Principle of Hydraulic Machines and System Design
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Lecture - 07
Stodola slip model, problems - II

So we will continue our discussion on Slips that is departure of Euler's theory. We have seen that the actual head developed by the pump is always less than that of the that of ideal head that is the head predicted by a Euler's theory. We have tried to quantify that what would be the possible reasons for which this is happening.

In fact, we have seen that slip is one of the slip is one important factor by which we always get I mean reduced head which is not the head developed by the pump you know we following the Euler's equation.

So, we have seen that if you calculate head developed by a pump using Euler's theory then of course, we will get ideal head and it is not the case that always we will get the ideal head because say losses, separation losses, a fictional losses and also the presence of non-uniform velocities a give rise to a deviation this deviation.

We have to quantified rather we have predicted the amount of you know slips because our discussion today our you know today our discussion was on basically a slip velocity and we have tried we have seen that mainly there are two reasons; one is the non-uniformity of the velocity because of the existence of the blades and of course, the formation of boundary layer there are you know separation losses, fictional losses and velocity to a formation.

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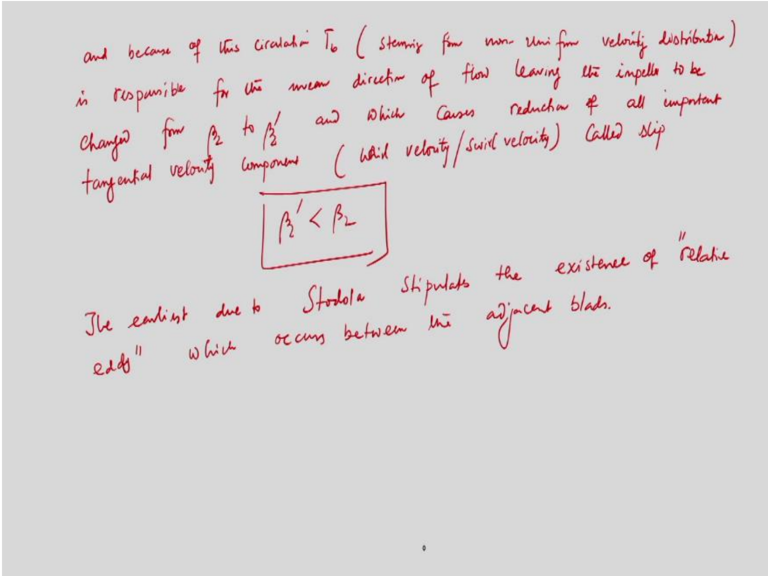

So, we have written that due to non-uniform velocity distribution there is a circulation around the blade and because of the circulations the mean velocity the mean direction of flow, the mean direction of flow leaving the impeller, changes from β_2 to $\beta_2{}'$ and β_2 and $\beta_2{}'$ the blade angle at the outlet I mean and due to the change in mean direction of flow at the impeller outlet, there is a reduction of very important there are all important velocity component that is swirl component or wall component which is called slip.

So, a change in swirl velocity at the outlet or change in wall velocity at the outlet say theta 2 at the outlet I mean the deviation or a change because of change because of the change of mean direction of the flow that is essentially because of the non-uniform velocities at the outlet of the impeller. There is a slip and that slip is responsible for the reduction of head which is predicted by Euler's theory.

So, we have understood that several reasons like, in a fictional loss you know and also the several losses it will recirculation separation and this is happening.

So, the earliest many we have seen that there are many you know formulas which have use to predict this amount of slip, but the Stodola who earliest I mean the earliest due to Stodola who is stipulates that existence of relative eddy I mean which occurs between the two which occurs between the adjacent blades and we have seen following Stodola.

