Principle of Hydraulic Machines and System Design
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Lecture – 29
Degrees of Reaction and Efficiency of Hydraulic Turbine

So, we will continue our discussion on Principle of Hydraulic Machines and System Design. Today, we will discuss about the Degrees of Reaction and Efficiencies. Probably, we have discussed about the degrees of reaction in a context of the discussion of pumps, but again we recapitulate that.

And, we will try to discuss that how velocity triangle is a function of degrees of reaction and for different magnitude of you know rather different values of degrees of reaction, how can we construct the velocity triangles. And of course, the component of velocities I mean component of absolute velocity and component of relative velocity in the tangential direction changes, which with the change in degrees of reaction.

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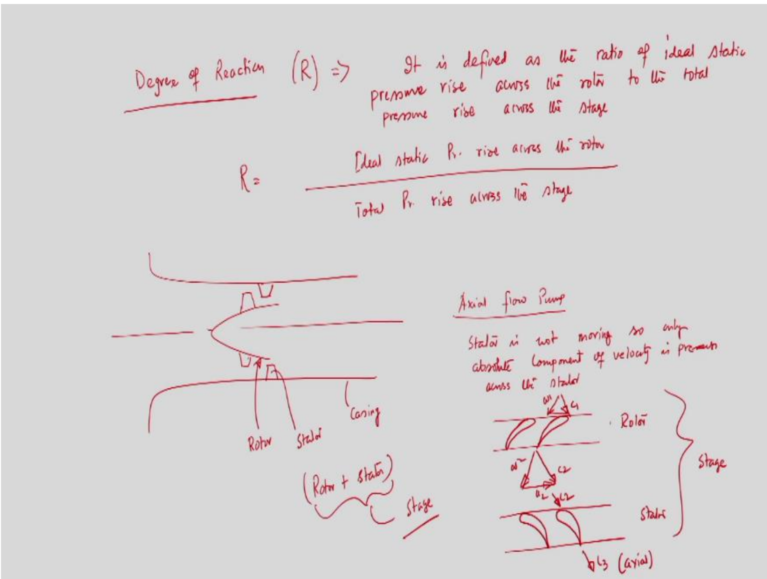

We have defined what is degrees of reaction? So, again I am writing that what is degrees of reaction? So, you know this is very important, important in the context of pump and turbine operation, because as I said you that the based on the magnitude of degrees of reaction velocity triangles changes. And, probably if velocity triangle changes then of course, the efficiency and of the total you know guide vane, blade, every design will change.

So, we will discuss about the degrees of reaction today, we also we will define what is what is degrees of reaction? So, probably I have defined again, but today again I will recapitulate what is degrees of reaction? So, I am writing degree of reaction degree of reaction, we sometimes call it R.

Which is defined probably I have written in the context of discussion on pumps again I am writing that it is defined, that it is defined as the ratio of, as the ratio of ideal static pressure rise, ideal static pressure rise across the rotor, across the rotor to the total pressure rise across the stage of the machines.

So, therefore, R which is defined that, total pressure rise across the stage to the I mean ideal static pressure rise across the rotor, across the rotor to the total pressure rise across the stage. So, now, before I go to discuss about you know rather I, if I try to drive the mathematical expression of degree of reaction in the context of operation of pumps or turbine.

Let us, first discuss about what is rotor and what is stage? So, we have discussed that rotor that is the rotating part of I of a turbo machines, it is impeller for a pump and it is runner for a turbine. So, whenever fluid is approaching rather fluid is passing through the rotor rather rotating part there is a static pressure rise. And, total pressure rise across the stage, because stage is not only the rotor stage is you know the combination of stator plus rotor. So, I will now discuss about what is stage? And, what is you know how the I mean how you can call it stage 2 stage, 2 stage, 3 stage machine like this.

So, if I talk about an axial flow machine. So, if I take an example of axial flow machine, suppose this is an axial flow machine. So, we have this is the impeller. So, let us this is axial flow pump or machine whatever it is. So, this is rotor and we may have another you know blade. So, this is hanging from the casing. So, this is casing, and this is rotor, which is rotating, but this is known as stator.

So, stator this is not rotating. So, rotor is a rotating equipment rotating components of the machine, if it if we talk about axial flow machine. So, the impeller is a rotor and we may have a stator the blades, which is hanging from the casing, but those are not rotating. So, stator I will write that stator is not moving. So, only since it is not moving. So, relative velocities are absent.

