Principle of Hydraulic Machines and System Design
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Lecture – 12
Radial equilibrium of axial flow machines

Today, we will discuss I mean we will continue our discussion on Principle of Hydraulic Machines and System Design.

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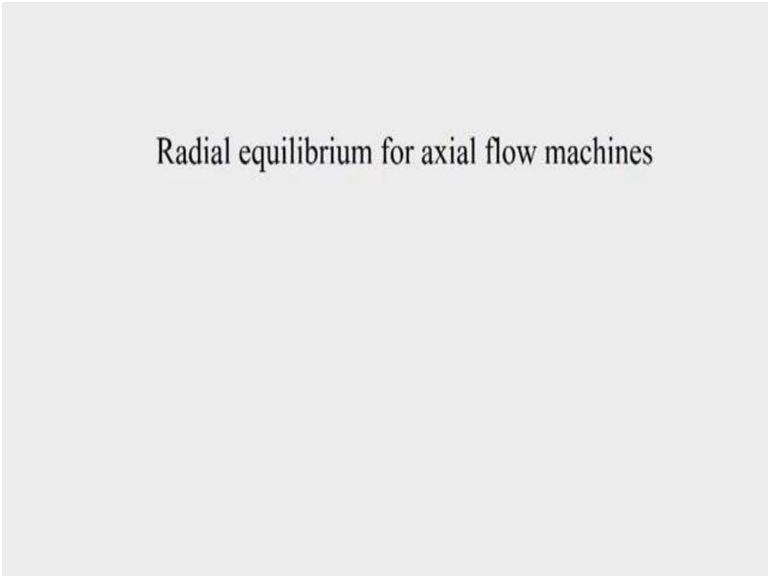

Today, we will discuss about Radial equilibrium for axial flow machines similar to what we have discussed like you know that for a radial flow machine, we have discuss about the slip which is an important phenomenon that is there and because of the presence of slip, I mean we have a reduction in head being develop by the pump and we have discussed how we can quantify this slip and that is essentially because of the presence of relative radial that according to (Refer Time: 01:02).

And today, we will discuss the radial equilibrium whenever in axial flow machines. So, when fluid is flowing through axial flow machines, if radial equilibrium is not attained, then what will be the case? And how we can have different cases like for free vertex and force vertex flow pattern even if radial equilibrium is attained, then what would be the velocity distribution at the inlet and outlet of the impeller? That we will discuss.

Before we go to discuss about this, today, we will solve one problem related to cavitation. We have discussed about the cavitation; probably one of my precious lectures and today, we will take up one problem and we will see that how we can avoid cavitation; I mean what will what will be the different you know steps, I mean whenever we are installing a pump in a pumping station. So, we should know about the different cases, different conditions by how we can avoid cavitation because it is not a desirable phenomenon at all.

So, we will solve one problem related to cavitation.

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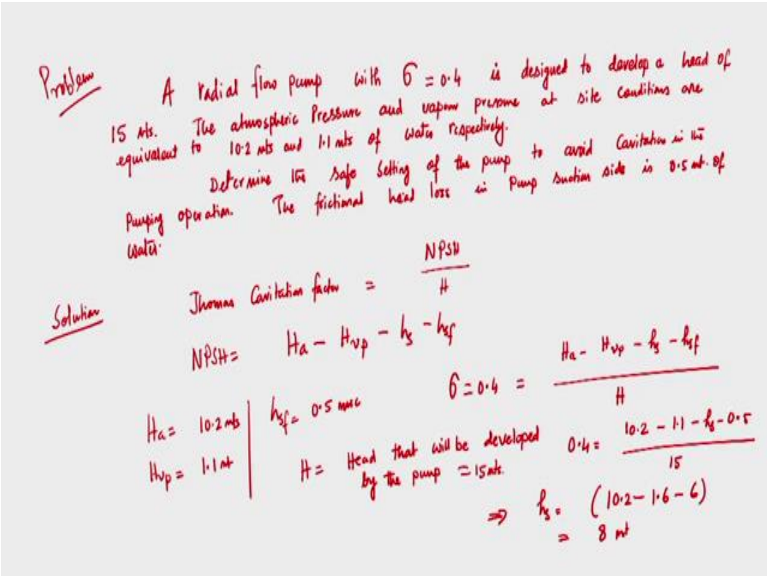

So, problem related to cavitation today will solve. So, let me write the problem first of a radial flow pump, A radial flow pump a radial flow pump with sigma; sigma is Thomas Cavitation factor. Sigma is equal to 0.4 is designed is designed to develop a head of a head of 15 meter.

