Principle of Hydraulic Machines and System Design
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Lecture – 08
Stodola slip model, problems – III

So we will continue our discussion on Principle of Hydraulic Machine and System Design ah. Today we will solve the problem that we have discussed in the last lecture. So, the problem is on slip I mean as we have discussed that because of the slip the actual head developed by the pump deviates from that predicted by the Euler equation.

So, the problem statement is an 8 bladed radial flow pump has an impeller exit diameter of 20 centimeter.

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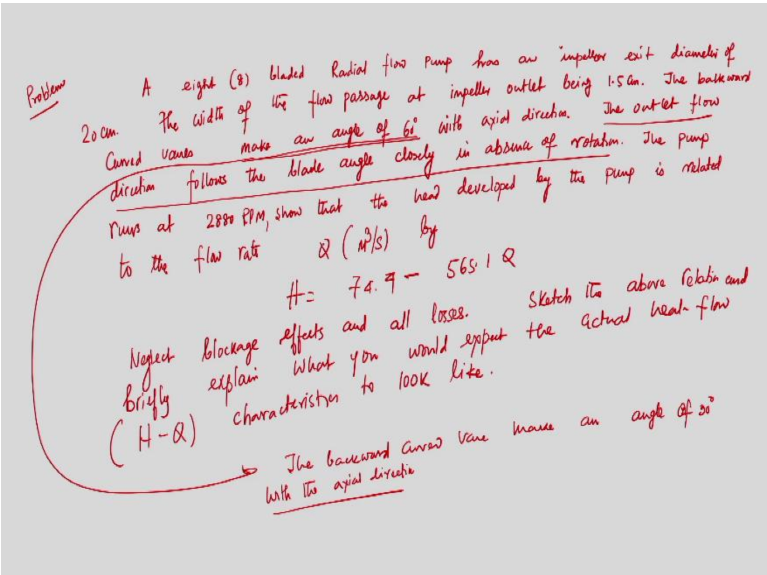

The width of the flow passage at the impeller outlet being 1.5 centimeter. The backward curved vanes make an angle 60 degree with the axial direction. There is a mistake in the problem statement. The backward curved vane makes an angle it will be the backward curved vane I am writing the backward curved vane; the backward curved vane makes an angle of 30 degree with the axial direction with the axial direction.

So, you know there is a mistake. So, this would be 30 degree that backward curved vane makes an make an angel 30 degree with axial direction. The pump runs at 2880 rpm, we need to show that the head developed by the pump is related to the flow rate H = 74.4 - 565.1 Q.

We need to ignore the blockage effect and all losses and finally, we need to sketch the relationships and again we need to make a comment on that relationship and how we can expect that the actual head flow characteristic will be will look like ok. So now, we will proceed to solve the problem.

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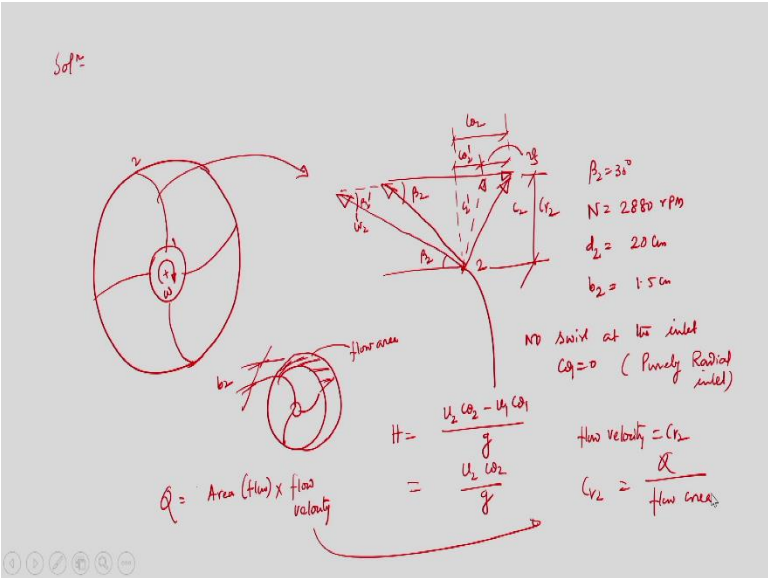

So, we will solve the problem. So, this is a radial flow pump. So, we need to draw one impeller schematic of the impeller of the radial flow pump. So, initially we should draw the impeller of a radial flow pump which is having backward curved vanes. So, these are the backward curved vanes. So, impeller is rotating in the clockwise direction at an angular velocity omega right.

