

Principle of Hydraulic Machines and System Design
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Lecture – 17
Affinity Laws, Specific Speed – II

We will continue our discussion on Affinity Laws and Specific Speed. But as I said that before I go to discuss about the affinity laws, we have tried to analyze one another important aspect of you know fluid flow through any closed conduit is the dimensional analysis that is Buckingham pi theorem. So, today we will work out a case when fluid is flowing through a pump and if we apply Buckingham pi theorem, then what are the pi terms whether dimensionless term we can obtain. And from there we will see that from those pi terms we will try to explain what the affinity laws are.

That means, if we know that one pump maybe it radial flow pump or axial flow pump, whenever one pump is rotating at a given speed and if we get a certain particular discharge and head at that speed or at that rated speed. Now if you would like to quantify the rate you know another I mean if you would like to quantify head and discharge head and discharge at another speed at a different speed than the rated speed, then how we can quantify without doing any experiment.

Because it is not possible to carry on you know true, you know rather to do experiments at each and every time, rather it is not possible to test at each and every speed to quantify the head and discharge. So, using affinity law, we can predict that what will be the head and discharge if we know the head and discharge at any rated speed.

So, as I we have understood that before we go to discuss this, we now need to know what the dimensionless term are we can you know form whenever liquid is or fluid is flowing through a pump. And to do that we have discussed about Buckingham pi theorem and we have yesterday we have discussed about the theories, I mean what are the procedures of applying the Buckingham pi theorem and also we have discussed a some useful relationship. So, today we will consider that whenever a fluid is flowing through a radial flow pump.

(Refer Slide Time: 02:28)

Say an incompressible fluid is flowing through the impeller of a radial flow pump

1) list of variables : $D, N, \rho, \mu, gH, E, P, Q$

Variable	Unit	Dimensions
Q (discharge)	m^3/s	$L^3 T^{-1}$
N (speed)	rpm	T^{-1}
gH	m^2/s^2	$L^2 T^{-2}$
D (diameter)	m	L
ρ (density)	kg/m^3	$M L^{-3}$
μ (viscosity)	$Pa-s$	$M L^{-1} T^{-1}$
P (Power)	$Watt$	$M L^2 T^{-3}$
E (Energy per unit mass of fluid)	m^2/s^2	$L^2 T^{-2}$

Fundamental dimension $M-L-T$

Repeating variables (none dimensionless and each with different dimension)

$n = 8$
 $m = \text{no. of fundamental dimension} = 3$
 $(n-m) = (8-3) = 5 \text{ } \pi \text{ terms}$

So, let us say a fluid or an incompressible fluid is flowing through the impeller of a radial flow pump or any pump rather of a impeller of a pump. Now what are the variables involve with this and out of this variable what would be the repeating variables and what are the what fundamental dimension we will consider obtaining the pi terms that we will work out today.

So, whenever an incompressible fluid is flowing through a impeller of a radial flow pump, we can list down the you know list up variables. So, if we recall that yesterday we have you know discussed about the procedures step wise. So, initially that is a first step we need to list down the variables involve with these you know case. So, number 1 is list of variables. So, list of variables we need we need to know. So, what are those? One is D that is impeller diameter then speed of the pump, we have discussed about an incompressible fluid so, maybe density of the fluid is ρ viscosity is μ then we can consider as I discussed that instead of considering h as a separate variable, because in most of the fluid machines fluid is flowing through a closed conduit.

So, it is not an open surface flow. So, we can club together g and H and that will help us. So, we can consider g in to H , also we can consider E that we have written that E is energy per unit mass of the fluid and of course, you know that is P power and this 3 variables, that is D, N, P ; D, N, ρ, μ, gH, E and P . So, these are the variables we can consider also for another important quantity that we did not write till now that is Q because Q is very

important we need to know that is discharge. So, head discharge speed and then diameter of the so, diameter of the impeller D. So, we now listed down the variables.

The next step is what is the next step if you can recall, that we need to find out the list of variables and as well as the dimensions. So, if I now write in a tabular form. So, we will write these. So, initially variable we are writing variables, then their unit and another important thing is as I discussed we need to follow fundamental dimensions either M L T or flt whatever it is so, that is dimensions. So, these 3 are very important before we try to figure out a few pi terms or dimensionless term using Buckingham pi theorems dimensions. So, the first quantity is Q.

