Theory of Rectangular Plates-Part 1
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Lecture – 10
Tutorial: Load Matrices Calculation

Welcome to our tutorial 3. In tutorial 1, 2, I have discussed that how to transform a vector or a tenser then how to transform the stresses, strains and how to evaluate that extensional stiffness, bending stiffness and so on. So during this course you have learnt that how differential equation is obtained for a case of a plate and different solutions, specifically the first part the Navier solution. In the Navier solution that bending solution, free vibration, buckling.

I have all discussed all that cases. Now we are going to get the numbers that how to use those solutions.

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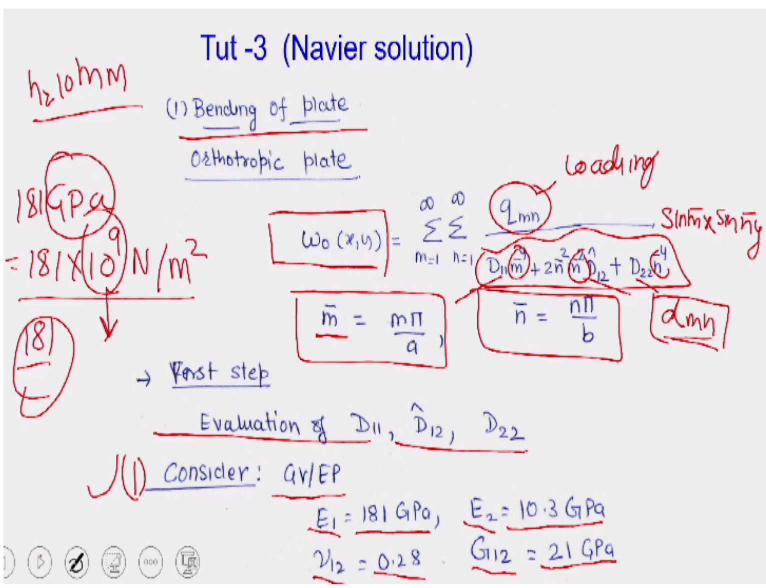

So very first, Bending of a plate. If we talk about Orthotropic plate in that case deflection will be Qmn, D11 m 4, 2 n bar square m bar square, D hat 2 n + Sin m bar of x and sin n bar of y. So during whole course that notation mr is nothing but m pi/a. m bar is nothing but m pi/b. so whenever I say that m bar means m pi/a whenever I say m bar it means n pi/b. So we know that deflection can be expressed like this where Qmn is loading.

And it means sometimes we call as a small Dnm, if not effective I would like say that stiffness bending, stiffness Dmn. So in the very first step, if you want to know the deflection of a plate let us say, there is a plate made of still, having length A 1 meter, B=2 meter, if I say that thickness is 0.01 mn or 0.01 meter then what will be my deflection for a particular set of loading. So before proceeding you must know D11, D hat 12 and D 22 and all these values.

So you have to evaluate Dmn first. For evaluation of Dmn we must know these things. So I am going to take, let us consider plate is made of graphite epoxy in which Young's modulus one direction is given 181 GPA, Young's modulus second modulus is given 10.3GPA, Mu 12 is taken as 0.28; G 12 is taken up 21 GPA. Here GPA means, if I say 181 GPA means 181*10 to the power Newton power meter square. So sometimes w-- students makes a mistake they do not consider this under the dimension, just they take 181 and divide, multiply then get the solution of deflection.

So in that case you will get a wrong deflection, because you not consider this 10 to the power 9 or sometimes that our dimensions are given in terms of thickness, let us say 10mm. So if you want to use this formula you must convert this thickness into meter then you have to use it here.

