Theory of Rectangular Plates - Part 1
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Lecture - 11
EKM and Buckling of Plates

Welcome to our class theory of rectangular plates part 1.

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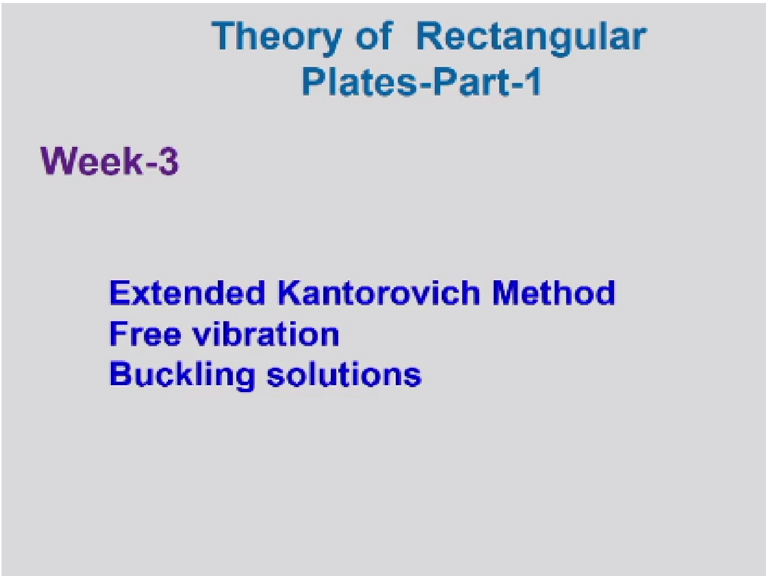

So today is week 3, second lecture. In this lecture I am going to cover extended Kantorovich method, free vibration and buckling solutions of rectangular plate. In the previous lecture I have told you that extended Kantorovich method is also an approximate method which can be used to develop solutions for the plates subjected to the arbitrary supports conditions.

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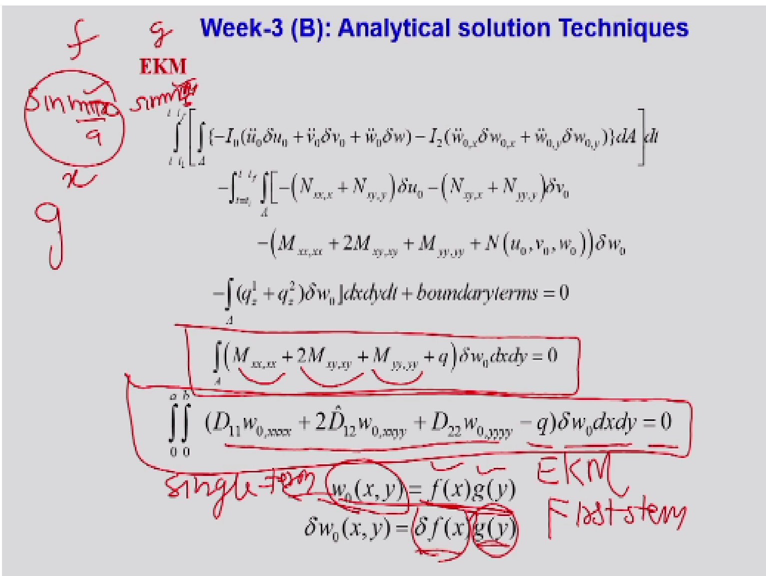

So in this slide I am going to explain that how you are going to apply this extended Kantorovich method like Ridge method I have told you, you have to develop first weak form of the equation then you substitute that approximation of w in that and that weak form is converted into a linear algebraic equation and which is solved using the standard matrix theory or matrix techniques like you can use the Cramer's rules or you may use some Gauss elimination technique and one can get the solution from different constants.

So for this purpose I am going to tell you that you are aware that when we are developing a plate solution, the last stage that we just say that del w, del u, del v0 are the arbitrary, so their coefficient must vanish, so this leads to a partial differential equation just before the partial differential equation if I am just explaining for bending case, one can go for even using this u0 v0.

So I am taking only for bending so let us say this one and loading case which is a coefficients of delta w0. So this is our equation on which we are going to work. Now you substitute using the plate constitutive relations like in the few lectures we are substituting this using the plate constitutive relations what will be that. If you substitute that it becomes like this okay. So this is the weak form of a plate under bending.

Now the EKM approach, extended Kantorovich approach, I am going to explain that you assume a solution into 2 bivariate functions f which is a solely a function of x, g which is a solely a function of y or I would like to say the simplest example when you are going to develop a solution for a simply supported case, there you have assumed sin m pi x/a, which is the function of x only sin series and similarly sin m pi y/b.

If you assume a solution like this it serves the boundary, satisfy the boundary condition exactly you get a Navier solution. So we will say that let us say this is f and this is g, not saying in terms of exactly sin series. We want to satisfy any boundary condition. So fx and gx f is the function of x, g is a function of y. So this is a single term I would like to say that approximation.

If you go and see in the literature you will find 90% of the work in which extended Kantorovich method or Kantorovich method is used that single series, single term solution is used, but recently when we tried for a 3-dimensional case we found that single term solutions does not provide the accurate solution for 3D case. So we went for a multi term solutions that f1 g1 + f2 g2 and so on.

So if I am going to explain the basic technique that how you are going to solve set of equation. So assume fx and gs and let us say in the first step like Kantorovich in book 1958 Higher Engineering Mathematics in that book it is given that let us assume priorly that gy is known, that you know that gy satisfy the boundary conditions or whatever a function is known to you along the y direction that solution initial guess.

Then you are going to solve for x so you will take the arbitrary variation along x in which that is the unknown part, unknown direction, so it will be the unknown one. So I am going to repeat it again, y first case known, priorly known, or chosen function whatever then solve for fx, so we will say that let us say arbitrary variation over the fx.

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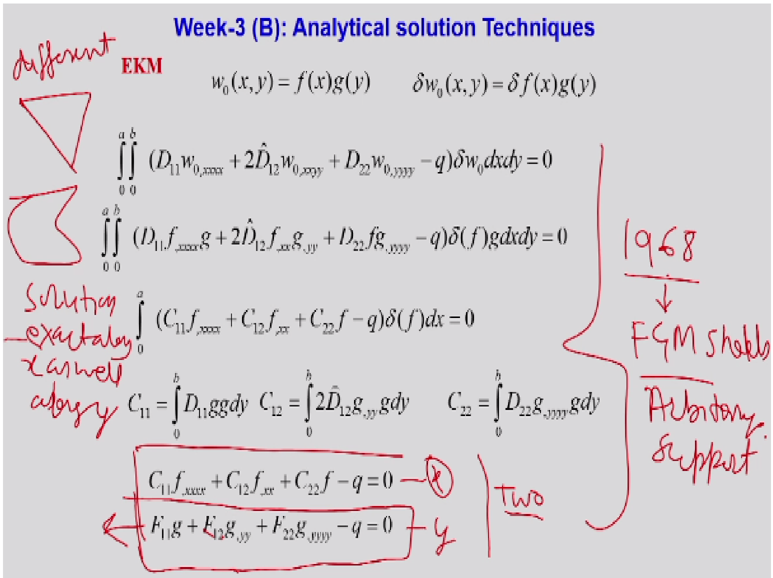

So in this equation if you substitute this values fx and ds, so where w0 is the function of derivative along x-axis. So you will get fs derivative for g remains as it is, here mixed derivative, so g double derivative y, fx double derivative y. Similarly, here w derivative y g 4 times of derivative by and del w not will be del f and gy. So here written like this, so you see del f and gdxdy = 0.

