Principle of Hydraulic Machines and System Design
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Lecture – 06
Stodola slip model, problems – I

We will continue our discussion on Principle of Hydraulic Machines and System Design. Today I will discuss on Stodola slip model and problems on based on this model. So, why slip is important when we are talking about operation of a pump?

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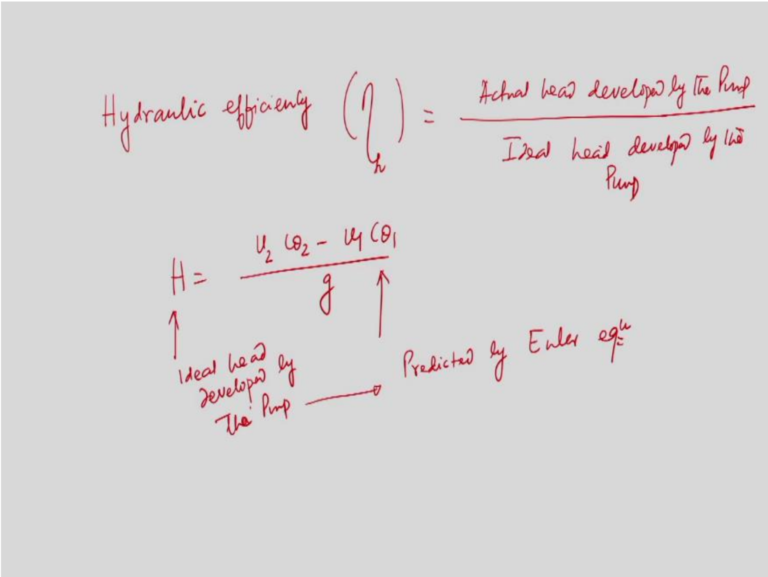

We have discussed that one efficiency of a pump is hydraulic efficiency and we have written that hydraulic efficiency of a pump

$$\eta_H = \frac{Actual\ head\ developed\ by\ pump}{Ideal\ head\ developed}$$

So, we can see from this definition there are some this efficiency that probably actual head developed by the pump is already lesser than that of the ideal head. So, why it is happening I mean why actual energy we have seen that pump absorbs energy. So, whenever pump in a pump we have discussed that mechanical energy from the moving part is converted to the to increase the energy of the fluid; stored energy of the fluid by increasing either a pressure or velocity.

So, there are certain losses; so actual energy transfer in achieved by the hydraulic machines I mean by the pump. So, actual energy transfer achieved by the pump is smaller than that of the ideal. Since ideal head developed by the pump if we again recall that ideal head developed by the pump H =$(u2\ C_{\theta 2} - u1\ C_{\theta 1})/g$; so, this is the ideal head developed by the pump that is predicted that is predicted by the Euler's equation.

So, these equations I mean whenever you are talking about ideal head developed by the pump this equation I mean this ideal head developed by the pump is predicted by Euler equation; predicted by Euler equation. But that means, the head predicted by Euler equation is not the actual head, but actual head developed by the pump will be always lesser than that predicted by the Euler equation.

Now, my question is why that I mean deviation is there? Why the actual head is always smaller than that of the ideal head predicted by the Euler equation? We have discuss that there are losses I means whenever we are talking wherever you are calculating ideal head from Euler equation; probably we did in take in to account those losses, but probably to take those losses into account of course, the actual head will be always lesser than that the ideal head. Now, today I will discuss an important topic that is slips.

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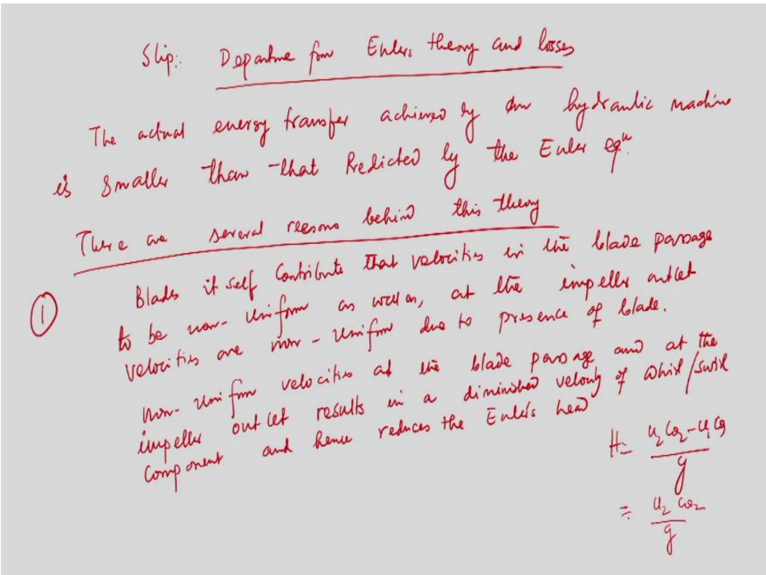

So, why slip? And probably slip is one of the important you know is an important phenomenon probably because of which the there are deviation of the energy transfer. So, while you are talking about slip in a pump we need to discuss there are so many models available to quantify this amount of slips that is there you know pump, but we will discuss one important model from which we will we will try to quantify the amount of slips.