So, if I write Q that is discharge of course, the unit is meter cube per second; unit is meter cube per second. So, Q is discharge unit is meter cube per second and what will be the dimension? If we follow M L T suppose we are following fundamental dimensions,

Variables	unit	Dimension
Q(Discharge)	m ³ /sec	L ³ T ⁻¹
N speed	rpm	T ⁻¹
gH	m ² /s ²	L ² T ⁻²
D	m	L
ρ density	Kg/m ³	M L ⁻³
μ viscosity	Pa-s	ML ⁻¹ T ⁻¹
P power	watt	ML ² T ⁻³
E energy	m ² /s ²	L ² T ⁻²

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So, we have listed down the variables we have written their units and we have written also the dimensions following a fundamental dimension M L T now. So, this is the first step that we have carried out. So, next what is the next step? If we follow the yesterday, we have clearly written the procedure of Buckingham's pi theorem then what are the next step? Next step is we now need to select 3 repeating variables.

We now need to select 3 repeating variables none are dimensionless and each with different dimensions. So, if we follow our you know previous lectures I know where we have listed down the procedures the steps, then after listing down the variables writing their unit and dimensions, now we need to select following the fundamental dimensions M L T, and we need to know what will be the repeating variables.

While you are selecting repeating variables, I am writing while you are selecting repeating variable this is very important, repeating variables none dimensionless none dimensionless and each having an each with different dimensions; each with different dimensions. So, this is very important.

So, while you are selecting repeating variables from these variables, we need to consider that none are dimensionless and each of them will be having different dimensions. Now from these we can select 3 repeating variables and these repeating variables appear will appear in each pi term. So, we need to now select repeating variables and then these repeating variables should appear in each pi terms.

So, here what are the total variables that if we know m. So, m is the number of variables total m is the sorry n is the number of total variables. So, n is the number of variables is equal to how much we are having total 8. So, here we are having total variable 8; and we need to select m that is repeating you know m is number of fundamental dimensions; number of fundamental dimensions.

So, we have 8 variables and number of fundamental dimensions are 3 M L T. So, we will get n minus m that is we will get 8 minus 3 that is we will get 5 pi terms or 5 dimensionless terms we will get 5 number of or 5 pi terms or 5 dimensionless terms. So, now, what you have to do that we need to select repeating variables and we will go to our next slide. So,

out of this 3 out of these 8 variables, we are now selecting a repeating variables criteria is that none are dimensionless and each with different dimensions.

(Refer Slide Time: 13:54)

Repeating Variables $\rightarrow D, N, \rho$

Geometric Variable
 fluid property
 fluid property

The first π term can be expressed as a product of repeating variable, each raised to an unknown component and another variable raised to a known component say π_1

π_1 (First π term) \Rightarrow

$$\pi_1 = \left(L^3 T^{-1} \right)^{x_1} \left(L \right)^{x_2} \left(T^{-1} \right)^{x_3} \left(M L^{-3} \right)^{x_4}$$

$x_1, x_2, \text{ and } x_3 \rightarrow$ are unknown components

$M^0 L^0 T^0 = \left(L^3 T^{-1} \right)^{x_1} L^{x_2} \left(T^{-1} \right)^{x_3} \left(M L^{-3} \right)^{x_4}$

Considering dimensional homogeneity of both sides, of eq (1), and equating the Powers of each M, L, T, we get,

M (equating Power)	$0 = x_4 \rightarrow x_4 = 0$
L (")	$0 = 3 + x_1 - 3x_4 \Rightarrow 3 + x_1 = 0 \rightarrow x_1 = -3$
T (")	$0 = -1 - x_3 \Rightarrow x_3 = -1$

So, we are selecting repeating variable repeating variables, we are selecting for this particular case as D diameter of the impeller, speed and rho density. Note that all these repeating variables will appear in each and every pi terms and here every variable I mean repeating variable is not dimensionless they are having dimension and their dimensions are not equal. So, diameter is meter speed rpm T power minus 1 density k v power meter cube. So, their dimensions are completely different, and all are dimension dimensional.

