Principle of Hydraulic Machines and System Design
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Lecture – 22
Positive displacement pump, indicator diagram - II

So, we will continue our discussion on the indicator diagram of a reciprocating pump, rather a positive displacement pump.

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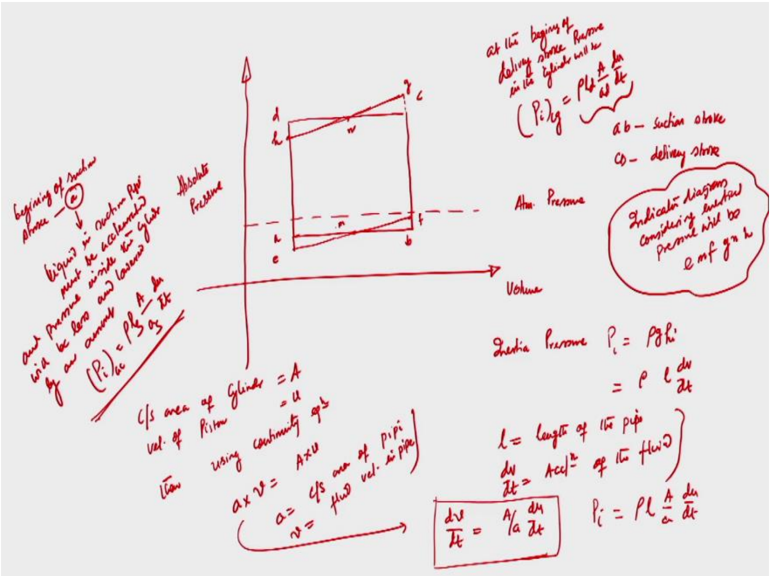

So, we have discussed yesterday the, a, b, c, d is the simplest on that the indicator diagram pressure versus volume. But, while we have you know described the a, b, c, d that is suction and delivery stroke and while we are discussing, that time we did not considered the effect of inertia that is because of the liquid volume in the pipelines and also the frictional effects in because of that fluid flow through the pipes.

And also we have discussed that if we take into account the inertial effects I mean then we have modified indicator diagram that is e m f. So, modified indicator diagram considering inertial pressure will be e, m, f, g, n, h that is the modified indicator diagram. Now, if we rank now again consider the frictional effect that is always there I mean we cannot ignore, so what will be the how the frictional effect will modify the indicator diagram, and that we need to look into I mean that that we should now discuss. So, you know that what we know; I mean how we can calculate frictional losses in a pipeline, whether it is suction pipe or delivery pipe.

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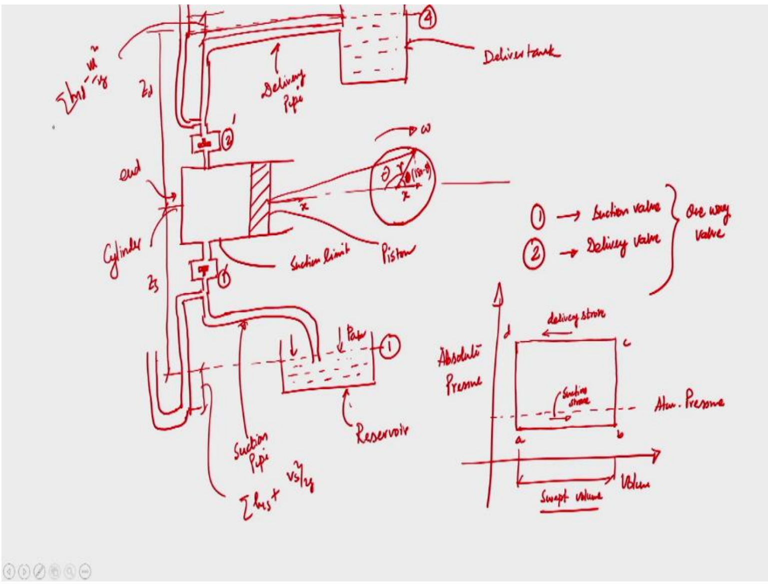

So, if we go back to my you know previous slide had where we have plotted, where we have described about suction pipe and delivery pipe through which fluid is flowing during suction and delivery stroke. Now, while fluid is flowing through suction pipe and delivery pipe, we cannot ignore the frictional effect.