The slip is an outcome; I mean probably that we have discussed that the actual head is not equal to the ideal head. Slip could be an one of the reasons which allow deviating actual energy transfer in a hydraulic machines.

So, we are writing that the actual energy transfer, the actual energy transfer, the actual energy transfer achieved by an hydraulic machine; I am writing an hydraulic machine is smaller than that predicted by the Euler equation.

That mean the energy transfer achieved by the hydraulic machine is always smaller than that predicted by the Euler equation. And slips is one of the factors probably that is responsible for this deviation we will discuss. So, there are many reasons why this behind this theory. So, this is a theory and that is departure from Euler theory; Euler's theory and losses.

So, when you are thinking about that actual head other even your we have seen that the actual head and that is true; that actual head developed for the pump is always smaller than that of the ideal head that is the head predicted by the Euler equation. So that means, there is a deviation and slip is one of the factors which is responsible for the deviation. That is slip is one of the factors which I mean always allow that the you know which is responsible for this you know departure I mean which is responsible for the deviation of energy transfer actual energy transfer in a hydraulic machine.

So, there are I am writing there are several reasons, there are several reasons behind this theory; there are several reasons behind this theory. I will write and I will discuss; first one is that blade of an hydraulic machines blade itself contribute that velocities in the blade passage; to be non-uniform. Yesterday we have discussed that velocities in the flow process are not uniform probably there are losses each circulation losses, separation losses.

So, departure of Euler theory departure from Euler theory and losses when you are talking about; there are several reasons first reason is that blade itself contribute that velocities in the blade passage to be non-uniform. As well as at the impeller outlet, pump impeller outlet velocities are non-uniform due to presence of blade right.

This non uniform velocities; non uniform velocities at the blade passage and at the impeller outlet results in a diminished velocity of whirl component or swirl component or swirl component and hence reduces the Euler's head.

So, we have discussed that there is a departure of Euler theory and; that means, actual energy transfer achieved by a hydraulic machines that is a pump is smaller than that predicted Euler's equation; there are several reasons I am I am going to discuss about slips. So, just I am discussing what could be the causes for that slips, there are several reasons behind this theory that when I mean whenever we are talking about pump or hydraulic turbine; there are blade there are blades the impeller or runner and fluid is flowing in the process between to guide bends or blades between 2 blades.

So, blade itself contribute that velocities in the blade passages to be non-uniform. And we have discussed that non uniform velocities because of this existence of non-uniform velocities, there are recirculation losses eddies formation all those things also at the outlet of the impeller velocities are non-uniform because of the presence of blade.

This non uniform velocities at the blade passage and at the impeller outlet results in a diminished component of swirl velocity or whirl velocity; which is very important for the development of at that we have discussed that the you know Euler's equation if you can recall. If we recall that Euler's equation H =$(u2\ C_{\theta 2} - u1\ C_{\theta 1})/g$.

So, this $C_{\theta 2}$ and $C_{\theta 1}$ are very important. So, $C_{\theta 1}$ is also important, but we have seen that we can make $C_{\theta 1}$ positive negative and 0; positive is not an important issue because positive $C_{\theta 1}$ always we lead to always we leads to a diminished strength of a development I mean it is always decrease the head development.

But if you can make $C_{\theta 1}$ is 0 that is by purely radial inlet or sometimes making c theta 1 negative we can increase the head development, but 0 is fine, but if you can make negative $C_{\theta 1}$ that we have discuss that negative c theta 1 will always lead to a higher relative velocity at the inlet. And if velocity increases pressure will fall and if pressure falls; the vapor pressure at that temperature then it will lead to an undesirable phenomenon which is known as cavitation.

So, one important issue; so if you make a $C_{\theta 1}$ 0 by making a purely radial inlet probably then in that case here developed will be u2 $C_{\theta 2}$/g. So, swirl component or whirl component of velocity at the outlet is a always an always the important for this because u 2 is fixed; since the outer diameter of the impeller is six ten impeller is rotating at a certain speed. So, $C_{\theta 1}$ is very important while you are talking about total head developed by the pump is in Euler's equation.

Now, because of this non uniform velocity at the blades [pass/passage] passage as well as the impeller outlet the swirl component of velocity of swirl component of velocity reduces eventually which results a deviation from the Euler's head. So, while you are calculating head using Euler's equation that we are getting u2 $C_{\theta 2}/g$, but we do not know a priory that what could be the exact value of $C_{\theta 1}$ in reality; so, in real case.