This in the clockwise direction and we need to draw the velocity triangles at the outlet of the impeller. So, if I identified a particular blade by inlet is 1, outlet is at 2, then if I draw this; if I take out this blade inside draw the velocity triangles at the outlet. And I we can obtain velocity components.

So, this is β_2 blade angle, this is u 2 and this component is absolute velocity c 2. So, this is point 2 at the outlet it is given that backward curved vanes make an angle 30 degree with an axial direction. So, this is β_2, these $\beta_2 = 30$ given. Pump runs at 2880 rpm that is also given, d 2 exit diameter is given 20 centimeter. And width is given b 2 is 1.5 centimeter.

So, if I draw the impeller again and this is the width; so, this width is given b 2, that is 1.5 centimeter. So, width of the impeller is given outlet diameter d 2 is given. And we have drawn the velocity triangle this is w 2, but an important point that we need to you know take into account is that, it is written that outlet flow direction follows the blade angle closeness sense of rotation; that means, whenever pumps is not rotating, in absence of rotation, the outlet flow angle follows the blade angle. But whenever pump will start rotating the outlet flow angle will deviate; that means, there is a presence of slips.

So, this sentence is an indication of that we need to take the effect of slip into account while you are solving the problem. So, here we need to take the effect of slip. So, in presence of slip you have seen that that if I can recall that the resultant force, will shift the absolute velocity and relative velocity to their new position.

So, if I draw this is the changed absolute velocity C2', and similarly this is the w 2 prime. So, this will be β_2 dash. So, having of discuss that the angle changes from β_2 to $\beta_2{}'$, because of this resultant force that is acting on the forward surface of the blade. And which is responsible for the deviation of I mean actual head from the Euler head, from the head predicated by the Euler equation.

So now this is the $C_{\theta 2}$ and this is component of absolute velocity while you know we are concentrating the slips. And this is known as the slip velocity Vs. So, we have to solve this problem, since no information is given about the effect of inlet swirl, we are assuming that no swirl at the inlet. No swirl at the inlet. That is $C_{\theta 1} = 0$.

Now we need to solve the problem, n is given beta 2 is given d 2 is given and b 2 is given. So, if I recall the Euler equation which will gives us information about the head developed by the pump; that is, H = $(u2\ C_{\theta 2} - u1\ C_{\theta 1})/g$. Since we are assumed that, that are no information is given about the effect of the inlet swirl, we have assume inlet swirl component is 0, there is no inlet swirl is purely radial inlet. That is purely radial inlet; that means, head developed by the pump will be u2 $C_{\theta 2}$/g.

Now, since we are going to consider the effect of slip probably the component of absolute velocity there is all very important component for the, you know, head which is been developed by the pump that is $C_{\theta 2}$ will be $C_{\theta 2}{}'$ for the present case. So now, we will look at the effect look at the velocity triangles at the outlet.

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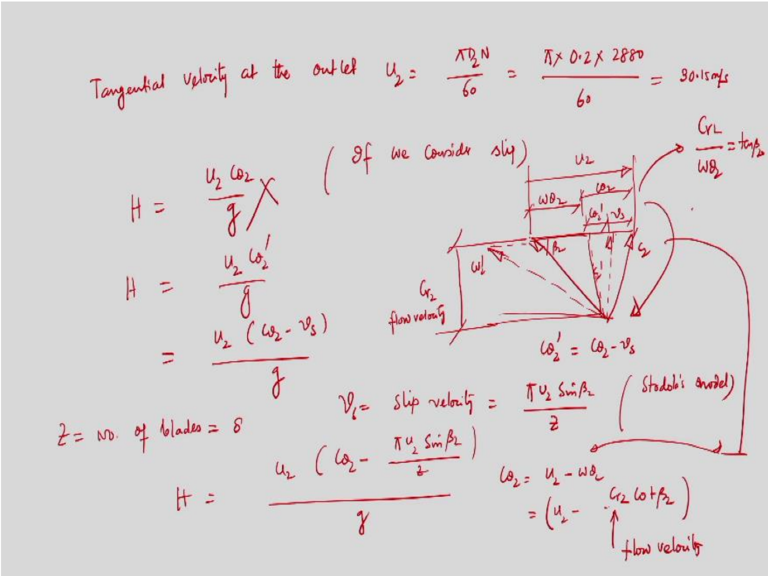