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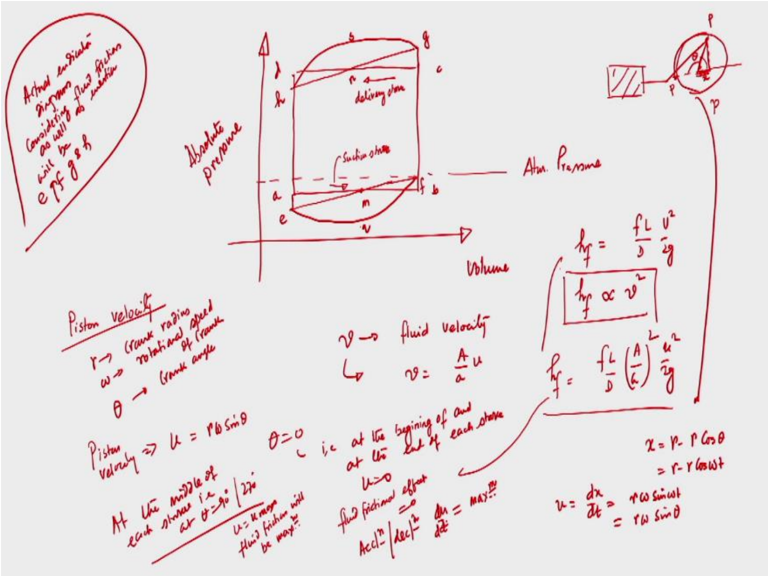

So, even if we draw the indicator diagram modified with the inertial effect due to the presence of liquid volume in the pipelines rather suction pipe and delivery pipe, so the modified indicator diagram will be like this. So, this is the volume, and this is absolute pressure absolute pressure right. So, this is volume, this is absolute pressure. And if this is the atmospheric pressure, so this is atmospheric pressure. Then this one was the suction stroke, so then there will be a rise in pressure for constant volume, this is delivery stroke, and this one is sudden falling pressure.

So, we have given name a, b, c, d. And then this is a simple indicator diagram, where we did not take into we did not take the effect of rather effects of inertia and frictional into account. Now, considering the effect of inertia, the modified indicator diagram that that is what we have discussed in my last lectures that that will be like this, so it will be like this. And e, m, f, similarly there will be rise in pressure, and the modified indicator diagram will be like this e, m, f, g, n, h. So, the modified indicator diagram considering the effect of inertia will be e, m, f, g, n, h that we that is what we have discussed my last day.

So, now we need to take the effect of frictional effect into account. So, whenever fluid is flowing through suction pipe or delivery pipe, normally we do calculate frictional losses following Darcy Weisbach equation. So, if we write down the Darcy Weisbach equation

$$h_f = \frac{f l v^2}{2gD}$$

Sometimes f is replaced by 4 f, because that depends upon whether you know fanning friction coefficient or the whether it is f = 64/RE or 16/RE that depends upon. Now that is not an important issue, so that is $\frac{f l v^2}{2gD}$ the frictional loss. So, from this expression, it is quite cleared that the frictional head loss h f will vary or will be proportional to V^2 that is a velocity square.

So, now if I try to modify you know this V^2 is the basically fluid velocity, now I can replace the fluid velocity in terms of the piston velocity. So, V is the velocity of the fluid, so it is fluid velocity fluid velocity. This fluid velocity can be replaced or can be written in terms of the velocity of the piston that is u that will be that, that is what we know how $V = \frac{A}{a} u$, where A is the cross section area of the cylinder; small a is the cross section area of the pipeline, whether it is suction pipe or delivery pipe and this u.

So, the indicator diagram whatever you are plotting that is essentially in the cylinder I mean how the cylinder pressure and volume is changing. So, we have now replaced velocity of the fluid in the pipe in terms of the piston velocity, where you know piston velocity is u. Now, L and A by a these are these are variable, because L maybe Ls or Ld, and A maybe As, Ad depending upon the suction pipe and delivery pipe, whatever it is what we can replace we can write in terms of the piston velocity.

So, what it is clear, in fact what is seen that Darcy equation whatever it is must Darcy equation varies h_f verses V^2. So, if I now try to superimpose that effect on the modified indicator diagram that that is what we have drawn that e, m, f, g, n, h. Then of course, now I can write something that how we can this piston velocity, piston velocity maybe this is very important. So, the frictional effect will be parabolic in nature that is quite obvious from the equation hf verses V^2. So, the hf $\propto V^2$ from where it is quite cleared that the variation will be parabolic in nature, but somehow, we need to calculate the piston velocity.

So, how do we calculate piston velocity? So, if we go back to my previous slide, where we have written that you know sorry this will be θ this angle will be θ instead of this angle will be θ, and this will be 180 - θ. So, if piston let us say is moving a distance x from its initial position. So, if piston moves a distance x from its initial position maybe piston was you know piston is moving let us say x distance in this direction let say x distance is moving, and because of what crank has connected let us say angle theta. And so I can now calculate this may be this distance is x.