So, we now need to proceed for or we now need to proceed towards obtaining a few pi terms, and the number of pi terms that we are expecting to get from this exercise is 8 minus 3 that is 5. So, now, question is while we are selecting these repeating variables note that, here these repeating variables are geometric variables flow property and fluid property. So, here we have selected 3 repeating variables one is geometric, you know geometric variable that is diameter, fluid property or flow property.

So, these 3 very important during diameter, density and the speed. So, these 3 repeating variables D N and rho and then you need to follow another step, what are what is the next step? Then first pi term can be expressed as the product of this repeating variables, if I can recall thus you know steps number 4 that the first pi terms suppose I would like to have first pi terms pi 1, this is the first pi term.

This is first pi term can be expressed as the product of repeating variables each with, but each rest to an unknown exponent. So, I am writing the first pi term again I am writing the first pi term can be expressed as a product of repeating variable and each raised to a component, each raised to an unknown component to an unknown component, and another variable that is very important raised to a known component raised to known component say 1; known component say 1. So, we had rather we have 8 variables and we have considered 3 repeating variables.

So, we are left with 5 variables that is we can form 5 pi terms. So, while you are having pi terms, this pi terms can be expressed as a product of repeating variables each raised to an unknown component let us say a 1 b 1 a 1 to 3 or x 1 x 2 x 3 whatever it is that is up to you and another variable we need to take out of this 5 variable we need to take another variable that will rest a known components let us say 1.

$$\pi_1 = (L^3 T^{-1})^1 L^{x_1} (T^{-1})^{x_2} (ML^{-3})^{x_3}$$

$$M^0 L^0 T^0 = (L^3 T^{-1})^1 L^{x_1} (T^{-1})^{x_2} (ML^{-3})^{x_3}$$

Now, what we need to do next is very important. So, we have now, you know expressed that what would be the first pi terms, and we have followed the procedure. So, what will be the now next step? Next step is considering dimensional so, this is very important this is very important. So, these is very important. So, what we have to see now I am writing considering dimensional homogeneity, because left hand side is M power 0 L power 0 T power 0, right hand side is also having M L T with some other unknown components.

Let us say x 1 x 2 x 3 or I do not know that we need to know. So, considering dimensional homogeneity of both sides both side, what we need to do? Both sides of let us say both sides of this is equation I am telling 1 or equation 1. So, considering dimensional homogeneity of both sides of equation 1 and equating the power of each of M L T we get. And considering dimensional homogeneity of both sides of equation also we are considering that equation 1 left hand side and in a right hand right hand side is having dimensional homogeneity, and equating the powers of; equating the powers of each M L T we get what we will get that is what we need to know.

So, how we can proceed? So, now, see if we need, we are considering dimensional homogeneity and then what we will do we will be equating? We will equate the power of

each M L T in both the sides. So, left hand side power of M is 0. So, if I now equating power of M; so, equating power of M equating power then left hand side is 0 and right-hand side is x_3 .

$$0 = x_3 \quad \text{by equating power of M}$$

$$0 = 3 + x_1 - 3x_3 \quad \text{by equating power of L}$$

$$0 = -1 - x_2 \quad \text{by equating power of T}$$

$$x_1 = -3, x_2 = -1, x_3 = 0$$

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$$\left. \begin{array}{l} x_1 = -3 \\ x_2 = -1 \\ x_3 = 0 \end{array} \right\}$$

$$\pi_1 = (L^3 T^{-1})^1 \cdot D^{x_1} N^{x_2} \rho^{x_3}$$

$$\pi_1 = Q D^{-3} N^{-1} \rho^0$$

$$\Rightarrow \pi_1 = \frac{Q}{ND^3}$$

$$\phi \text{ (flow coefficient)}$$

$$\pi_2 = (L^2 T^{-1})^1 \cdot L^{x_2} (T^{-1})^{x_3} (ML^{-3})^{x_4} = M^0 L^0 T^0$$

$$\left. \begin{array}{l} M \text{ (equating power)} \Rightarrow 0 = x_4 \\ L \text{ (") } \Rightarrow 0 = 2 + x_2 - 3x_4 \Rightarrow x_2 = -2 \\ T \text{ (") } \Rightarrow 0 = -1 - x_3 \Rightarrow x_3 = -1 \end{array} \right\}$$

$$\pi_2 = (\rho^H) \cdot D^{-2} N^{-2} \rho^0 \Rightarrow \pi_2 = \frac{\rho^H}{ND^2}$$

We have obtained x_1 is equal to minus 3, x_2 is equal to minus 1 and x_3 is equal to 0.