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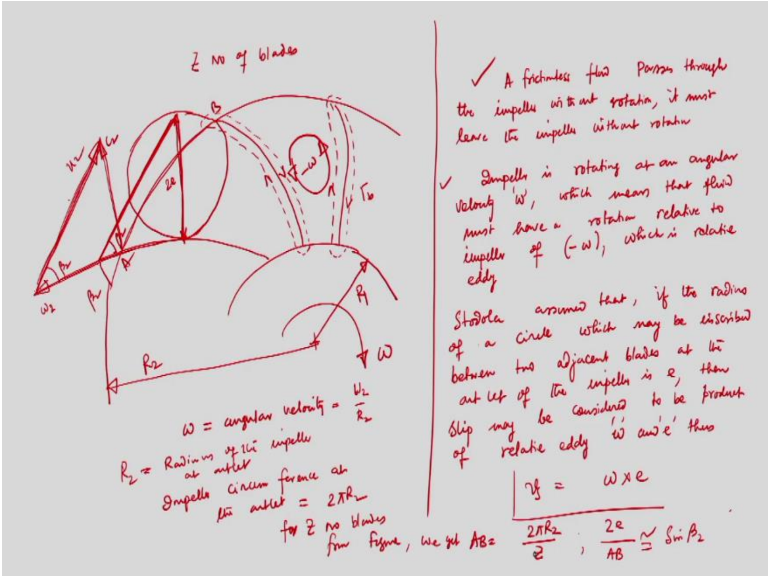

We have tried to quantify the amount of slip and Stodola assume that, whenever a frictionless flow is flowing through the passage of the impeller; that means, it must have leave the it must leave the impeller without rotation I mean if a frictionless flow passes through the impeller without rotation; that means, it must leave impeller without rotation.

And if we assume that the impeller is rotating at an angular velocity ω; that means, related to the impeller fluid must have a rotation in the negative omega, I mean which is the relative Eddy and Stodola assume that if a velocity I mean we would like to calculate the slip velocity; that means, this is happening at the impeller outlet.

So, diminished rather reduction of slip reduction of swirl component or wall component velocity at the outlet is responsible for these you know deviation I mean deviation of the actual energy transfer. Then Stodola, assume that if a circular of radius e can be inscribed at the outlet of the impeller, then from the geometry and from very simpler trigonometric calculation we have calculated that the diameter of the you know circle can be expressed in terms of the blade angular the outlet, radius at the radius of the impeller at the outlet and the number of blades and the slip velocity which is according to Stodola's model is the omega that is the angle of velocity to the angular velocity of the impeller multi input.

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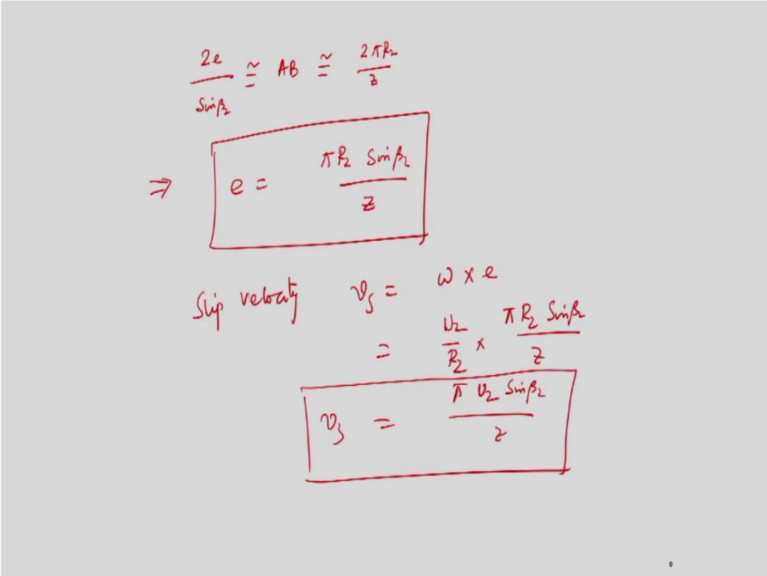

I mean the slip velocity can be calculated following Stodola’s model that is the product of angular velocity omega and the radius of the circle which is inscribe at the outlet of the impeller. So, I am following that we have arrived at this expression of slip velocity at the you know at the outlet of the impeller.

