Principle of Hydraulic Machines and System Design

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Lecture - 16
Affinity laws, specific speed

So, we will continue our discussion on Principle of Hydraulic Machines and System Design. Today we will discuss about Affinity Laws and Specific Speed. So, affinity laws and specific speed which are very important, at least someone should know about this whenever pumping system design or pump operation is important, in fact whoever is designing pump and operating pump, they should know about this.

So, before I go to discuss about affinity laws and specific speed, it will be nice if I discuss about one another aspect that is you know that Buckingham's pi-theorem who was very important which gives us a few pi terms; that is how we can make few dimensionless term, how we can club together few variables to form a dimensionless term and which will give us an important clue about the affinity laws for a given values of other parameter.

So, now question is whenever we talk about fluid machines, in all the fluid machines flow is occurring through a closed conduit, not over the free surface. So, instead of considering there are so many other variables. So, when you are talking about an affinity, you know Buckingham's pi-theorem there are so many variables when we talk about let us say if we talk about a pump. There is a diameter of the pump, then speed of the pump, density of the fluid, viscosity of the fluid, fluid properties, flow properties and there are many other properties like pressure and also the energy per unit mass.

So, these are very important parameters and also the head developed by the pump. Now, since in almost all the fluid machines, fluid is not occurring over the free surface. So, it is always advisable to consider g h term together instead of considering h as the separate variables because acceleration due to gravity does not appear as a separate various variable.

So, if we consider g h as a, this if I club together, these two terms pair it acceleration due to gravity and head developed by the pump since acceleration due to gravity and in that case acceleration due to gravity won't appear as a separate term and this is quite logical because pump on hydraulic machine flow is not occurring over a free surface.

So, now we will discuss about what do you mean by Buckingham's pi-theorem and how using Buckingham's pi-theorem, you can form a few dimensionless term rather pi terms and from there we will discuss what is the, what is the affinity laws, what is the affinity law and how using this affinity law we can predict the different other parameters. As I said you that, in a pump most important parameter discharge and the head being developed by the pump so may be whenever pump is running at a given speed, we can have one certain, we can have certain discharge and certain head development. If someone closes the valve of the pump, operant pump, pump speed might be it decreases.

So, if the pump is operating different other speed, then what would be the head and what would be the discharge? It is not possible that we should carry out test at each and every speed. So, someone should know that if we know the quantity that at a rated speed and using the affinity law, we can predict what would be the head discharge at different other speed than that rated speed. That is what the affinity law predicts.

So, before you we go to discuss about this, let us first discuss about the Buckingham's pi-theorem. How we can create, how we can form a few dimensionless term from a few variables which are very important. When we are talking about pumping system or pump operation, so that there are a few parameters. So, if I write a few parameters, I will write in later, but let me discuss about what Buckingham's pi-theorem is.

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So, this is very important Buckingham's pi-theorem.

What is this? This is very important. So, I am writing not only for the flow through of machines in any case any fluid flow phenomena we can use Buckingham's pi-theorem to create to form a few dimensionless term. So, I am writing that if in any flow phenomenon, in any flow phenomenon if there are n variables n variables. Let us say a1, a2 up to an and those are connected by relations connected by relations. Let us say function of a1, a2, up to an = 0 containing these variables, containing these variables and containing these variables are nothing else that pure numbers and nothing else, but pure number pure numbers and if these variables. If these variables are described with m described with m fundamental described with m fundamental dimensions, m fundamental dimensions, then they can be grouped into they can

be grouped into (n- m) dimensionless term dimensionless term or these dimensionless terms are known as π terms.

These dimensionless terms are known as π terms, such that such that $\varphi(\pi 1, \pi 2 \ldots .. \pi n - m) = 0$ and each of this π term, each of these π terms contains not more than m plus 1 variables. So, this is Buckingham's pi-theorem. So, in any flow phenomenon, it is not necessary in that flow has to occur through a pump. What are any flow phenomenon? If there are n variables like a1, a2 upto an connected by the relations that f a 1 to upto a n is equal to 0 continuing this variables and nothing else, but pure numbers and if these variables are described with m fundamental dimensions and then, they can be grouped into n minus dimensionless term or π term.

Each of this π term contain not more than m +1 variables very important is when you talk about pump. So, when you talk about pump flow through a pump, then what are the variables? So, here I am listing down flow through a pump. Since, before I go to discuss about affinity laws, I would like to see how I can create a few π terms, how dimensionless term whenever fluid is flowing through the pumps.

So, let us first see what are the whenever fluid is flowing through the pumps, what are the variables? So, flow through a pump, then what are dimension, what are the variables? As I said you see flow through a pump is not a open surface flow or free surface flow. So, instead of considering g as the separate term, I can club together g and h.

So, acceleration if I do that, then acceleration due to gravity should not appear as a separate term. Of course, diameter of the impeller will come into the picture. So, this variable is diameter, diameter of the impeller, then specific speed of the pump speed of impeller diameter speed of pump, then rho is very important that is density of the liquid being handled by the pump density of fluid μ viscosity of fluid being handled by the pump is very important, because these are the important quantity that we should know.

Then, as I said we will instead of considering h, I can club together g h. This term is very important and then, head developed by the pump club booked with clubbed with the acceleration due to gravity, then what are the terms these are the diameter. Then, p is what that is the power, this is the power. Then we can consider what that is. You can consider e that is energy per unit mass, energy per unit mass. So, this p is the power, this energy power unit mass. So, these are the terms we can club together, I mean these are the variables.

So, these are the variables and we should now apply Buckingham's pi-theorem to group to form a few dimensionless term or pi terms out of these variables and how we can apply pi terms that we that we will now discuss. What are the procedure of applying Buckingham's pi-theorem? So, out of these variables there are some fundamental dimensions are there. So, we will have may be here we are having 7 variables.