So, these 3 exponents we have calculated from this Buckingham pi theorem and then pi term which is dimensionless that we expressed in terms of M power 0 M power 0 T power 0 that equal to L cube T power minus 1 to the power 1 this is nothing, but Q that we took while calculating or while we are trying to obtain the dimensionless pi term that in to dn rho to the power x. So, if I go back to my previous slide, D to the power x 1 that is L to the power x 1.

So, that is D to the power x 1 then this is N to the power x 2 and rho to the power x 3. So, that is what we obtain, that is what we wrote in a in a previous slides. So, this first pi term pi 1 now x_1 is equal to minus 3. So, I can write this is Q in to D 1 to the power minus 3

N 1 to the power minus 1 rho to the power 0 so; that means, pi term the first pi term will be Q divided by N D cube.

$$\pi_1 = \frac{Q}{ND^3}$$

So, this is the first pi term that we obtained from this Buckingham pi theorem that Q is equal to pi 1 is equal to Q by N D cube. This is having one particular name, we will discuss what is called D series this is called you know 4 coefficient this is Q you know Q by N D cube that is called flow coefficient this term known as pi term is known as phi or it is known as flow coefficient. So, the first pi term that we obtained pi 1 that is Q by N D cube that is expressed in terms of that we called phi that is flow coefficient.

Similarly, we can again follow the same step and we can arrive with the other pi terms. So, let us now try to do how we can obtain another 3 pi terms I mean 4 pi terms because we will obtain total 5 pi terms. So, first pi terms you have obtained, for the second pi terms I am not going to explain each and every again in a step details, but let us say for second pi term pi 2 I am writing that second pi term will be how which one? That will be which one. The second pi term will be, if I go to my previous slide we have taken Q D N rho are the repeating variables we need not to take. So, we can take g in to H. So, I am taking g in to H.

$$\pi_2 = (L^2T^{-2})^1 L^{x_1} (T^{-1})^{x_2} (ML^{-3})^{x_3}$$

After equating powers

$$x_1 = -2, x_2 = -2, x_3 = 0$$

$$\pi_2 = \frac{gH}{N^2D^2}$$

So, we have obtained first pi terms and we have obtained second pi terms gH by N square D square we have to proceed again to obtain another 3 pi terms that is of course, we have taken Q and g H. So, we are left with mu E and P. So, we have to obtain another 3 pi terms mind it we have said that it is always possible that we will get pi terms may be multiplication of you know each pi terms you need to multiply with another.

So, we will get pi terms that some useful relationship is yesterday we told that any pi term can be expressed in a function of other pi terms by multiplying in the numerical constant also. So, for the time being you have obtained you know 2 power pi terms one is Q by N D cube, that is flow coefficient another pi term that we have obtained that is gH by N square D square this is you know we have expressed. So, we will now proceed to obtain another pi terms let us say if we consider E or P.

(Refer Slide Time: 33:15)

$$\pi_3 = (ML^2T^{-3})^1 L^{x_1} (T^{-1})^{x_2} (ML^{-3})^{x_3} = M^0 L^0 T^0$$

$$\begin{aligned} M \text{ (equating Power)} &\Rightarrow 0 = 1 + x_3 \rightarrow x_3 = -1 \\ L \text{ (" ")} &\Rightarrow 0 = 2 + x_1 - 3x_3 \Rightarrow 2 + x_1 + 3 = 0 \Rightarrow x_1 = -5 \\ T \text{ (" ")} &\Rightarrow 0 = -x_2 - x_3 \Rightarrow x_2 = -3 \end{aligned}$$

$$\pi_3 = P D^{-5} N^{-3} = \frac{P}{\rho D^5 N^3}$$

$$\boxed{\pi_3 = \frac{P}{\rho N^3 D^5}} \quad \text{Power coefficient}$$

$$\pi_4 = (L^2T^{-2})^1 L^{x_1} (T^{-1})^{x_2} (ML^{-3})^{x_3} = M^0 L^0 T^0$$

$$\begin{aligned} M \text{ (equating Power)} &\Rightarrow 0 = x_3 \rightarrow x_3 = 0 \\ L \text{ (" ")} &\Rightarrow 0 = 2 + x_1 - 3x_3 \Rightarrow x_1 = -2 \\ T \text{ (" ")} &\Rightarrow 0 = -2 - x_2 \Rightarrow x_2 = -2 \end{aligned}$$