So, whenever fluid is flowing through the blade passage and then this $C_{\theta 1}$; actual $C_{\theta 1}$ will not be equal to the ideal $C_{\theta 1}$ that we are calculating from velocity triangle. So, to take that non uniform velocity at the blade passage as well as the impeller outlet into we need to take into account to calculate the actual head. So, this is the one of the important reasons and another reason could be I am writing. So, one reason is that the non-uniform velocity that is there because of the blade and the because of the presence of blade in a blade passages as well as the impeller outlet.

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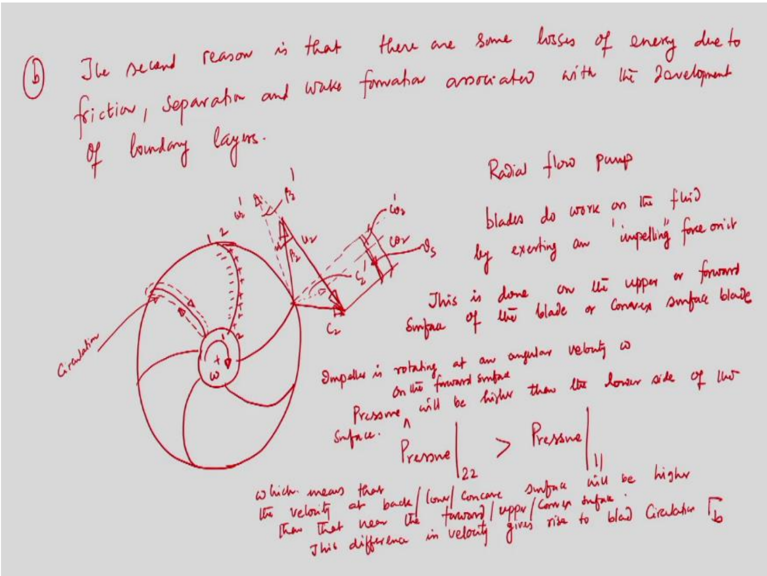

Second reason is that the second reason is that; there are some losses of energy due to friction separation and wave formation associated with the development of boundary layer development of boundary layers.

So, as I discussed that whenever fluid is flowing over a solid surface, we cannot ignore the development of boundary layer will develop. And whenever boundary layer is being developed there will be a loss because of friction separation and wave formation.

So, whenever fluid is flowing between the blade passages I mean the through the impeller that is between the process in a process between 2 blades boundary layer will develop and because of this development of boundary layer there will be I mean there will be losses because of the friction separation and wave formation. So, these are the 2 important reason from which we can say that the actual energy transfer in a pump will be always less than that predicted by the Euler equation.

So, now we need to quantify I mean there are several models I mean try to predict the actual energy transfer and what could be the actual swirl component of velocity at the impeller outlet. And which can give us and actual estimate about the head development by a pump there are several models. So, I will discuss one model which was you know predicated by Stodola; who first you know predicted. So, before I go to discuss about that model, again I will try to recapitulate what is happening whenever fluid is flowing through a impeller that is through a process between 2 blades.

So, if I draw again an impeller. So, this is an impeller this is hub; so, this is rotating clockwise direction and we are assuming there are backward cup bands. So, this impeller is rotating in a clockwise direction and impeller is having backward cup bands.

You know this is $C_{\theta 2}$ and this is $C_{\theta 2}$ this is $C_{\theta 2}$' and these velocities is known as slip velocity Vs. So, we have drawn with velocity triangle at the outlet and we will discuss that this is high pressure less than; I will discuss about why this is high pressure, and this is low pressure reason. As a result of which we are having one circulations around the blade right.

So, in case of a reason impeller of a radial flow pump; so, for a radial flow pump I am writing; for radial flow pump blades for a radial flow pump blades do work on the fluid by exerting and impelling; by exerting and impelling force on it impelling force on it.

And whenever; so in a radial flow pump blades do work on the fluid by exerting and impelling force on it. And because of this and this is done, this work is done and by exerting and impelling force on it this; this is done on the upper or forward surface of the blade or convex surface of the blade right.

So, in a centrifugal pump I am trying to estimate the amount of slips; as I discussed that there are several reasons because of these deviation of Euler theory. And we have discussed that actual head developed by the pump will be always lesser than that predicted by Euler's equation and even if you can make a purely radial inlet at the entry.

So, there will be losses of friction, separation and wave formation with a development of boundary layer and also we have discussed that because of this non uniform of the velocities in the flow fluid you know flow passage because of the existence of blade; there will be a diminish strength of swirl component at the outlet since the actual head predicated ideal head predicted by the Euler's equation is use to $C_{\theta 2}/$ g in case of a purely radial inlet.

