Principle of Hydraulic Machines and System Design

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Lecture – 02
Euler equation for turbomachines: Net head developed by the pump/turbines

Today, I will continue, our discussion on Principle of Hydraulic Machines and System Design. So, the topic for today's discussion is Euler's equations for turbomachines. We will discuss that from Euler's equation of for turbo machines, how we can quantify the net head being developed, for pump or turbines.

So, to start with, we will just recall that in the last lecture, we have you know classified turbo machine.

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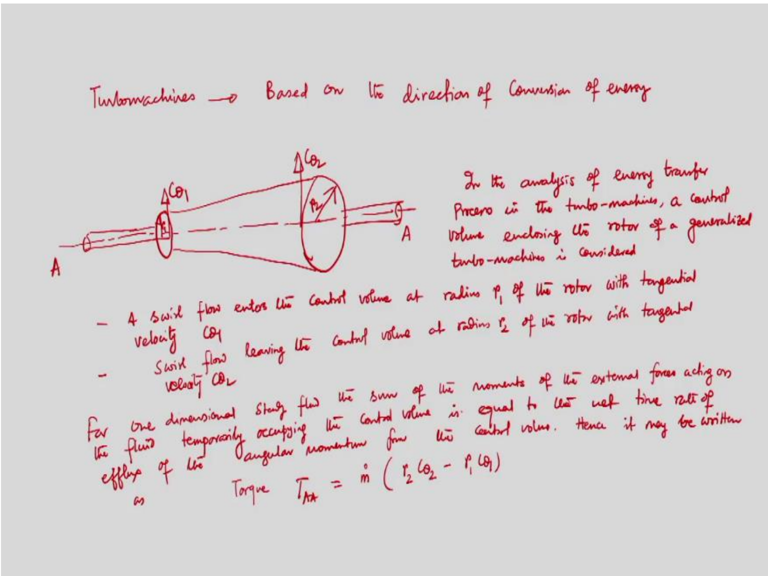

Last, last lecture, we have, you know classified turbo machines which is based on the, which is based on the direction of, based on the direction of conversion of energy.

So, we have seen conversion of energy. So, we have seen that, in a turbo machines the mechanical energy from the moving part is transferred to the fluid, to increase its stored energy by increasing either it's pressure velocity or, specific enthalpy in case of a pump compressor or blower or sometimes the kinetic or potential energy or intermolecular energy of the fluid is transferred to increase the mechanical energy of the moving parts, which is,, known as turbine.

So, whatever it is energy is being transferred from mechanical energy to increase the stored energy of the fluid or the in kinetic or potential energy of the fluid to increase the mechanical energy of the moving parts. So, now, today we will discuss the Euler's equation for turbo machines, it is a generalized, it may be a for pump or may be a turbine.

So, what I will do? I will draw a schematic. So, I will draw a schematic on rotor and which is connected with shaft and then there is a axis of the shaft and. So, radius at the outlet is r_2 and radius at the inlet is r_1.

Now, it is again, the shaft is like this and I have taken a cross section A A. So, now, the radius at the inlet is r_1 and maybe we are having one component $C_{\theta 1}$ here, we are having another component, $C_{\theta 2}$. So, in the analysis of energy transfer, in the analysis of energy transfer process, in the turbo machines, a control volume consisting the rotor of a generalized turbo machine is considered.

So, here we have consider, I am writing so, in the analysis of, in the analysis of energy transfer process in the turbo machines a control volume enclosing the rotor of a generalized turbo machine is considered.

So, what we do? So, we consider a rotor. So, we have discussed that in a turbo machines either mechanical energy is going transferred to increase the stored energy of the fluid either increasing its pressure or volume or sometimes the kinetic or potential energy of the fluid is transferred to increase the mechanical energy of the moving parts.

So, what we are doing now? We are considering a control volume enclosing the rotor of a generalized turbo machines. So, it is a generalized equations initially, I will drive by generalized equation, which is Euler equation for turbo machines.

Next, we will go for that, what form can I give that Euler equation, if it is a pump or if it is a turbine? So, now, this is what we have considered that control volume and closing rotor of a generalize turbo machines, also we are assuming they are swirling swirl flow.

So, we are assuming that swirl flow. A swirl flow entering the control volume by enters or enters the control volume, a swirl flow enters the control volume at radius r_1 of the rotor at radius r_1 of the rotor at with tangential velocity with tangential velocity $C_{\theta 1}$.

So, we are assuming that a swirl flow, which is entering the control volume, a swirl flow enters the control volume at radius r_1 of the rotor with, a swirl flow enters the control volume at radius r_1 of the rotor with a tangential velocity $C_{\theta 1}$.

And we also assume that this swirl flow, swirl flow, leaving the control volume at radius r_2 of the rotor with tangential velocity $C_{\theta 2}$.