So now what is the tangential velocity?

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi * 0.2 * 2880}{60} = 90.15\ m/s$$

H = u2 $C_{\theta 2}/$g

Because if we consider slips, if we consider slips and that we need to take into account because it is written, that the when that when pump is rotating, the blade angle does not the flow direction does not you know [vocalize-noise] in presence of rotation, the flow angle you know blade angle the outlet flow direction, you know may not match or does not match with the you know blade angle.

So, it is given that in whenever know the rotation is there, that is in absence of rotation, flow direction will match the blade angle, I mean closely, but whenever pump will start rotating there might be a deviation. So, we need to take the effect of slip into account from this information, ok. If you consider the effect of slip into account, then we can write this expression that the component of absolute velocity in the tangential direction will be $C_{\theta 2}{}'$.

So, that is the head that will be developed by the pump which is given in the problem statement. So now, so, if we consider the velocity triangle again, just again I am drawing velocity triangles here

H = $\frac{u_2 C_{\theta 2}}{g} = \frac{u_2 C_{\theta 2}'}{g} = \frac{u_2 (C_{\theta 2} - Vs)}{g}$

Vs = slip velocity = $\frac{\pi\, u_2 \sin \beta 2}{z}$

H = $\frac{u_2 (C_{\theta 2} - Vs)}{g} = \frac{u_2 \left(C_{\theta 2} - \frac{\pi\, u_2 \sin \beta 2}{z}\right)}{g} = \frac{u_2 \left(u_2 - W_{\theta 2} - \frac{\pi\, u_2 \sin \beta 2}{z}\right)}{g}$

$\{C_{\theta 2} = u2 - W_{\theta 2}\}$

$\frac{C_{r2}}{W_{\theta 2}} = tan\beta_2$

$C_{\theta 2} = u2 - C_{r2} cot\beta_2$

So, we have obtained the expression of $C_{\theta 2}$ in terms of known quantities, but here again we are having another unknown quantity is the C r 2 that is flow velocity. So, we need to know again what the magnitude of flow velocity will be. Otherwise everything we know from this expression.

So, if we again it is given that the b d 2 is equal to 20 centimeter, that is 0.2 meter.

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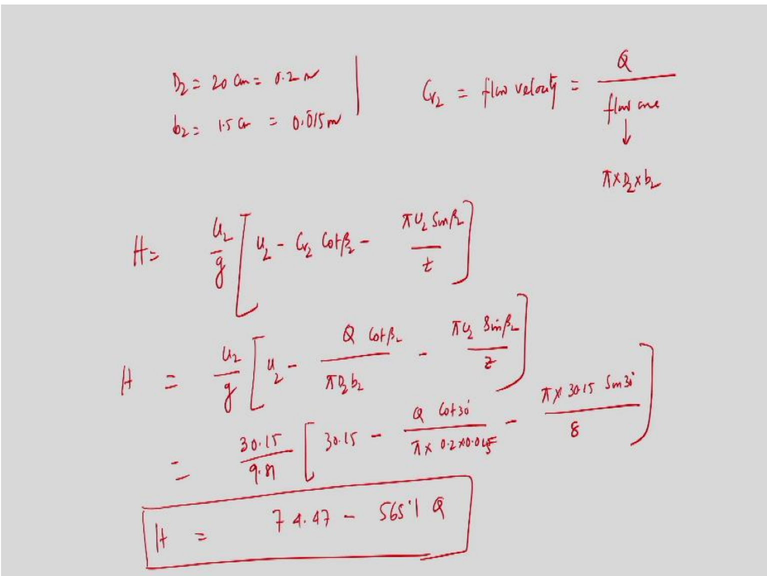

B 2 is equal to give an 1.5 centimeter, that is 0.15 meter, that is equal to given. Now we have to calculate Cr2, very important; mind it the problem statement is giving us I mean is asking to relate to establish a relationship between H and Q.