So, whenever we are talking about in fact, I would like to discuss in this context here that when I am applying Bernoulli equation in the rotating frame of reference. Always, I write always we should write that you know relative velocity, because their absolute velocities are replaced by the relative velocities. Since stator is not rotating. So, stator is not moving so, only absolute components of velocities. Absolute component of velocities are there velocity is present, velocity is present. Stator is not moving so, only absolute component of velocity is present across the stator. So, if I draw, I mean a rotor and stator combines.

So, it may be a 1 stage rotor and this rotor plus stator these 2 called a stage. So, this is 1 stage single stage. So, I have 1 rotor and we have a stator which is hanging from the casing. So, this is a single stage machine. So, this rotor plus stator combined is called stage. Stator is not moving only the absolute component of velocity is present over across the stator. Now, if I write the, if I draw the combination of stator plus rotor. Suppose this is if I write, suppose this is rotor and here we have 3 components let us say this is absolute velocity, this is tangential velocity, this is relative velocity. So, this is c1 this is w1 and this is u1.

But the you know again from outlet from the rotor we are having 3 component of velocities 1 is c2 this is w2 and this is u2, but when we have only you know stator. So, stator so, this is stator. So, maybe this c2 is approaching the stator and the velocity is coming out from the so, this is c2. So, this is c3 which is axial. So, this is combinedly it is called stage, this is called stage.

So, total pressure drives across the stage including rotor plus stage rotor plus stator, while ideal stator rise pressure rise across the rotor, the ratio of these 2 quantities is known as degree of reaction. So, we have now we have drawn a particular case and we have shown that what is stator, what is rotor.

And, as I said you that stator is not moving. So, only absolute velocity component is present there. So, c3 which is coming out from the rotor from stator is in the axial direction ok. So, now it is very important to note that, whenever liquid is fluid is approaching to the stator, we may have different angle. So, that will be very much important while we will try to have a mathematical expression of the degree of reaction. So, now if I draw say 1 rotor. So, if I draw 1 rotor.

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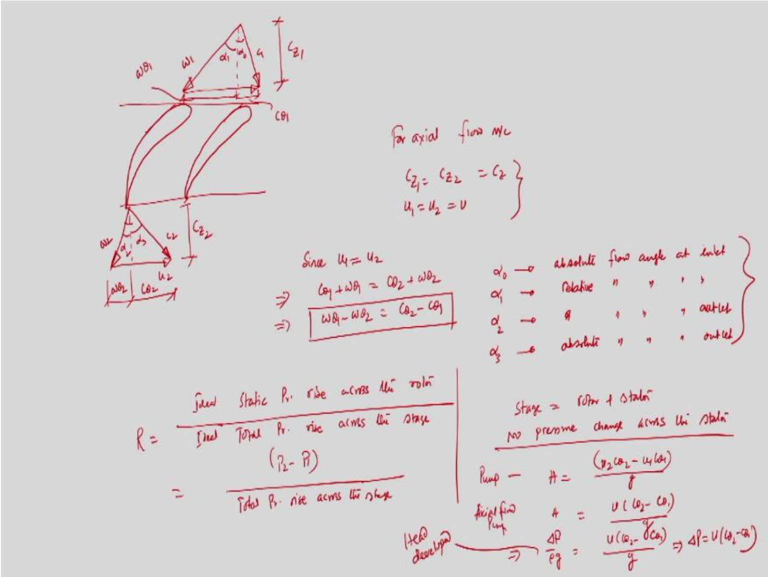

So, this is you know rotor. So, if I draw the velocity component. So, maybe this is the tangential velocity, this is absolute velocity, this is relative velocity, and this is absolute velocity c1 and this is w1. So, this component is $C_{\theta 1}$ and this is $W_{\theta 1}$. Now, this angle is let us say alpha naught and this angle is alpha 1 and this is you know if it is if I talk about axial flow machine this is this is definitely c z 1 right.