$$\pi_4 = E D^{-2} N^{-2} \rho^0 \Rightarrow \pi_4 = \frac{E}{D^2 N^2}$$

So, another pi term p 3 pi 3 that is equal to let us say we are considering power is equal to what? That is equal to how much M L square T power minus 3 and known is variable raised to the power 1 in a N square D power minus 3 that will be L 1 to the power x 1 in to T power minus 1 to the power x 2 and that is you know ML to the power minus 3 to the power x 3 that is rho. So, and this is again a dimensionless term. So, this is M power 0 L power 0 T power 0 and if we equate each and every terms I mean we have to consider dimensional homogeneity at both the sides, and we have to equate powers of M L T and if we equate that.

$$\pi_3 = (ML^2T^{-3})^1 L^{x_1} (T^{-1})^{x_2} (ML^{-3})^{x_3}$$

$$x_3 = -1, x_1 = -5, x_2 = -3$$

$$\pi_3 = \frac{P}{\rho N^3 D^5}$$

So, we have already you know established 3 different pi terms pi 1 coefficient and then gH by N square D square and P by rho N cube D to the power 5 that is power coefficient. So, we have expressed you know 3 different pi terms. So, we are left with another 2 pi terms that we need to establish. gH pi 2 by pi 1 can be expressed as a you know some other quantities as a I said you that yesterday we have written some useful relationship any pi terms can be expressed by power of other pi terms.

Any pi terms can be multiplied by a numerical value to obtain another pi term, any pi term can be replaced as a function of another pi term and then we will obtain pi 4 if we consider now e. So, what is the unit of e; L 1 square T to the power minus 2 L T T power minus 2 2 to the power 1 and L to the power x 1 T 1 minus to the power x 2 and M L minus 3 to the power x 3 and that will be again dimensionless M power 0 L power 0 T power 0 sorry T power 0 that is T power 0, as I said that we need to consider the dimensional homogeneity as and then we have to equate power.

$$\pi_4 = (L^2 T^{-2})^1 L^{x_1} (T^{-1})^{x_2} (ML^{-3})^{x_3}$$

$$X_3 = 0, x_1 = -2, x_2 = -2$$

$$\pi_4 = \frac{E}{N^2 D^2}$$

that may not be we need to define we need to multiply with another you know numerical value or we need to express in terms of any other pi terms. So, other pi terms. (Refer Slide Time: 39:46)

$$\pi_5 = \frac{E}{N^2 D^2}$$

$$\pi_5 = (ML^{-1}T^{-1})^1 \cdot (L)^{x_1} (T^{-1})^{x_2} (ML^{-3})^{x_3} = M^0 L^0 T^0$$

$$\begin{aligned} M \text{ (equating power)} &\Rightarrow 0 = 1 + x_3 \Rightarrow x_3 = -1 \\ L \text{ (" ")} &\Rightarrow 0 = -1 + x_1 - 3x_3 \Rightarrow -1 + x_1 + 3 = 0 \Rightarrow x_1 = -2 \\ T \text{ (" ")} &\Rightarrow 0 = -1 - x_2 \Rightarrow x_2 = -1 \end{aligned}$$

$$\pi_5 = \mu \cdot D^{-2} N^{-1} \rho^{-1} = \frac{\mu}{\rho N D^2}$$

$$\pi_5 = \frac{\mu}{\rho N D^2} = \frac{1}{\frac{\rho (ND)^2}{\mu}} = \frac{L}{Re}$$

$$f(\pi_1, \pi_2, \pi_3, \pi_4, \pi_5) = 0$$

$$f\left(\frac{\rho}{N D^2}, \frac{gH}{N D^2}, \frac{P}{\rho N D^5}\right) = 0$$

↓ Friction Coeff
 ↓ Head Coeff
 ↓ Power Coeff

So, we have obtained you know pi 4 is equal to E by N square D square and we are left with only mu. So, I am writing pi 5 that equal to mu; mu unit is M L power x 1 T power minus 1 to the power 1 into D L power x 1 T power minus 1 to the power x 2, and ML power minus 3 that is rho to the power x 3 and that equal to M power 0 L power 0 T power 0. So, again we are considering dimensional homogeneity and equating power of each M L T and both the sides.