So, here we do not know an explicit magnitude of you know we do not know the magnitude of Cr2, but I can relate Cr2 in terms of the flow rate. So, if I go back to my previous slides, and if we see that here the flow area will be this is the flow area. So, whenever liquid is coming out from the impeller, whenever fluid is flowing in the passes between 2 blades. So, this is the flow area hashed portion is the flow area, this is the flow area.

$$C_{r2} = \frac{Q}{flow\ area}$$

Flow area = $\pi D_2 b_2$

$$H = \frac{u_2}{g}\left[u_2 - C_{r2}\cot\beta 2 - \frac{\pi\, u_2 \sin\beta 2}{z}\right]$$

$$= \frac{u_2}{g}\left[u_2 - \frac{Q}{\pi D_2 b_2}\cot\beta 2 - \frac{\pi\, u_2 \sin\beta 2}{z}\right]$$

$$= \frac{30.15}{9.81}\left[30.15 - \frac{Q}{\pi\ 0.2\ 0.015}\cot 30 - \frac{\pi\ 30.15 \sin 30}{8}\right] = 74.47 - 565.1\ Q$$

So, if we calculate this, we will get the final expression is 74.47 minus 565.1 Q. So, this is what we need to establish that the heads can be related in with the discharge following this relationship; that means, if you take the effect of slip into account, axial head developed by the pump may not be equal to the head predicted by the Euler equation. So, here although we are following the Euler equation, but we are considering the effect of slip and we have calculated the magnitude of slip following the Stodola’s model and we got the expression of that 70 H can be related in terms of 74.47 minus 560 51 Q.

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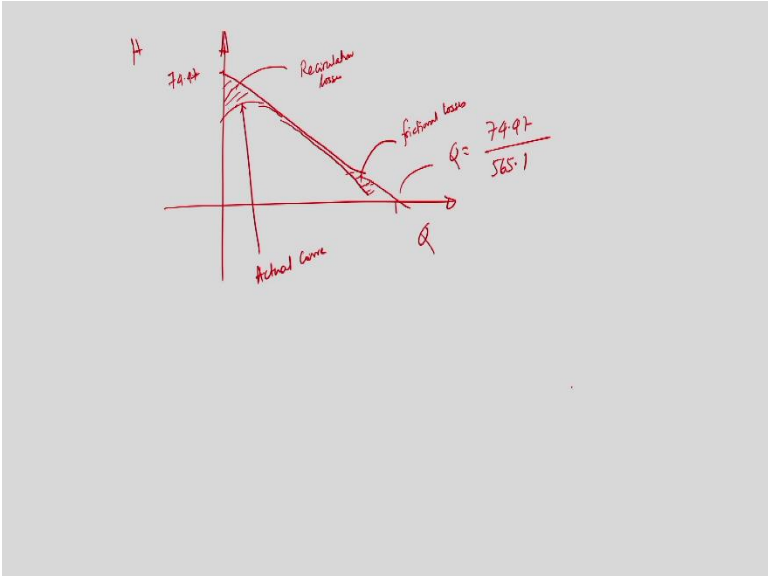

So, this is. So, this is actual H Q curve, I mean, following the relationship that we have obtained. But I may not, I would expect a different curve from that what we can obtain from the H Q relationship because as I said you although we have taken into account the effect of slip. But there will be a recirculation losses in the suction side that we have discussed that because of non-uniform from velocities, because whenever liquid is flowing through the pipe boundary layer start following and velocity may not be uniform, along the pipe I mean in a given construction.

So, velocity will change from one point to another point in a given construction. So, because of this non uniformity is the velocity distribution, there will be a recirculation loss in the pipe you know suction side. As a result of which this head I may not get and the actual curve will be like this. Actual curve will be like this. So, here we will have some losses.