So, what we are assuming that we are assuming a swirl flow, which enters the control volume at radius r_1 with a tangential velocity $C_{\theta 1}$ and the flow, the same flow is leaving the rotor at radius r_2 with a tangential velocity $C_{\theta 2}$ right. Now, what we can see that for one dimensional flow. So, for one dimensional for one dimensional steady flow, steady flow the sum of the moments of the external forces acting on the fluid temporarily occupying the control volume, so, control volume. So, temporarily a certain fluid volume of fluid mass is there within the control volume. So, if I consider the one dimensional steady flow and then sum of moment of all external forces acting on the fluid, which is there temporarily on the control volume is the net time rate is equal to the net time rate of efflux of the angular momentum from the control volume, angular momentum from the control volume.

Hence, it may be written, it may be written as, it may be written as toque A A acting torque A A on the shaft. I mean we are, we have considered a control rotor and shaft is a then

T_{AA}= m (r2$C_{\theta 2} - r1C_{\theta 1}$)

So, from the above paragraph, from the above statement that for one dimensional steady flow we have considered that flow is steady, an one dimensional flow that for one dimensional steady flow, sum of moments of all external forces acting on the fluid, occupying temporarily on the control volume is equal to the net efflux, net time rate of efflux of the angular momentum from the control volume. So, accordingly you can write torque T A that is on this, shaft a cross section T_{AA}= m (r2$C_{\theta 2} - r1C_{\theta 1}$)

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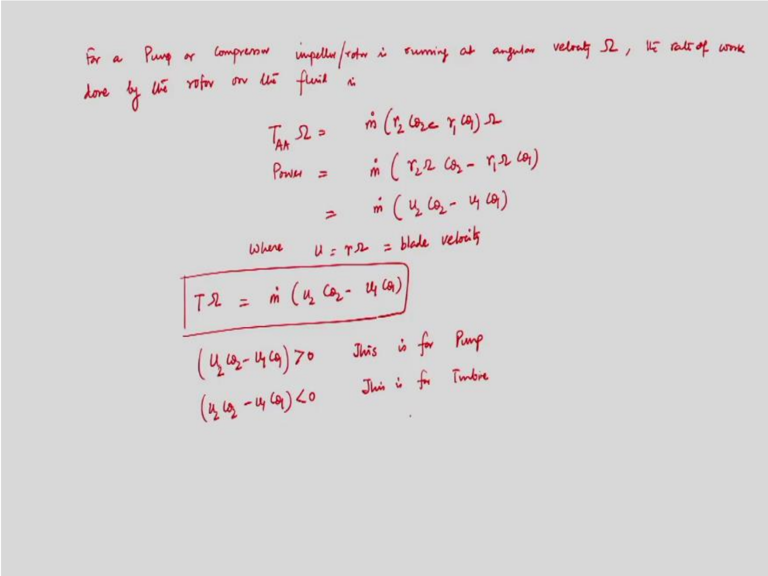

Now, if I consider the pump. So, for a pump or compressor, for a pump, for a pump or compressor, for a pump or compressor rotor, for a pump or compressor rotor, I mean this moving, element rotating element rotor, I mean it should be impeller or rotor for pump, it is impeller or compressor hub plus blade is rotor. So, for pump and compressor, this if impeller or rotor is running at angular velocity omega, at angular velocity omega we, I mean the rate of work done, the rate of work, the rate of which the rate of work no done on the fluid rate of work done by the rotor on the fluid, by the rotor on the fluid is T A A into capital omega, that is T_{AA}= m $(r2C_{\theta 2} - r1C_{\theta 1})$ into capital omega.

Now, so, this is essentially power torque into omega that is power. So, this is power is equal to

T_{AA}= m $(r2C_{\theta 2} - r1C_{\theta 1})$

$T_{AA}\,\omega$ = m $(r2C_{\theta 2} - r1C_{\theta 1})\omega$

Power= m$(u2C_{\theta 2} - u1C_{\theta 1})$

Where $u=r.\omega$

$u2C_{\theta 2} - u1C_{\theta 1} > 0 \quad this\ is\ for\ pump$

$u2C_{\theta 2} - u1C_{\theta 1} < 0 \quad this\ is\ for\ turbine$

So, this is the generalized equation we have derived. Now, we are going to see, if we apply, if we write the Euler equation of Euler equation particularly for the pump or radial flow, pump then what form, I can get, I mean from this equations?

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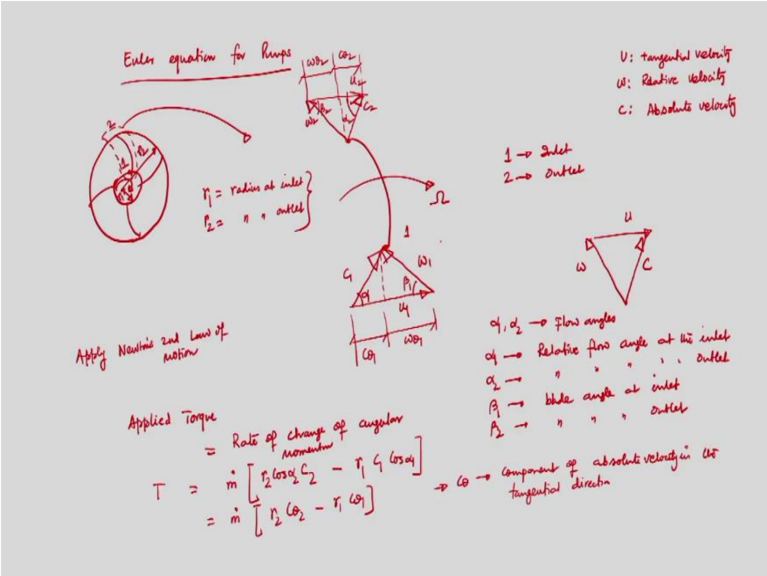

So, for that I will go to you know discuss this Euler equations. Now, will discuss Euler equations for pumps.

