Principle of Hydraulic Machines and System Design
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Lecture – 13
Radial equilibrium of axial flow machines II

We will continue our discussion on Principle of Hydraulic Machines and System Design.

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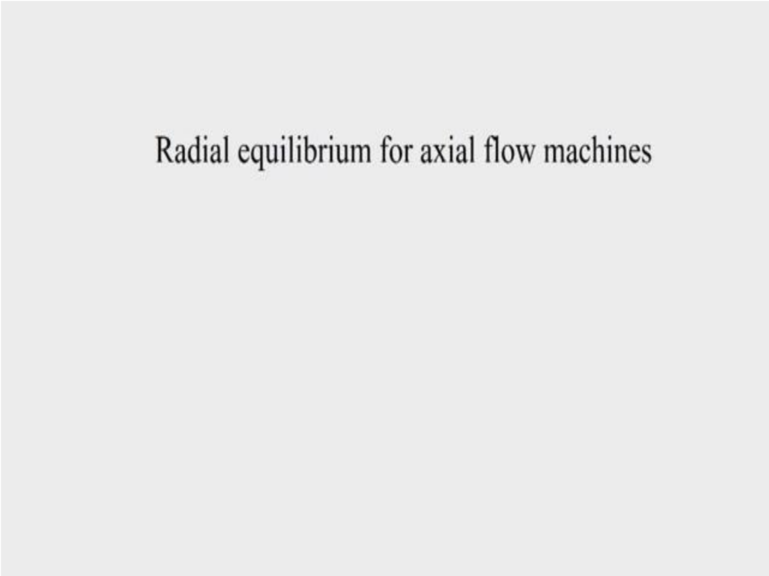

We have discussed about Radial Equilibrium for axial flow machines and we have derived the equation when radial equilibrium is attains considering incompressible flow as well as compressible flow. And eventually, we have arrive at that when pressure is not changing, total pressure is not changing along the radius, then what is the radial equilibrium equations. And from there we have discussed a case, when the flow is a free vortex flow; that means, whenever flow is flowing through an axial flow machines and flow is free vortex flow, what are the velocity, what is the velocity distribution? That is I mean axial velocity distribution in the axial direction as well as the radial direction.

Today, we will discuss another case where the flow is for vortex flow and we will solve the radial equilibrium equation to see the velocity distribution whenever fluid is flowing through an axial flow machine. So, yesterday, we have discussed about a case when the flow is free vortex flow.

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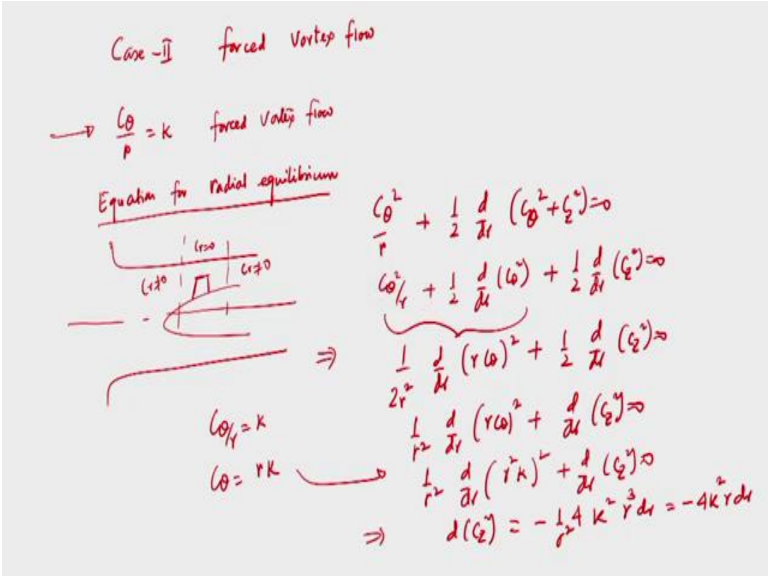

So today, we will see; what would be the velocity distribution whenever flow is passing through an axial flow machines for a forced vortex flow. So, today we will discuss about another case that is case 2; that is forced vortex flow forced vortex flow.

So, for a forced vortex flow, what will be the velocity distribution? Whenever flow is passing through the impeller of an axial flow machines that is what we are we are going to in analyze. So, now, for a forced vortex flow, we know that $\frac{C_\theta}{r}$ is equal to constant. So, for a forced vortex flow, we know that $\frac{C_\theta}{r}$ is equal to constant; let us say K that is for the forced vortex flow that is for the force vortex flow.