Similarly, whenever liquid is coming out from the rotor; so, again we will have tangential velocity, we will have absolute velocity and then relative velocity. So, this is w2 this is c 2 this is u2 and we will have $W_{\theta 2}$ $C_{\theta 2}$ and this is Cz2. So, Cz1 and Cz2 are the flow velocities. So, for axial flow machine, axial flow machine it may be a pump or turbine if

For axial flow m/c

Cz1 = Cz2 = Cz

u1= u2 = u

$$C_{\theta 1} + W_{\theta 1} = C_{\theta 2} + W_{\theta 2}$$

$$W_{\theta 1} - W_{\theta 2} = C_{\theta 2} - C_{\theta 1}$$

So, we need to know what is this, what are those angles? So, alpha naught I am writing alpha naught is equal to alpha naught is identified by the you know absolute flow angle, absolute flow angle at inlet and alpha 1 is essentially relative flow angle, relative flow angle at inlet. Similarly, alpha 2 is the relative flow angle at the outlet and alpha 3 is absolute flow angle at outlet.

So, these 4 angles are very important. So, absolute flow angle at the inlet relative flow angle at the inlet relative flow angle at the outlet and absolute flow angle at the outlet. So, these are very important angles which we need because while we will try to derive the mathematical expression for the degree of reaction fine. So, now, with these let us move to derive the mathematical expression for the degree of reaction that is very important, because or as I said you that we will today we will try to draw, the velocity triangles for different magnitude of degrees of reaction. And, not only that we will see that, how blade angle and I mean these angles are changes with the change in magnitude of degrees of reaction.

So as I said you that R that degree of reaction which is defined as the ideal static pressure rise across the rotor to the total pressure rise across the stage, we have now seen that what is called stage and what is rotor and what is stator? So, rotor plus stator this 2 combined is called as a stage. And, since stator is not moving only the absolute component of velocity is there. So, this is the total degree of reaction. So, now, R is if I try to recall the definition what is a R?

$$R = \frac{Ideal\ static\ pressure\ rise\ across\ the\ rotor}{ideal\ total\ pressure\ rise\ across\ the\ stage} = \frac{P_2 - P_1}{total\ pressure\ rise\ across\ the\ stage}$$

Now, see as I said that the stage is equal to rotor plus stator. So, no pressure changes across the stator. So, across the stator no pressure change.

For pump $H = \frac{u_2 c_{\theta 2} - u_1 c_{\theta 1}}{g}$

For axial flow pump $H = \frac{u(C_{\theta 2} - C_{\theta 1})}{g}$

$$H = \frac{\Delta P}{\rho g} = \frac{u(C_{\theta 2} - C_{\theta 1})}{g}$$

$$\Delta P = \rho\, u(C_{\theta 2} - C_{\theta 1})$$

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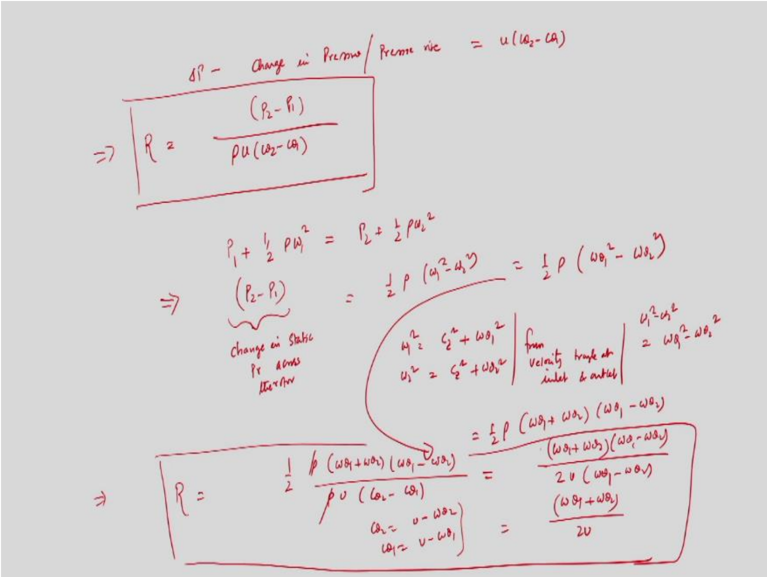

$$R = \frac{P_2 - P_1}{\rho\, u(C_{\theta 2} - C_{\theta 1})}$$

$$P_1 + \frac{1}{2}\,\rho {w_1}^2 = P_2 + \frac{1}{2}\,\rho {w_2}^2$$

$$P_2 - P_1 = \frac{1}{2}\,\rho({w_1}^2 - {w_2}^2) = \frac{1}{2}\,\rho({w_{\theta 1}}^2 - {w_{\theta 2}}^2)$$