So, we have derived till now, we have derived the equation, Euler equation for turbo machines, which is valid for the pump and turbine and we have seen that depending upon the magnitude of the quantity $u2C_{\theta 2} - u1C_{\theta 1}$, if it is greater than equal to 0 of course, then work is done by the rotor on the fluid, then it this is for the pump, this is for the pump and when $u2C_{\theta 2} - u1C_{\theta 1}$ is less than 0 then of course, work is done by the fluid element to increase the mechanical energy of the moving component, then this is for the turbine.

So, now we will discuss that if I write Euler equation, for the pump then what would be the expression and, the net head being developed by the pump. This is very important, because in a pump we have seen that, mechanical energy of the moving part is, you now transferred to the fluid to increase it, pressure I mean velocity or you know or sometimes specific enthalpy in case of a pump compressor blower or fan.

So, we will see, if we apply Euler equation for a pumps, particularly for a radial flow pump then what would be the final expression and from that expression, how can quantified that how we can quantify, the net head being developed by the pump and to do that what I will do? I will draw a radial flow impeller. So, if I draw a radial flow impeller, if I draw a radial flow impeller, so, this is as I said that this is hub and this is pump impeller.

So, hub ready, this is point 1 and this is point 2. So, if I take out a particular blade, if I take out a particular blade and if I draw, the zoomed inform so, if I take out this blade and if I draw rather if I take, I enlarge of this particular blade then it look like this. So, this is point 1 and this is point 2 and 1 and 2; 1 is inlet and 2 is outlet.

So, 1 is inlet and 2 is outlet. So, we have drawn a radial flow impeller, impeller of a radial flow pump and we know that the there are two radial. So, one is r 1 right and another is r 2. So, this, r 1 and r 2 are the radius inlet and outlet radius of the impeller and we have discussed the velocity triangles in the last lecture. So, we will discuss again the velocity triangles.

So, blade and impeller rotating a certain angular velocity, I mean so, there will be a absolute velocity. Let us say if I draw the velocity triangle again, there is, this is a tangential velocity u and there is a absolute velocity w. So, the resultant velocity will be like this and that is, we call it absolute velocity c. So, whenever the pump impeller is rotating at certain angular velocity, then we will have a tangential velocity u.

So, u is the tangential velocity, w is the relative velocity and velocity relative to the blade relative velocity and there will be an resultant velocity c according to the Lami's theorem and we call it absolute velocity of the flow absolute velocity. Now, if I draw, so, if I just include suffix 1, it would be that the triangle will be at the inlet and similarly, if I suffix 2, it will be at the outlet.

So, if I now, draw the inlet and outlet velocity tri, triangle which is very important to obtain the, expression for the, you know power that we have discussed in the last slide that net I mean torque into omega that is power, which is coming from Euler equations and we will see that how we can give a different form of that equation, in terms of head being developed by the pump. So, the inlet velocity triangles is like this.

So, this is, absolute velocity, this is relative velocity and this is tangential velocity. So, this is u 1, this is w 1 and this is C 1 Similarly, I, if I draw the velocity triangles at the outlet I will get like this. So, this is tangential velocity at the outlet, this is absolute velocity C 2, this is u 2 similarly, I will get the relative velocity at the outlet is w 2.

Now, I am giving, the different angle. Let us say this is $\alpha 1, \beta 1$ and component of absolute velocity at the inlet in the, in the tangential direction is $C_{\theta 1}$ that we have discussed. So, and this is $W_{\theta 1}$. Similarly, if I draw the component of absolute velocity at the outlet, this is alpha 2 and this is beta 2.

So, this is $C_{\theta 2}$ and this is $W_{\theta 2}$. So, what we have done, we have drawn, the velocity triangles at the inlet and outlet. We have identified different velocities that is the tangential velocity, absolute velocity, velocity I mean and the relative velocity. The velocity related to the blade and then you have given different angle alpha 1, beta 1, alpha 2 and beta 2, note that, that alpha 1 and alpha 2, these are known as the flow angle.

So, I am writing alpha 1 and alpha 2 are the flow angles that is alpha 1 is the relative flow angle at the inlet, relative flow angle at the inlet, an alpha 2 is the relative flow angle at the outlet. Note that all angles are measured with a tangential directions. So, similarly beta 1 is the blade angle, is the, is the blade angle blade angle at the inlet and beta 2 is the blade angle at the outlet.