Now we see that since g is known to you, known function so definitely you can also this integration known, some constant will come, so this D11 and g and multiply with g and Dy is known to you. So you can treat as a constant. So next step you will see that we can write this equation C11 of f derivative 4 times, we will say that some constant C12 f double derivative x, some constant C22 only f and so on.

So what are these C11 is nothing but D11 of g from here and g from here and dy. So 2 times of g. So this gives you C11. Similarly, C12 two times of D12 and g, yy if g is known it is derivative is also know. So you can find out that C12 similarly you can find it out C22. Now we will say that since del f is arbitrary. So its coefficients must vanish, or you can say using the fundamental of variational principle that gives to an ordinary differential equation of fourth order.

And this you know how to solve, I already explained in the Levy solution that fourth order differential equation for a static case, the same way you will get the solution of this equation, that you will have 4 roots on the basis of roots, you can define that my solution will be harmonic function or some constant and satisfying the boundary conditions along x axis exactly like levy type you can solve it.

Now in the next step, this till now this is Kantorovich that y known x all exactly. Now extended Kantorovich, Kerr, he proposed that. Let us say we know this after solving this we get a f solution. Now we assume f is known and g is unknown so for that case del w will be a f * del of g. By substituting same thing here following the same procedure you will get another set of equations like this along the Y direction.

So which is the fourth order differential equation along y direction and along x direction. So next you repeat it by solving y, again go for x. So in this way around 2, 3 iterations you will get that boundary conditions or the solution will satisfy exact boundary conditions along x as well as along y. So the initial choice whatever you choose gy need not to satisfy actual boundary condition when you go for iteration.

In first step okay, the solution may be slightly inaccurate, but the next step which is the OD in y there it will satisfy the boundary condition exactly. So you will get accurate solution further. So in this way whatever case you will choose it will satisfy later on, so in this way we will saying a 2 set of OD. So it is very fast since we are solving OD. So it is accuracy is very high and convergence rate is very high.

So this method since 1960 onwards has been used to analyse various problems starting from the bending of a plate to free vibrate, these days it has been extended to even the FGM shells, you see FGM cells with arbitrary support conditions, piezoelectric shells with arbitrary support conditions. So this, I would like to say that promising future, even these days some of the very little one or two I would like to say two, three papers has been, researches has tried that for different geometry basically.

Different boundary condition is proved but different geometry maybe some triangular or maybe some hexagonal or something can be tried for that some of the researchers has proved that this can be done, but it has to be proved further so there is a very promising future for this method that we can apply to different kind of problems and we can see that whether this method is able to give the solution or not.

So basically after the simply supported people just go for a numerical solutions, but this method is very simple, easy to apply and so one can use instead of a numerical when a plate is subjected to, simple plate is subjected to any arbitrary boundary conditions. In the next lecture also I will help you that how to develop a plate model in the abacus. So there will be a video so you will understand that how to make a composite plate in abacus software and get the solution for that.

Similarly, some of the students may go for developing some Matlab course. Later on if you got the idea you may develop some Matlab code or you may just for a simply supported you may get some solutions and you can check verify whatever the solution you are getting from the numerical approach and from analytical approach, what is the difference or how much the computational cost like this.

So the solution is given in basically A. Kerr paper that is in 1968 Acta Mechanica and someone another author and one more I forgot his name, so one can try these solutions.

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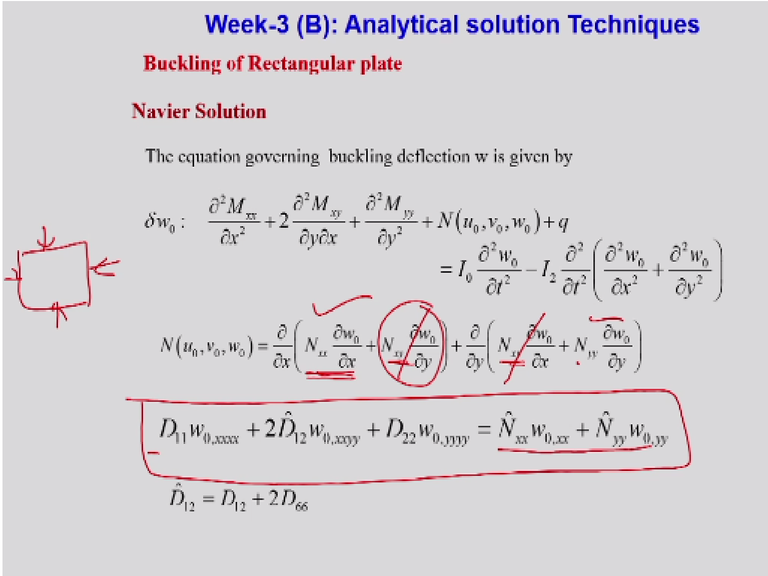

Now come to our course topic that buckling of the rectangular plate, I think most of the undergraduate students are aware about the buckling of beams, what is that buckling of beams. When you are saying buckling, basically buckling of columns not even aware about the buckling of beams, they are not just when there is a column like this and if you apply a load axially compressive load and you can get some critical buckling load.

Even for different boundary condition sometime this hinge and free. So they have some standard formulas, analytical formulas out there. Apart from the columns we have beams, we have plates, we have shells. So they may also have a buckling kind of thing. So if you talk about a beam. So let us say this is a beam and buckling will be like this, if you apply a compressive load like this.

What do you call Nxx? now you talk about plate buckling, a plate means when there is a plate and if you apply a compressive load along the axis, along x-axis as well as along y-axis. So in the plate you have 2 option, uniaxial buckling when the buckling or I would like to say that compressive load is applied only in one direction, you may have a biaxial loading, biaxial buckling when the compressive load in both 2 directions.

Maybe some other case that in one direction it is compressive another direction it is stretching. So we may solve for that cases. So what will be the governing equation for that. So if you remember or I am just going to give you this was your main governing equation for the static case I told you I am going to neglect this nonlinear case and this dynamic portion, the time dependent terms.

For the static I have choose only this thing. Now we are interested in the buckling of the plate then what are the terms are required we will consider this and m terms. We will not take care dynamic part of this. Some of the student may ask same time you take the time dependent terms and buckling terms and loading terms, yes, that can be taken, but the solutions will be very complex.

So one by one we will solve and we know that it is under the linear case or later on we can add the solutions if you are interested to that object is subjected to a transverse load as well as compressive load and time dependent terms are also there, so that results can be combined or combined effect can be analysed, that will be very complex case.

