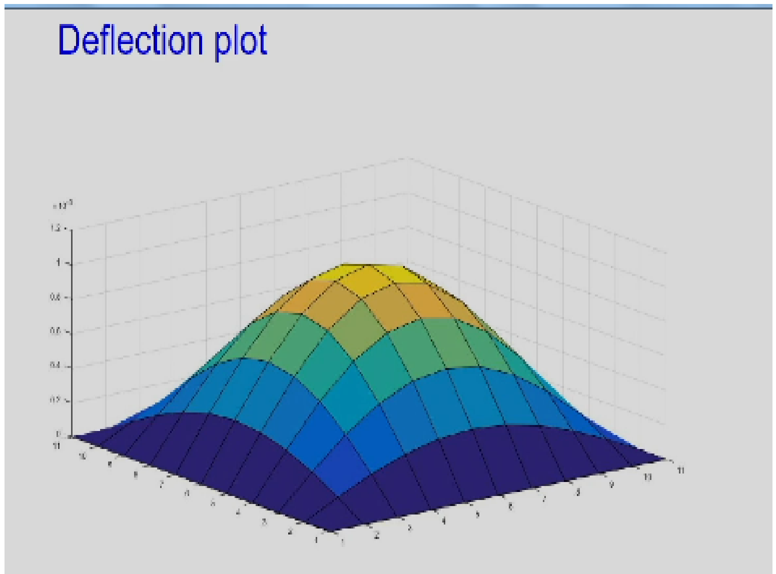

Now you see that along actually this is to be .1, .2 in that form if divided by 10 then similarly here alone y axis and x axis and z axis. This is the plot of w so this is the 3 dimensional plot or sometimes we call it as surface plot. Then how the moments varies from over that area so this is he graph of a moment it is plotted in MATLAB you can go through.

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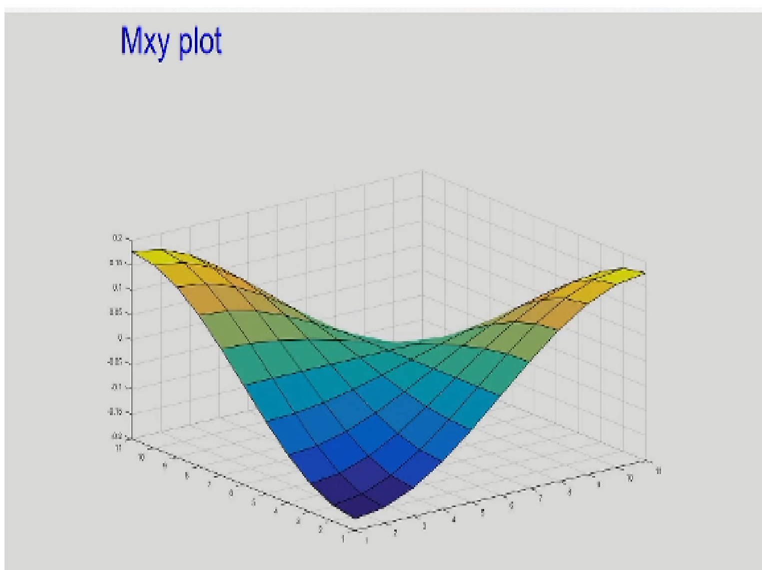

Now you see this is the moment Mxy so student come up over and this looks entirely different then I say no it is fine because when x is 0 y is 0 it must have some value. Because it is cos cos function. So, definitely it will have some non-zero value there. If you go to previous what should be there moment is 0 along this these all are simply supported similarly here and in the center of the plate moment will be highest.