The atmospheric pressure and vapour pressure at the site condition, the atmospheric pressure and vapour pressure at the site condition, the atmospheric pressure and vapour pressure at site condition are equivalent to 10.2 meters and 1.1 meters of water respectively.

We have to determine the safe setting. So, determine the safe setting of the pump to avoid cavitation to avoid cavitation in the pumping operation. So, we need to determine the safe setting of the pumps to avoid cavitation in a pumping operation; that means, whenever pump is in operation, then what would be the safe setting I mean why pumps will be installed so that cavitation can be avoided.

It is given that the frictional losses or frictional head loss, frictional head loss in pump suction side, in pump suction side is 0.5 meters of water. So, this is the problem, we have to solve that a radial flow pump is installed in a in a station which is with Thomas Cavitation factor 0.4 and it is designed to develop a head of 1.15 meters.

The atmospheric pressure and the vapour pressure at the site condition that means, [atmospheric pressure at the site is given and a vapour pressure at that temperature is given. 10.2 meters and 1.1 meters of water respectively. Determining the safe setting of the pump to avoid cavitation in a pumping operation and it is also given that the frictional losses frictional head loss in the suction side is 0.5-meter of water.

So, we have to solve the problem. So, we have write that the Thomas Cavitation factor Thomas Cavitation factor that we have write that equal to NPSH that Net Positive Suction Head available at you know site because it is a dimension less factors. So, it is divided by H. What is a NPSH? So, now, NPSH, if we can recall it is Ha the atmospheric pressure head minus vapour pressure minus static suction heights minus frictional head loss in the static suction in the suction side.

Thomas Cavitation factor $\sigma = \frac{NPSH}{H} = \frac{Ha-hs-hf-hv}{H}$

So, this is the net positive suction head available in the suction side. Atmospheric pressure heads minus Vapour pressure minus Static suction heights minus the Frictional head loss in the suction side. So, this is Thomas Cavitation factor. Now, so, if it is given from the problem we can see that Ha is given 10.2 meters; H vapour pressure is equal to 1.1 meters and we need to calculate safe setting that means, where pumps will be installed. So, if we have a look at the expression of a NPSH rather Thomas Cavitation factor, we can see one unknown is h s; that means, static that is a suction you know heights.

$0.4 = \frac{NPSH}{H} = \frac{Ha-hs-hf-hv}{H} = \frac{10.2-1.1-hs-0.5}{15}$

hs = 8 m

So, we can see from this calculation that hs should be 8 meter. So, this is the typical condition. So, I mean if you see that in a NPSH available, we have written the h vp vapour pressure, you have written h vp that is vapour pressure. It is not a suction pressure at the suction side. So, putting the value of vapour pressure at the expression of NPSH and we know the Thomas Cavitation factor is 0.4, if pump needs to develop a head of 15 meters; then we can see that the static suction height should be 8 meters.

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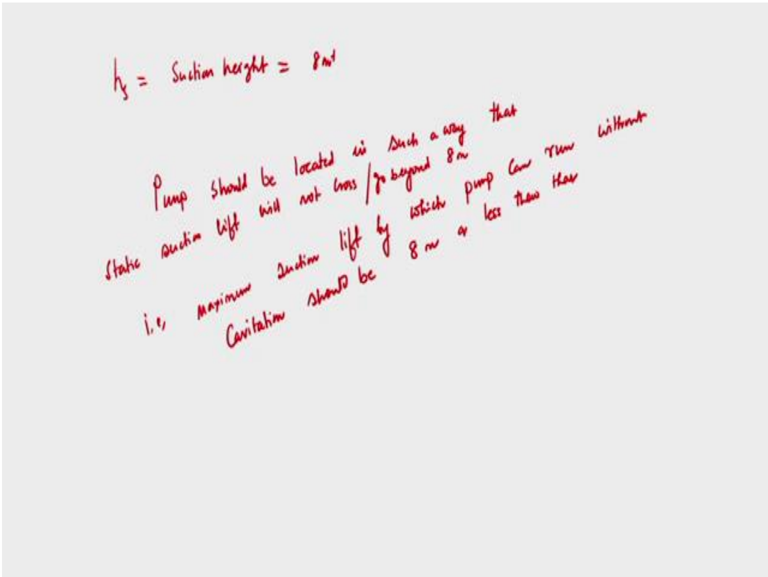

So, that means from here, we can we can conclude that pump should be that pump should be; so, I am writing hs that is suction height suction height will be equal to 8 meter; that means, pump should be located.