All angles are measured as I said all angles are measure with the tangential direction, alpha 1 and alpha 2 are not the blade angle; mind it, alpha 1 and alpha 2 are not the blade angle, because the flow direction does not match or co inside with the when at the inlet and outlet. So, these are we call it the flow angle. So, now, we have obtained the and this blade or the impeller is rotating let us say with an angular velocity omega in the clockwise direction.

So, that we have drawn in the impeller, that the blade angle is impeller is rotating at the at an angular velocity omega in the, clock wise direction. Now, from Euler equation of motion, we have seen that, total power is essentially mass flow rate into u2 $C_{\theta 2} - u1\ C_{\theta 1}$

So, here we have drawn the velocity triangles and blade is rotating at an angular velocity omega. Now, if I apply Newton's second law of motions, so, if I apply Newton's second law motion, to for this particular blade then applied torque, then applied torque will be equal to what?

So, if I apply Newton's second law of motion to this particular case. So, if I apply Newton's second law of motion then for this particular case blade, then applied torque will be equal to rate of change of angular momentum. Then rate of say if I applied Newton's second law of motion for this rotating system, then applied torque will be equal to rate of change of angular momentum, rate of change of angular momentum, then applied torque that is, see is a prime mover, I mean to, we need a prime mover to run this pump.

So, which is a radial flow impeller, impeller of radial flow pump? We are interested to find out the Euler equation for this pump in a bit different form and from there we would like to quantify rather we will try to quantify the net head being developed by this pump and to do so what we are doing? We have drawn the velocity triangles at the inlet and outlet, we have identified different component of velocities then we have given different angles, different flow angles and blade angles and we have assume that the impeller as well as the entire, l blades are rotating at angle of velocity omega. Then applying Newton's second law motion for this rotating system, we can quantify, applied torque is equal to rate of change of angular momentum. Then see applied torque is let us see, if we assume the applied torque is T that is coming, because we are we are rotating this impeller using a prime mover. It maybe, electric motor or it may be a diesel engine.

So, this torque is equal to rate of change of angular momentum is essentially equal to m to a mass flow rate into I, I identified r 1 and r 2 are the radius at the inlet and outlet. So, r 1 is equal to radius at the inlet, radius at inlet and r 2 is equal to radius at the outlet. So, in radius at the inlet and outlet of the impeller, this are we have, you know identified r 1 and r 2, then I can write rate of change of angular momentum is equal to applied torque from the Newton's second law of motion. So, the rate of change of angular momentum will be

T= m (r2 cosα2 C2 – r1 cosα1 C1)

T = m (r2 $C_{\theta 2} - r1\ C_{\theta 1}$)

Power = T ω = m ω (r2 $C_{\theta 2} - r1\ C_{\theta 1}$) = m (r2 $\omega C_{\theta 2} - r1\ \omega\ C_{\theta 1}$)= m (u2 $C_{\theta 2} - u1\ C_{\theta 1}$)

So, we have quantified torque is equal, we have written quant torque in, in terms of the component of absolute velocity in the tangential direction and radius. So, now, will write then total power will be equal to what? Torque into omega.

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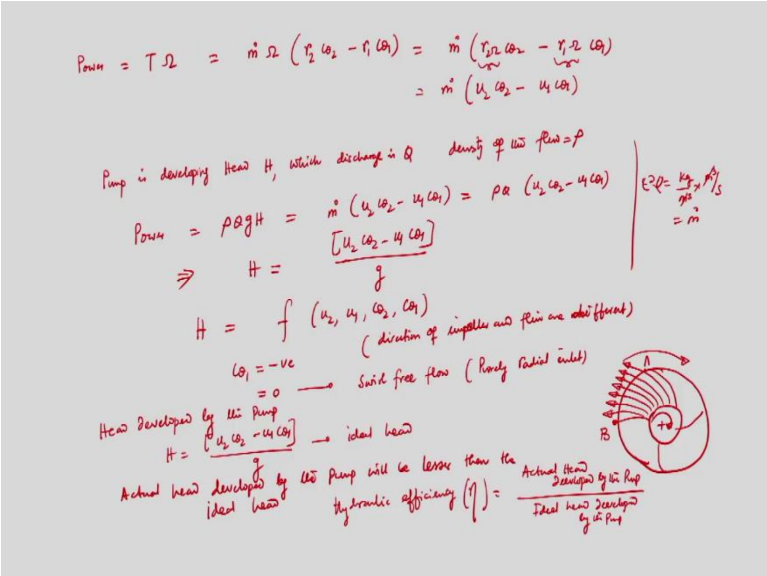

So, now power will be torque into omega since, the impeller is rotating at an angular velocity omega that is equal to mass flow rate into omega into r 2 c theta 2 minus r 1 c theta 1.