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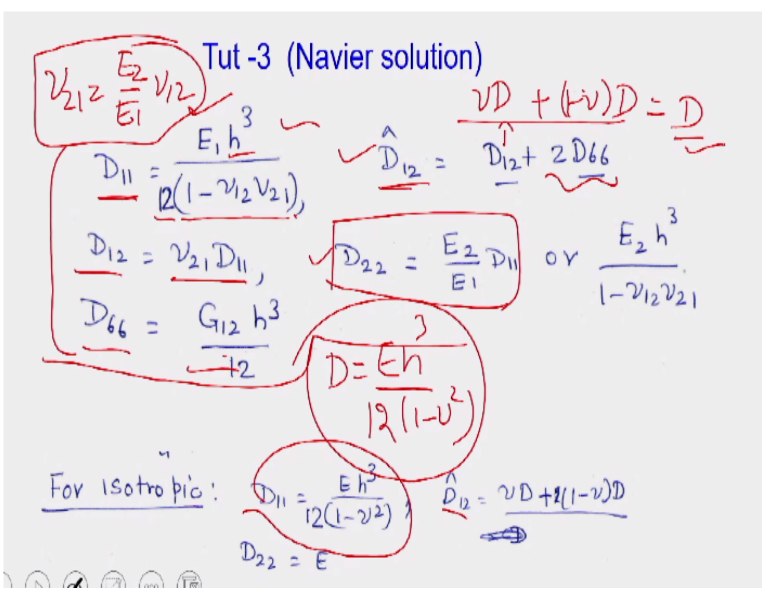

So we will evaluate what is D11. Young's modulus in one direction taken as cube, 12 times 1-V12 and V21. Already I have discussed that, V21 is nothing but E2/E1 and V12. Similarly, D12, D66 and D22. I would like to say that, that every course or every program has its own syntax, like if you are going to learn a C language or MATLAB, if you know that how to represent a matrix their or how to or what is the syntax of applying a control loop, if loop, for loop, while loop.

Similarly, in this course I would like to say that these are the basic syntax if you, or basic alphabets D11 D22, if you do not remember these things, so try to remember these things what are all bending stiffness for calculation of deflection, movement, stresses, okay. When you have a open book exam then you can use these as in initial talk these are the very simple formulas. Even for the case of isotropic En cube/12 1-Mu square.

Once you try to remember these thing, so that calculation during the exam or during the tutorial or during the assignment will be easy. So for the case of isotropic D11 is this thing. Now D hat 12. Most of the time we are going to convert this D hat 12 because this is generally used in the equation of motion in partial differential equation or ordinary differential equation. So this is your Mu times of D and this is you 1-Mu times of D.

So overall result is that D hat 12 for the case isotropic is reduces to just D. Okay. Then D22 is again same D11.

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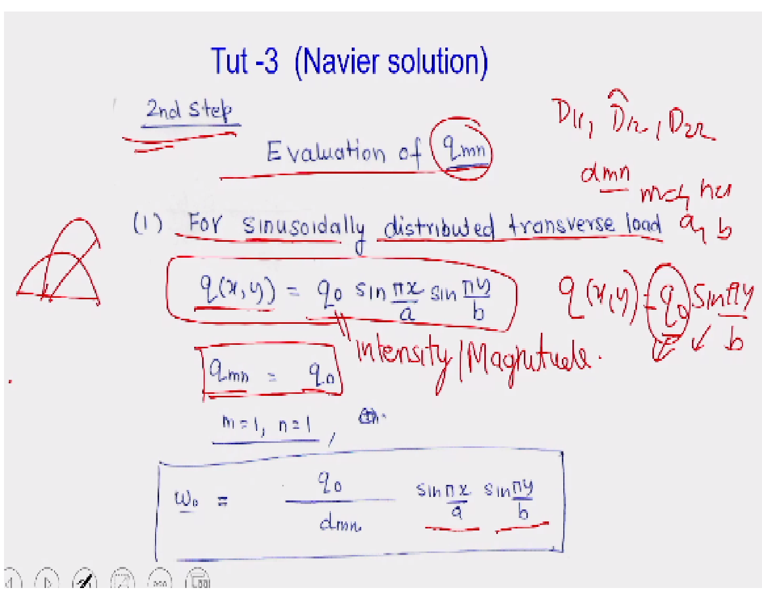

So if we know all D11, D12 hat or D22, so dmn can be calculated for a particular set of m=1 or n=1, a and b. Then the next step would be the evaluation of Qmn. So basically this Qmn depends upon the type of load. For example, if we say that plat is subjected to sinusoidally distributed transverse load, in that case if function q x,y is represented like this where this is the intensity or sometimes the magnitude.