So, if there are 3 di, the fundamental dimensions, then 7 – 3 = 4 pi terms we will obtain. After having this 4 pi terms, we will now discuss about the affinity laws and will discuss that if pump is let us say 1 pump is tested at a given speed and there and then, if we obtain let us say the head being developed by the pump is h and pump is discharging a few amount of product at that speed. If I would like to run pump at a different speed from that rated speed, then how without having any more testing of the pump how I can predict that what will be the head and discharge at that speed. That we can calculate in using the affinity loss.

So, now I will before since our objective is to now obtain a few dimensionless terms out of this variables flowing variables, so what are the procedures for applying Buckingham's pi-theorem? That is very important that we should know. So, I will now write what are the procedures of applying Buckingham's pi-theorem.

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So, now I will write procedure of applying Buckingham's pi-theorem. So, this is very important. So, as if first one list the list the n variables list, the n variables and note their dimensions and note their dimensions this is very important. So, first of all that is what I have discussed just now that if we have a, if we consider flow through a pump, then what would be the variables whatever you have listed down that is d rho mu n p e all those things and we need to know their dimensions, then 2 is very important.

Find out the number of fundamental variables, find out the number of fundamental variables m to be used to describe the variables. So, we have to find out the number of fundamental variables m and these fundamental variables will be used to describe the variables. Now, if either MLT, right MLT that we know mass, length and time or FLT we know that is used to describe these two or different dimensionless term are used to describe used to describe used to describe the variables, used to I mean if either MLT or FLT system is used.

This system either in MLT or FLT system is used system is used. M is equal to 3 and because fundamental dimensions are 3, either NLT or FLT, so either can either you can use either MLT or FLT. If we use this and m will be fundamental variables will be m as I see and the number of and the number of pi terms to be formed will be n – m = n - 3 . So, I can use either MLT or FLT.

So, in that case if n is a variable and then, the number of pi terms that will be formed will be n minus 3. Number 3 is very important. Select 3 of these variables select 3 of these variables this is very important. Note that non-dimensionless and each with different dimensions such that all of the, such that all of all of the 3 fundamental dimensions. All of the 3 fundamental dimensions fundamental dimensions either MLT or FLT dimensions should be included collectively by the selected variables should be included collectively by these variables. These variables are called repeating variables.

These variables are called repeating variables and they appear in each term and they appear in each pi term 4. So, this is very important. Select 3 of these variables, none will be dimensionless, and each will have different dimensions, such that all of these 3 variables. All of these 3 fundamental dimensions MLT FLT should be included collectively right by these variables.

So, we have to select 3 of these variables and these variables are called repeating variables and then, they appear in each pipe terms. So, that means we have to select 3 variables such that none should be dimensionless and each with different dimensions while at the same time and the fundamental dimensions should be included collectively by these variables. We will discuss this issue again when we will solve the, when we will use this theorem to form a few pi terms whenever flow is flowing through a pump.

4 is the first pi term. The first pi term can be expressed can be expressed as the product of as the product of as the product of 3 repeating variable as the product of three repeating variables each raised to an unknown exponent and another variable raised to a known power say 1. So, again we will see this very important, the first pi term can be expressed product of the repeating variables each raised to an unknown exponent and another variable raised to a known power say 1. So, this we will discuss while again we are executing this thing to obtain a few pi terms while you are taking example of a flow through pump.

Number 5, this is very important. Repeat the process 4 for the other pi terms. So, this was 4 was for the first pi terms that we have to repeat it, because you may have few pi terms. So, we have to repeat process 4 for the other pi terms each time taking a different variable, different variable along with the 3 repeating variables along with the 3 repeating variables. So, we have to repeat, because we may have a few pi terms. So, the first pi term can be expressed product of this repeating variables each raised to exponent and variables raised to another, variable raised to a raised to a known power let us say 1.

We repeat the process 4 for other pi terms by taking different variables along with 3 repeating variables. So, may be whenever the variable what variable we have considered to obtain the first pi term, we should not consider. So, we have to consider other variables and we have to repeat the process and along with the repeating variables. And finally 6 is very important.

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6 is very important that for each pi term for each pi term, for each pi term solve for the unknown exponents, unknown exponents by invoking by invoking dimensional homogeneity dimensional homogeneity. So, we have to take for which pi terms we have to solve the unknown expression by taking dimensional homogeneity. I am writing some helpful relationship. Any pi term may be replaced by any power of that term including pi to the power minus 1. Any pi term may be replaced by multiplying it by a numerical constant. Any pi term may be expressed as the function of the other pi terms.

So, here I am writing some helpful relationship what is those any pi term may be replaced by any power of that term that term including pi minus pi 2 pi to the power minus 1. Number 2 any pi term may be replaced by multiplying it by a numerical constant. These are very important helpful relationship and the number 3 is any pi term may be exposed as a function of other pi terms, other pi terms.

So, here are few helpful relationships that any pi term may be expressed as a function of other pi terms. So, we will execute to apply this same and to form a few dimensionless term whenever fluid is flowing through a pump, and we have listed down the variables when for a case. And we are discussing a flow pump and we will try to discuss; what are the fundamental dimensions

either taking MLT or FLT of a fundamental system. And then, if these 3 are fundamental dimensions, I mean then what will be the pi terms.

We have to out of this, we have to consider few repeating variables and then, we will proceed to obtain a dimensionless term. And then, having or out of having this pi term, we will discuss about how affinity laws help us to obtain pump head or pump discharge whenever pump is operating a pump is running at a speed which is or which is not exactly at a rated operated speed.

With this, I stop here today and we will continue our lecture in the next class.

Thank you.