Power = $\rho\, QgH = \text{m}\,(\text{u2}\ C_{\theta 2} - u1\ C_{\theta 1}) = \rho\, Q(\text{u2}\ C_{\theta 2} - u1\ C_{\theta 1})$

So, now, what is the total power? Suppose, as I said that from this Euler equation now, I will try to have, I will try to get an expression of the head, which is being developed by the pump. So suppose that if I use the radial flow pump if I use the radial flow pump in a place, where pump is developing say, pump is developing pump is developing head, developing head H while discharge is Q. So, pump is place that particular pump is placed to develop a head H against a discharge of Q. So, then what will the power? Then power will be equal to if density is rho, discharge Q and density of the fluid is equal to rho, then power is equal to rho Q g in to H.

H= $(\text{u2}\ C_{\theta 2} - u1\ C_{\theta 1})/\text{g}$

H= $\text{f}(\text{u2},\text{u1}, C_{\theta 2}, C_{\theta 1})$

So, now what we can do? That by changing the magnitude of c theta 1; that means, whenever we are installing a pump, in a particular place to increase its efficiency pump, efficiency are always, always our target will be, to have a higher head from the particular pump. So, if we would like to have a higher head for a given pump, if I would like to develop a higher head for a particular pumping system or for a particular pump then our, the target should be to increase, to make $C_{\theta 1} = 0$ or $C_{\theta 1} < 0$, because, a positive component of c theta 1 will always decrease the net head developed by the pump.

So, our target from this, what we can see from this expression is that if we would like to develop a higher head from that particular radial flow pump, we need to make there two cases; either we can make $C_{\theta 1}$ is equal to negative quantity or $C_{\theta 1}$= 0; that means, there is no swirl, that is called swirl free flow, that is swirl free flow that is purely radial unit, radial inlet purely radial inlet or if I can make $C_{\theta 1}$ negative that I have discussed in the last class, just we can make the component of absolute velocity at the, in the tangential direction at the inlet to be negative, if the direction of rotation of the impeller and fluid are in the different, at different.

So, when direction of, of impeller or rotating, rotating element and fluid are different. So, when direction of the impeller and fluid are different then we can have a negative component of C_θ if we can somehow have negative component of C_θ then that develop the head will, will be developed by the part by that same pump will be higher or else we can make $C_{\theta 1} = 0$ that is there will be no swirl at the inlet, the swirl free flow.

Now, free world and in that case also head developed by the pump will be higher, important thing is the head develop by the pump, whatever expression I have got that is (u2 $C_{\theta 2} - u1\ C_{\theta 1}$)/g essentially, the ideal head. So, head developed by these head developed, by the machines or by the pump, this H = (u2 $C_{\theta 2} - u1\ C_{\theta 1}$)/g

is ideal head.

So, if I now draw this again impeller of this radial flow pump. So, suppose, I am drawing the impeller of a radial flow pump, this is hub and this rotating in the clockwise direct, direction and there are blades, which are falling lower than spiral and whenever pump is rotating in this direction, maybe water will try to follow this direction, if it handles water as a working fluid.

So, water is coming out from the impeller and we are assuming that the when suppose, this is passes A B. So, this is A and this is B. So, when water flowing through the passes A B through the impeller, it is assumed that, it is guided by large number of wends. So, whenever water is you know passing through the passage, water is flowing through the passes AB in the impeller.

Then it is assumed that it is guided by large number of wends and as if there are no loss, which is there between to fluid, earlier. So, whenever water is flowing out of the impeller then of course, there will be a losses, because one fluid layer is moving over the other fluid layer, fluid frictional losses we cannot ignore, but if it is assumed, it is assumed that, that the water is moving other water is flowing through the passes A B between large number of wends, but this is not the case in actual case, because there are solid boundary. So, whenever water fluid will start flowing what the solid boundary will start develop and we cannot ignore the frictional losses.

So, the head developed by the pump, which is coming from the Euler equation for pumps rather this is the theoretical head. This is the ideal head, but the in actual case, the head developed by the pump will be even lesser than the theoretical head. So, in actual head, the actual head, actual head developed by the pump will be lesser than the ideal head.

So, what so; that means, whatever expression of head that is (u2 $C_{\theta 2} - u1\ C_{\theta 1}$)/g that we have derived from Euler equation that is the ideal one, because, it is assumed that the water is passing through the passes, water is flowing through the passes, in the impeller, between two wends are assumed to be guided by large number of wends.

So, there will be no frictional losses I mean between two fluid layers, but this is not the case of course, there will be a losses between the fluid layer also there are solid boundary. So, whenever fluid will start flowing over the solid boundary, the boundary here will form and we cannot ignore the frictional losses and all those things. So, the actual head developed by the pump will be always lesser than the head predicted by this expression, (u2 $C_{\theta 2} - u1\ C_{\theta 1}$)/g and thereby, one efficiency is defined, one efficiency is you know defined that is known as hydraulic efficiency of the pump. So, hydraulic efficiency, of the pump eta, it is defined. So, hydraulic efficiency eta is defined as the ratio of actual head developed by the pump to the ideal head developed by the pump that means, always actual head developed by the pump will be less than the ideal head.