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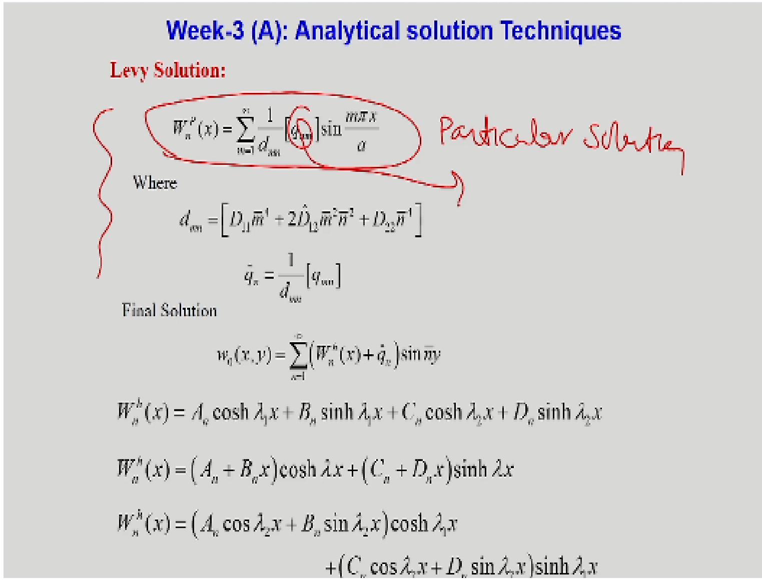

Now I am going to give you a brief code or I would like to say that beginners code that how to obtain the Levy solution and you all know that this is your particular solution and where qmn is you can evaluate any time 4/ab so something function of k sin and sin dxt like that and so on. Now dmn is this qn hat we have already defined these things in our previous lectures. And the final solution can be written as homogenous solution and q hat particular solutions.

Now Homogenous solution depend upon the characteristic equation that lambda 4, 4 roots whether they are real, different real or same or a complex conjugate. Based on that this homogenous form will be varying. Now you see that how to write a program to evaluate this W0, whatever you want all the things are here. Now again that here lambda 1 if you talk about a for a complex conjugate lambda 2 is the imaginary root and lambda 1 is the real root.

Real part of that basically is like lambda 1+- lambda 2 of I then -lambda 1 +-lambda 2 of like that for a complex, here lambda 1 +- lambda 2 like this where it is a real case. So, you must to be careful the meaning of this so most of the books it is given or you can write in better way lambda 1 lambda 2 lambda 3 ,4 in that way you can also write. But if what you are writing in this form it is given in the Professor J. N. Reddy book that lambda 2 is this and lambda 1 is this.

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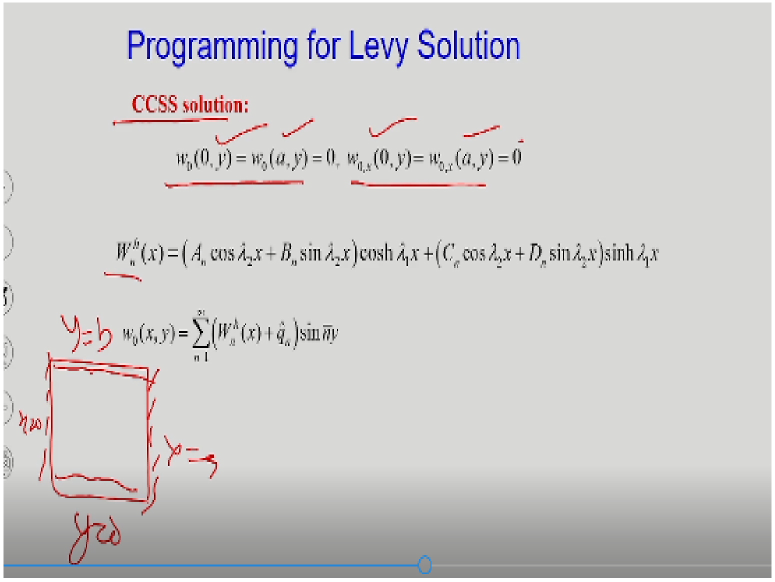

So, we are interested to solve a plate where y=0 and y=b are simply supported and x=0 clamped and x=a clamped What is the condition for a levy type solution there are 2 opposite must be simply supported. So, along the x axis x0 and xa deflection must be 0 and slope must be 0 and your homogeneous solution most typical case when it is a complex conjugate. For a real system it is very simple but when our roots are in a complex conjugate form then it is a difficult one.