So, I can calculate; what is this distance x linear distance linear distance. So, because of the crank revolution by an angle theta if there is a linear displacement of the piston is x, then I can calculate x in terms of the frequency omega crank radius r. If the crank radius is let us say r crank radius is r, then I can replace the linear displacement x of the piston for you know angular displacement by an amount theta in terms of crank radius r and frequency omega.

Now, if I try to express that is very important that is piston velocity. So, how I can calculate piston velocity is very important, because that is important to obtain. Maybe grossly the effect of frictional effect of friction will be parabolic in nature I mean if would like; if you would like to modify the indicator diagram, then of course need to have will be parabolic in nature frictional effect. But how we will calculate, where the velocity will be 0, where acceleration will be maximum to know that we need to calculate the piston velocity.

How we can calculate piston velocity is very important. So, piston velocity this is very often. How we can calculate piston velocity, this piston velocity may be obtained from the displacement equation. So, if I recall that if this is my piston and piston is moving like this, so this angle is theta, so this is 180 - θ. So, let us say this is the axial displacement x. So, I can calculate, and radius is r. So, r is the crank radius, ω is the frequency of the rotation of the crank, and then piston velocity can be obtained from the displacement equation as in simple harmonic motion.

And it can be written in terms of so we are assuming that the displacement equation following the simple harmonic motion. And we can calculate piston velocity in terms of the crank radius r, and also the ω that is a frequency of the crank rotational speed of the crank, rather we can write that is rotational speed of the crank rotational speed of crank.

So, assuming the simple harmonic motion that the displacement equation follow follows the simple harmonic motion, and we can calculate piston velocity in terms of crank radius, rotational speed of the crank, and crank angle θ by how. So, then how we can calculate this is very important. So, you know that as I said you from this figure, I can write that the linear displacement x for an suppose initially when piston was at the and cylinder end, then initially this point let us say this is P, this point P was here. So, now P has moved a distance by an amount theta by an angle θ and for that we have a linear displacement x. So, this linear displacement x can be calculated from this triangle. So, this x = r - r cosωt.

So, now this $\theta = r - r\ cos\omega t$. So, I can obtain dx dt that is velocity of the piston is equal to r omega sin omega t. So, how we can calculate? So, if I know the linear angular displacement, suppose piston has a linear displacement x, and for that crank has rotated by an angle theta, from there we can calculate angular linear displacement x in terms of crank radius and crank angle, while ω is the rotational speed of the crank. From there, we can calculate what will be piston velocity dx/dt = r ω $sin\omega t$. This is very good.

So, piston velocity that is piston velocity will be u = r ω $sin\omega t$. So, it is quite clear from this equation is that that when $\theta = 0$, I mean that is at the beginning. So, for is $\theta = 0$; that is at the beginning of at the beginning of at the beginning and rather at the beginning of and at the end of each stroke $\theta = 0$. U = 0. So, at the beginning as well as at the end of ends at the beginning and end of each stroke $\theta = 0$. So, velocity will be 0, if velocity is 0 right. Then fluid frictional effect will be 0 following this Darcy equation.

So, fluid frictional effect will be equal to 0 from the Darcy equation, this is very important. So, what we obtain that at the beginning and at the end of each stroke $\theta = 0$, so velocity will be 0, so fluid frictional effect will be 0, but the acceleration or deceleration whatever it may is it will be maximum right. So, but the acceleration, deceleration du dt, but the acceleration or deceleration that is du dt will be maximum. So, this is very important that at the end and beginning of each stroke fluid frictional effect is 0, but the acceleration deceleration may acceleration and deceleration is maximum.

Similarly, at the middle I mean at the middle of the when the velocity is maximum that is at the middle of the stroke. Similarly, at the middle of each stroke that is at $\theta = 90$ degree or 270 degree right, so $\theta = 90$ degree or 270 degree, fluid frictional effect will be maximum because velocity will be maximum, but the acceleration and deceleration will be 0. So, u is equal to u max. So, fluid friction will be maximum fluid friction will be maximum, but the acceleration deceleration will be 0.

So now, if I try to superimpose the effect of friction on the modified indicator diagram: then how it will how it will look like. So, what we obtain that at the end and beginning of each stroke, so $\theta = 0$, and fluid friction effect will be no fluid friction effect rather acceleration deceleration will be maximum. And at the middle, velocity will be maximum while the acceleration and deceleration will be 0, no acceleration deceleration. So, and it we need to mind, we need to keep in mind that that frictional head loss varies you know it is parabolic in nature.