And also here we will have some losses at the delivery side. So, this is losses in the suction side because of recirculation losses. And here you also have some losses that is frictional losses at this or separation losses at the delivery side. So, the actual curve that I would like to expect is the actual curve look like this, but if we draw the H Q curve following the relationship, that we have established that will be a straight line, but the actual curve will be look this. So, this will be the actual curve.

Because we need to take the effect of recirculation losses also the frictional losses recirculation losses in the suction side and the frictional losses in the delivery side into account. So, this is all about the problem that we have solved. So, may be may be if we normally our centrifugal pump testing, we will discuss where we vary the flow rate and we calculate the H that is developed by the pump. From they are if you tabulate the H Q data we can obtain the H Q curve. And H Q curves will be a straight line from whatever we have obtained from this problem also.

But in actual case it should not be like this, but sort of head should be reduced. So, because of the recirculation losses and at the suction side also there will be a some losses at the pump delivery side. So, the actual curve will be will not be a straight line, ok.

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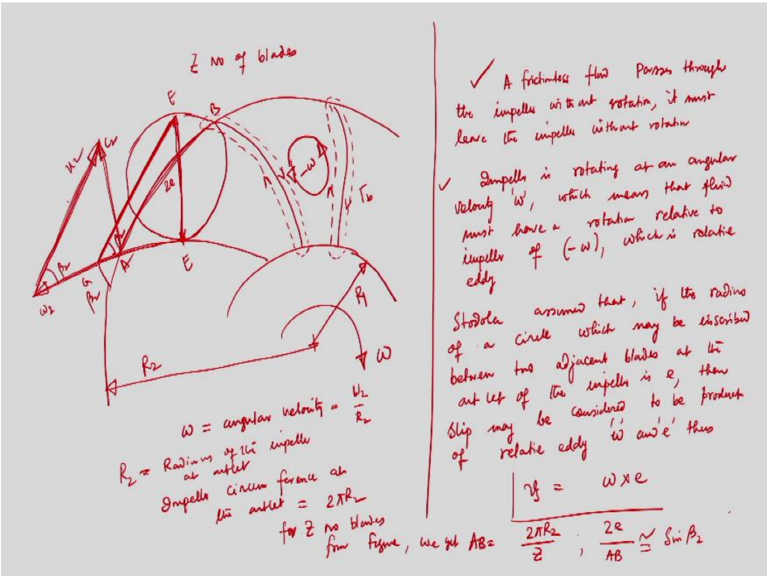

With this I will proceed to next part. But I will like to discuss another important point in this lecture; that when you have derive the expression of slip velocity following Stodola's model, and we have, you know Stodola.

Assume that, if a velocity can be inscribed you know if a circle can be inscribed at the outlet of the impeller is having radius e, then probably slip velocity is the product of omega that is the angular velocity at which impeller is rotating into e. So, rather the product of relative eddy into e; where if as I said that e to impeller is rotating at an angular velocity omega, then the fluid must have a rotation related to the impeller is minus negative omega.

So, this omega product is the slip velocity. So, we have try to calculate we have try to obtain the new you know a numerical expression of the slip velocity using a Stodola's model. And if we go back we have seen that this is a circle so, here this is you know you know portion of the length along the periphery between 2 blades. A portion of the length along the periphery of the impeller between 2 blades which is nothing but $\frac{2\pi\, r_2}{z}$.

And we are approximating this length will be equal to the sides of this triangle; that is, if I give a name E F G, then the side F G where approximating the side of this you know length the you know the magnitude of the magnitude of. These you know length that is G F is almost equal to the length of the periphery of the impeller between 2 blades adjacent blades. And that is 2 you know $\frac{2\pi\, r_2}{z}$.

So, a portion which is you know length of the portion between 2 adjacent blades H $\frac{2\pi\, r_2}{z}$ that is equal to the length G F in a triangle E G F that you are approximating and that is the approximation. We have done in the Stodola's model to obtain the numerical value of slip velocity.

And this is 2, because the blade angle for a given blade angle and probably, we have assume that the circle the maximum largest circle which can be inscribed between the you know at the outlet of the impeller E, then probably this approximation is not very bad approximations. So, with this we will proceed to the next part of this course.