So, for that case homogenous solution can be written as this and qn and sin ny this is your final solution.

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```
clc
clear all
E1=210e9; % longitudinal modulus of elasticity in Pa %
E2=210e9; % transverse modulus of elasticity in Pa %
mu12=0.3; % poisson's ratio %
mu21=(mu12*E2)/E1
G12=E1/[2*(1+mu12)] % shear modulus in Pa %
h=0.05; % for thin plate thickness(h)>= 1/20,(in meter)%
a=1; % plate length(a) is in meter %
b=1; % plate width(b) is in meter %
s=b/a; % plate aspect ratio %
% for orthotropic plate for n=1 %
D11= E1*h.^3/(12*(1-mu12*mu21))
D22= E2*h.^3/(12*(1-mu12*mu21))
D12= mu12*D22
D66= (G12*h.^3)/12
Dcap12= D12+2*D66
% roots of lambda %
lambda=[D11 0 2*[(pi/b).^2]*Dcap12 0 D22*(pi/b).^4];
A=roots(lambda)
B=real(A)
C=imag(A)
% when roots are complex conjugate and (B>0 & C>0) %
```

Now you see in the code you may for an isotropic plate or a orthotropic plate because we have input all the data E1 E2 mu12 mu21 and G12 later on recently I have modified also that you can also enter G12 also and then thickness and all these things. Then evaluate D11 D22, D12 D66 Dcap now you have a characterize sketching equations. Which is that lambda 4 D11-twice pi n/b whole square D cap12 lambda square+D22 pi n.

Or sometimes I would like to say that 4 times=0 this is the equation. So, you write A lambda 4 th order equation interested to find out the roots. If you are interested to find out the root of a quadratic equation suppose in a calculator you give co efficient of x square, co efficient of x ABCD like that. Similarly, this is the fourth order so you are giving just a coefficient of that. So, lambda 4 (FL) co efficient D11.

There is nothing lambda cube 0 for lambda 2 it should be – of this thing then again lambda 0 and this is the constant. So, I can say roots of this so it will give you the all the roots 4 roots lambda 1 lambda2 lambda 3 lambda 4. So, if they are complex roots so it will write as we divide into 2 parts real part and imaginary part.

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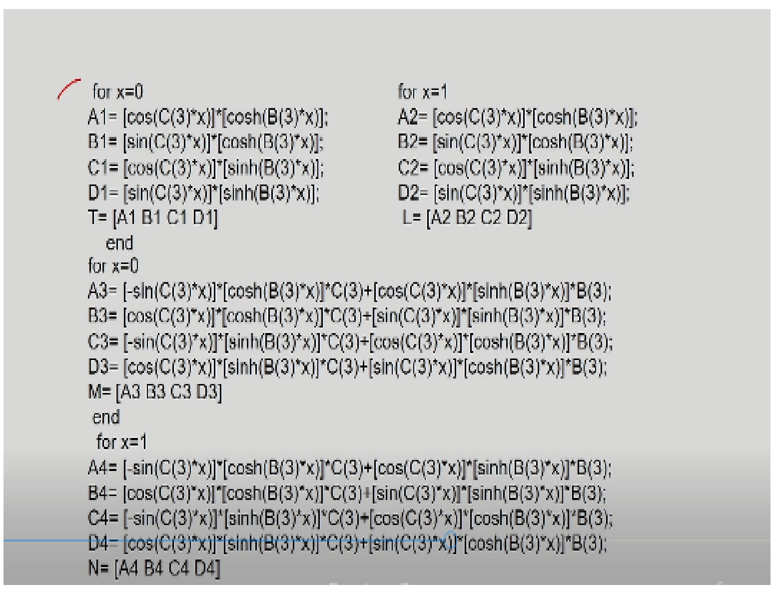

Then you write a boundary condition so before coming to the boundary conditions I would like to come here that these are the unknown constants C1 D1 that we have to find it out. So, when x is 0 w0 is 0 so if you put here cos 01 this will become 0 then again so basically An and Cn= + some qn=0. When x=1 then again deflection is 0. So, same thing we have written in terms of a code for x=0.