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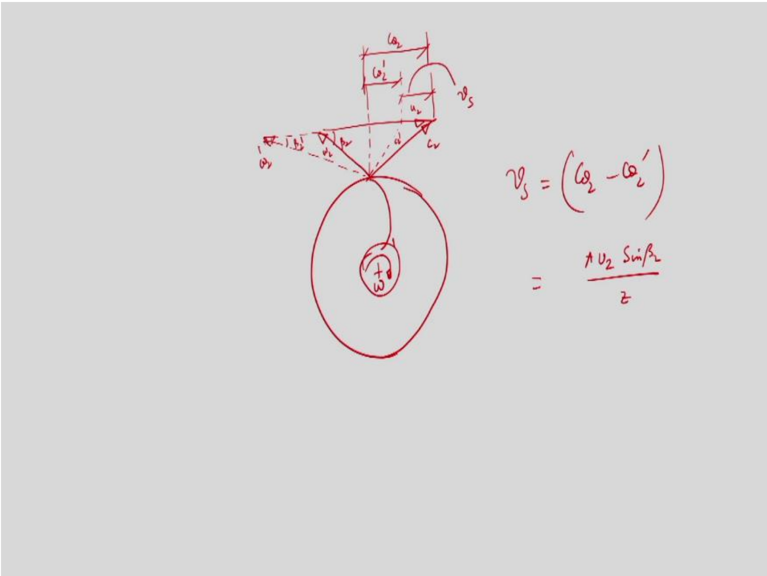

If we draw the velocity angles and the outlet again, suppose you are having an impeller pump impeller. So, if we have a impeller and if we draw suppose, this impeller is rotating at angular velocity omega and if we just draw a single blade 2 2 and if I draw the velocity angles at the outlet, so, we have seen that there is a change in mean direction of the flow at the outlet which changes β_2 with a changing β_2 from $\beta_2{}'$ that is what we have seen that because of the non-uniformity of the velocity distribution which is responsible for the mean direction of flow leaving the impeller to be changed from β_2 to $\beta_2{}'$.

So, if I draw the you know velocity triangle at the outlet, if this is the absolute velocity at the I mean this is a relative velocity at the outlet, but it changes because of this non uniformity and this is w 2 prime, this is w 2. So, this was β_2 and this is $\beta_2{}'$. So, the angle changes from β_2 to $\beta_2{}'$ because of this you know non uniformity of the velocity distribution which is essentially we call as the circulation and these changes of blade angle from β_2 to beta 2 is you know responsible for the changing I mean it reduces the all-important swirl component velocity at the outlet.

So, if we draw this is the absolute velocity at the outlet c 2 and this is u 2. So, the absolute velocity changes from C2 to C2' and this is known as slip velocity v s.

So, this was $C_{\theta 2}$ and this is $C_{\theta 2}$ and this is $C_{\theta 2}{}'$. So, we have seen that the main reason is that because of non-uniform velocities, of course, solid surfaces of the earth blade and existence of blade formation of boundary layer the direction of you know mean direction of flow leaving the impeller changes which also reduces the wall component of velocity by changing the absolute velocity at the outlet.

So, the absolute velocity of the outlet changes from C2 to C2' as a result of which it reduces the wall component of velocity at the outlet that is from $C_{\theta 2}$ to $C_{\theta 2}{}'$. So, now, this Vs is essentially $C_{\theta 2} - C_{\theta 2}{}'$.

So, a changing swirl component velocity at the outlet because of this changing that mean direction of flow at the outlet of the impeller will got the slip velocity.

So, now this is the quantification of slip velocity and we have calculated that slip velocity can be expressed in terms of

$$\text{Vs} = \frac{\pi\, u_2 \sin\beta_2}{z}$$

that is the expression of slip velocity at the for the pump operation.

So, this slip velocity because of the velocity slip we do not get the actual I mean ideal head. So, we always get the actual head which is always less than this ideal head. Apart from this there I mean I will now try to explain slip from different prospective ok.