So, if I try to superimpose that effect, so here you know it will be like this and this will be like this right. So, this will be the actual modified diagram. So, this is the suction stroke, this is suction stroke, and this is the delivery stroke. So, the actual indicator diagram modified with the effect of inertia as well as effect of fluid friction, it will be let us say it will be you know e e, q, f, g, s, h. So, I am writing that the actual indicator diagram considering fluid friction as well as inertia will be e, q, f, g, s, h. So, this is very important. The actual modified actual indicator diagram considering the effect of fluid friction as well as inertia will be a e, q, f, g, s, h, this is very important fine. So, this is what is very important that we have calculated.

Now, we need to know a few important aspects to solve a few problems, I mean when you are going to solve numerical problem that mean what will be the flow rate from that positive displacement pump. And for that we have to work out a few exercises I mean we need to do some exercise to calculate that what would be the volume flow rate, and then what will be the efficiency like this ok. So, if we go to the next slide, then we can write that whenever piston is moving then the instantaneous pressure in the cylinder p is p. Suppose, how we can calculate work done, so this is important.

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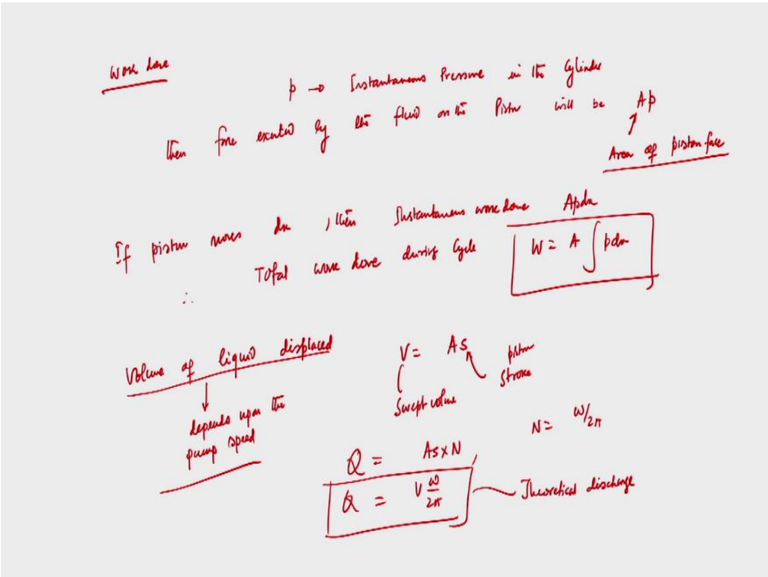

So,If p is the instantaneous pressure in the cylinder, then the force exerted by the fluid on the piston; of course, then how we can calculate, then force exerted by the fluid on the piston will be how much, that will be how much, A* p, where A is the area of the piston face, area of piston face. So, this is the work done. So, force exerted by the fluid on the piston will be A*p, then and for that if piston has a displacement dx, then work done will be right.

Instantaneous work done = A*p*dx

Total work done $= \int A * p * dx$

Swept volume Vs = A s {s = piston stroke}

Discharge Q = A s N = $\frac{\omega}{2\pi} V$

Now, the volume of the liquid displaced will depends this will depend upon the pump speed upon the pump speed right. So, volume of liquid displaced will depend upon the pump speed and that will be A into s, where s is the piston stroke, and A is the area, and V is the swept volume. Then Q that is the discharge, I do not know whether it is theoretical or ideal, or theoretical or actual.

This is theoretical discharge. Why, it is theoretical discharge because there will be certain amount of as I said you that positive displacement pump rather reciprocating pump is a kind of you know PD pump which is normally used to have a deliver high pressure while flow rate will remain constant.

So, we have calculated that volume of liquid displacement that is very important quantity that we need to know, because essentially, we are installing one pump only to obtain high flow rate, of course discharge head will be important. So, we will work out again how do we calculate total head that is another issue. But the volume, so in one stroke length I mean one stroke how much volume that is a swept volume that is area into s, A is the piston area.

So, if this is the case, then I have calculated this theoretical discharge, but this theoretical discharge this discharge is theoretical one, because you would not get this amount of discharge. Whenever piston is moving piston is moving towards the end of the cylinder, and it will try to develop high pressure there will be certain amount of leakage through glands through valves. And so we should not get this amount of discharge rather actual distance should be always less than that.