$${w_1}^2 = {C_z}^2 + {w_{\theta 1}}^2$$

$${w_2}^2 = {C_z}^2 + {w_{\theta 2}}^2$$

$$R = \frac{\frac{1}{2}\rho({w_{\theta 1}}^2 - {w_{\theta 2}}^2)}{\rho\, u(C_{\theta 2} - C_{\theta 1})} = \frac{w_{\theta 1} + w_{\theta 1}}{2u}$$

So, now if I draw the velocity triangle or if I go back to the velocity triangle that is what we have drawn in the previous. So, what I can write as I said you that if I like if I apply Bernoulli equation of course, in a rotating frame of reference then instead of absolute velocity I need to write the relative component of velocity. So, if I write Bernoulli equation

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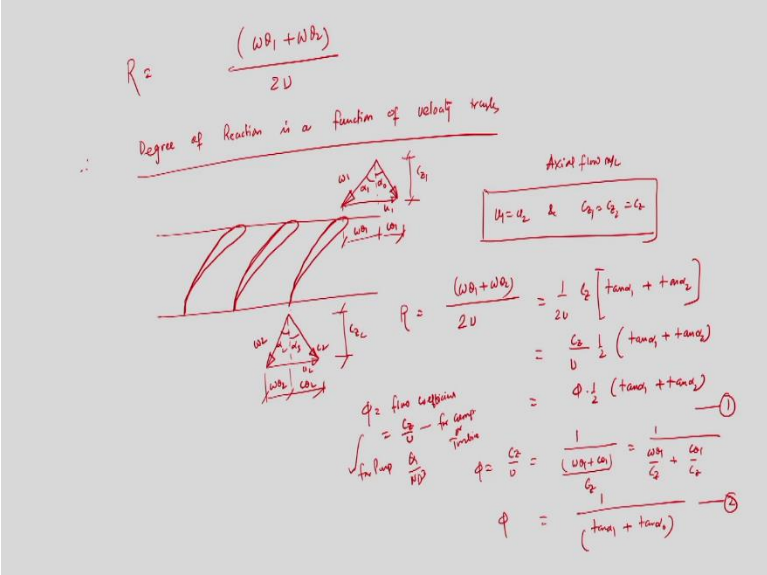

Now, we have to proceed to see, how we can you know mathematically you know how we can drive, that the that is what I said that the degree of reaction is a function of velocity triangle. And, how we can get the velocity triangles for different magnitude of degree of reaction?

So, now if I try to express, suppose we have a you know rotor, suppose we have blade, we have blades and if we velocity triangles say this is u1 this is c1 and this is w1. So, this is let us a alpha 0 this is alpha 1 that is what we have discussed and this component is $C_{\theta 1}$ and this component is $w_{\theta 1}$ and what is the velocity triangles at the outlet?

So, this is Cz1 and velocity triangles at the outlet is this is this is w2 this is c2 this is u2 this is $w_{\theta 2}$ this is $C_{\theta 2}$ and this is Cz2. So, this is alpha 2 and this is alpha 3. So, that is what alpha naught alpha 1 alpha 2 alpha 3, that we have already identified what are those we have given their name. So, relative and absolute flow angles in the at the outlet and inlet respectively ok.

Now, if I talk about axial flow machine. So, u1 = u2 and Cz1= Cz2 = Cz. So, far the axial flow machines we are talking about axial flow machines axial flow machine. So, this is the case.

$$R = \frac{w_{\theta 1} + w_{\theta 2}}{2u} = \frac{1}{2u} C_z \left[\tan \alpha 1 + \tan \alpha 2\right] = \emptyset \, \frac{1}{2} \left[\tan \alpha 1 + \tan \alpha 2\right]$$

$flow\ coefficiant\ \emptyset = \frac{C_z}{u}$

$$= \frac{1}{\frac{w_{\theta 1} + C_{\theta 1}}{Cz}} = \frac{1}{\frac{w_{\theta 1}}{Cz} + \frac{C_{\theta 1}}{Cz}} = \frac{1}{\tan \alpha 1 + \tan \alpha o} \qquad \text{eq2}$$