So, the swirl component or whirl component of velocity at the impeller outlet $C_{\theta 2}$ will be reduced because of these losses. Now, we need to quantify and the reduction of the swirl component velocity at the inlet is known as the slip velocity. So, we need to quantify what could be the amount what would be the amount of slip velocity at the impeller outlet and that is why I am now trying for a for a radial flow pump.

So, for a radial flow pump blades do work on the blades do work on the fluid by exerting and impelling force on it. And this is done on the upper or forward surface of the blade or the convex surface of the blade the; if the blade is rotating in a clockwise direction. Suppose blade is rotating with an angular velocity ω. So, impeller blades or I mean blades are rotating or impeller is rotating at an angular velocity omega.

So, if blade or impeller rotates with an angular with an angular velocity omega; then and we have drawn the velocity triangle at the outlet and we are trying to quantify what will be the amount of slip velocity that is we have seen that the actual; you know absolute velocity C_{θ}' will be I mean c theta the actual absolute velocity will be c c 2 prime not c 2.

And actual relative velocity will be w 2 prime not the w 2. So, these you know blade angle will be reduced from beta 2 to beta 2 prime and these whenever we are reducing the actual absolute velocity; c 2 prime then this post this velocity is known as slip velocities. So, you need to quantify what is the magnitude of the slip velocity and what are the reason of the slip velocity I mean why this is you know why I mean of course, you have seen that losses and non-uniform velocities; the there will be a diminished strength of swirl component of velocity.

But now I need to know what are the reasons because of which the slip you know swirl component of velocity at the outlet you know decreases. So, that is what now I am trying. So, whenever blades to work on the fluid by exerting and impelling force on it and this work is done on the upper surface or forward surface.

Because here the forward surface, suppose this is 1, this is 1, this is 2, this is 2. So, 2; 2 is a forward surface one another backward surface whenever blade impeller rotating in a clockwise direction. So, the work is done on the forward surface that is on the surface 2 2; as a result of which the pressure at the upper surface since work is done on the upper or forward surface of the work from the surface of the blade.

So, pressure will be higher than the pressure; you know pressure on the forward surface will be higher than the you know lower side of the surface, lower side of the surface right. So, pressure will be higher than the lower side of the surface, see pressure is equal to force into area.

Because so force is exerted on the upper surface or forward surface or convex surface; so, pressure at surface that is pressure at surface 2 2 will be greater than pressure at surface 1 1; pressure at surface 1 1. So, because of this impelling force exerted by the blades on the fluid; as a result of which pressure at the forward surface or convex surface will be higher than the that of the lower surface.

So, whenever pressure at the upper surface or forward surface is higher that is pressure at surface 2 2 will be higher and pressure surface 1 1 will be lesser and since pressure is higher. So, velocity at surface 2 2 will be higher and velocity at surface 1 1 will be higher than the velocity as 2 2, as a result of which there will be circulation. So, that is what I have you know drawn.

So, this is a circulation that is stemming from the stemming because of the differencing because of the pressure difference. So, since pressure at the surface 2 2 is higher than the pressure surface 1 1; that means, this which means that which means that the velocities velocity at back or lower or concaved surface will be higher than that near the forward, upper or convex surface; convex surface. This difference in velocity; this difference in velocity gives rise to blade circulation, you know capital γ_b.

So, pressure difference gives rise to a velocity difference and this velocity difference between the back and up forward surface gives rise to a blade circulation you know gamma b. We will discuss that this blade circulation plays at the role for the formation of a eddy and because of the formation of eddy there is a diminished strength of whirl component of velocity at the outlet and because of which actual head is always less than the ideal head.

Now, there are many other reasons one reason is that that the circulation gives rise to a formation of eddy. And that eddy is responsible for the diminished strength of swirl velocity at the outlet and which will definitely reduce the actual head developed by the pump. But another reason is that you know there are other reasons we will discuss.

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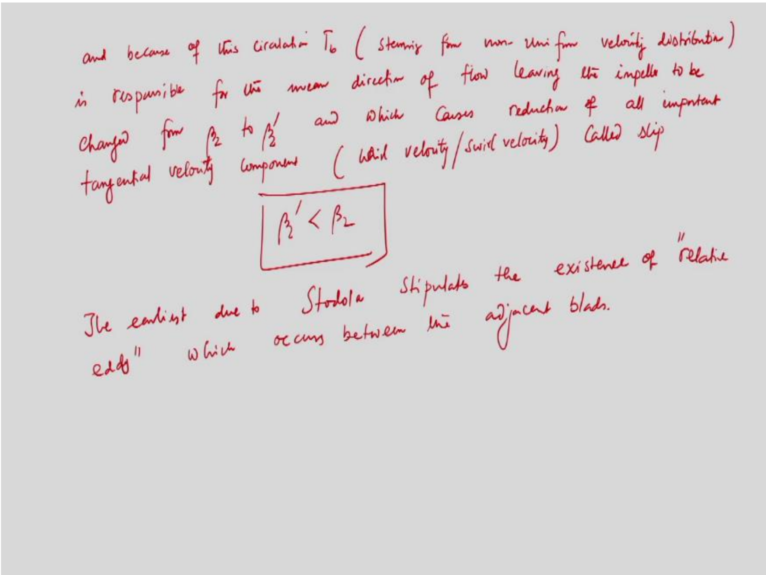