So for study point of you or explaining that basics that how to solve a buckling equation, how to solve by bending question, how to solve a free plate under a free vibration case. So I am going to explain that. So this will be our buckling equation okay. We are going to use this equation out of that Nxx and this Nxy because it does not call any, we do not consider that Nxy as a buckling load.

So this term will not contribute, only this term will contribute. Now you see inside that even we are considering only this when you talk about nonlinear analysis of your plate then definitely you will consider Nxy, but when will you say that the buckling that compressive load like this and like this. So we are applying only this now your governing equation will be this. So what is the next step before going to the next just I would like to explain.

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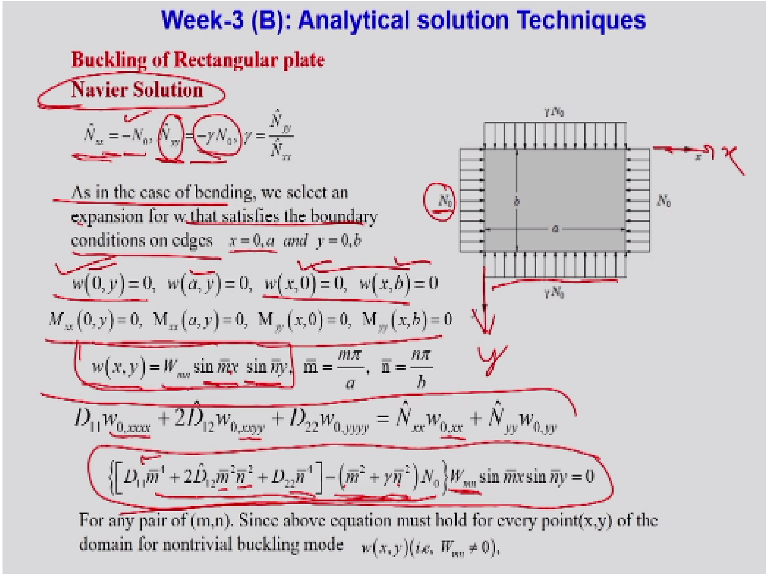

Let us say a plate, these are my coordinates along x-axis, this is along y-axis and we have applied a buckling load along y direction. In first apply buckling or a normal in plane stress N0 along x direction and along y direction and we say that let us say that along y direction this is a gamma times of N0. You take any load definitely you can take rest of that whatever whether it is 2 times or 5 times or < 0.1 times, 0.2 times and in times you can find out the ratio so that our solution will be slightly easy.

If you take Nx as it is Ny as it is then solution will be slightly difficult, we cannot get the common terms. So we are saying that along x axis and not along y-axis gamma times of N0. So what is your Nxx which is the compressive load so –Nx0, what is you Nyy, it will be – gamma times of N0. Now we are talking about Navier solution. So that satisfy the boundary conditions on the axis x0 and xa y0 and yb, what will be those.

So deflection definitely along x 0 and a, and along Y 0 and b similarly the movements along x axis normal movements, and normal movements along y-axis, this has to be satisfied. So our aim is to choose if w0 such that which satisfies this boundary conditions. So this is our function that wmn, sin m bar x sin m y if we choose that this will satisfy boundary conditions as well as displacement boundary conditions and moment boundary conditions.

Next to substitute this into the governing equation if you substitute so double derivative, 4 time derivative of x it will become m bar 4 square then double derivative x, double derivative y, m bar square, m bar square then 4 derivative y, you see that double derivative will become

- and m0 is -, -, - + then again -, - + and if you take this side so basically m bar square + gamma, n square n0, put like this, becomes a question = 0.

So one solution is that ywmn is 0, but if you say that if wmn 0 there is nothing, you cannot do anything so nontrivial solution will be that you put it is coefficients must vanish. When we say this wmn cannot be 0 so its coefficient must vanish for a nontrivial buckling load.

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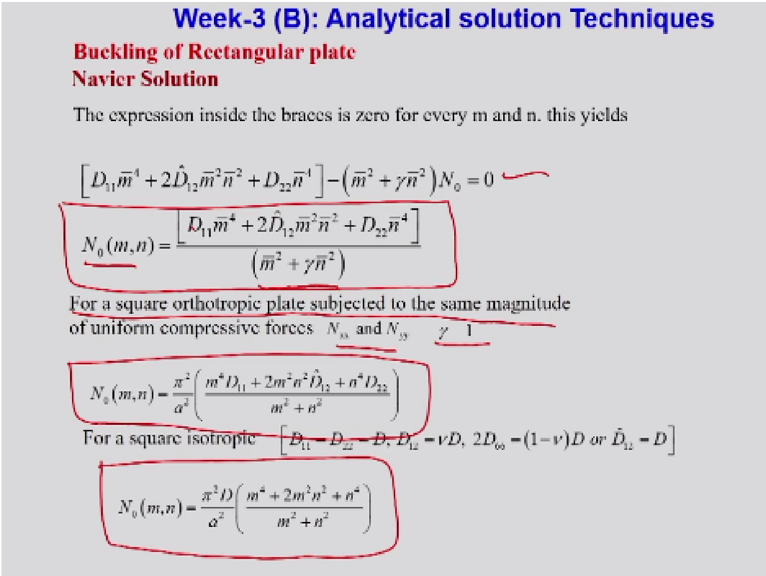

So If you are putting equal to 0, n0 becomes D11, this term divided by Mn square + gamma and bar square. So this is your buckling load, but you do not know whether it is a critical lowest buckling load or not. So this term that value of mn and the value of D1, D2, D22. Based on that only if you substitute those values and evaluate for every m and n and you find which one is the lowest that will be the first critical buckling load that first lowest.

Now we are talking about a square orthotropic plate, so for that case and we say that equal uniform compressive force gamma is 1, it reduces to like this. For the case of isotropic it becomes like this. So based on m and n you can find out the value of n0. So basically then you see when m1, 2, 3, 4 number of infinite kind you can take. So a system or like a plate or a beam which are continuous system they have infinite sets of loads.

Or they have a infinite sets of if we talked about the frequency also, similarly infinite set of buckling loads first, second, 3, 4, 5, 6, 7, 8, 9, 10 and so on. So the first we say that lowest buckling load is known as first critical buckling load. So you can find out for a particular geometry, for a particular D ratio, it looks very easy for the case of when all edges are simply supported.

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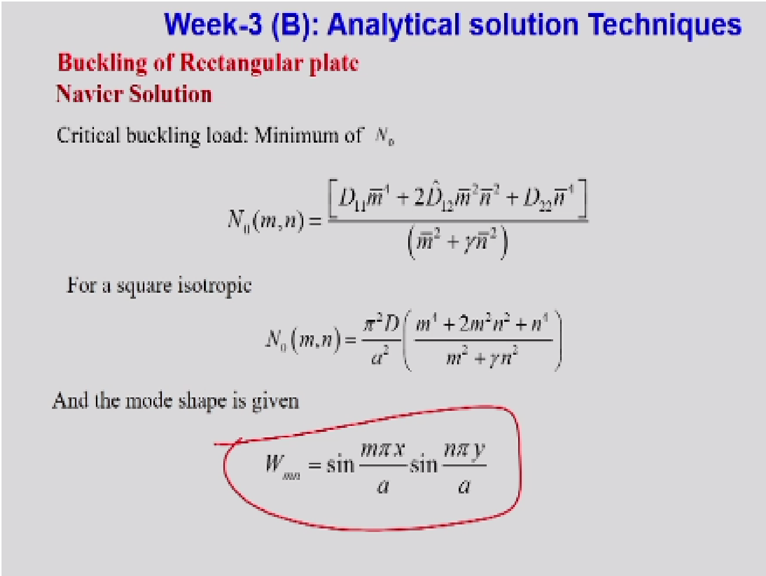

And the mode shape can be written as, so if you take about n11, so you put n11, n112, instead of m so you will get the mode shape for that.