So you may say that 1,10,20 and it follows that sign variation like this along x-axis; along y-axis so on. So what D will be Qmn for this case. So when a plate is subjected to a sinusoidally, W sinusoidally distributed load then Qmn will be just Q0. Sometimes I may say that along y-axis it is sin but along x-axis it is udl loading. So it will be a combination of sin and udl load. So for a present case when m=1; n=1 W0 will be Q1 0 upon Dmn and sin pi x/a, sin pi/b.

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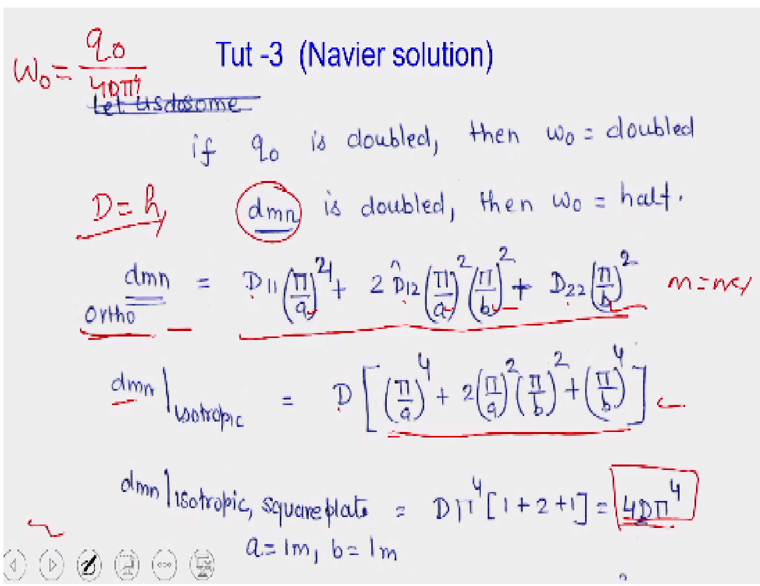

Now we are going to some conceptual analysis like if you say that very obvious that if your load is doubled so your deflection will be also doubled, clear. Can you tell me that if your Dmn or the bending stiffness or your thickness for is different like let us say a plate made of an isotropic plate, square plate, is thickness is H and a second plate of a same materials same length and width but thickness is half, so how deflection will be change?

So in this case how to analyze such cases, it is not just you can say by intuition you have to come up with some formula. So for the case of orthotropic material Dmn will be like this for m=2;

n=1. Here see b is the width of the plate, a is the length of plate. Then for the case of isotropic instead of these things D will come out and like this. Now I am talking about square plate where a is 1 meter, b is meter then it reduces to only 4 pi 4 D pi raise to power 4.

So your Dmn is reducing to 4D pi 4. So now your deflection will be for isotropic square plate under sinusoidal loading will be 4D pi 4, okay.

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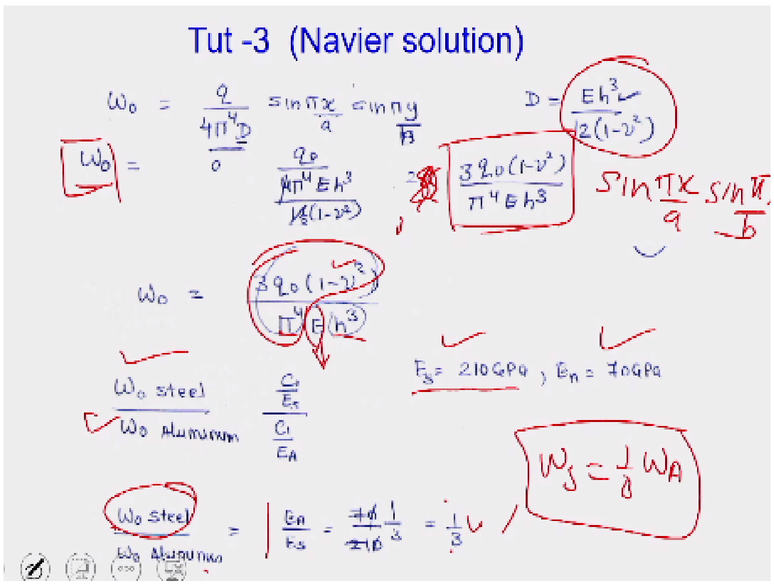