And because of the circulation because of the circulation; gamma b that stemming form, non-uniform velocity; you know velocity distribution rather; velocity distribution is responsible for the; is responsible for the mean direction for the mean direction of flow leaving the impeller leaving the impeller to be changed from β_2 to $\beta_2{}'$ and which causes reduction of all-important tangential velocity component that is known as whirl velocity or swirl velocity called slip. So, whatever we have seen that we have seen that we have started our discussion on the topic of slip what is slip?

So, there are I mean losses whenever fluid is flowing through the impeller that is process between 2 blades. And whenever fluid is flowing between the process of the blades non uniform the existence of the blade itself will provide non uniform velocities at the you know outlet of the impeller; as well as that whenever fluid is flowing through blade passage.

Since non uniform velocity is responsible for the you know; as I said you the non-uniform velocity responsible for the deviation, I mean for the losses because of the formulation of you know eddies in circulation losses. And also, we have discussed that there several losses because of the friction, separation wave formation and which are associated with the development of the boundary layer.

Now, whenever there are losses then of course, the actual head predicted by the head predicted by the Euler's equation is not I mean this is not correct I mean head predicted by the Euler's equation will not be the actual head develop by the pump; there will be certain deviation.

We have tried to quantify that deviation and we have tried to we have seen that the head developed by head predicted by the Euler's equation is due to H =$(u2\ C_{\theta 2} - u1\ C_{\theta 1})/g$ is the absolute velocity component of absolute velocity is the inlet that is swirl velocity at the inlet.

If I make $C_{\theta 1} = 0$ that is the best possible case because $C_{\theta 1}$ positive will always it is a head c theta; negative $C_{\theta 1}$ always we will try to at head development by the pump, but it will lead to a different other problem. Because negative component of $C_{\theta 1}$ although it will increase the head developed by the pump.

But at the same time it will invite another problem of you know a cavitation that we have discussed that relative velocity will increase as a result of which the pressure will fall. If pressure falls below the (Refer Time: 31:01) pressure at that temperature then local wailing will takes place and it may lead to another undesirable phenomenon of cavitation; so, known as cavitation.

Now, we have discussed that because of these losses there will a circulation I mean in a in radial flow pump; whenever impeller is rotating the blade blades do work on the fluid by exerting and impelling force on it. So, and this is done on the convex or the upper or the forward surface of the blade. So, whenever work is done on the upper surface or forward surface, then the pressure will be high and at the lower surface pressure will be less.

So, because of the pressure difference there will be difference in the velocities and because of this non uniform velocity; I mean velocity distribution there will be a circulation around on the blade which is we which we have seen. And that circulation it $\gamma_b$is responsible for the mean direction of flow mean direction of flow, leaving the impeller sends to β_2 from a β_1.

So, that is why if I go back to my previous slide; we can see that the circulation, the circulation is responsible for the change of mean direction of flow leaving the impeller will be pumped will be $\beta_2{}'$ from β_2. And this β_2 this $\beta_2{}'$ is always less than β_2; so, you know we have seen that the β_2 is changed to β_2 and $\beta_2{}'$ is always less than β_2.

So, because of this deviation of this mean you know direction of flow leaving the impeller; there is a reduction of all-important tangential velocity component, that is a component of absolute velocity in the tangential direction that is swirl velocity or whirl velocity which is called slip.

Now, I can say that many attempts have been taken; many attempts have been taken to predict to quantify the amount of slip and a number of stages have been formulated. But the earliest due to Stodola the as I said that the earliest due to Stodola; stipulate the existence of relative eddy; the earliest due to Stodola stipulate the existence of relative eddy you know which occurs between the adjacent blades.

So, you have seen that the non-uniform velocity distribution which is arising because of the difference in pressure between the upper and lower surface of a blade give rise to a circulation; whenever there is a circulation this will leads to a formation of eddy. So, as I said that many attempts have been taken to quantify the amount slip and number of things have been formulated. For the earliest, due to Stodola stipulate the existence of eddy which causes which occurs between the adjacent blades.