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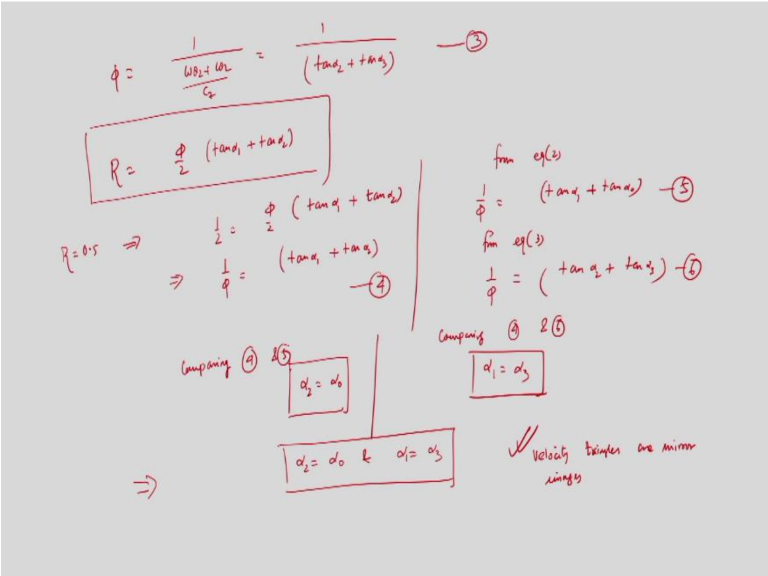

$$\emptyset = \frac{1}{\frac{w_{\theta 2} + C_{\theta 2}}{Cz}} = \frac{1}{\tan \alpha 2 + \tan \alpha 3} \qquad \text{eq3}$$

$R = \emptyset \frac{1}{2} [\tan \alpha 1 + \tan \alpha 2]$

If R = 0.5

$0.5 = \emptyset \frac{1}{2} [\tan \alpha 1 + \tan \alpha 2]$

$\frac{1}{\emptyset} = \tan \alpha 1 + \tan \alpha 2$

$\frac{1}{\emptyset} = \tan \alpha 1 + \tan \alpha 0$

$\frac{1}{\emptyset} = \tan \alpha 3 + \tan \alpha 2$

$\alpha 2 = \alpha o \quad , \quad \alpha 1 = \alpha 3$

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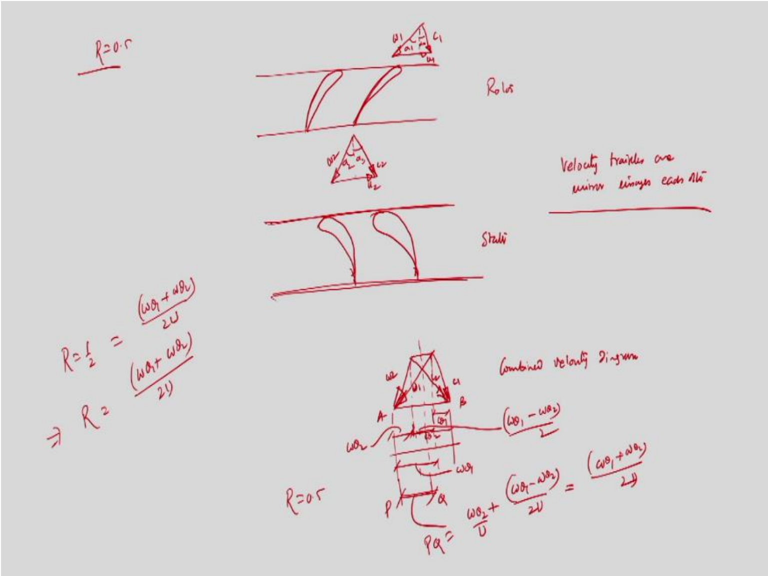

If, I draw the velocity triangles we have a rotor and we have stator. So, if we have a rotor. So, this is rotor and then we have this is c 1, this is w 1, this angle is alpha 0, this angle is alpha 1. Similarly, this is let us say outlet velocity triangle. So, this is w_2, C_2 this is alpha 3 this is alpha2 this is u2 this is u1 and we have a rotor. So, we have a rotor. So, this is stator. So, stator plus rotor is a stage. So, the very triangles are mirror images; if, degree of reaction is equal to pint 5 that is what we have seen. Now, question is velocity triangles are mirror images for each other. So, velocity triangles are mirror images each other for degree of reaction 0.5.