Then inside this D you will have Eh cube upon 12 1-Mu, so you will substitute all these things it become a formula like this, the deflection can be represented like this. Now I am taking only m=1 and n=1. So now if you say that if thickness is half or double or Young's modulus if plate is made of steel or plate is made of aluminium what will be the deflection, can you guess this. If you are saying that, let us say my plate of same length, same width, same thickness; one plate is made of steel, another plate is made of aluminium.

So the steel deflection, aluminium deflection, so these things are just called same loading, same Mu everything same thickness, so only Young's modulus changing. So for this case the deflection in the steel will be 1/3 times of deflection aluminium. So steel deflection will be 1/3rd of aluminium deflection. So this type of, if we have this close analytical formula we can directly tell that okay, if you made a plate of steel what will be the deflection.

If you made a plate of aluminium what will be the deflection. Even you can tell if your plate is made of orthotropic material, substitute those values and get the deflection.

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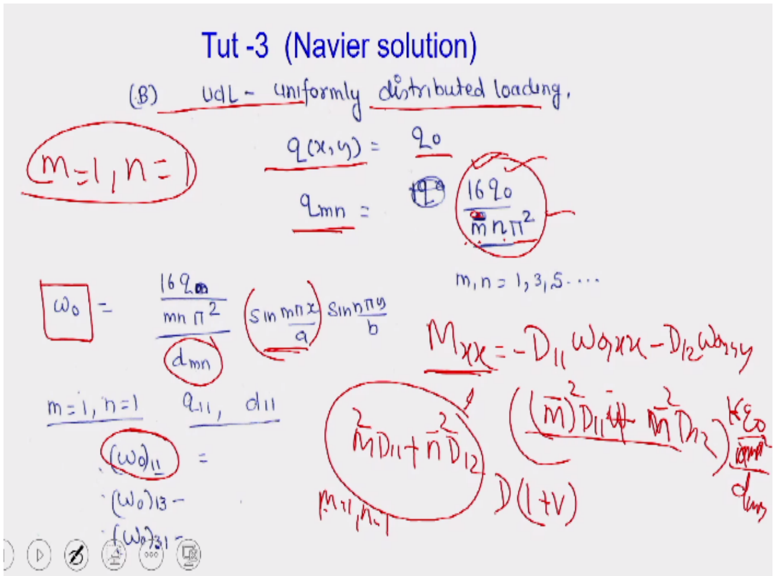

Now we are talking about if a plate is subjected to the uniformly distributed load, that q x,y is Q0, Qmn will be 16 times of Q0 mn/square. Here mn is just mn numbers 1 and 2 not m pi/a. So this will be your Qmn then substitute that Dmn. Here mn takes only odd values 1, 3, 5 like that, so these term combinations like deflection of a first term one when I say m=1; n=1. Then m=1; n=3.

When a plate is subjected to the sinusoidal loading in both the directions then only single term solution is their only m=1 and n=1 for rather term it will be 0. But when it is a udl then you have to add at least for a good accuracy for a deflection tantrum, non-zero tantrums or 11 trums like W0 1, W0 13 W0 31, 51, 17 like that you have to add in that series and you will get a final deflection of that plate.