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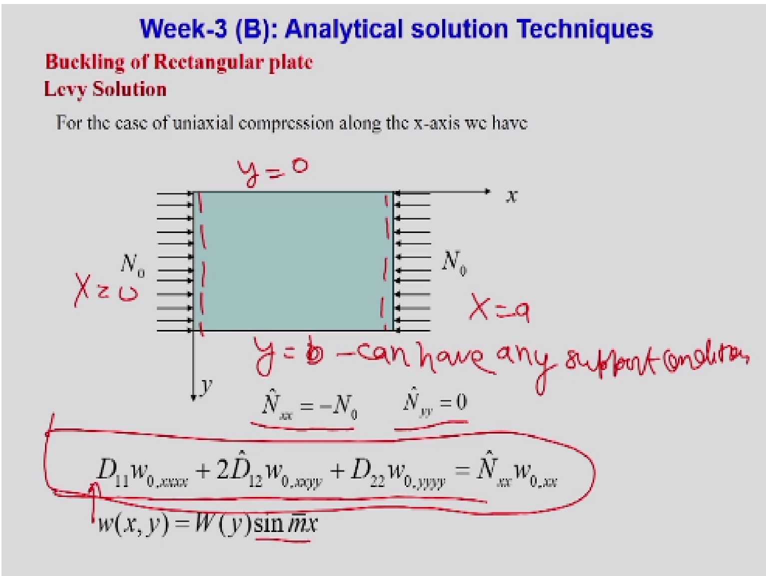

Now you talk about Levy solution which is like you have seen for the case of bending Levy solution was okay, slightly complicated not much complex like you get ordinary differential equation of fourth order, now you are solving based on the roots you can get the solution and applying the boundary condition you can solve that, but here when we say Levy solution 2 opposite edges must be simply supported.

So basically x0 and xa, x is 0 and x is a are simply supported so we can apply only loading here, y0 and yb can have any support conditions. Some of you may be curious about that why not I am taking along y direction. So you just give me some time I will explain even if you take this it becomes equation very complex to solve.

If you want to consider I do not know analytically I think you cannot solve it, there will very complex case, so we substitute and yy is 0 then the solution will be along x-axis will satisfy the boundary conditions w and mxx so we can assume a solution sin mx and wy.

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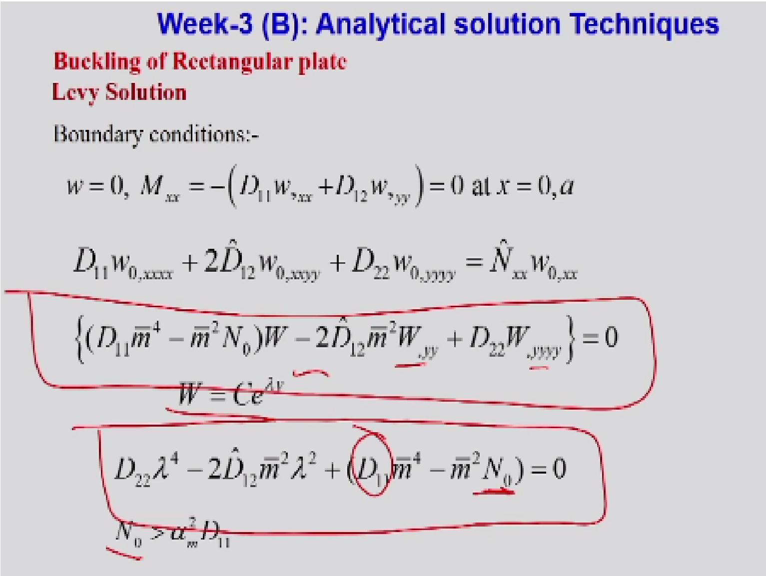

If you substitute in to this equation it becomes, this is just I am telling you, equation like this, okay, which is fourth order differential equation, ordinary differential equation of fourth order like this. Now you assume a solution similarly W = C Lambda y so it reduces to a equation like this. So it will have 4 roots. Sidewise you do not know what is m0, this is also a function of N0, since you are applying.

But our aim is to find out that minimum load that n0 were interested in the magnitude of that, we are saying we have applied but that is unknown, we are interested to find out. If you know that okay, this much has been applied and then you can solve it then it is easy, but if you are interested to calculate from the system that what is my n0 critical then this equation becomes difficult. So it is saying this N0 is greater than D11 for that case, you can write the solutions.

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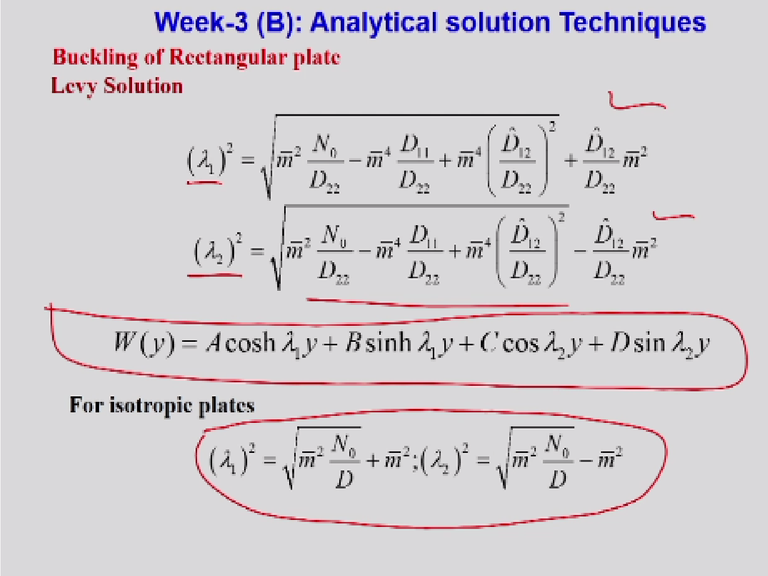

So basically you will have four roots lambda 1 square, lambda 2 square like this. So if that condition satisfies, so you can write a solution like this based on the roots thus I have already explained for the bending case, if my roots are complex, real or linearly dependent then we can write the solutions. So for the case of isotropic D11 is D, D here 2 is D so this lambda 1 square, lambda 2 square reduces to like this.