So, coefficient of A1 so they have written and B1 C1 and D1 and we have arranged in terms of a row vector. We evaluate similarly x=1 you have to go to end here so A2 B2 C2 like that when x=1 then we are talking about w, x has to be 0. So, evaluate w, x so this is the form okay similarly B3 C3 D3 you evaluate and arrange in this form and arrange. When x=1 so same things just you have to put x=1.

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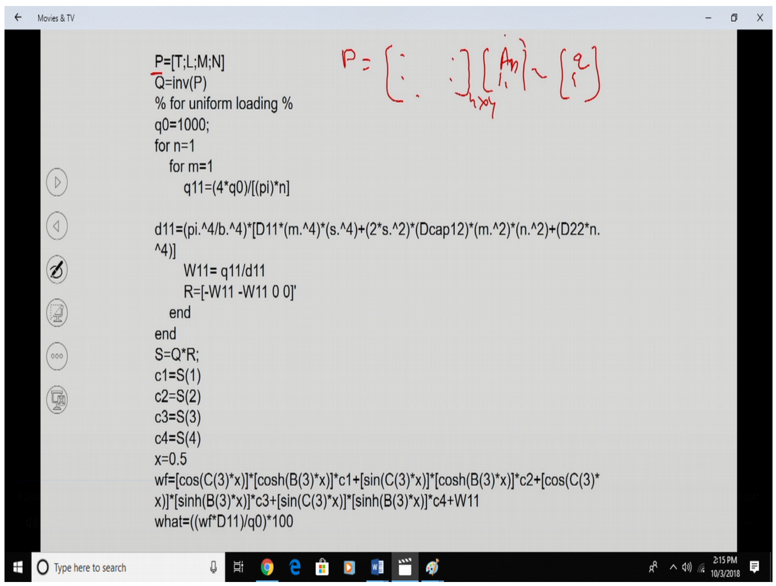

Now you say that a matrix p will condense all the coefficient 4/4 matrix and ultimately solution is that *A1 basically A and B and so and load vector. So, basically this can be find out p inverse of this metrics, so we have contained q inverse of p then we obtain a R matrix loading vector. So, for a Udl case this will be like this and ultimately when you take a derivative since that is a constant so it loading along that will become 0 and recontribution in this form.

So, S the solution will be Q. or Q*R and then you can arrange this C1 C2 C3 4 unknowns S1 S2 S3 S4. Now you can evaluate x=0.5 that is maximum where deflection will be maximum that wf can be written in terms of final form. So, something w11 yes we would like to say that particular summation and again y=0.5 for that it will be sin n/pi will be 1. In this way, one can write the simple code for themselves or for understanding point of view.

Or later on if you are interested to develop for a solution for a composite plate or some difficult geometry or boundary conditions similar way one can write a simple code in MATLAB. So, for programming point of view we do not rely only one source. Suppose we have written a code in your MATLAB but for a simple case you can check okay every step no problem but sometimes code is very big.

And you do not know where is the mistake and how to verify that. So, basically you can verify the end result the final solution with whom either you check with the adjusting papers in the literature in which the results are given from that you can validate for simply supported case at least results are available. Sometimes we choose a boundary condition or the loading in such a way that nobody has done that.

So, how do you verify whatever you have written here your analytical study or your own finite element. So, you need a second reference these days as a second reference we choose a commercial softwares. Sometimes we used to do get the solutions from ABAQUS or in sense it depends upon the instructor choice that to in which he or she is comfortable. So, we used to seam model that plate and get the deflection plot or the stresses.