Is somebody is interested to know that what is the bending movement? So based on this deflection bending movement will be D11 or W, xx-D12 W, yy so along x direction derivative I would like to say that m bar square – m bar so it will become + and this Qmn upon Dmn what are the constant you have. Similarly, m bar square D12 and (()) (14:24) dmn. So basically if in the deflection term if you add m square D11 + n square D12 that will give you the movement.

Sometimes this Mu times of D, this is D so basically if M is 1, n is 1 and plate is square so it is giving you 1+Mu D kind of things. If you multiply that deflection with this much of constant you will get the movement. Similarly, you can get the movement in y direction and the stresses.

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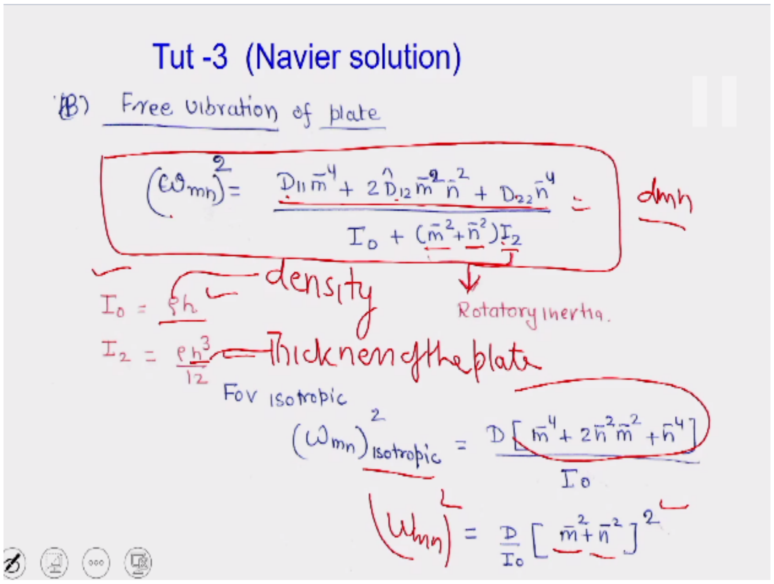

Now I am talking about the Free vibration of a plate. Can we do the similar kind of analysis as done for a redounding that if a plate thickness is change, if a stiffness of a plate is change, what will be my new frequency or if plate length and width changes what will be my new frequency or how much will increase or decrease. So for that purpose you must know that the fundamental frequency of a Navier plate will be look like this. So it is again Dmn kind of thing.

And here I0 + m bar square; n bar square * I2 where I2 is a Rotatory inertia. So most of the time we neglect the rotatory inertia. Sometimes the question maybe that if your plate is thick, if rotatory inertia is becoming twice or four times; how frequency will change in that way one can do the analysis. So I0 is Rho times of h where Rho is a density of the material, and that is the thickness of the plate. So this is the definition for an Orthotropic plate.

That fundamental frequency of a Navier Orthotropic plate subjected to a all round simply supported. If we talk about an isotropic plate, so it becomes stay, it becomes stay, it be stay so D times m square + n bar square + mn square + m bar 4. So basically what is this, a+b whole square. So here m bar square + n bar square whole square d/d0 considered the I2. So Wmn square for isotropic can be written as.

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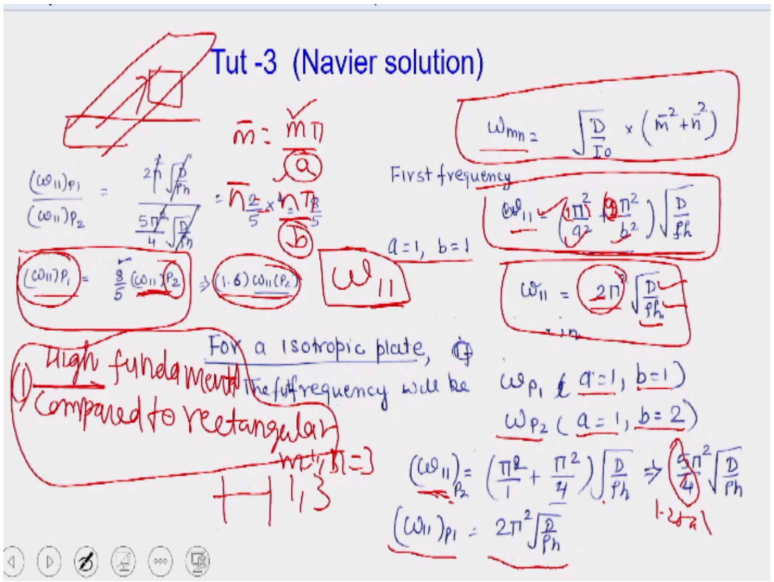