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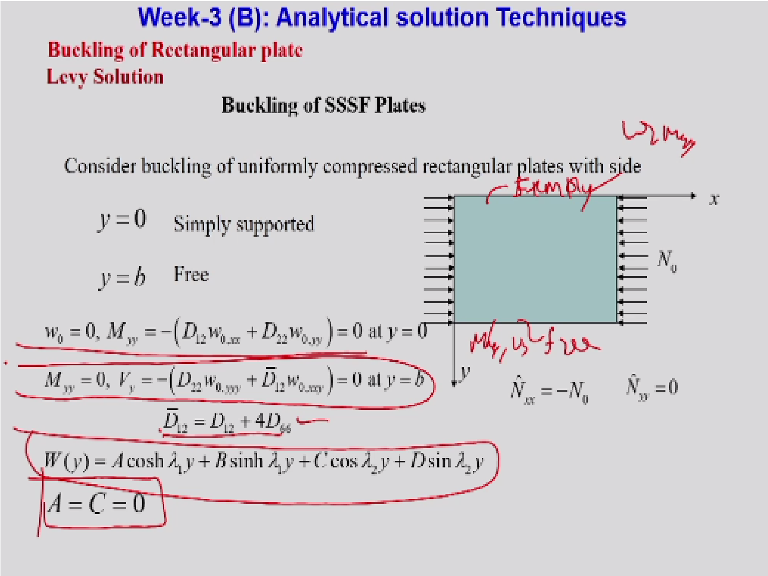

So you have returned w solution lambda 1, lambda 2 are written like this, where A, B, C, D are arbitrary constants, which need to be find out, satisfying the boundary condition along y edge. So first of all let us say y0 is your simply supported, and yb = free. So for that case w and myy and for this case myy and vy is to satisfy 0. Where D bar 12 is this, so this is our solution, if you say that w is 0 when y is 0 if you put cos hyperbolic this becomes 11 and this becomes 0.

So A + C = 0 similarly if you put in the terms, some terms will come A + C again = 0 so from that 2 equations A + C = 0 plus some constant A + C = 0, it leads to both A and C are 0. When you apply that y = b movement is 0 and shear force is 0, that lead to give 2 equations, one is this, another is this.

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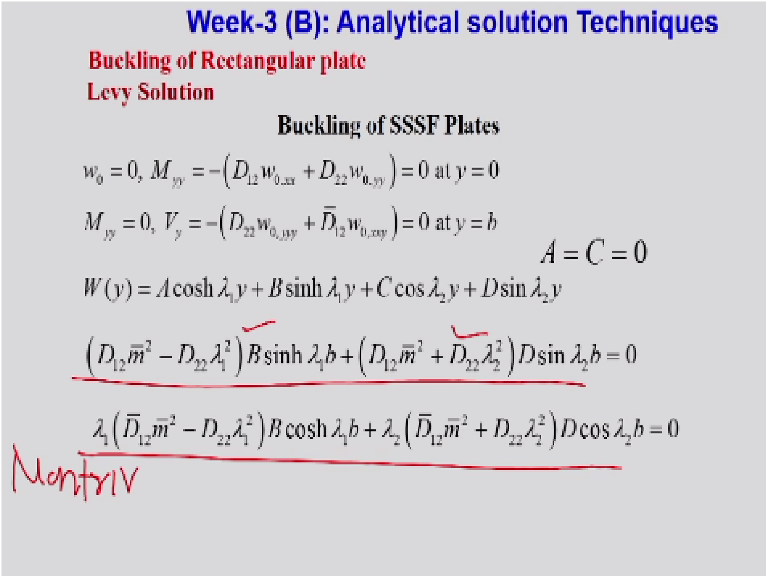

Where B and D are unknown constants. So basically the one trivial solution that B is 0, D is 0 but we are not interested in that, so what is the fun if all constants are 0, so W is 0, so we have to find out the nontrivial solution.

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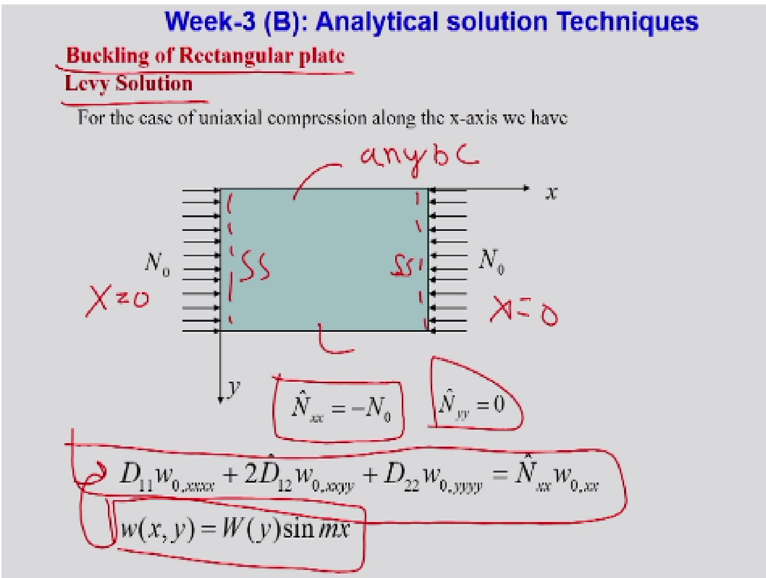

Buckling of rectangular plate for Levy type boundary conditions. So we have solved for the Navier case it looks very easy, but for the case of Levy it is not so easy. So first of all let us see the geometry, x = 0 and x = a we have assumed that simply supported. For a Levy 2 as this must be opposite as is first be simply supported, other 2 edges can have any boundary conditions.

So our loading along x direction along y directions we are taking 0. So this governing equation becomes like this, so a function which satisfy the 2 opposite edges as the simply supported we can assume W like this and substitute it to here gives you this equation.

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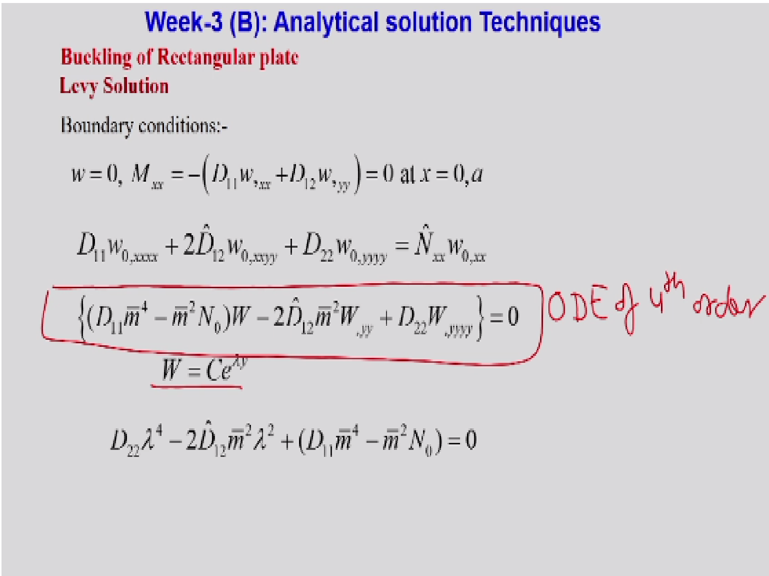

Which is an ordinary differential equation of fourth order? (()) (31:38) you know like a bending case that you assume W = C times of e raise to the power lambda y and substitute it here, it gives you a fourth order equation and if you say that lambda square = R so you know that solution is a quadratic equations lambda 1 and lambda 2.