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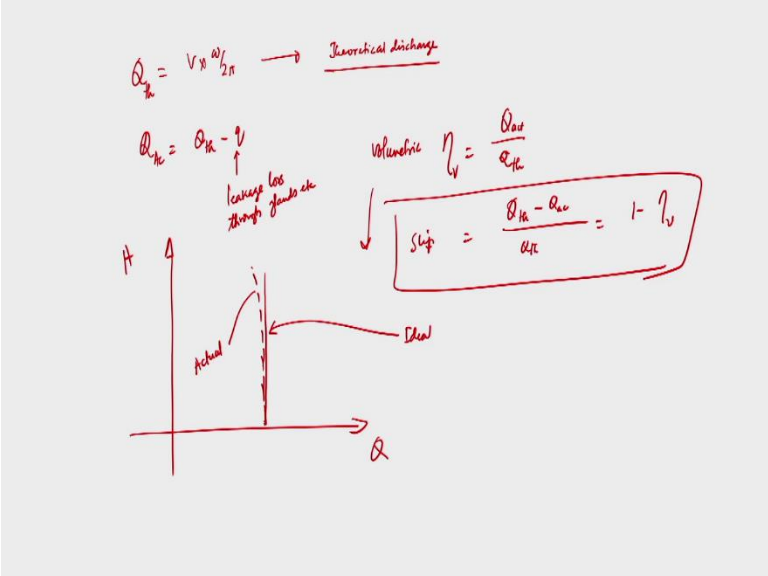

So that is why if I go to next slide, so what is important, so this

$Q_{th} = \frac{\omega}{2\pi} V$

$Q_{ac} = Q_{th} - q$

small q amount this will be loss leakage loss, leakage loss through glands etcetera. So, there will be certain some amount of loss rather through the gland etcetera, because whenever cylinder is put you know moving towards the end, there will be high pressure, because of that high pressure there will be certain amount of leakage loss through the glands etcetera

Volumetric efficiency $\eta_v = \frac{Q_{act}}{Q_{th}}$

So, Q actual will be always less than the theoretical one and that is why one efficiency is defined that is known as you know per efficiency and that rather volumetric efficiency that is known as volumetric efficiency of the pump.

Slip = $\frac{Q_{th} - Q_{ac}}{Q_{ac}} = 1 - \eta_v$

Actually if I now try to draw the indicator you know h Q curve of the positive displacement pump, so if I try to draw that if this is head developed by the pump and this is the discharge by the pump, ideally we have higher discharge the product will remain constant. So, ideally the curve like be this. So, this is the ideal one. But to take into the to take the effect of slippage into account rather considering the effect of slippage, slip will be there, because whenever fluid you know a piston is pushing that a piston is moving towards the end, it is always trying to pushing the liquid. As a result of which of course, you will have a high pressure and because of that high pressure some amount of liquid will leak through the glands and also the valves. As a result of which we may not get this amount of discharge rather actual discharge will be less than the theoretical whatever you have predicted. So, the actual curve will be like this. So, this is the actual curve.

So, existence of slip that is the leakage of fluid through glands ok. So, till now we have calculated the volumetric efficiency, and we have an idea about the discharge through a PD pump, so that is one aspect.

And now how do we calculate the head developed by the pump that is another important point we need to take into account. So, how do we calculate you know head developed by the pump this is very important again. And for that what we have to do, we have to calculate you know we have to go back to a slide where we have drawn the schematic. So, if we go back to my you know previous slide where we have drawn the schematic, and from here we can tell that what should be the total h. So, this is very important. And total head developed by the pump will be how much and I am telling now.

So, here if now try to energy balance right. So, the pump pressure will depends upon the system it is a important. The pump pressure the pressure that will be developed by the pump that or the head developed by the pump will depends on the system in which it is acting. So, if I apply energy equation between that is why I am trying to go back to my previous slide, if I apply energy equation between point let us say 1 and 4 that is you know. If I go back to my previous slide, and if I let say this is my point, this point 1 and this is point 4.

So, if I now try to write energy question between the point 1 and 4, what I get and that is what I will try to write over here. That if I said you the pump pressure that will depend upon the system in which it is working and if I try to write energy equation between point between point 1 and 4, then what can I obtain. So, now we will discuss about the head developed by the pump.