$$\pi_5 = (ML^{-1}T^{-1})^1 L^{x_1} (T^{-1})^{x_2} (ML^{-3})^{x_3}$$

$$x_3 = -1, x_1 = -2, x_2 = -1$$

$$\pi_5 = \frac{\mu}{\rho ND^2} = \frac{1}{Re}$$

So, reciprocal of this is essentially 1 by Reynolds number. So, all this pi terms that is against a dimensionless terms form here it is cleared that it essentially indicates that in it is Reynolds number. So, with this we have arrived a few pi terms. So, that is what I have exercised only to know how we can obtain a few dimensional term when for from a flow through a pump or it need not to be flow through a pump whenever fluid is flowing through a pipe or any closed conduit.

If we know the variables and from there knowing the fundamental dimensions and considering a repeating variables we can you know form a few dimensionless term. And as I said you that pi 5 can be expressed in terms of 1 by Reynolds number. So, it is a dimensionless term. So, our interest is to obtain that out of this dimensional analysis; out of this dimensional analysis we have obtained a few pi terms so, but our interest is only to obtain 3 important pi terms that is this is important for this analysis of course, f pi 1, pi 2, pi 3, pi 4 and pi 5 are 0 that is what we have or we have written that that will be you know functional relationship. And that will follow such that that has to be 0.

we can write that you know pi 2 we can call as this is a head 2 coefficient, this is flow coefficient this is called head coefficient, and this is power coefficients.

So, this is very important power pump because flow head and power. So, now, we have obtained 5 pi terms, but out of these 5 pi terms we have expressed these 5 pi terms in terms of a few you know 3 important one coefficients flow coefficient power coefficient and head coefficient. Now knowing this we can now proceed obtain or with another

arrangement of or we can arrange with this pi terms we can multiply each another or divide another and we can obtain another of few pi terms.

So, now that I mean if data obtained from the test or model machines, if we obtain data from a model machines and so, that the variation of this dimensional parameter with one another. And suppose we are carrying we are you know carrying out a test in a model machines to obtain the variation of these 3 quantity and if we plot the graph, and that graph can be used you know that graph whatever you obtain that can be applicable to predict or the you know performance of any other homologous series machines of belongs to homologous series.

So, what I am telling? This 3 coefficient flow coefficient head coefficient power coefficient so, if we carry out test in a particular machines in a model machines, and if we can obtain these 3 coefficients and if we plot a graph that graph can be used to predict the performance of machines of homologous series. So, this is very important, now as I said you that now we will slowly move to obtain the affinity laws.

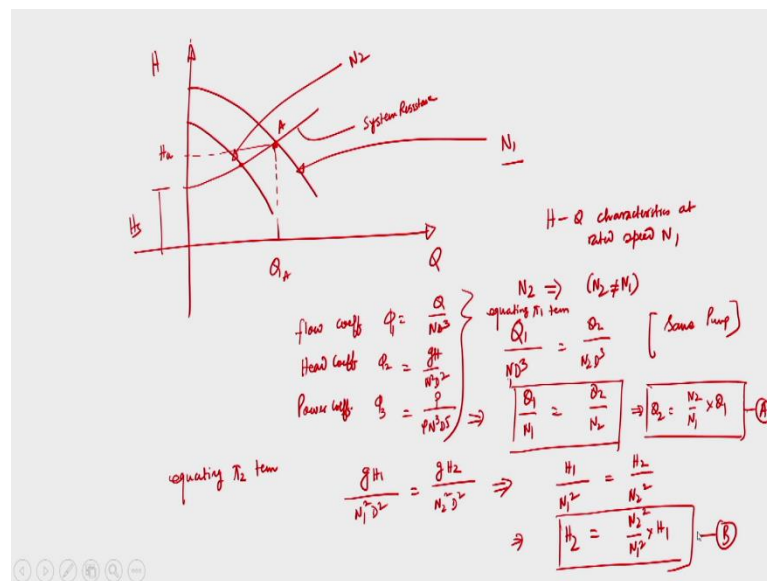
What is affinity laws? As I said you that you know whenever we carryout test in a model machines it is not possible that we should carry out test at each and every machine. So, when you carry out test at particular machines rather we consider a particular speed and that we call at rated speed.