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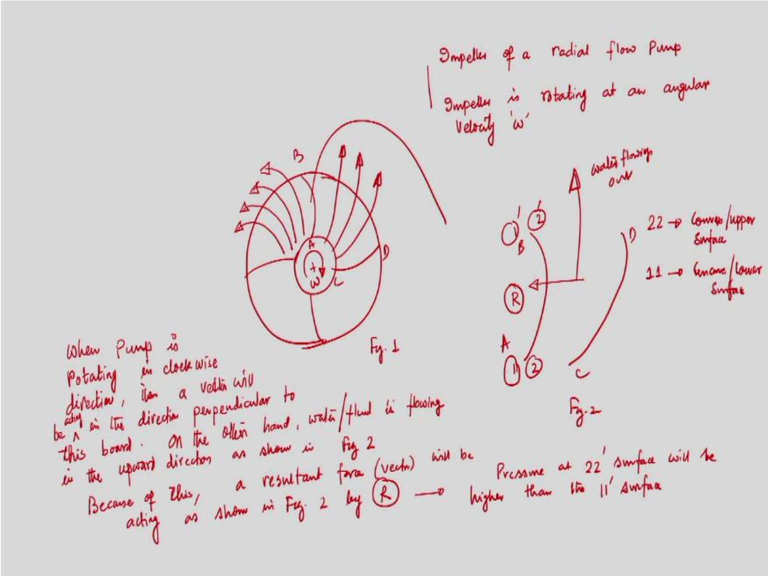

Stodola for slip blade the existence of relative eddy at the pump in which occur heating the blade passage and because of what we always get the diminished strength of swirl velocity at the outlet as result of which we always get I mean reduction in the head.

Now, for radial flow pump there is another you know argument from which we can explain that the reduction of slip velocity at the outlet. So, if I now draw an impeller I mean if a blade if I draw a again an impeller of a radial flow pump and suppose this impeller is rotating at an angular velocity omega.

So, impeller of a radial flow pump, this impeller is rotating at an angular velocity omega. So, this is impeller of a radial flow pump and impeller is rotating at an angular velocity omega.

And so whenever this impeller is rotating at an angular velocity omega, we have seen that this liquids are this is on that liquids are moving like this liquids are flowing out like this. So, liquids are moving like this and this is in housed in a casing.

So and there is a flow passage and while it is moving through the casing and it is ultimately going out. Now, there is an argument then when it is rotating suppose if I take this blade this blade and if I take out this blade and if I draw the enlarge view.

So, there are two surface 1 1 and 2 2; so, 2 2 is the convex or upper surface, 1 1 lower surface; concave or lower surface.

So, now, when this pump is rotating right, so, when pump is rotating some writing; when pump is rotating in clockwise direction then a vector will be in the direction perpendicular to this board.

On the other hand, water is or water or fluid vector will be acting, water or fluid is flowing in the upward direction.

So, if I take this particular two blades; one is A B another is C D. So, this is 1 1 surface lower surface, 2 2 surfaces is the upper surface; 1 1 is the lower surface and this is A B and this is C D, two adjacent blade we have considered.

So, whenever these implies the rotating of course, a vector will be acting in the direction perpendicular to this plane of this board and on the other hand water is flowing out? So, water is flowing out? So, this is water flowing out.

So, of course, will be in this direction that is perpendicular to the plane of the board and water is flowing out in the upward direction. So, this is the water is flowing out in the upward direction, upward direction as shown in figure let us say 2. So, this is figure 2. So, a force is acting in the direction perpendicular to the plane of the board. On the other hand, water is flowing out in the upward direction.

So, there will be a reason because of this because of this right because of this as per you know Fleming's rule resultant force or vector will be acting.

So, this is water flowing out in the upward direction and a resultant will be acting in this direction as let us say this is resultant force R, resultant force will be acting as shown in figure 2 by R right.

So, we have discussed in the previous lecture that the non-uniform velocity distribution and we have seen that velocity at the upper side will be always higher sorry lesser because of the higher pressure because pressure will be higher we have seen the blades to work on the blades do work on the fluid by exerting and impelling force on it and because of which pressure at the upper side of the blade will be higher than the lower side and since the pressure at the upper side of the blade is higher than the lower side there will be a you know mismatch of the velocity.