So, the conclusion is velocity triangles are mirror images each other. So, that is what we have seen for degree of reaction 0.5 fine. We can have a combined velocity diagram and probably normally what do we you know dimensional zed every pump. So, if I draw the combined velocity diagram combined velocity diagram. So, let us say as I said you the angles are mirror images. So, if I draw that combined velocity diagram; so, velocity triangles are mirror images. So, this is c1, this is c2, this is w1, this is w2.

If R=0

Ideal static pressure rise =0

P1 = P2

Example pelton wheel

$$P_1 + \frac{1}{2}\rho {w_1}^2 = P_2 + \frac{1}{2}\rho {w_2}^2$$

w1 = w2

$$R = \frac{w_{\theta 1} + w_{\theta 2}}{2u} = 0$$

$$w_{\theta 1} = -w_{\theta 2}$$

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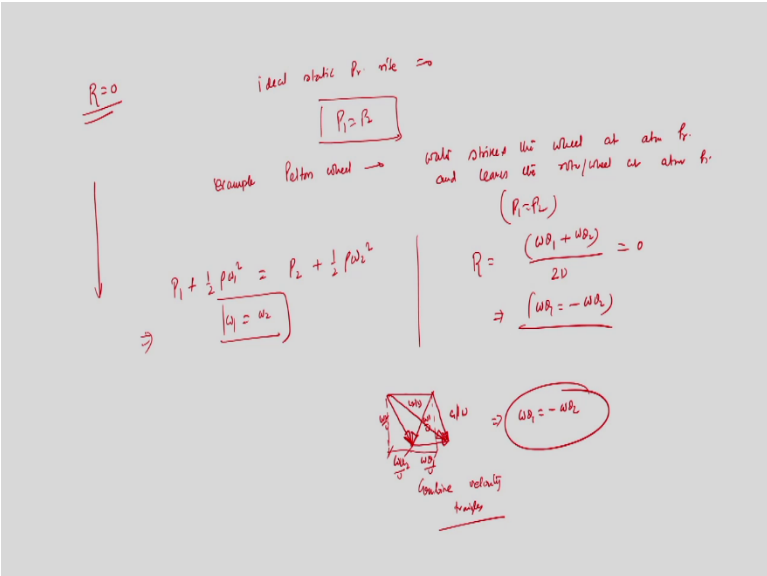

Now, we will take another example. So, we have taken example of R is equal to 0, that is a special case we have seen we have drawn the combined velocity triangles and we have identified that, how w theta 1 plus w theta 2 by twice U is the R is equal to 0.5, then we have tried to a special case R is equal to 0 example is Pelton wheel, there I no change in pressure across the rotor. And, we have taken and for that we have drawn the velocity triangles. Now, we will try to draw the velocity triangles for R is equal to 1, that is very important. So, R 0.5 and 1 there may be another different cases but so, now, R is equal to 1.

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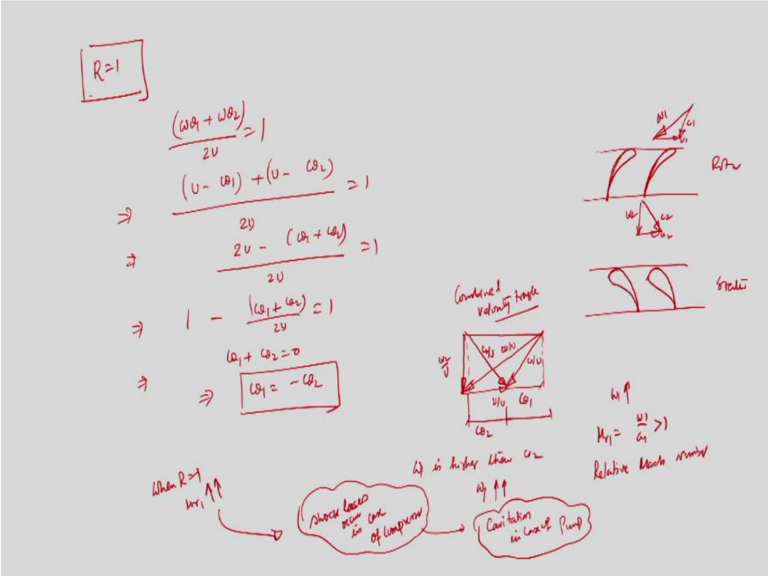