So, Stodola suggested that the existence of eddy between 2 blades as I said as I as we have discussed that of course, there is a circulation. So, there will be a eddy and because of the formation of eddy the whirl component velocity will be diminished; because of the change of mean direction of flow leaving the impeller. And that diminished strength of swirl component or whirl component velocity will give rise to a relatively lesser amount of head that is developed by the pump and because of which there will be a deviation from Euler's equation.

So, now we will try to quantify following Stodola you know model; what would be the amount of actual slip in a radial flow pump.

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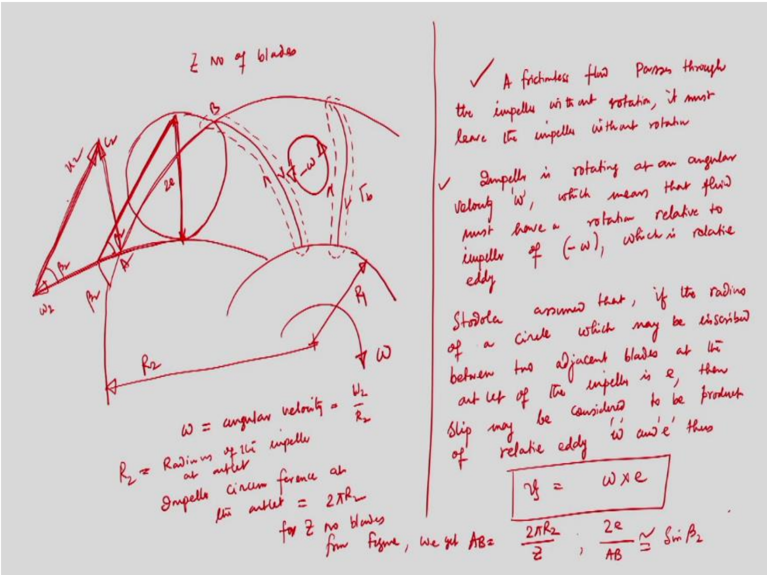

So, again I need to draw if I draw impeller of radial flow pump. So, we have seen that there will be a circulation; gamma b and there will be a. So, Stodola suggested that the relative eddies between which occurs between 2 blades; who first you know stipulates I mean the existence of relative eddies between which occurs between the 2 blades.

So, we have seen that because of the pressure difference there will be a non-uniform velocity distribution and because of which we will there will be a circulation around a blade. So, gamma b and because of the circulation; there will be a eddy which will rotate. If the impeller is rotating with an angular velocity omega that we have seen; so, there will be a eddy existence of eddy between 2 adjacent blades.

And the eddies will rotate with the same angular velocity omega, but in the opposite direction. And that is quite obvious because if you see if you look at this a diagram we have we can see that between 2 adjacent blades because of this you know the development of because of this circulation; in one side of the blade that is you know in the lower side of the blade fluid will try to move from inlet to outlet while in other in the backward side; when in the you know forward surface of the b blade in a back side blade in the rather in the residue blade at the forward surface fluid will try to move from the outlet to inlet.

Because of this you know mismatch in the direction of fluid. So, in between 2 adjacent blades there will be a circulation and that is what you know there is a there will be a eddy. And that eddies is having rotation which is the same angular speed, but in the opposite direction.

Now, suppose the outlet diameter of the impeller is R2 and inlet diameter is R 1 right. So, just we need to write and if we can; So, am I writing of some things important things that is very important.

A friction less flows a friction, less flow process through the impeller without rotation. It must leave the impeller without rotation. The first one is a friction less flow process through the impeller without rotation; it must leave the impeller without rotation.

Since we have assumed that the impeller is rotating at an angular velocity omega which means that fluid must have a rotation; relative to impeller of negative of omega, which is relative eddy.

So, Stodola the first one we have seen that many attempts are being taken to quantify, to predict the amount of slip and number of things have been formulated Stodola who first stipulate the existence of relative eddy relative eddy between the between 2 blades. And we have seen that because of this non uniform velocity distribution which is arising because of the pressure difference; between the upper and lower surface of the blades, there will be a circulation and circulation is quantified by omega b.

Second thing friction and Stodola stipulates; the existence of relative eddy, but he suggested that whenever a friction less flow process through the impeller without rotation. So, whenever a friction less fluid passing through the impeller without rotation; that means, it must slip the impeller without rotation.

Now, we are we have assume that the impeller is rotating at an angular velocity omega; which means that relative to the impeller I mean the fluid must have a rotation related to the impeller minus omega negative of omega which is relative eddy; so, since this is friction less flow. So, Stodola assumed that if the radius of a circle which may be inscribed; which may be inscribed between 2 adjacent blades at the outlet of the impeller is e.