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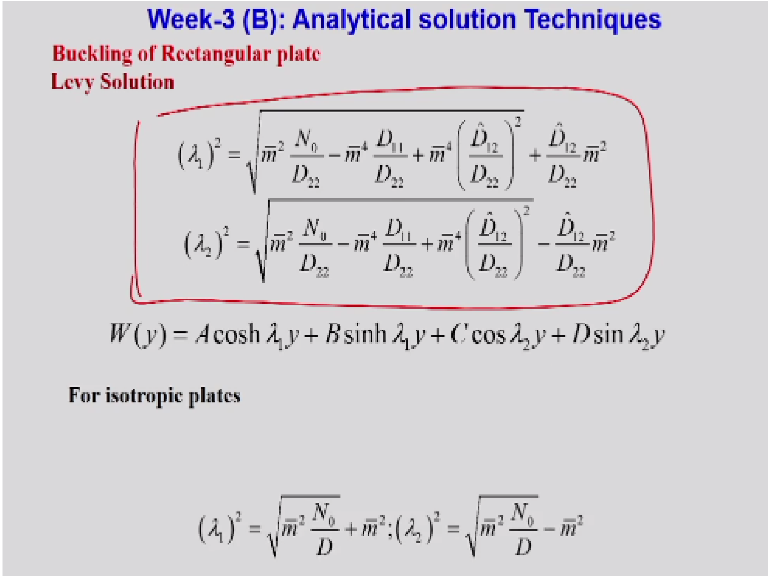

So you can find out the roots of this equation lambda 1 and lambda 2 can be like this for a orthotopic plate. For the case of isotropic plate, the lambda 1 and lambda 2 can be written like this. Now there is a case when this n0 > D11, so you see that this is the positive term, only this is the negative one, this is positive one. So if this is > so your under root will have a positive value.

But if N0/D22 < this and sum is really this value may be some combination of that. so it may have a minus of under root so you may have a complex root. If this is more than that that means you may have real root only. If you prove that this is greater than of this so it will be having a real root for the case of real roots you can say that solutions can be written like this.

Or you need not to say that okay you just assume that my roots are real or equal solution will be like this, my roots are complex the solution will be some different kind like that you may write all 4 cases. Now A, B, C, D are unknowns, arbitrary unknowns. Here also your roots are also known in terms of N0, these are known to you, but N0 is not known to you that we are going to find it out.

There may be cases that somebody is giving you that load under this load this plate will buckle or not, then it is easy, just you put that load and check the set of equations, but you are interested to find out the minimum for a particular geometry, for a particular material you are interested to find out the minimum critical buckling load, so for that it is unknown kind of thing.

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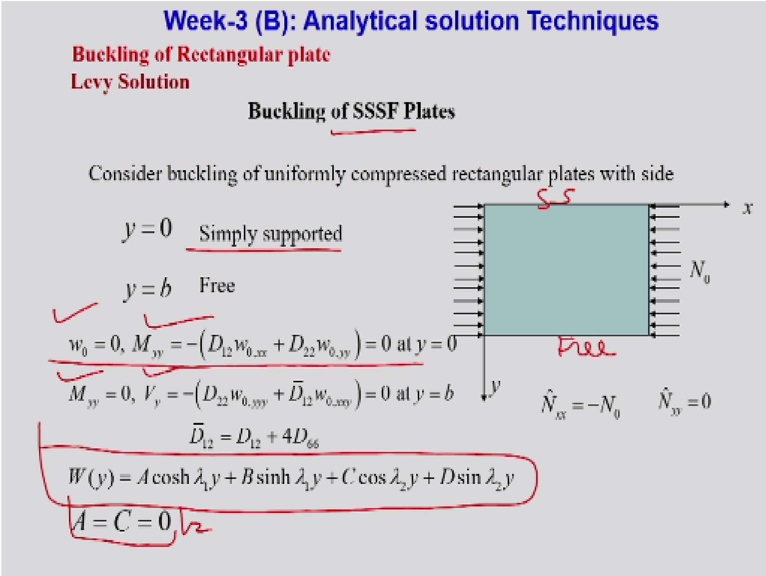

So now you are going to solve a plate which may have y = 0 simply supported and y = b2 free. So when simply supported boundary condition w must be 0 and moment along y direction must be 0. For the free case moment and vy has to be 0. So the solution is written in this form for a particular case then if you substitute that when y is 0, w is 0 and y is 0 moment is 0. So from that you will get A and C constants are 0. Now the next one y = 0, my moment is 0 and shear force is 0.

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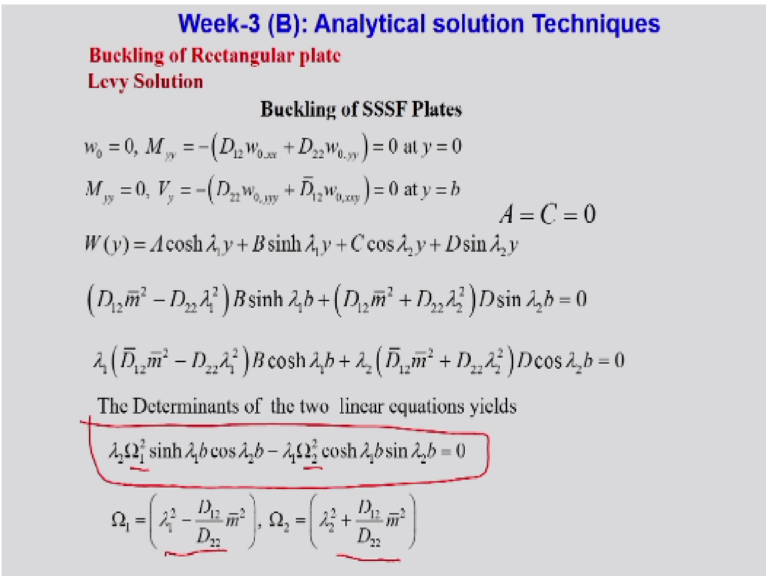

Applying those conditions, you will get these 2 set of equations. Now the set of equations the trivial solution is that B = D =0, B or D both are 0. If A and C is already 0, if you say that this solution is, one solution is that B and D = 0 then there is no buckling, no load, nothing, no function. So non trivial solution for such cases before proceeding further you may hear about that let us say 2 x 2 matrix x1 and x2 and righthand side is 00.

Let us say a11, a12, a21, a22, these are my elements. So trivial solution, x1 = 0 and x2 this is also 0, nontrivial solution. So non trivial solutions for a such a system is that if a nontrivial solution exists so this determinant must vanish okay, similarly here we can write in a matrix form B and D = 00 and you had some constant this sin hyperbolic and this constant and sin hyperbolic lambda B and so on.