There velocity is will be higher at the lower side velocity to be lesser at the upper side and because of this changing velocity there will be a circulation that is represented by γ_b and whenever velocity is since pressure is higher, so force will be higher on the upper side.

Now, we have seen that because of the circulation I mean which is I mean stunning from the non-uniformity in a velocity distribution there is a I mean reduction in the in all important swirl velocity due to a change in direction of mean flow at the outlet of the impeller and ultimately we have calculated that because of this change in swirl component of velocity there is a you know change in head developed by the pump.

Rather, the to precise the absolute velocity at the outlet of the impeller is changing from C 2 to C2' and as a result of which we are getting a lesser amount of swirl velocity at the outlet. So, it reduces the head.

Now we are trying to you know explained otherwise you trying to understand this from a very from a different prospective and it is from our you know this is not that Stodola's model, many theories have been predict.

Many theories have been you know formulated to predict the rather to quantify the slip, but this is what we are discussing to for our own understanding that how we can understand the reduction of absolute sorry swirl component of velocity at the outlet of the impeller.

So and in order to do so what we have done that we have considered a radial flow pump, the impeller of a radial flow pump which is rotating in a clock wise direction at an angular velocity omega. We have drawn figure 1 and figure 2. Figure 1 which show the impeller of a radial flow pump and figure 2, just I have taken two adjacent blades A B and C D, we have seen the 2 2 at the upper surface of the blade and 1 1 at the lower surface of the blade.

So, if I take two adjacent blades then of course, water is flowing out in the upward direction because of this rotation of the impeller. Now, whenever impeller is rotating, so, a force vector will be acting in the direction perpendicular to the plane of the paper and water is flowing out in that upward direction. So, there will be a resultant vector as per Flaming's rule, there will be a resultant vector.

So, because of this there will be a resultant vector which will be acting in the direction as shown in a figure 2 that is R and because of this resultant force pressure at pressure at 2 2 pressure at 2 2' and 1 1' 2 2 dash surface will be higher than the 1 1' surface.

So, the resultant force is acting on the upper surface of the blade. So, pressure will be higher on that surface that is pressure at 2 2 surfaces will be always higher than the 1 1 surface.

That is pressure at the convex surface will be higher at the convex surface. So, pressure at the convex surface will be higher and pressure at the concave surface will be lower. If that is the case because of this change in pressure, I mean pressure at the convex surface is higher and pressure at the concave surface is lowered, in the last case we have discussed that of course, pressure difference you know we have seen that.

So, since it is rotating of course, it is blades are working on the fluid by exerting force. So, of course, pressure will be higher. From here the same thing, but we are trying to you know discuss from different prospective.

So, we have seen that this is not you know wrong, but what you have seen that whenever impeller is rotating. So, a vector will be acting in a direction perpendicular to the paper and since water is emerging out. So, there will and that is in the upward direction.

So, as for Flaming's rule there will be a resultant force which will be acting on the upper surface that is a denoted by R. So, the force which is acting on the upper surface naturally pressure at the upper surface will be higher and at the lower surface will be pressure will be lower less.

Now, since the pressure at the upper surface is higher and then the lower surface. So, it will try to if I draw again one velocity triangles.

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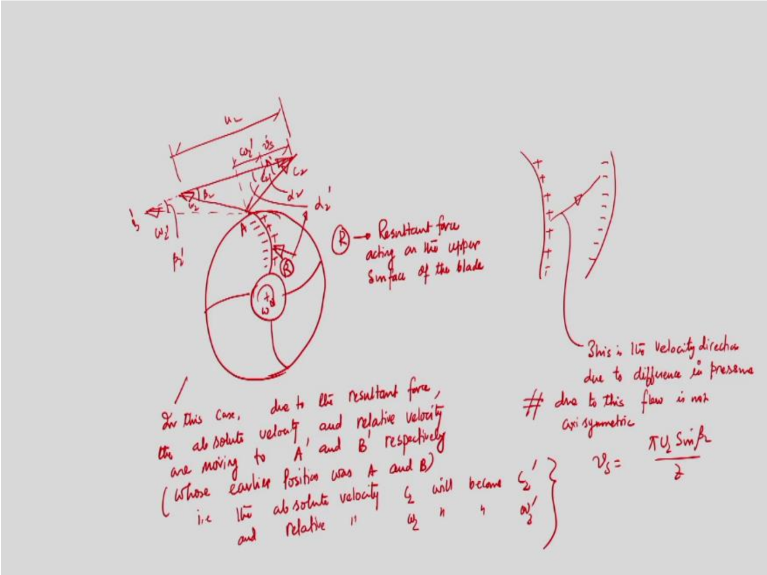