Then, finally your Wmn will be look like this where m bar contains m pi/a; n bar is n pi/b. They contain the information about the length and width of the plate and its number like first, second because it is a continuous plate, so m and go to infinite. So 11,22,33,44,13,14 number of fundamental frequency will be there as per the different mode shapes. So in a case of first frequency if I say m=1; n=1 so generally we use to write same 1,1.

So whenever you find a symbol omega 11 it means m is 1; n is 1 and this is the formula of first fundamental frequency. If I say that my plate is square and its dimension is 1meter, 1 meter or something accordingly it reduces to 2 pi square under the root D upon the Rho h. So accordingly now one can analyze that is isotropic plate, square plate first fundamental how it will behave a D and how do you change that natural frequency if H is changing, Rho is changing.

Or sometimes a and b are changing. So I am just going to a plane. Let us say I have a plate 1, whose length is 1 meter, width b is 1 meter and I have another plate, plate 2 length is 1 meter, width is 2 meter. So I am interested to know that the frequency of ratio of a plate 1 to plate 2. So I will evaluate first omega 11 of plate 2 using this formula which is coming phi pi square upon a 4 d and rho h., same material. And then plate 1 which I have calculated.

Now, if you put it there so it becomes a/ five times of omega 1 of plate 2. So basically plate 1 frequency will be 8/5 times of plate or 1.6 times of plate 2. So if you have change from square plate to a rectangular plate so basically square plate width will have high, I would like to say that fundamental frequency. You see that for a rectangular plate 5/4, so basically it is coming 1.25 pi square. For the case of square plate, it is two times.

So basically, a square plate have high fundamental frequency compare to a rectangular plate made of same material. So this is a very, very important conclusion for a design point of view. Suppose if you are interested designing a machine element or a structure or a component then you can think that where you need that you want to avoid the resonance you try to make that square plate, so it will have a for the same material it will have a high fundamental frequency.

If you try to make rectangular plate then it will have a less, so the resonance takes place thus lower frequency thus bend is there. So by doing the simple analysis we may draw the very important design criteria, for that that is—similarly, for different thickness for different density for different bending stiffness when we come up with some kind of this basic inputs that the fundamental frequency for a same plate made of slightly different thickness how it will be behaving or what will be the change in that frequency.

So you have seen that a square plate and a rectangular plate. Similarly, if you change this a what will be that for this case, but if you say that 1/3 m=1 and n=3, in that case here 3 square 1 whole square you see that 9 times, so the second one had 1/3 frequency. So you can come up or sometimes this type of questions may ask in the assignments; may ask in the tutorials or it is also very useful you can do you can prepare your own diary.

That if my thickness is this what will be the natural frequency and so on.