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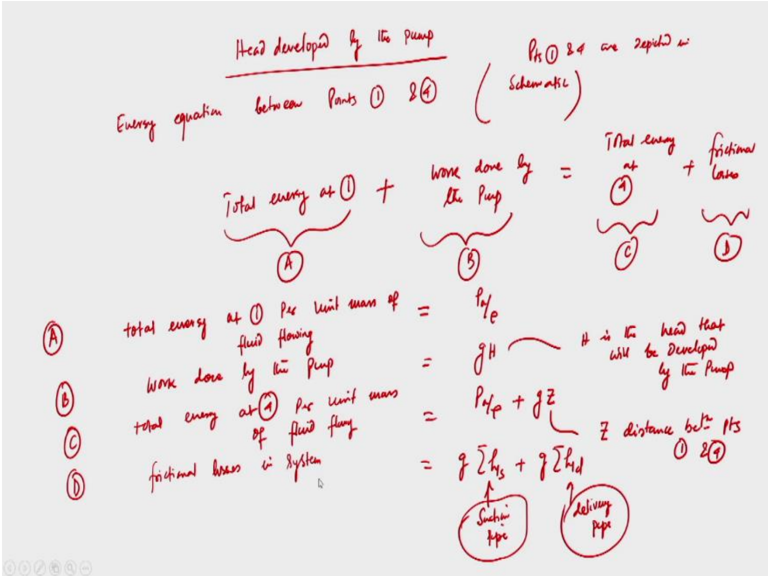

So, head developed by the pump. Last slide we have discussed about the discharge and there will be a theoretical discharge, and actual discharge theoretical discharge is predicted by that quantity $\frac{\omega}{2\pi}V$ while the actual distance will be less than that. But how what will be the head developed by the pump right, what will be the head developed by the pump this is very important. So, if I apply energy equation between points 1 and 4 in may the schematic in the schematic points 1 and 4 are defected in schematic right.

This is quite simple, because energy at 1 is the atmospheric pressure that we know. And we are taking water in the pump rather we are sucking water in the pump. And then, we are working on that fluid that is work done by the pump, and whatever energy we are getting at the point 4 that will be equal to the energy at point twelve plus energy plus work done by the pump. And of course, while we are lifting water let us say from point 1 to point 4, we cannot ignore the frictional losses, so that amount of energy that we need to overcome using energy at 1 plus pump you know work, so that is the total equation.

Total energy at 1 + work done by pump = total energy at 4 + frictional losses

Total energy at 1 per unit mass of fluid flowing = $\frac{Pa}{\rho}$

Work done by the pump = g H

Total energy at 4 per unit mass of fluid flowing = $\frac{Pa}{\rho} + gZ$

Frictional losses = g ($\sum h1s + \sum h1d$)

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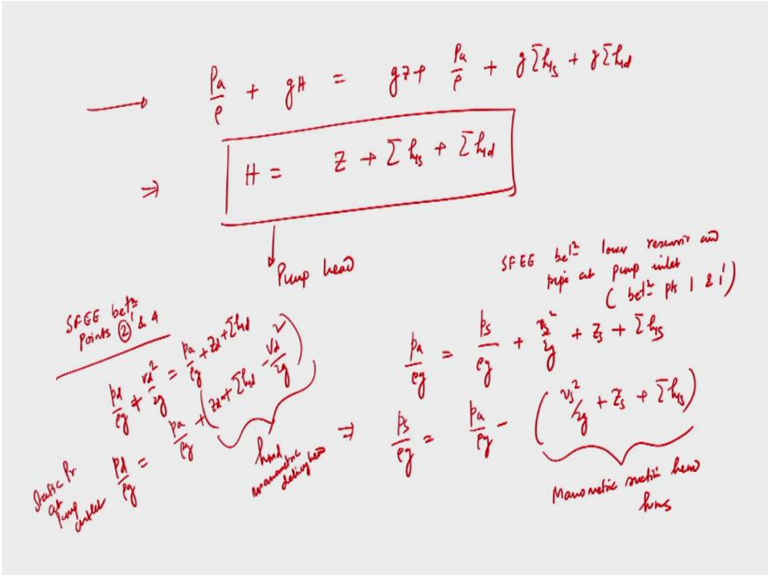

$$\frac{Pa}{\rho} + gH = gZ + \frac{Pa}{\rho} + g\left(\sum h1s + \sum h1d\right)$$

$$H = Z + \left(\sum h1s + \sum h1d\right)$$

Now, this is very important that capital Z is the height between the rather distance between points 1 and 2, this is very important. So, while we are calculating total pump head, then I can split this equation into different parts, I can obtain that what would be Z. Z will be equal to the suction lift, and plus delivery height I mean we are sucking water from the reservoir suction sump, and then we are discharging to the delivery sump. So, there will be a you know height difference between suction sump to the pump axis that is Z s, suction lift suction lift as well as there will be a delivery lift. So, suction lift as well as delivery lift these two will be the total height that is Z.