And the deflection from our program and try to verify that. SO, in this way we used to verify our codes. So, one has to be when a person is working in a computational mechanics he wants to be aware of the second reference. It is very, very much required whatever code you have written remember lot of mistakes like recently I found that it should be here it should be -1 otherwise this will be some issue will be there.

So, this kind of things and there be some of more mistakes may be there so when you are writing and going to check that each step. So, for that purpose we need to have so currently we are using.

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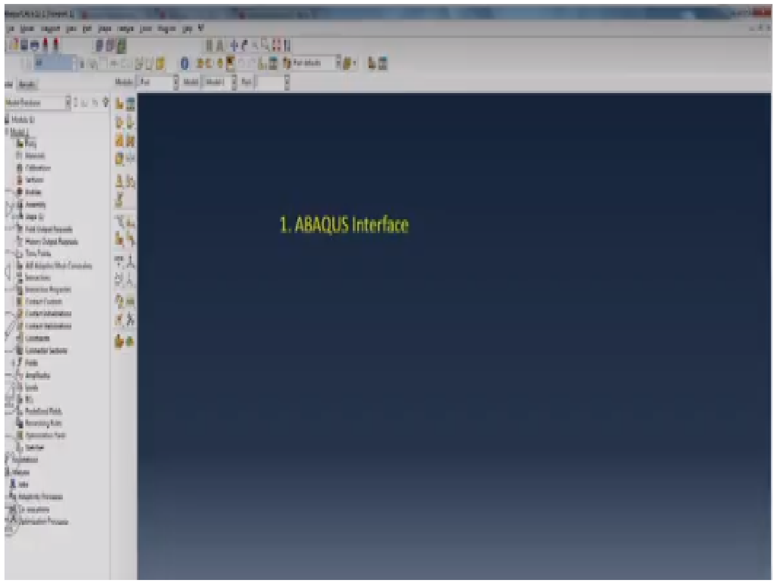

Like I used to write a code in Fortran and get the solution and went to verify with the other solutions. So, there are standard ABAQUS software is there so in this slide basically the very first are there is sometimes some of the students may be aware of how the window look like that. So, I am just going to tell you in that you have to first come to a path module then the material then you have to assign the sections and so on.

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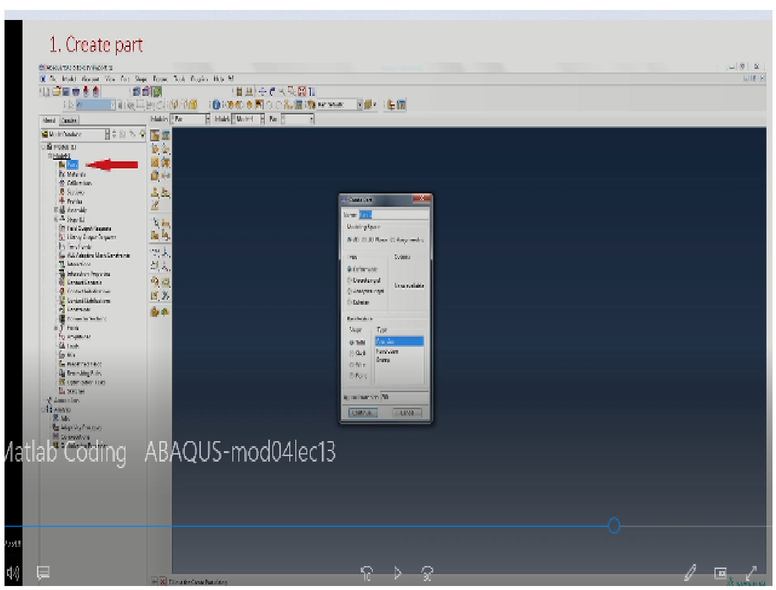

Now in this slide I am going to get this slide so that how to make a part of all the stuffs are given in this slide. At first we have to choose a path whether you want to go for a solid, shell, wire or a point, Deformable do you see the Discrete rigid, Analytical rigid, Eullarian. So, these are the things which right now you are not aware you are working in a small zone. So, may be for some other cases you may have to choose this kind of.