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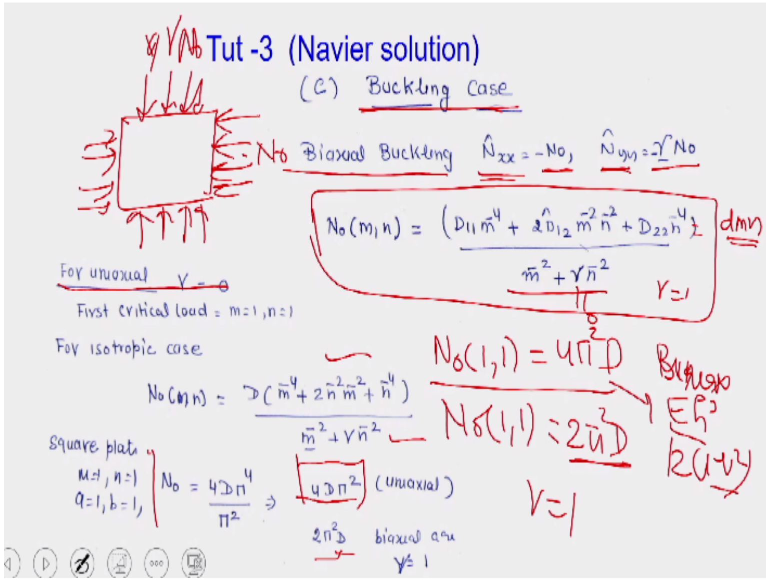

Now I am talking about that conceptual analysis of Buckling plate, buckling of a plate. So this is the buckling case when a plate is subjected to a Biaxial loading like this, I will call it not then like this gamma times or sometimes r0. So basically N xx hat is –N0, N yy is –gamma N0. For that case this is my formula for calculating the buckling load for a plate. So this is again Dmn.

Now you see that for the case of bending; for the case of re-vibration and for this buckling you have to evaluate Dmn every time. And here m square + n bar square. So for different m and n one may calculate or for a different D1 D2 one can calculate the things. So if we talk about a uniaxial buckling you put gamma = 0 then this /m square. If you are interested to find out for a biaxial buckling of a isotropic plate, thus it becomes D like this.

If you say a particular very special case m is 1, n is 1, a is 1, b is 1 then this load is reduced to 4 times pi square D N0 11 becomes 4 pi square D, biaxial buckling. If you talk about uniaxial buckling then gamma will be 1 and same load, sorry this is for uniaxial buckling, I forget that and your biaxial buckling where gamma is 1. Then you see that it is directly with the bending stiffness and bending stiffness is Eh cube/ 12 1-Mu square.

So that critical buckling load is directly to the stiffness of the material or directly proportional to the h cube thickness of the plate, and inversely proposed into the Poisson ratio. So based on that when we develop that okay if my Poisson ratio is changing or if my E is changing H is changing so what will be the buckling load.

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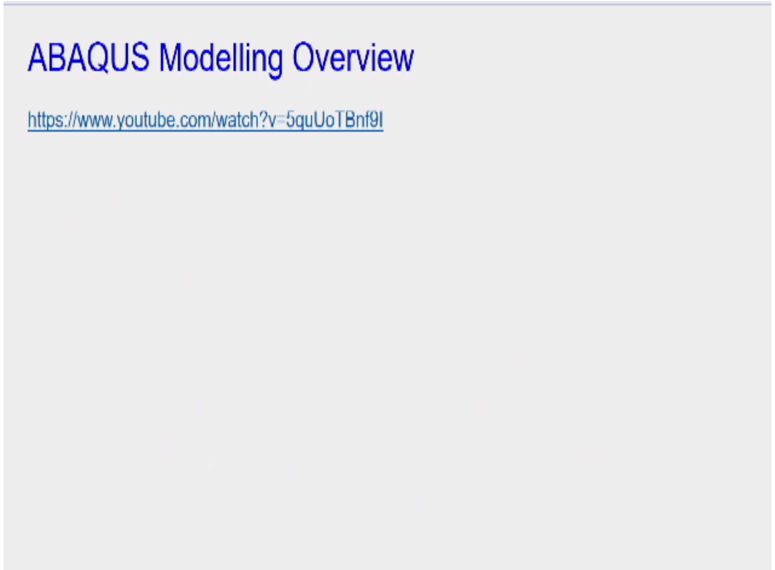

The last tutorial 1 and 2, I would like to give that this is the link of a YouTube video that one of my Ph.D student develop this, you can go through that how to model an orthotropic plate in ABAQUS.