So, now how we can calculate the suction lift and the delivery lift in terms of other quantities like you know pressure and frictional losses. And for that we need to apply again steady flow energy equation between of a few points. So, if we do not exercise, then I may obtain that what will be the actual head developed by the pump and you know then what will be that overall efficiency, this is important. You know that if I if I go back to my previous slide schematic rather, this is very important. So, what I am trying to write, see suppose the suction side say this is point one and this is point and from these, I am telling that this is the suction height, so Zs. So, this is whatever I told that is Zs. And similarly, this point will be Zd that is delivery lift suction lift plus delivery lift.

And this is the amount which is very important that is below atmospheric pressure. I mean if I now try to write the suppose I try to write you know steady flow energy equation between lower reservoir, and the pipe at the pump inlet. So, suppose I am writing Bernoulli equation between this point 1 and this is 1' and this 2'. So, I am now writing that steady flow energy equation between point 1 and 1', then what I will get that is the lower reservoir as well as the pipe at the pump inlet. Then what I can obtain, so I can obtain again I have to go to the next slide.

Steady flow energy equation between lower reservoir and pipe at the pump inlet-

$$\frac{Pa}{\rho g} = \frac{Ps}{\rho g} + \frac{{V_s}^2}{2g} + Zs + \sum h1s$$

Manometric suction head hms = $\frac{{V_s}^2}{2g} + Zs + \sum h1s$

Steady flow energy equation between point 2 & 4

$$\frac{Pd}{\rho g} + \frac{{V_d}^2}{2g} = \frac{Pa}{\rho g} + Zd + \sum h1d$$

Manometric suction head hmd = $Zd + \sum h1d - \frac{{V_d}^2}{2g}$

Pump manometric head Hm = hms + hmd

$$= \frac{{V_s}^2}{2g} + Zs + \sum h1s + Zd + \sum h1d - \frac{{V_d}^2}{2g}$$

$$= Z + \sum h1s + \sum h1d - \left(\frac{{V_d}^2 - {V_s}^2}{2g}\right)$$

Energy equation H = Z + ($\sum h1s + \sum h1d$)

H- Hm = $\frac{{V_d}^2 - {V_s}^2}{2g}$

If delivery and suction pipe of same diameter.

H= Hm

So, the pressure at that point should be and this is known as the manometric sorry this quantity will be obtained by connecting an u tube manometer at the pump delivery point (Refer Time: 41:56) outlet of the pipe at the pump delivery point, and another end will be the (Refer Time: 42:01) ambience the atmospheric pressure. From these two expressions how what can I write that we have obtained. The manometric delivery head manometric static head, and then from there the pump manometric head is defined.

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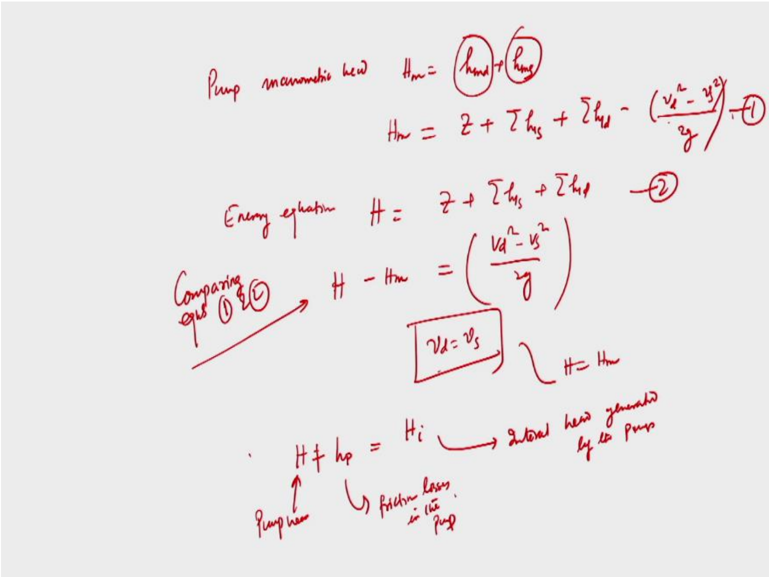

Now, what is the question is the head developed pump H. The head developed the pump H should be always less than rather H plus some quantity some quantity is let us say frictional losses h frictional losses h p, will be equal to the Hi that is the internal head generation internal head generated by the pump, where h p is the friction inside the piston cylinder right. But, the pump is greater than pump head H define (Refer Time: 45:31) inter loss in the pump. So, this is the frictional losses in the pump.