So, whether if you are going for like a wire like a beam so when we ~~ra~~ are interested to - modular beam we sometimes for a 1-dimensional beam you just model as a wire.

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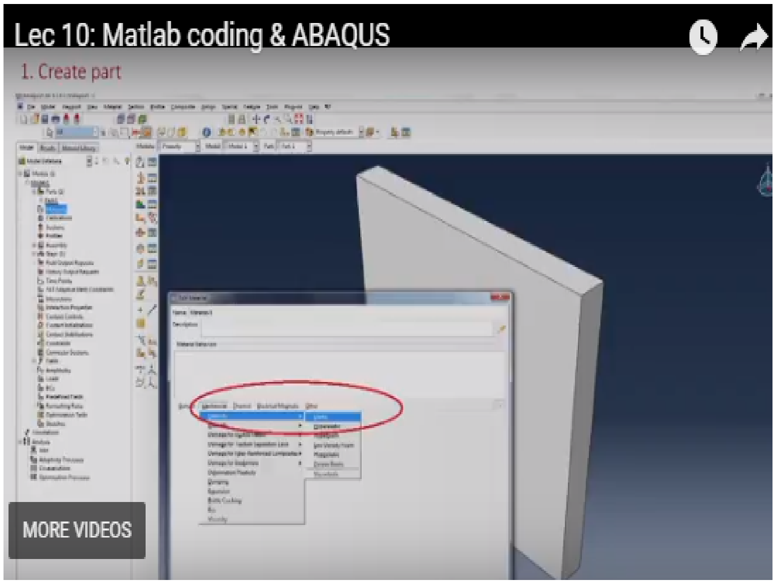

Sometimes it is 2 dimensional also then you have to choose if you are talking about rectangular plate or circular or different kind. So, you have to go to this kind and I am giving this dimensions and creating the part, in the next step you have to go to the material sections and there one has to choose elastic material or hypo elastic material Poruss material you see that damage of those blah blah blah.

So, there are number of variations is there so it depends upon your area. In which area you work you may use those kind of thing for the present in this course I have explained you the basic how to model an isotropic plate or a composite plate under a only the static loading. So, may be later on one may try for a different kind of.

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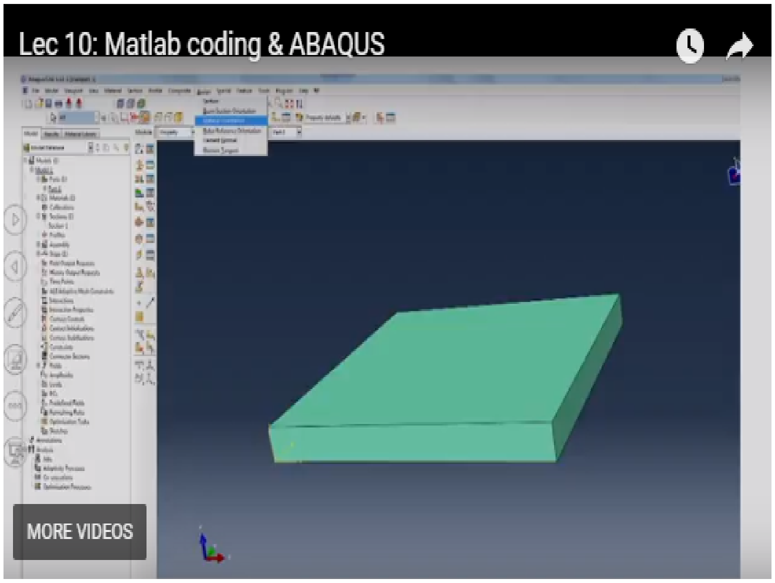

And then you can put the Youngs modulus and then you have to decide the homogenous section. Then you have to apply this sectioning then create a datum reference. These are the some when you talk about it generally that orthotropic material then you have to provide an orientation. But for the case of isotropic you need not to.