So, whenever we run a machine at a rated speed, we obtain Q and H if you would like to predict Q and H when machine will be operating different other speed then the rated speed then is it possible to carry out test at again no and that is why affinity laws are used. So, that we can predict the head discharge capacity or performance of a pump when pump is running at a speed which is not exactly the rated speed where rather when the pump testing was done.

So, whenever the pump testing was done at a speed that is rated speed, we have obtained Q and H and we have plotted a graph and other we have obtained flow coefficient power coefficient, head coefficient and from and that graph we can predict what will be the head and discharge whenever machine is operating different rather speed than the rated speed. So and how we can predict and that is governed by the affinity laws.

So, now I will discuss about the affinity laws by knowing these 3 you know coefficient only to obtain these 3 coefficient very important for the pump you know pumping system pump operation, that is flow coefficient head coefficient power coefficient and that is why I did this exercise that is Buckingham pi theorem how we can proceed how we can you know analyze or how we can you obtain a few pi terms whenever fluid is flowing through a pipe And now I am discussing about the affinity laws. So, what is the affinity laws? Suppose if I draw H Q curve of the pump, suppose I am drawing H Q curve of a pump H Q curve.

(Refer Slide Time: 48:19)



So, and whenever HQ curve I am obtaining let us say this is the HQ curve I have obtained at a the rated speed let us say N_1 .

So, this curve H and Q head discharge relationship, head discharge relationship that I obtained at this rated speed N_1 . Now, may be if we have obtained another head discharge characteristics which is a when pump is operating at different rated speed N_2 . Suppose I have obtained another head discharge characteristics when machine is running at different speed at N_2 suppose this is at N_2 .

So, if I know the flow coefficient head coefficient and power coefficient, only we need to predict, now if we vary the speed may be if we close the discharge valve or somehow or some reason we need to you know change the it is the speed of the pump, then shall we get the same discharge or head no there will be a change.

So, how we can predict whenever pump is running? It is possible that whenever pump is running in a field or in a industry how we can measure the head and discharge? But if we know this information then we can predict fine if we reduce the speed of the pump by this much amount by this amount. Then what by this magnitude then what would be the head what will be the discharge whether that head or discharge is capable of meeting the demand of that system or not that that is very important.

And you need we needed to know whether that the head and discharge is capable to meet the demand of the system or not because sometimes it might happens that whatever discharge we are getting whatever head you are getting that it is not possible to supply that amount of fluid in that desired location. As a result of which they are you know some undesirable phenomenon might occur.

So, we need to know that even if you reduce the speed that speed will be capable of having developing head and discharge from that pumping station so, that system demand can be satisfied. So, now, if our system resistance curve is like this, suppose system resistance curve is like this. So, here is operating point here is operating point. So, now, suppose this is very important. So, if I know the head discharge capacity at a rated speed.

So, suppose if I know the head discharge characteristics; head discharge characteristics at a rated speed then each speed deviates from the rated one then what would be the head and discharge developed by the operating pump? So, now, what I will do? I will like to apply affinity laws what is the affinity laws? Suppose we have obtained a few coefficient that is known as flow coefficient, head coefficient and power coefficient.

So, I now would like to see suppose I know the when operating point is here let us say this is point A I know the head and discharge whenever speed is N_1 . So, suppose speed is N_1 , I know the head and discharge at this point it let us say Q_A and this is head developed by the pump H_A . And this is the H static and this is the system resistance curve; this is system resistance curve right. So, static height plus system resistance curve and let us say rated speed let us N_1 .

So, N_1 is the rated speed at which I have tested the pump and I have drawn the HQ characteristics. And from the Buckingham pi theorem I know there are a few coefficients that you have obtained flow coefficient that is ϕ that is Q/ND^3 then what you obtain? Head coefficient that is gH by N^2D^2 and power coefficient.

Let us say ϕ_1 , ϕ_2 and ϕ_3 that is P by $\rho N^3 D^5$ to the power 5. So, this 3 coefficient we have obtained and when you are carrying out test of a model pump at a rated speed, if we calculate this 3 coefficients if we plot the graph as I said and then that graph can be used to predict the head discharge or performance of machines of homologous series what do you mean by homologous series? Now, how we can predict?