If R=1

$$\frac{w_{\theta 1} + w_{\theta 2}}{2u} = 1$$

$$\frac{u - C_{\theta 1} + u - C_{\theta 2}}{2u} = 1$$

$$\frac{2u - (C_{\theta 1} + C_{\theta 2})}{2u} = 1$$

$$C_{\theta 1} + C_{\theta 2} = 0$$

$$C_{\theta 1} = -C_{\theta 2}$$

Similarly, our outlet velocity triangle will be w 2 c 2 u 2 and we have a stator part. So, this is stator part. So, this is stator this is rotor note that for degree of reaction R = 1, w 1 becomes higher. So, if w 1 becomes higher. So, w1 increases this is very important. So, relative Mach number Mr1 = w1/a1 > 1. So, relative Mach number that is relative velocity at the inlet increases for a degree of reaction 1, we have drawn this is a combined velocity triangle, this is combined velocity triangle. So, w 1 becomes higher relative Mach number this is relative Mach number will be higher.

So, when R = 1, relative Mach number increases. And, if relative Mach number increases it will lead to on several problems, I mean it will leads to shock losses shock loss occur, in case of compressor in a case of a compressor. And, and we have seen that if w 1 increases probably you have discussed this issue in the context of inlet us valve in a pump. If relative velocity at the inlet increases pressure will fall and if pressure falls below the vapour pressure local boiling takes place and it will lead to an undesirable phenomenon of cavitation.

So, we have seen that for degree of reaction 1 velocity triangle will change in such a way that, the relative velocity at the inlet will be higher. And, it will lead to a higher value of relative Mach number, that it will create a shock loss in case of a compressor or it will leads to a cavitation in case of a pump or cavitation in case of a pump. So, this is very important. So, we have taken 3 different cases of R = 0. 5, R= 0, R = 1 example is that Pelton wheel there is no change in pressure across the wheel I mean static pressure no change P1 = P2 , because water strikes the bucket at atmospheric pressure and leaves atmospheric pressure.

We have taken example of R is equal to 1 for which and we have taken excel flow machines rotor. And, stator then what we see that from the R is equal to 1? R is equal to 1 itself will allow us to construct a velocity triangle in such a way that w 1 has to be higher. If, w 1 increases then relative Mach number will be higher, it will create you know it will leads to several undesirable phenomenon in different machines, in case of a compressor shock loss there will be a shock loss and in case of a pump there will be cavitation.

So, we have discussed about the degree of reactions and how the velocity triangle changes with change in magnitude of degree of reaction we have constructed velocity triangles in a in a combined plane. Also, we have constructed velocity triangles in a at the inlet of the rotor outlet of the rotor.

And, in a stator there is no as I said there are only absolute component of velocities are present. So, there are no this since it is in a moving ok. So, with these now we will discuss about efficiencies of turbine that is although we have discussed about the efficiency of a turbine, because we have worked out 2 example, but today I will discuss about efficiencies.

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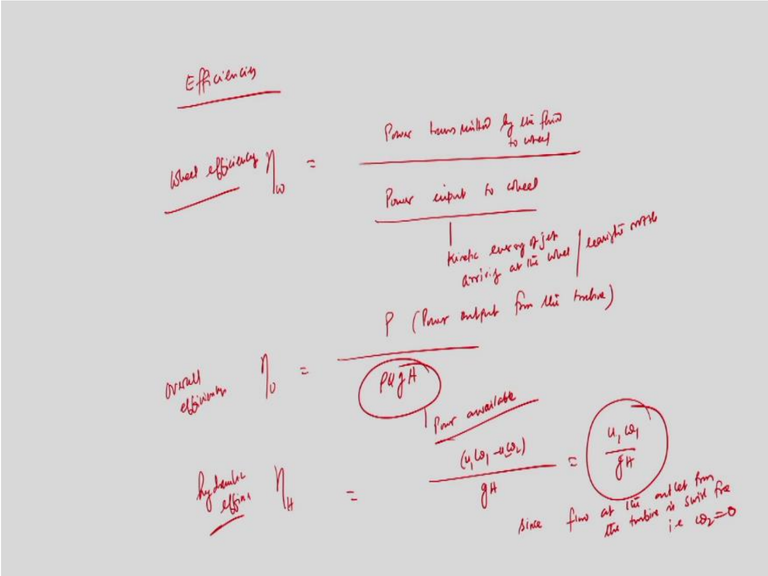