So, this is important that for forced vortex flow $\frac{C_\theta}{r}$ is equal to constant. Now if I try to recall the equation for the radial equilibrium, I mean whenever in a total pressure is not changing along the radius, then if I try to recall the equation for the radial equilibrium. So, equation for radial equilibrium; equation for radial equilibrium for a case that is for a incompressible flow and when pressure the total pressure is not changing at the total pressure is not varying along the radius.

$$\frac{{C_\theta}^2}{r} + \frac{1}{2}\frac{d}{dr}\left({C_\theta}^2 + {C_z}^2\right) = 0$$

we know that if we draw the again an axial flow machine, then this is the blade and this is that means, whenever radial equilibrium is attain, then the centrifugal force is getting balance by the radial pressure gradient and the radial you know c r become 0. So, this is the radial equilibrium equation. Now if I apply that if we try to obtain the velocity distribution that is C_θ and Cz; of course, we can obtain for a forced vortex flow; what should be the distribution of c theta, but what will be distribution of Cz from this radial equilibrium equation, then now I am going to consider $\frac{C_\theta}{r}$ is equal to constant.

$$\frac{{C_\theta}^2}{r} + \frac{1}{2}\frac{d}{dr}\left({C_\theta}^2\right) + \frac{1}{2}\frac{d}{dr}\left({C_z}^2\right) = 0$$

$$\frac{1}{2r^2}\frac{d}{dr}\left(rC_\theta\right)^2 + \frac{1}{2}\frac{d}{dr}\left({C_z}^2\right) = 0$$

$$\frac{1}{r^2}\frac{d}{dr}\left(r\,k\right)^2 + \frac{d}{dr}\left({C_z}^2\right) = 0$$

$$d\left({C_z}^2\right) = -\frac{1}{r^2}\,4\,k^2\,r^3\,dr = -4\,k^2 r dr$$

$${C_z}^2 = -4\,k^2 r^2 + A$$

$$C_z = \sqrt{A - 4\,k^2 r^2}$$

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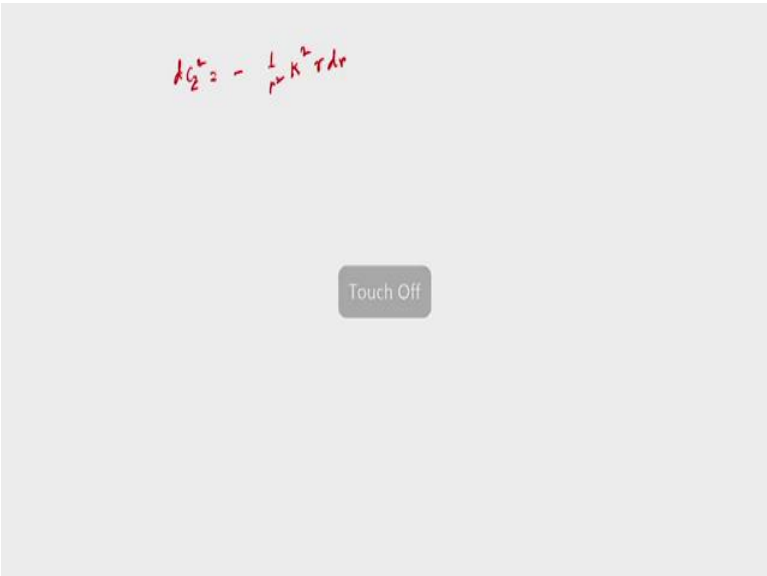