Let us say C1 constant, so determinant of that must vanish. So we will get the determinant, multiply this minus of this, so you will get this is the form of the determinant where omega 1 and omega 2 is again function of some material property in the roots. Sometimes we called it as a characteristic equation. Here lambda 1, lambda 2 is there and omega 1, omega 2 is also a function of lambda 1 and lambda 2.

Now you know that lambda 1 and lambda 2 contains n0, n critical so while solving this, so how do you solve such kind of equations. So very first technique is the iterative, that you are assume let us say sum n0, initial and evaluate all the roots and come to this characteristic equation and equate it to 0. So from there if it satisfies it means this N0 is the root of the equations and part of this.

Otherwise it is not going to be 0, so that remnant will not be 0 for that particular case. So in this way one can find out and so that is why it is very complex in the literature or the research area in the field of mechanics, composite mechanics or in the structural mechanics you will find only maximum 10 papers related to the Levy type solutions. Most of the papers are devoted to Navier type solution, just to check the formation, but Levy type solutions very rare.

I would like so that in structural mechanics or composite mechanics not more than 20 if you say, basically 10, basic 10 or 20 not more than that. So the reason is behind that solution getting the solution and writing the algorithm for that is very trivial. So it is very difficult one can write, used to write but it is not very easy, not straightforward.

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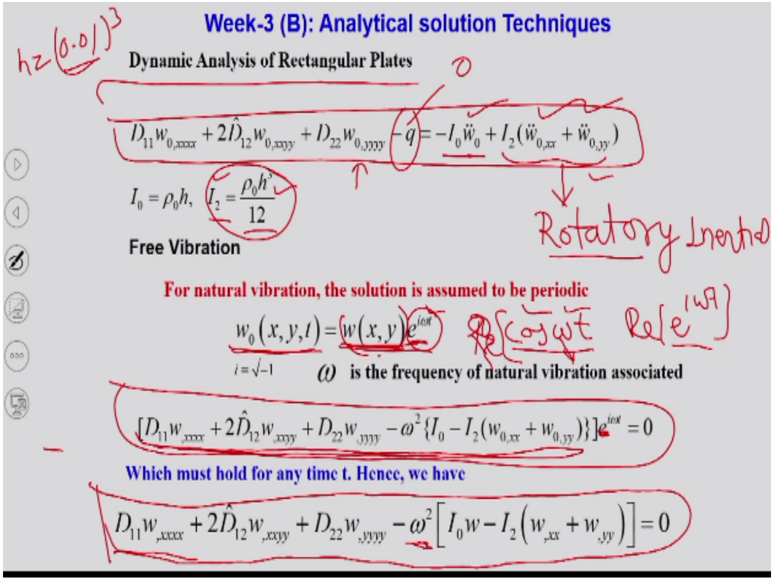

Next come to the dynamic analysis of a rectangular plate. Now for the dynamic analysis we are going to consider this equation. When we talk about free vibration then again this term will vanish further, sometimes this term is known as rotatory inertia. If you say that my plate is thin, so h cube becomes further thin, means order will be further less, so I2 will be negligible.

If my plate thickness is 0.01 so this cube will be little contribution. So for the case of thin plates we neglect this rotatory inertia, but if you are talking about thick plates definitely we have to consider this. Otherwise there will not get some of the vibration modes or the effects of these, okay. Then here w is a function of time as well as a function of space. So first of all this general function is divided into 2 parts.

One, let us assume a function w which is solely a function of a space and another variable e raise to power i omega t which is a function of time. We are assuming that along time, over the time my function varies like this. Sometimes just you assume a cos omega t in actual this is, so this is the real part of basically e raise to the power i omega t. So in general case you may assume that e raised to the power i omega t, it contains both sin and cos sin terms.

But most of the cases if you take only cos sin that also works, so where omega is the frequency or the natural frequency, fundamental frequency associated with that, t is the time. So you have assumed and you substituted here. So time derivative will vanish, only space derivative will remain. So you see this equation like this.

Now you are saying because it is valid for all time, so we can say that this has to be this coefficient must vanish, so it is = 0. So from there omega 0 can be find it out so this is your now the governing equation.

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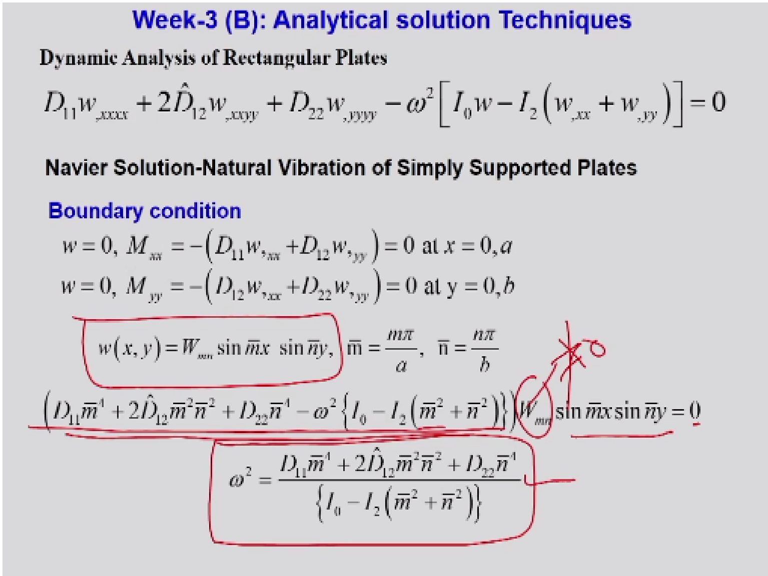

Now the next step Naiver solution are simply supported case, assume w like this, substitute it here, it leads to this equation, this cannot be 0 or I would say this cannot be 0, so this has to be 0. So from there one can get omega square S like this for a plate that natural frequency for an orthotropic plate is this. You may be aware about the fundamental frequency of a beam, fundamental frequency of a discrete system spring and mass, but what is the fundamental frequency of a plate, for a simple supported plate all round, this is this.

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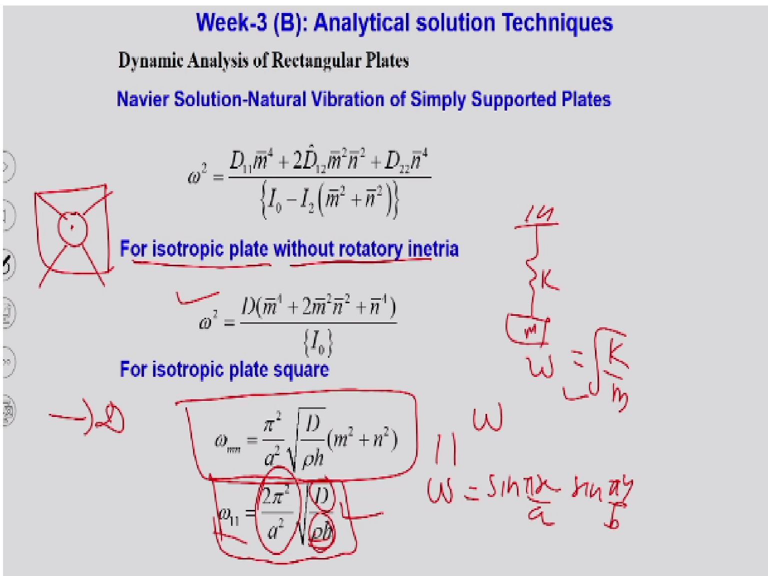

The next for the case of isotropic plate without rotatory inertia it leads to like this. If you say the plate is square, then it reduced to like this. If you say the lowest one m = 1, n =1 then it gives you next, twice of this. So the first frequency of an isotropic plate is related like this 2 pi square plate a square under root d/rho h. If you remember for the case of a spring mass system under root K/M.