So, there is a efficiency, which is not the hydraulic efficiency apart from this efficiencies there are so many, there are two other efficiencies, we can define in the context of pump operation, we will discuss in our subsequent discuss, you know, lectures, I mean one is known as mechanical efficiency other is known as hydraulic efficiency it, overall efficiency.

So, apart from this hydraulic efficiency, there are two other efficiencies, the one is known as mechanical efficiency other is overall efficiency that will discuss in our, you know subsequent, lectures. But for the time being, we will discuss that, what is hydraulic efficiency, because the head predicted by this Euler equation is the ideal head, because, this does not take, takes into account of the frictional loses that is there in reality. So, the actual head developed by the pump will be always lesser than this ideal head.

So, we define one efficiency, which is known as hydraulic efficiency that is the rate, that is defined as the ratio of actual head developed by the pump to the, you know ideal head developed by the pump. Now, this is the case, this expression can be written in different form, for axial flow machine and as well as the mixed flow machine.

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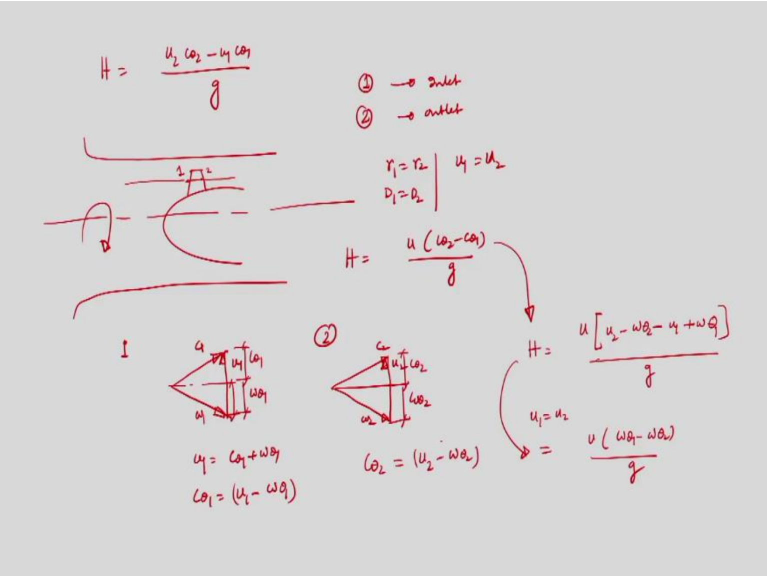

So, if I try to write this expression in terms of a for a axial flow machine what would be the expression that I will write now, and, of course, we will discuss these if I write this total head, that is head developed by the pump is (u2 $C_{\theta 2} - u1\ C_{\theta 1}$)/g. This is the head developed by the pump. If I try to write this expression for the axial flow machine then what will be the expression right?

That we will discuss for the axial flow machines, because for the axial flow machines, for the axial flow pump and for that, what we have to do? We have to draw the velocity triangles. So, if I draw the axial flow machine again schematic, as I said in the last class and last lecture. So, there will be a impeller and this is the blade and this is inlet 1 and this is outlet 2.

So, this is inlet 1 and this is outlet 2 and this is rotating like this. So, if I draw, so 1 is the inlet and 2 is the outlet, this is 1 1 is the inlet, is the inlet and 2 is the outlet. So, if draw the velocity triangle at the inlet, so, velocity triangles at the inlet, I mean mind note that here, r1 = r2 ,D1 = D 2.

That means u 1 is equal to you know, u1 = u2 for the axial flow machines. u1 = u2 that is not the case for the radial flow machines. if I draw the velocity triangles at the inlet and outlet then how even without drawing the velocity triangles at the inlet outlet. I can express H that is, H= (u2 $C_{\theta 2} - u1\ C_{\theta 1}$)/g, if I draw that velocity triangles at the inlet. So, the velocity triangle at the inlet, if I draw the velocity triangle at point 1 then it will be like this.

So, this is u1, this is c1, this is w1 and similarly, velocity triangle at the outlet will be like this, because u1 and u2 are equal. So, this is c2. This is w2 and this is u2, these are the velocity triangles at the inlet and outlet, but for the axial flow machines.

u1 = u2 , r1= r2, D1 = D2

H = u($C_{\theta 2} - C_{\theta 1}$)/g

So, this is the expression of head developed by the axial flow machines or axial flow pump, I can express you know this component in, in a bit different from, because what is C_θ, U, u 1, because if I consider this velocity triangle at the inlet. So, this is w theta 1 and this is $C_{\theta 1}$. So, this is $C_{\theta 1}$ and this is $W_{\theta 1}$. Similarly, this is $C_{\theta 2}$ and this is $W_{\theta 2}$.