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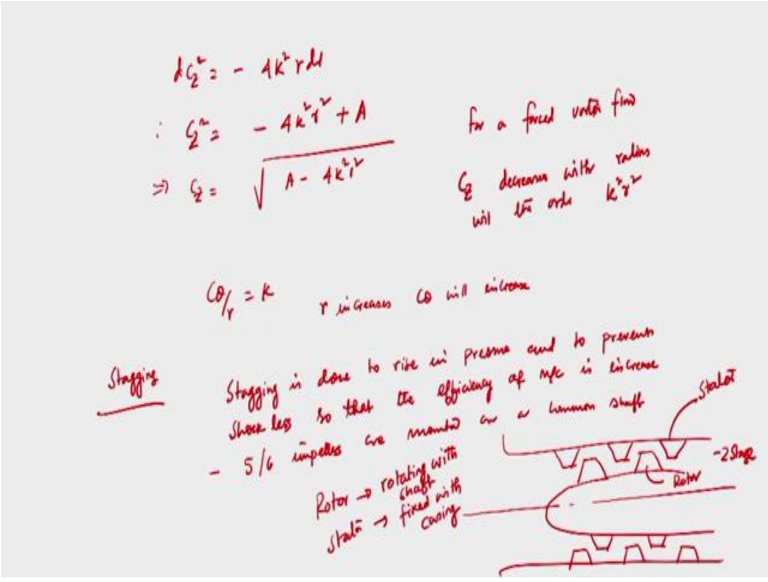

So, it is it is seen from this expression it is seen that therefore, a forced vortex flow say for a write that for forced vortex flow; for a forced vortex flow for a forced vortex flow c z that is the component of axial velocity which decreases with radius with and with the order of; with the order of. So, for a forced vortex flow Cz decreases with radius with radius with the order of with the order of $-4\ k^2 r^2$.

So, that is in a forced vortex flow c z decreases with radius with the order of K square r square, but you have seen that in case of a free vortex flow that we have discussed in the last class that for a free vortex flow you know that c z is not function of r that is c z is equal to constant.

So, for a free vortex flow, we have seen that if radial equilibrium is attained then forced when axial flow machine c z is not a function of r, but c z might change along the axial direction, but in case of a forced vortex flow is a decrease with radius with an order of K square r square. While since c theta and c theta what about the tangential component since $\frac{C_\theta}{r}$ =k ; that means, if r increases $\frac{C_\theta}{r}$ is equal to constant so; that means, if r increases c theta will increase c theta will increase.

So, this is the variation of C_θ and cz for a forced vortex flow pattern that is for a forced vortex flow is a decreases with radius while c theta will increase with increasing r now I will briefly discuss something about the you know stagging. So, you know that in a sometimes we need to do staging; what is staging?

So, if I write the what is stagging very very important, sometimes, we need to go for stagging that is when a particular you know pump is not in a position to meet the demand, sometimes we require higher you know head or something then we need to stagging is done to rise in the pressure. So, this stagging is done; stagging is done to rise in pressure to rise in pressure and to prevent shock loss this is very important.

So, sometimes we know that that I will discuss today again about this what again for axial flow machines sometimes we need to go for a stagging; that means, only to increase or rise the pressure and to prevent shock loss; so, that efficiency; so, that the efficiency of the machine can be increase. So, that the efficiency of machine is increased. So, what is staging?

Basically 5 to 6 impellers are mounted on a shaft, on a common shaft 5 to 6 impellers are mounted on a common shaft and there will be a one rotating stator and rotor just like this I am drawing. So, for a axial flow machine, sometimes, you go for staging, sometimes, we need to go for say radial flow machine sometimes we go sometimes we need to go for a serial parallel operation that we will discuss in my next lecture. But stagging is done to rise in pressure whenever it is very difficult to meet the demand of the system.

So, we need to go for stagging that is 5 to 6 impellers are mounted on a shaft while there will be a stator part that is hanging from the casing. So, if I draw the schematic again now suppose this is axial flow machines. So, there will be some stator and there will be on; similar thing will be here that we will have. So, this is known as stator and this is rotor. So, rotor is rotating with shaft. So, rotor is rotating with shaft and stator is fixed with casing, as I said fixed with casing. So, as I said that sometimes stagging is done; that means, you know this a 3 stage this is called 3 stage.

So, impellers are you know mounted in a common shaft. So, that this is let us say this is 3 stage or 2 stage, this is 2 stage, there will be a stator which will be hanging from the casings or fixed with the casing and then rotating rotor with rotating with the shaft. So, this is I mean impellers which few impellers are mounted on a shaft only to rise the pressure and to prevent shock loss so that efficiency of the machine can be increased.

We will discuss a few cases I mean this is of reaction whenever we are having stagging, 2 stage or 3 stage and I mean and what is also known as cascading probably in one of my next lectures. So, with this I stop here about these discussion about radial equilibrium of an axial flow machines and we will continue our discussion on the next topic that is the series and parallel operation of a pump and will try to work out few at least few problems.

Thank you.