H + hp = Hi

So, the internal head generated by the pump will be always higher than the head developed by the pump H rather the pump head. So, this is the pump head, we cannot ignore, because fluid we are putting pressure by piston, so there will be a fluid friction. So, if you take that effect into account, then the head internal head developed by the pump will be always higher than the pump head by an amount h p which is h p is the frictional losses. So, what will be the internal?

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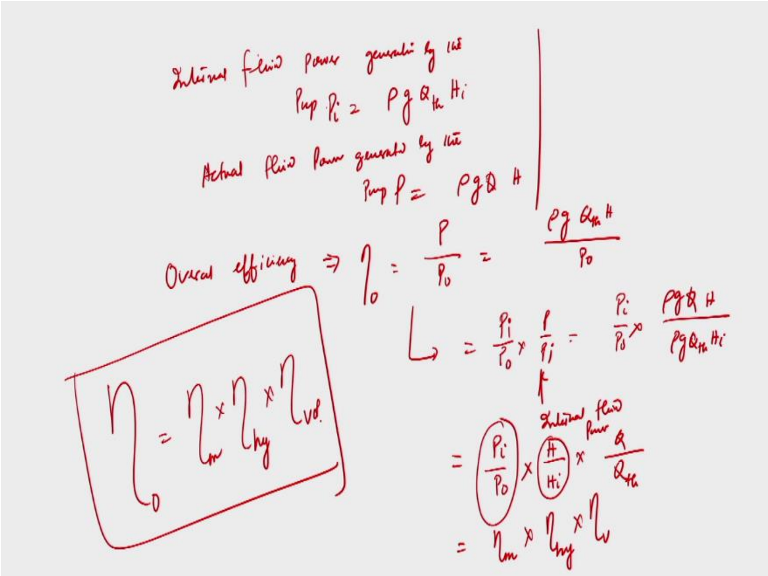

So, what will be internal you know fluid power generated by the pump will be equal to will be equal to rho g Q, Q theoretical or Q of course, Q theoretical we have to consider into H i right. And actual pump this in actual fluid power actual fluid power generated by the pump equal to will be equal to rho Q rho g Q theoretical into H. So, this is the internal fluid power generated by the pump, and this is the actual fluid power generated by the pump.

So, now we are taking pump on the prime mover that is electrical motor. So, the overall efficiency of this is the now so this is internal fluid power. So, internal fluid power will be always higher than the actual fluid power generated by the pump fine. So, to run the pump normally what do we do, we take power from electrical motor. So, overall efficiency overall efficiency we have defined volumetric efficiency, overall efficiency of the pump will be defined as eta overall is equal to P/ Po, where P = $\rho g\ QH$. So, this is the actual you know this is the overall efficiency.

So, what I obtained Pi/P naught into H by H naught into Q by Q theoretical, so that what I can write that eta overall this is what Pi/Po. Pi/Po is known as mechanical efficiency. So, this is known as mechanical efficiency time. , H/Ho is equal to how much the actual this is the internal fluid power generation, and this is the actual fluid power generation. So, H/Ho is known as the hydraulic efficiency. So, this is eta hydraulic. And this is Q by Q theoretical is equal to you know eta overall volumetric efficiency, so eta volumetric.

Overall efficiency $\eta_o = \frac{P}{Po} = \frac{\rho Qth\ g\ H}{Po}$

$$= \frac{Pi}{Po}\ \frac{P}{Pi} = \frac{Pi}{Po}\ \ \frac{\rho Qth\ g\ H}{\rho\ Qth\ g\ Hi} = \ \eta_m \eta_v \eta_{hy}$$

So, I can write that eta overall is equal to eta mechanical into eta hydraulic into eta volumetric. So, this is very important. And we have we have been able to prove that the overall efficiency can be written in terms of three other efficiencies that in mechanical efficiency that is Pi/ Po, where P i is equal to internal fluid power generation, because the internal power generation will be always higher than the actual power. Because, internal head developed should be higher than the actual head, because there will be some losses in the pump itself. So, H / Ho I known as the hydraulic efficiency, and eta V is the theoretical, Q/ Q theoretical is the volumetric efficiency.

So, rearranging these three terms, I mean I can write eta overall efficiency eta o that is overall efficiency in terms of hydraulic efficiency, mechanical efficiency, and the volumetric efficiency. So, this exercise will help us to solve out to solve the numerical problem. And so far whatever I have discussed, I think it will help us to you know solve numerical problem as well as to have to help and it will help to have a you know fare idea about how the reciprocating pump is working, and how the head is being developed, and what should be the volume flow rate. Also we have discussed that the h q curve, why the actual q flow rate is deviating from the ideal one.

So, with this, I stop here today, and we will continue the next topic in the next lecture.

Thank you.