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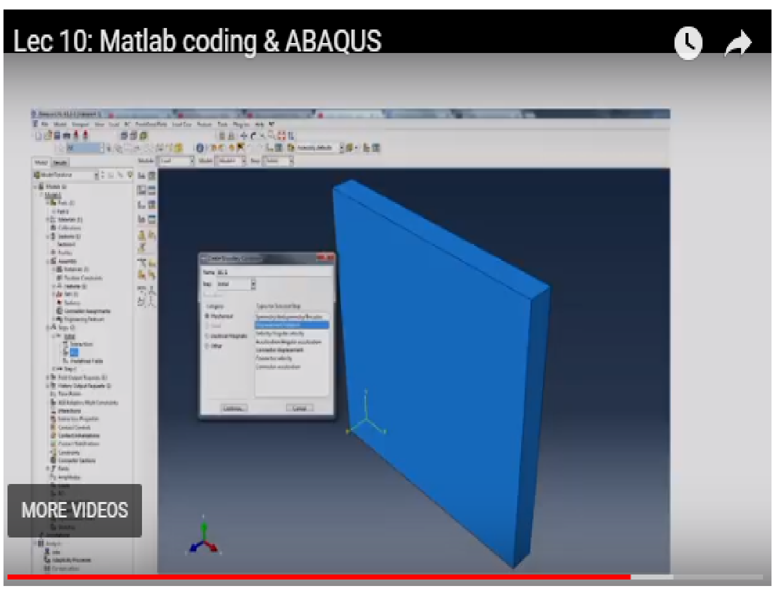

And then finally this part is ready and then you have to go to the boundary conditions. How do you apply mechanically, electric boundary condition or other type?

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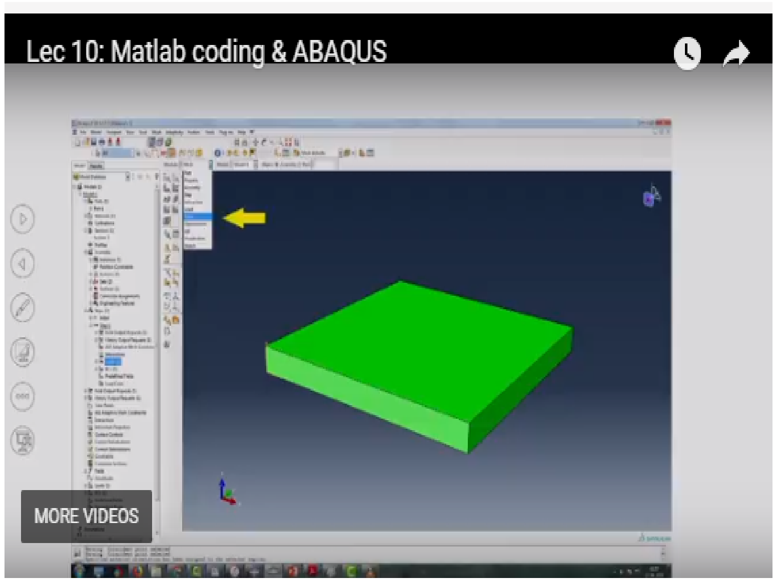

So, from the section one can decide apply the boundaries you see that U1 U2 U3. All these are clamped you put it BC1 BC2 and so on and then load pressure you have to apply you see UDL pressure then the last is the meshing so you have to mesh. So, it is my suggestion here whenever we have a regular part like we have a rectangular plate or a cylindrical shell or a beam we try to map a meshing not default or free meshing.

We should not go for a free meshing until unless we have a very varied geometric or a complex geometry. So, that we cannot define the structural mesh then we can go for a free meshing. But for a regular path we should go for a map meshing to a structural meshing.