$C_{\theta 1} = u1 - W_{\theta 1}$

$C_{\theta 2} = u2 - W_{\theta 2}$

H = u[u2-$W_{\theta 2} - u1 + W_{\theta 1}$]/g

H = u [$W_{\theta 1}$ - $W_{\theta 2}$]$/g$

We can express the head develop by the pump using Euler equation for pumps, we can express h in different, form since, u 1 is equal to u 2 that is r 1 since, r 1 and r 2 are equal. So, u 1 will be equal to u 2. So, I can write only in terms of the swirl component of velocity that is the, component of absolute velocity in the tangential direction, at the inlet and outlet otherwise, I can convert that expression in terms of only the, component of relative velocity in the tangential direction, at the inlet and outlet.

We have discussed that, we can express, net developed by the pump in terms of absolute swirl component of velocity or we can express. We have seen that we can express the head developed by the pump for axial flow machines.

In terms of relative component of, you know component of relative velocity in the tangential direction, at the inlet and outlet. Now, we will see that, from the velocity triangles, for the radial flow machine if I draw the velocity triangles at the inlet and outlet.

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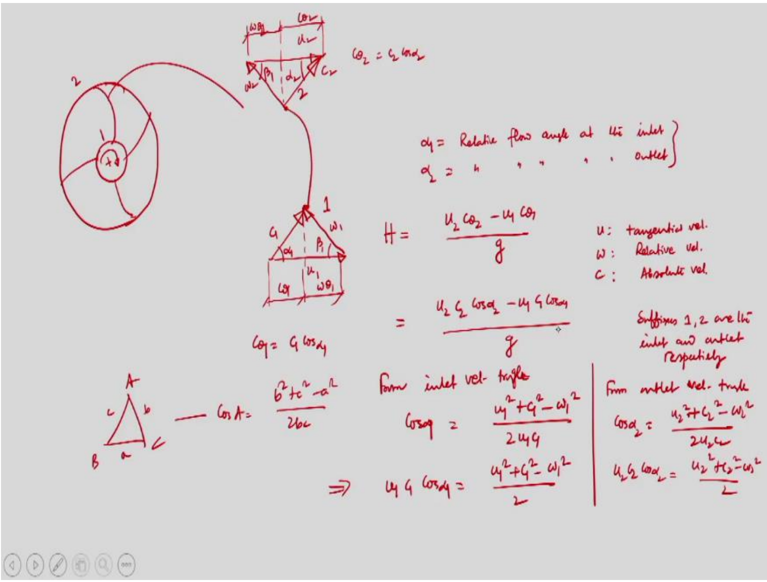

Again, I have drawing an impeller for the radial flow machines I mean, if you radial flow pump rather.

So, this is the hub and it is rotating in the clockwise direction at an angle of velocity ω and there are blades. So, if I take out a particular blade and if I draw the in, in enlarge view that is 1 and this is 2. So, if I take the take out this particular blade 1 and 2 and if I draw the velocity triangles at the inlet and outlet. We have seen that there are three components; one is, tangential velocity, which is u 1 another is the relative velocity w 1.

And the there will be an resultant, which is the absolute velocity at the inlet c 1, which absolute velocity makes an angle $\alpha1$ with in the tangential direction and this is relative velocity makes an angle $\beta1$ in the tank with the tangential direction. So, $\alpha1$ is the relative flow angle at the outlet relative flow angle, at the inlet sorry, at the inlet and similarly if I draw the velocity triangles at the outlet I will get like this.

So, this is the tangential velocity u 2, this is the absolute velocity c 2 and this is the relative velocity, w 2 at the outlet. So, $\beta1$ and $\beta2$ are the blade angle at, at the inlet and outlet and this angle is alpha 2 at the outlet and we have seen there are components of I mean, absolute velocity and relative velocity, in the tangential direction $W_{\theta2}$ and $C_{\theta2}$. Similarly, I may have a component of absolute velocity and relative velocity at the.

In the tangential direction at the inlet as well. So, this is $C_{\theta1}$ and this is $W_{\theta1}$ component of absolute velocity at the inlet and component in the tangential direction component of relative velocity in the tangential direction in the inlet. So, $\alpha1$ is the relative flow angle, inlet and $\alpha2$ is the relative flow angel, at the outlet and $\beta1$ and $\beta2$ are the blade angle at the inlet and the outlet respectively.

Now, I will try to have another expression. So, we have seen that head developed by the pump from the Euler, from the Euler equations can be written (u2 $C_{\theta2} - u1\ C_{\theta1}$)/g. So, from Euler equation, we have seen that the net head developed by the pump can be expressed that in terms of blade, velocity at the inlet and outlet, and the component of absolute velocity at the inlet and the outlet for the axial flow machines. We have given form of this head developed by the axial flow pump.

In terms of, component of relative velocity at the inlet and the outlet, in the tangential direction. Now, can I express this ex quantity in terms of three different velocities rather. So, there are three different velocities, one is u that is, tangential velocity, tangential velocity, I mean velocity of the blade in the tangential direction. w is the relative velocity, velocity relative to the blade, relative velocity and c is the absolute velocity of the flow, absolute velocity.