Then slip may be considered to be product of relative eddy omega and e. Thus, this slip velocity is a product of omega into e; this is what Stodola predicted. So, we have seen that whenever a friction less flow; we have seen that there is a circulation around a blade which is coming because of the non-uniform velocity distribution.

Because blades do work on the fluid by exerting and impelling force on it; as a result of which there will be a higher pressure at the back upper side and lower pressure at the at the lower side. And because of this pressure difference will have a non-uniform velocity distribution that give rise to a circulation.

So, whenever there is a circulation around the particular blade and whenever there are 2 adjacent blades. So, because of these you know mismatch of the direction of flow near the surface of a blade still there might be a circulation, also Stodola assume that if impeller is rotating at an angular velocity omega then if the fluid is friction less. So, whenever fluid is flowing through the impeller without rotation that; that means, it must leave the impeller without rotation.

Now, if the impeller is rotating with an angular at an angular velocity omega which means that relative to the impeller; you know fluid must have a again you know you know if the impeller is rotating at an angular velocity omega, which means that fluid must have a rotation relative to the impeller that is negative omega which is known as relative eddy.

So, fluid is friction less since whenever it is flowing through the process of an again I am telling; I am repeating fluid is a friction less and whenever fluid is flowing through the process of the impeller without rotation; that means, it must leave the impeller without rotation. Now, if we assume that the impeller is rotating at an angular velocity omega; that means, relative to the impeller fluid must have a rotation at negative omega that is known as relative eddy.

Now Stodola assume that the if the radius of a circle if we can inscribe the circle whose radius is e at the outlet of the impeller. Because you are always talking about slip and that is or of course, because of the deviation because of the diminished strength of swirl component or whirl component velocity at the outlet because the component of swirl velocity at their outlet is responsible for the total head developed by the pump for a purely radial inlet.

Because we have seen that we can have positive negative and 0 value of $C_{\theta 1}$ that is swirl component velocity at the inlet. Positive is not desirable at all because it always reduces head 0 is best possible option but negative we cannot go for because a negative value of swirl component velocity which is always lead to an undesirable phenomenon of cavitation.

Now, if the if for a purely radial inlet; the radial flow pump with a purely radial inlet, the head developed by the pump using the Euler's equation will be into u2 $C_{\theta 2}/g$. But we have seen that in the reality the swirl component of velocity $C_{\theta 2}$ at the outlet will be always less than that you know predicted by velocity triangles.

So, and there are so many factors like write you know frictional losses separation losses because of the formation of boundary layer. Also because of the non-uniform of velocities this axial component of swirl velocity will be less than that will be always less. And Stodola attempted to quantify the amount of actual slip velocity in a pump operation.

And what he assumed that if the radius of a circle which is which may be inscribe at the outlet between 2 adjacent blades; at the outlet of the impeller between 2 adjacent blades is g then the slip velocity or slip maybe consider the product of omega ne where omega is the angular velocity of the impeller.

So, now we need to vs is equal to omega into find omega into e. So, where e is the radius of the circle you know the circle which can be inscribe at the outlet of the impeller. Now we need to know exact expression of vs because we have to calculate e and for that we need to dos again some mathematics. So, if I draw the outlet velocity is are the relative velocity at the outlet here.

So, this is the omega this is w2 and if I assuming this is straight and if this is you know this is the diameter of the circle is inscribing is 2 e. And if this is the tangential velocity at the outlet e 2; then probably this is the beta 2 right blade angle at the outlet.

So, assuming that there are losses because of the friction separation and also the non-uniform velocities and because of this non-uniform velocity there is a circulation that we have discuss many a times. Now, if this is the tangential velocity at the outlet and then this is beta 2; now if I draw a triangle again to quantify. So, if this is the absolute velocity c2 and this is the u2 where these are parallel; these 2 are parallel and then this is beta 2 also.

Then from this triangle we can quantify what will be the amount of slip velocity because you have seen the slip velocity is product of omega into e. So, we have drawn a velocity triangle at the outlet; so, this is the radius of the diameter of the circle which is inscribe at the in outlet of the impeller between 2 adjacent blades.

And if this is the tangential velocity u2 then probably this is the blade angle beta 2 and this is the relative velocity at the outlet. And if I draw another triangle where this u 2 is parallel to this and this is again will be beta 2. Now, considering this velocity triangle if you trying to quantify. So, now omega is angular velocity at which impeller is rotating.

So, this is angular velocity; so, this is cleared that we have inscribe the circle which radius is e at the outlet of the impeller which into adjacent blades. And if this is the absolute velocity at the outlet and if this is the tangential velocity at the blade outlet, then this is the blade angle beta 2 and this is the diameter of the circle which is inscribed.