So, turbine efficiencies so, we have discussed that there are one is in a Pelton wheel we have discussed about the wheel efficiency, or hydraulic or hydraulic efficiency is very important, that how we can discuss about the hydraulic efficiency or wheel efficiency? So, one is wheel efficiency, that is we have discussed in the context of Pelton wheel. What is that? We have discussed that maybe what is power transmitted by the fluid to the wheel.

So, maybe power input to the wheel is not equal to so, as I discussed that power input to the wheel; power input to the wheel may not be equal to the power transmitted, you know by the fluid to the wheel, by the fluid to the wheel. Rather, whatever amount of power input to the wheel that is obtained from kinetic energy of the jet power input to the wheel that is obtained from kinetic energy of the jet leaving a nozzle. Rather array kinetic energy of the jet arriving at the wheel or leaving the nozzle arriving at the wheel or leaving the nozzle.

So, the kinetic energy leaving the nozzle that is power input to the wheel, but whatever amount of power input to the wheel is not equal transmitted by the fluid to the wheel. So, there will be a change I mean difference because of it is losses. So, this is known as wheel efficiency. And, that is what is known wheel efficiency eta w and we have also discussed about that what is overall efficiency, you know overall efficiency is very important rather that, what is overall efficiency?

Eta o what is power output from the turbine. So, this is power output from the turbine is very important. Now, what is a power input to the turbine? Suppose, if I have Q beta cube per second flow rate, g and head available g H. So, g is the power input rather power that is available at the shaft while. So, this is the power available and this is a power output from, this is a power output from the turbine.

So, these 2 are not same because frictional losses windows losses varying frictional losses all those things. So, this is known as overall efficiency. And, single leak and define another efficiency known as hydraulic efficiency just like a farm hydraulic efficiency. In most of the cases that that is very important that how I can define hydraulic efficiency.

$$\eta_H = \frac{u_1 C_{\theta 1} - u_2 C_{\theta 2}}{gH}$$

$= \frac{u_1 C_{\theta 1}}{gH}$ if outlet from the turbine is swirl free

So, now, in most of the turbine the flow at the outlet from the runner is soil free. So, if flow at the outlet from the runner is swirl free So, in most of the cases the flow at the outlet of the arise wall free.Since, flow at the outlet from the turbine since flow at the outlet from turbine is swirl free.

So, that this is known as the hydraulic efficiency. So, these 3 efficiencies probably we have discussed while your solving the problem, but now we have defined that these 3 are the efficiencies in the context of power operations.

So, today what you are discuss we have discuss about the degree of reaction. We have defined degree of reaction and then we have discussed about that the stage what is called stage? We have taken an example of axial flow, machine axial flow farm and then we have tried to drive the mathematical expression for the degree of reaction. From there, we have taken 3 different cases that 3 different magnitude of degree of reaction and for that how velocity triangles are changing.

So, I mean with the change in magnitude of degree of reaction velocity triangles are changing. And, since we have say taken example of R is equal to 0 1 and 0. 5, we have seen the R is equal to 0 just case where there is no change in I mean no rise in static pressure across the router. And, example is Pelton wheel and for all 3 cases we have defined we have you know we have drawn the combine velocity diagram. And, we have seen that for R is equal to 1 the velocity triangles are velocity triangles takes us such takes us certain such a way that, the relative velocity at the inlet becomes higher.

And, if relative velocity of the inlet becomes higher, relative Mach number will be higher and it will leads to a I mean 2 different undesirable phenomenon for 2 different machines. If it is a compressor then shock losses will be there and if it is pump then cavitation might occur. And, then we have discussed about efficiencies for the turbine 3 different efficiencies. And, probably we have discussed rather I have taken the formula of efficiency while you have solve the problem, but today we have discuss about the definitions.

So, with this I stop here today and I will continue a discussion in the next class.

Thank you.