So, pressure at the upper surface is always higher and pressure is at the lower surface is always lesser I have I am denoting by positive and plus and minus sign.

So, now, what we can say that is pressure at the convex side is higher and the pressure at the concave side is lower and because of this the velocity due to difference in pressure will be form because of these if I would now draw the two adjacent blades again.

So, here pressure is higher here pressure is less. So, pressure is higher velocity is less here pressure is. So, this surface pressure is higher velocity is less this surface pressure is less velocity is high. So, there will be a velocity direction like this.

So, this is the velocity direction due to difference in pressure and due to this flow is not due to this velocity distribution because pressure at the convex side is higher and pressure at the concave side is lower. So, velocity will be just reverse.

So, we have a velocity distribution like this and due to this flow is not axis symmetric due to this flow is not axis symmetric. So, suppose this is the direction of water leaving the impeller and

So, this is the direction of water leaving the impeller the resultant force is acting in this direction. So, this is R resultant force, R is the resultant force acting on the upper surface of the blade. So, the resultant force acting on the upper surface of the blade.

That we have seen that is on the because of this two components of forces at one is because of the rotation of the impeller which is in the direction perpendicular to the plane of this board another is the water leaving or water emerging out of the impeller in the upward direction because of this Stodola assume that the existence of relative eddy when he suggested that if an frictional if a frictional less flow.

Is flowing through the impeller without rotation rather then it must leave the impeller without rotation and if we assume that the impeller is rotating at an angular velocity omega; that means, relative to the impeller fluid must have a rotation which is negative omega and he predicted the slip velocity will be you know angular velocity times the e if e is the circle can be inscribed at the outlet of the impeller.

Now, in this case we can say that due to this resultant force the absolute velocity and relative velocity. So, here the resultant force is acting on the upper surface of the blade; that means, if I now think that may be the impeller is rotating at an angular velocity omega, but because of this resultant force.

The blade will fill a retardation I mean as if some forces trying to retried the movement of the blade and as a result of which the absolute velocity and relative velocity are moving rather I can say that absolute velocity and relative velocity are moving to let say this is if this is a and this is A prime and this is B prime are moving to A prime and B prime respectively whose earlier position was A and B. So, we have seen that.

Of course, pressure is higher pressure is responsible we have seen that higher pressure because of the higher pressure there is a circulation around the blade and according to Stodola who first you know who first you know predicted the slip velocity which is angular velocity times these are radius of the circle which may be inscribed at the outlet of the impeller and we have understood that the non-uniform velocity distribution changes the mean direction of flow leaving the impeller and because of this change in mean direction of the flow.

The all-important velocity component that is swirl component in swirl component of velocity reduces. Here we have tried to understand the same phenomenon from a different prospective, but the you know mechanism is remaining almost same.

So, the resultant force whenever it is acting on the upper surface of the blade. So, as if it is trying to retire the movement as if it is as if the blade is filling from retardation effect because of this resultant force not only it fillings the retardation this retardation.

Because of these retardations I mean because of the resultant force the relative velocity and the absolute velocity that absolute velocity and relative velocity are moving to a new place A prime and B prime whose earlier positions was A and B.

That is that is the absolute velocity absolute velocity C2 will become C2' and relative velocity w t2 will become w2' right.