Now, I have drawn another triangle where this is the; this is the actual absolute velocity c 2 this is the blade velocity u2 because these 2 are parallel and this is relative velocity this is beta 2. Now omega is the angular velocity at which impeller rotating that is nothing, but u2/ R2; where R 2 e; R 2 is the radius of the out-outlet radius of the impeller at outlet right.

So, impeller circumference at the outlet; impeller circumference at the outlet is equal to 2π R2. If we have a Z number of blades; so for suppose we have Z number of blades; so, if we have Z number of blades; then this is I mean then this distance velocities is Ze then from figure for Z number of blades from figure we get AB is equal to AB is equal to 2πR2/ Z.

Now, from figure can we say that 2πR2/ Z.So, I can write from this figure; so, this is sin beta 2 this one. So, I can write from the figure again from the figure I can write that twice e from the figure I can write $\frac{2e}{AB} \cong sin\beta_2$ almost equal to.

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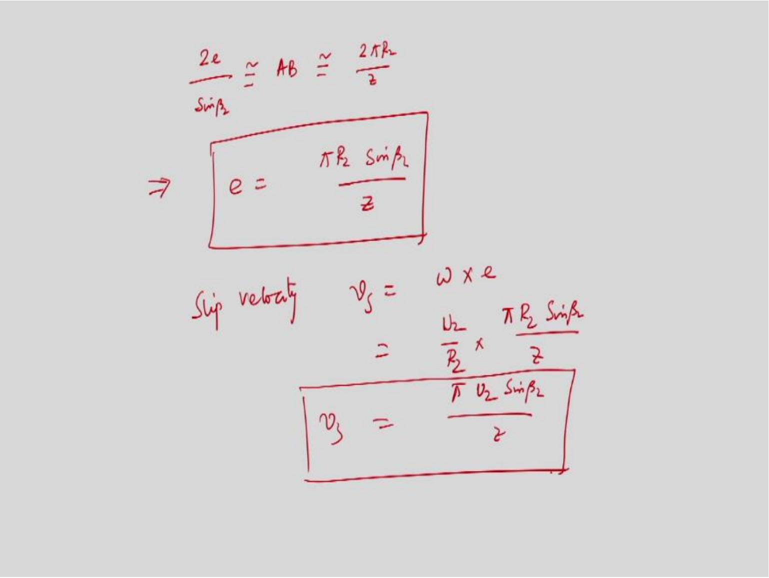

$\frac{2e}{sin\beta_2} \cong AB \cong \frac{2\pi R_2}{Z}$

e = $\frac{\pi\, R_2 \sin\beta 2}{Z}$

slip velocity Vs = $\omega * e = \frac{u_2}{R_2}\ x\ \frac{\pi\, R_2 \sin\beta 2}{Z}$

So, we have what we have discussed today? We have discussed that actual head developed by the pump will be always lesser than predicted by Euler's equation because of several reasons existence of non-uniform velocities whenever fluid is flowing through the impeller between the between the process in the process between 2 blades or there second reason there are losses because of friction separation wave formation because of the form formation of boundary layer the development of the boundary layer.

And we have seen that in a radial flow pump rather in most of the in a pump impeller do work on the blades and the blades do work on the fluids, I mean whenever blade is blades are rotating blades do work on the fluids by impelling by exerting and impelling process on it. And this is done on the upper surface; if it is a backward cup band I have we have drawn a backward cup band upper surface that is a convex surface.

And pressure at that surface will be higher than the lower side and because of the pressure difference there will be a non-uniform velocity distribution. And non-uniform velocity distribution give rise to a circulation and many attempts have been taken to quantify. And we have discussed that non uniform velocity distribution is responsible for the change of mean direction of flow leaving the impeller; from beta 2 to beta 2 dash or beta 2 dash is less than beta 2 that is a blade angel reduces; actual blade angle reduces because of the non-uniform velocity distribution. And this change of mean direction of flow leaving the impeller causes the reduction of all-important tangential component of velocity that is whirl velocity and which is known as slip.

Many attempts have been taken to quantify to predict the amount of slip and number of things have been formulated, but Stodola who first stipulates the existence of a relative eddy which occurs between the blades. We have tried to quantify that what could be the diameter what that if now Stodola supports propose that; if a friction less fluid a fluid in a friction less fluid is passing through the impeller without rotation; that means, it must leave the impeller without rotation.

Now, if we assume that the impeller is rotating at an angular velocity omega; which means that relative to the impeller fluid must have a rotation at negative omega and which is relative eddy; I mean that that is what you know Stodola proposed.

And then Stodola assumed that if a circles can circle can be inscribed at the outlet of the impeller is having radius e; then from the velocity triangles we have calculated that diameter can be calculating in terms of the blade angle beta 2, outlet radius R 2 and number of blades and then slip velocity can be calculated which is product of omega into e. So, we will continue in the next class.

Thank you.