You used to write so this factor extra for the case of plate plus rho h is basically related to a mass, D is basically what is bending stiffness, hear K is also stiffness, M is mass because here for a continuous system it becomes inertia terms. So you can relate with that. So I think with this you know that what is the first frequency done you can go 22, 12, 33, 44 infinite set of you will get bending frequency.

Then the more will be accordingly if you talk about 11 modes it will be sin of pi x/a and sin of pi y/b so first if somebody is interested to plot so it will be the maximum kind of thing. I will show these things that how it looks in the Abacus or you may also plot two dimensional plot of that mode shapes along thickness and using all these things that how the plate bends in the first mode when you talk about second, three and so on.

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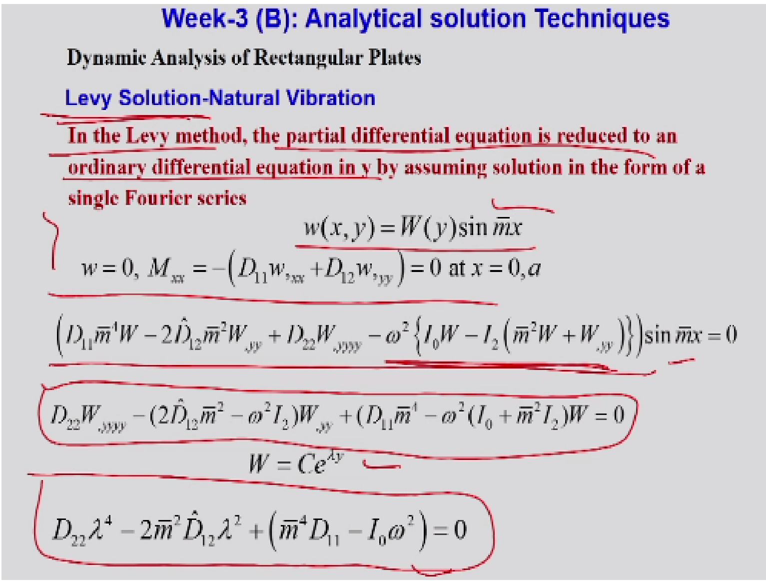

Now again for Levy type, so I am discussing this I think third time, first for the bending case, second time for the buckling case and third for the last is for vibration case. So Levy method converts a partial differential equation into an ordinary differential equation by assuming it to just simply supported. In Kantorovich you are doing or assuming a solution in ny and converting a partial differential equation into an ordinary ny.

And similarly assuming ny you can convert that partial differential equation ordinary differential equation along x-axis. Similarly, here again assuming a solutions like this, same way like your buckling way you have this kind of things equations and finally you write so fourth order ordinary differential equation assume a solution write down here, you will have four roots.

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Week-3 (B): Analytical solution Techniques

Dynamic Analysis of Rectangular Plates

Levy Solution-Natural Vibration

$$D_{22}\lambda^4 - 2\bar{m}^2\hat{D}_{12}\lambda^2 + \left(\bar{m}^4 D_{11} - I_0\omega^2\right) = 0$$

$$(\lambda_1)^2 = \sqrt{\omega^2\frac{I_0}{D_{22}} - \bar{m}^2\frac{D_{11}}{D_{22}} + \bar{m}^4\left(\frac{\hat{D}_{12}}{D_{22}}\right)^2} + \frac{\hat{D}_{12}}{D_{22}}\bar{m}^2$$

$$(\lambda_2)^2 = \sqrt{\omega^2\frac{I_0}{D_{22}} - \bar{m}^2\frac{D_{11}}{D_{22}} + \bar{m}^4\left(\frac{\hat{D}_{12}}{D_{22}}\right)^2} - \frac{\hat{D}_{12}}{D_{22}}\bar{m}^2$$

$$W_n(x) = A\cosh\lambda_1 y + B\sinh\lambda_1 y + C\cos\lambda_2 y + D\sin\lambda_2 y$$

$$W_n(x) = A\cosh\lambda_1 y + B\sinh\lambda_1 y + C\cosh\lambda_2 y + D\sinh\lambda_2 y$$

Then again you can write the solution similarly and then you substitute as per the boundary conditions.

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Week-3 (B): Analytical solution Techniques

Free vibration of Rectangular plate

Levy Solution

Buckling of SSSF Plates

$w_0 = 0,\ M_{yy} = -\left(D_{12}w_{0,xx} + D_{22}w_{0,yy}\right) = 0$ at $y = 0$

$M_{yy} = 0,\ V_y = -\left(D_{22}w_{0,yyy} + \bar{D}_{12}w_{0,xxy}\right) = 0$ at $y = b$

$$A = C = 0$$

$$W(y) = A\cosh\lambda_1 y + B\sinh\lambda_1 y + C\cos\lambda_2 y + D\sin\lambda_2 y$$

$$\left(D_{12}\bar{m}^2 - D_{22}\lambda_1^2\right)B\sinh\lambda_1 b + \left(D_{12}\bar{m}^2 + D_{22}\lambda_2^2\right)D\sin\lambda_2 b = 0$$

$$\lambda_1\left(\bar{D}_{12}\bar{m}^2 - D_{22}\lambda_1^2\right)B\cosh\lambda_1 b + \lambda_2\left(\bar{D}_{12}\bar{m}^2 + D_{22}\lambda_2^2\right)D\cos\lambda_2 b = 0$$

The Determinants of the two linear equations yields

$$\lambda_2\Omega_1^2\sinh\lambda_1 b\cos\lambda_2 b - \lambda_1\Omega_2^2\cosh\lambda_1 b\sin\lambda_2 b = 0$$

$$\Omega_1 = \left(\lambda_1^2 - \frac{D_{12}}{D_{22}}\bar{m}^2\right),\ \Omega_2 = \left(\lambda_2^2 + \frac{D_{12}}{D_{22}}\bar{m}^2\right)$$

If you talk about it 3 S simply supported and free y 0 simply supported y0 b free again you will get A C and 0 and this will be the characteristic of the equation and same it is solved by iterative way, you assume first omega and whether that the system is satisfied or not and you, so the solution technique or procedure is finding the roots basically of this equation.

So in the next lecture I will explain some how to model a plate in Abacus and then finally the 3 dimensions solution for a plate which we are going to use for analysing very thick plates. So I am going to give you just one exposure that okay these kind of solution have just one can go for that.