So, now, I will try to express this quantity in terms of three components of, velocity that is absolute velocity, tangential velocity and the relative velocity. So,

H= (u2 $C_{\theta 2} - u1\ C_{\theta 1}$)/g

= ($u2\ c2\ cos\alpha 2 - u1\ c1\ cos\alpha 1$)/g

From inlet velocity triangle

$$cos\alpha 1 = \frac{u1^2+c1^2-w1^2}{2\ u1\ c1}$$

From outlet velocity triangle

$$cos\alpha 2 = \frac{u2^2+c2^2-w2^2}{2\ u2\ c2}$$

$$u1\ c1\ cos\alpha 1 = \frac{u1^2+c1^2-w1^2}{2}$$

$$u2\ c2\ cos\alpha 2 = \frac{u2^2+c2^2-w2^2}{2}$$

So, now if I put the value of u 1 c 1 $cos\alpha 1$ and u 2 c 2 $cos\alpha 2$, the expression of head developed by the pump, that is Euler equation, for which is coming from Euler equation of, Euler equation for pumps.

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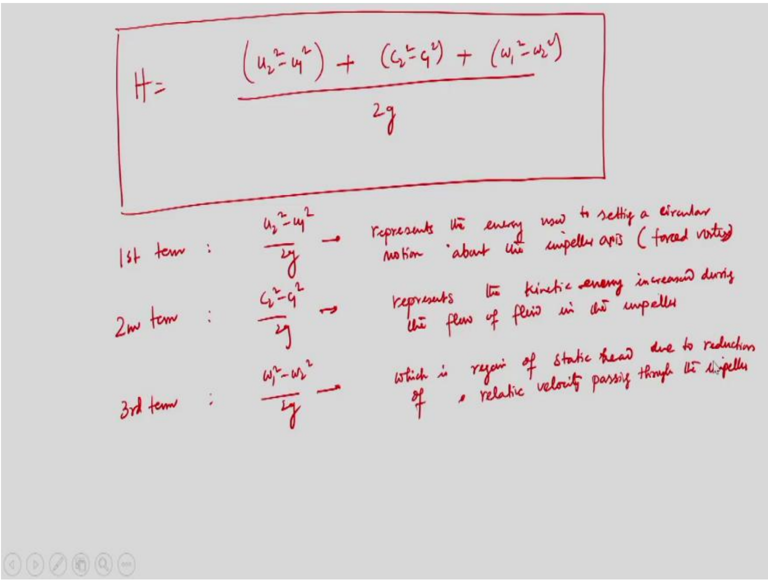

$$H = \frac{(u2^2 - u1^2) + (c2^2 - c1^2) + (w1^2 - w2^2)}{2g}$$

So, after doing some manipulation are not just algebraic manipulation, if I put the value of c 1 u 1 c 1 cos alpha 1 and u 2 c 2 cos alpha 2 in the expression of H, obtain the expression of H in term, in terms of three component of three velocities, rather tangential velocity, absolute velocity and the relative velocity. Now, I will discuss that there are three component of velocities, I mean absolute velocity, relative velocity and the, tangential velocity.

So, all these velocities are contributing to the net, net head develop by the pump. So, what is the significance of each and every term that is there in this expression. So, there are three terms, first term, first term is $(u2^2 - u1^2)/g$. So, this is the term which represents. So, this is very important, this is the term.

This term represents the energy used to setting a circular motion, circular motion in the impeller, circular motion to this $(u2^2 - u1^2)/g$. First term which represents the energy, used to setting up a circular motion about the impeller axis, about the impeller axis, which is nothing but forced vortex right. So, the first term which replace the energy, used to setting a circular motion about the impeller axis, which is forced vortex. Second term $(c2^2 - c1^2)/2g$.This represents, you know this represents the kinetic energy, kinetic energy increasing, increased during the flow of fluid in the impeller. So, whenever fluid is flowing through a impeller, there is a increase in kinetic energy.

So, $(c2^2 - c1^2)/2g$ that is the second term, which represents increase in kinetic energy of the fluid, when it is flowing through the impeller, third term that is $(w1^2 - w2^2)/2g$. This represents rather, this is the regain of static head.

This represents regain of static head due to reduction of, due to production of relative velocity, relative velocity passing through the impeller. Relative velocity of the fluid passing through the velocity, passing through the impeller, which is quite obvious, because; so, first term represents the energy which is used to setting a circular motion about the impeller axis about the forced vortex, second term which is essentially the gain in kinetic energy, whether increasing kinetic energy, when fluid is flowing through the impeller. And third term which is, which represents or the, which is regain of static head due to reduction of relative velocity passing, when fluid is passing through the impeller, because, whenever fluid is passing through the impeller, we have seen that it is a, you know convergent diversion part.

So, at the outlet, that at the outlet of the impeller where, few area is a larger, so, velocity will decrease, the relative velocity will decrease and pressure will increase.

So, that is why the third term which is essentially the increase in the gain in static head, because of the reduction of relative velocity at the, whenever fluid is flowing through the impeller, there is a reduction and because of this reduction there is a regain of the static head. So, that is the represented that is represented by the third term ok. So, we stop here today and we will continue in the next lecture.

Thank you.