So, here you have understood that the resultant force is trying to provide a retardation effect of the blade as a result of which the absolute velocity and relative velocity are now changing to a new and new relative velocity and new absolute velocity C2' and w2' and since we have identified that the slip velocity can be expressed Vs $=\frac{\pi\, u_2 \sin\beta_2}{z}$ I can write in a bit different form that it can be since I have written in terms of blade angle at the outlet.

I also can write in terms of flow angle at the outlet now we can define one factor which is known as slip factor. So, if I go to next slide.

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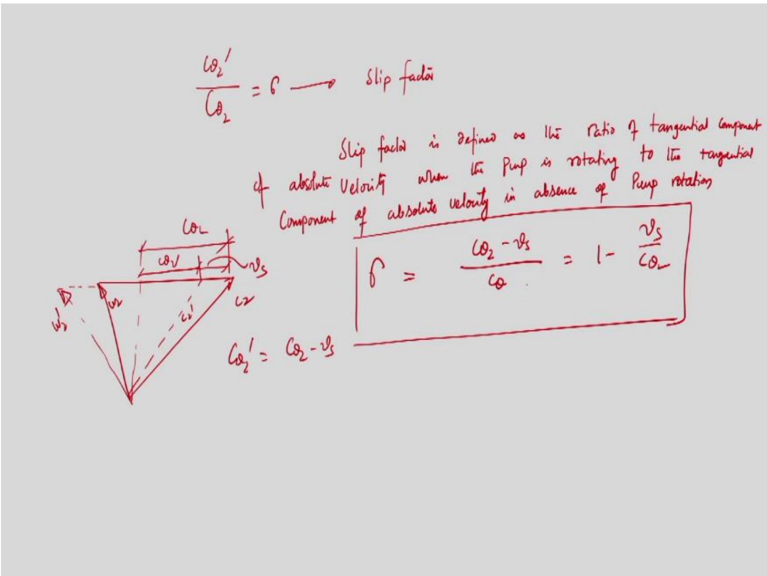

So, we have seen that whatever it is the main mechanism of having this you know undesirable phenomenon slip it is not desirable because it always reduces the head of the pump is only because of non-uniform velocity distribution.

We have try to quantify it using a Stodola's model which you know predicted the existence of relative eddy between two adjacent blades, but and also we have tried to understand it from a different prospective all though the mechanism.

Is remaining same that whenever an impeller is rotating, and water is emerging out in the upward direction. So, it is rotating means we have a force in the direction perpendicular to the plane of the board and whenever water is emerging out from the impeller then we are having an upward force in the upward direction.

So, as per Flaming's rules there will be a resultant force of vector which is acting on the upper surface of the blade and it is providing higher pressure at the upper surface of the blade and this resultant for because of this resultant force as if the blade is filling a retardation effect.

So, that the absolute velocity and relative velocities velocity are now taking at there are different position and since the absolute velocity is taking a different position the swirl component which is very important will be a new swirl velocity $C_{\theta 2}'$.

So, we have seen that the swirl component of velocity was $C_{\theta 2}$ earlier now it is change into $C_{\theta 2}'$ and the ratio of these two components is known as Slip Factor σ. So, this is known as slip factor right.

$$\sigma = \frac{C_{\theta 2}'}{C_{\theta 2}}$$

So, this slip factor is defined as the ratio of as the ratio of tangential velocity that the tangential component of absolute velocity, when the pump is rotating to the tangential component of absolute velocity in absence of pump rotation.

$$\sigma = \frac{C_{\theta 2} - Vs}{C_{\theta 2}} = 1 - \frac{Vs}{C_{\theta 2}}$$

So, from here you can understand that the amount of slip velocity divided by the actual swirl component of velocity is very important which gives an idea about the slip factor.

So, we have understood the you know slip and why it is you know you know originating while pump is rotating we have try to explain it from two different prospective I mean from two different angles rather using Stodola's model we have quantified the amount of slips and also from we have understood the existence of slip while pump is rotating.

That is mainly because of the non uniform of the velocities only presence of blades and also the development of the boundary layer.