So, now if we if I would like to calculate that what would be the discharge and if the machine is running at a speed which is not the rated speed let us say N_2 . Suppose I would like to obtain the head discharge characteristics when machine is running at N_2 , at the speed N_2 which is not equal to where N_2 is not equal to N_1 that is when we are deviating from the rated speed then how we can predict the head discharge characteristics.

So, from equality of pi terms so, I know that if we require the pi terms Q/ND^3 . So, Q_1 homologous series machine of same size you know not you know impeller of same shape may be size is different. So, this is called homologous series I mean when you are considering may be impeller of you know you know circular shape impeller. So, we might change diameter, but its shape is remaining same. So, it belongs to the homologous series.

So, if you know for a given pump, I mean even if it is not given pump if these changes then you can calculate but for a given pump. So, for the time being what the problem we are going to discuss may be the given pump, it is operating in a same system.

$$\frac{Q_1}{N_1 D^3} = \frac{Q_2}{N_2 D^3}$$

$$\frac{Q_1}{N_1} = \frac{Q_2}{N_2}$$

So, this is we are obtaining from the you know flow coefficient because that will remain same for a homologous machines, the machine belongs to homologous series here the same pump is not going to change rather shape is remaining same. So, I can apply this and I am obtaining. So, now, if I know Q_1 at a speed N_1 , now if I change the speed to N_2 , then I calculate what would be the Q_2 at which is running at different speed N_2 that will be N_2 by N_1 in to Q_1 . So, this is the head that will be developed by the pump, when pump the same pump or pump of homologous series is allowed to run at a speed which is now not equal to the rated speed very good.

So, we have obtained about the information about the Q now what would be the H? So, now, again we have obtained another coefficient that is called head coefficient this is head coefficient and this is called power coefficient; this is called power coefficient. So, we have obtained flow coefficient head coefficient and power coefficient P by rho N cube D to the power 5. So, now, if I apply again you know g is remaining you know D is remaining same. So, if I apply pi 2 terms.

$$\frac{gH_1}{N_1^2} = \frac{gH_1}{N_2^2}$$

Because as I said you that pump belong, if we can obtain the this 3 coefficient from a model testing machine from a model machine, tested at a rated speed N 1, then we can predict the performance rather other quantities any machine belongs to homologous series. So, here diameter is not changing. So, machine is remaining same.

So, what we can predict is that, now I can predict suppose whenever machine is running at the n speed suppose this machine is running at n speed and developing head Ha, what would be the head being developed by the pump one it is running at N 2 that we can calculate. Because it is not always possible that the pump operator should have this curve and he will calculate from the curve itself.

$$\frac{H_1}{N_1^2} = \frac{H_1}{N_1^2}$$

So, this is the another relationship that we can obtain head developed by the pump H 1. So, what will be the new head when pump is allowed to run at a speed which is not equal to the speed of rated one.

So, this is relationship 1 and this is relationship let us say or this is relationship say A and this is relationship B.

$$Q_2 = \frac{N_2}{N_1} Q_1$$

$$H_2 = \left(\frac{N_2}{N_1}\right)^2 H_1$$

$$\frac{H_2}{H_1} = \frac{Q_2^2}{Q_1^2}$$

$$H \propto Q^2$$

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The image shows handwritten notes on a light gray background. At the top, two equations are written in red ink: $Q_2 = \frac{N_2}{N_1} \times Q_1$ (labeled A) and $H_2 = \frac{N_2^2}{N_1^2} \times H_1$ (labeled B). A bracket on the right side of these equations is labeled "Affinity laws". Below these equations, a boxed equation shows $\frac{H_2}{H_1} = \frac{\omega_2^2}{\omega_1^2}$, which is followed by an arrow pointing to a box containing $H \propto \omega^2$.

So, that is if we carry out test of a model pump at a rated speed if we calculate this 3 coefficients, and now if you would like to allow and if we graph, if you plot this you know coefficient in a graph and then that graph can be used to predict the performance of the same pump or the pump becomes to homologous series, when if the pump is allowed to run at a speed which is not equal to the rated speed.

And how we can calculate? The head discharge when pump is running at a different speed and that can be calculated using the laws which are known as affinity laws that we have tried. And we will discuss about a few important things from these expression.

Thank you.