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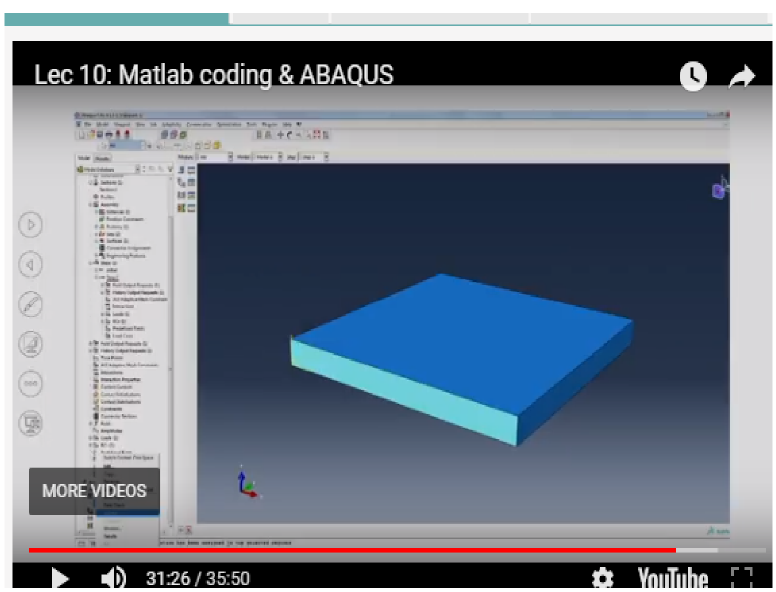

So, there you see you divide each edges into 10 parts just as the standard way of doing and through the thickness. So, it is a regular kind of a meshing then you have to choose the element when you are going for a three dimensional element so C3D20R. It depends you go for a hexagonal edge tetrahedral basically it depends upon your geometry. So, there may be a geometry which is a cone type of shape then you go for a tetrahedra rigid type of element.

And then you have to give hat type of analysis and so on. So, in this way one can summate the analysis and later on post processing those kind of things. So, there is a link I will give I think if you go to the YouTube. And you check that free abrasions of a composite plate you will find videos tutorials are there from there you can learn. Nowadays learning ABAQUS not just by reading a book.

Or going through you can go to YouTube type your whatever you are interested in that field that suppose you want to do a static bending. So, you can say that bending of a composite plate that video you can go through you can learn. So, basically we use this as a second reference when as a computational to a structural mechanic's researcher we used to develop our own code and want to verify our result with that.

Sometimes in the industry or somewhere they do not write their own code they directly model in the software sorry commercial software when to analyses. The important thing which I would like to share here that the structure let us say a bottle or a whole building. The very first mistake that people think let us say an automobile body a car or a scooter you want to analyze it. You need not model as it is that thing for a analyses point of view.

For a manufacturing point of view, you may have to exactly okay there should be bold there should be slightly curved or slightly thick. For analyses point of view, you need not require all these small information. You require a whole in general sense okay how to model that. So, for that analyses point of view your model may be looking entirely different than the actual. So, these thing require some kind of experience.

When you go okay can we model as a rotating line in a thin cylinder and rotating with that in this you have to first think or put some brainstorming that okay can we model we need all these stuff together to analyze or later on. First you go for a simplified model and then adding some complexity. At the very step if you try to model a very complex geometry and whatever data you give it will give you some numbers.

But whether this numbers are right or wrong or whether you are believable or not believable. To check for that, you first go for a simplified one and then verify with the standard whether they are in literature or in some deign codes. And then try to include the complexities let us say first you try to have a plate with all square edges then later on you try to let us say fill it radius what is the fact on that stress behavior.

Or try to put small hole or a bolt or try to put some welding beams later on. But first make a simple structure analyze that verify that which is giving all material properties are working fine and then you try to include the complexity. So, here I end this rectangular plate solutions part 1 so if you have any doubts you can mail to me or you can just when the course is starting basically in the later also.

If you have any doubt regarding the modelling or the coding or any issues related to rectangular plate you may write to me in, my official mail id, thank you thank you very much.