So, we will work out one example on this and probably that is only because of the slip very important is that we need to solve a numerical problem to calculate because the head developed by the pump will be always less than I mean the head predicted by the Euler equations. I will like to solve one numerical problem on this aspect that is slip.

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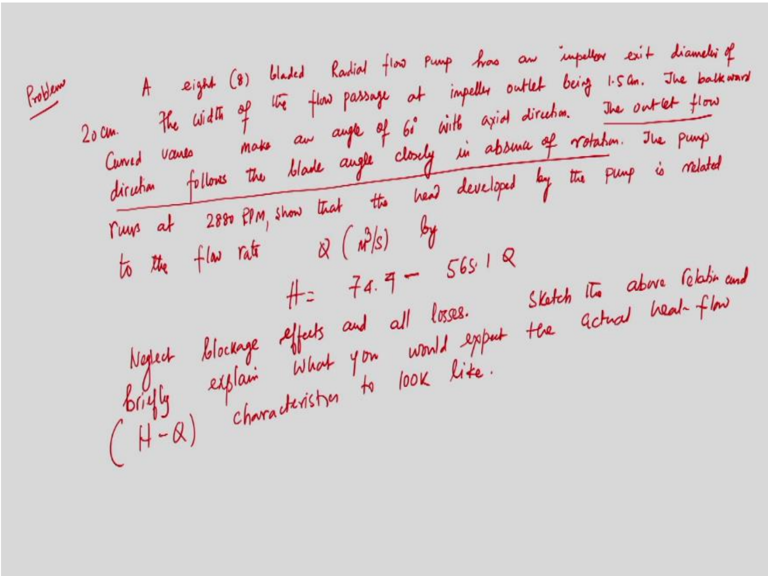

So, I am writing the problem, a 8 bladed radial flow pump has an impeller exit diameter of 20 centimetre. The width of the flow passage at impeller outlet being 1.5 centimetre. The backward curved vanes make an angle of 60 degree with axial direction, with axial direction; very important is the outlet flow direction.

The outlet flow direction follows the blade angle closely in absence of rotation. I am underlining this sentence. This is very important; this sentence gives us a clue that the outlet flow direction follows the blade angle closely in absence of rotation; that means, whenever pump is rotating the flow direction may not follow the blade angle.

So, whatever we have discussed today that because of these you know circulation because of the non-uniformity of the velocity distribution around the blades.

There is a mean there is a change in mean direction of flow from β_2 to $\beta_2{}'$ which is responsible for that reduction in reduction in all important swirl component velocity $C_{\theta 2}$ and which is called as slip.

So, the sentence is giving us a clue that the outlet flow direction follows the blade angle closely in absence of rotation, but whenever there is a rotation the flow direction may not follow the blade angle. So, there might be a slip. So, rotation the pump runs at 2880 rpm.

So, that the head developed sorry. So, that the head developed by the pump is related to the flow rate Q which is given by meter cube per second by H = 74.4 - 565.1 Q neglect blockage effects and all losses. Sketch the above relation is very important because we have discussed.

So, far the H Q curve pump and we have seen that the actual H Q curve is always is not the ideal one because to take the effect of recirculation losses in the section side and also the separation losses in the delivery side it takes a new shape.

So, sketch the above relation and briefly explain what you would expect the actual head flow that H cube characteristics to look like.

So, this is the problem we need to solve, this is a problem only on the slips. I would like to discuss this problem because we have understand we have we need to understand if really slip is there then what would be the actual head and we have understood the mechanism of slips in a pump operation and because of the slip the there is a deviation of energy transfer that is predicted by Euler's equations and the real case.

So, this is the problem where clue is given, the problem is not directly telling you that you need to consider slip while you are solving the problem, but there is a clue which is given that the outlet flow direction follows the blade angle closely in absence of rotation; that means, when rotation is there the outlet flow angle will deviate from the blade angle that is it will change from β_2 to $\beta_2{}'$ and which is responsible for the changing for the reduction in the reduction of important component of velocity is swirl velocity which is responsible for the development of the head. So, we will solve this problem in the next class and we will continue our discussion on the slip in the next class ok.

Thank you.