So, here I am writing pump should be located in a in such a way in such a way that static suction lift will not cross or a go beyond 8 meter. So, that means, we can install pump in a pumping session such that that the static suction lift or static suction height should not go beyond 8 meters; otherwise there will be a problem.

So, rather I can write that is the maximum suction lift, maximum suction lift by which pump can be operated by which pump can be operated or pump can run without cavitation should be 8 meter or less than that. So, this is the completion from the given data, we have calculated that if pump run pump has to run without cavitation; then static suction height or static suction lift should not go beyond 8 meter. It should be 8 meter either 8 meter or less than that. It is advisable that suction heights should be always less than 8 to avoid cavitation in the pump during the pumping operation ok.

So, we next discuss about we will with this we have solved the problem of relative to cavitation. Next, we will go to discuss about the Radial Equilibrium in an Axial Flow Machine.

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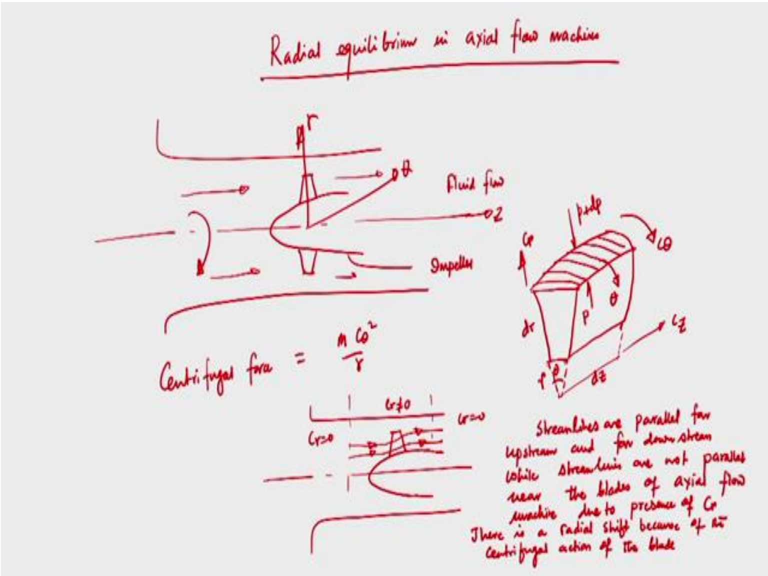

So, this is Radial equilibrium in axial flow machine or axial flow pump. So, I will draw a schematic of axial flow pump and if I draw a blade; so, this is impeller that is blade plus half and water is or a liquid fluid is flowing like this, is an axial flow machine.

So, this is an axial flow machines and we have drawn the impeller of a axial flow machine and this is the direction of fluid flow. So, this is the direction of fluid flow. What do you mean by radial equilibrium that we will discuss? So, whenever an axial flow machine is running, then a how you know water is you know I mean a how we develops a and how it transfer you know transport to water.

So, we will draw our schematic from where we will be having that what are the different components, we will we will we will come to know that what are the different velocity components whenever fluid is flowing through the axial flow machine. So, if I draw a schematic again, suppose this is the. So, this is Cz. So, I am taking small dr; this is r and this angle is theta; this is dz and this is C_θ this is Cr and p+ dp and this is P and this is angle theta.

So, we have taken you know sectional view of this particular you know blade and we have seen that whenever liquid is or fluid is flowing through the axial flow machines, there are 3 different component of velocities; c z which is in the axial direction rather we can have coordinate system over here also. This is z, you know this is r and this is theta.

And we have seen that c z c r and c theta are the components of velocities in the radial, tangential and the axial direction and we have seen that the pressure is changing within a small elemental volume dr; I mean the elemental in the sense dr; it is P and d p. So, pressure at the bottom may p and r at the top p+dp. So, here these centrifugal forces, it is rotating. So,

Centrifugal force = $\frac{m\, C_\theta{}^2}{r}$

Again, if you draw another schematic of an axial flow machine, we will we will come to know what the pressure distribution are whenever liquid is as the fluid is flowing through the blades. So, if I draw the impeller again; so, this is the blade and if you take two different sections; one is far away from the blade, another is far you know behind far behind of the blade, another is far you know away from the blade. So, here if you draw you know streamline, like this.

we can see that the streamline are not parallel stream lines are not parallel you know near the blades, but for upstream and for downstream stream lines are parallel. So, you can write that streamlines. So, whenever fluid is the impeller is rotating centrifugal force is M c theta square by r because it is rotating in the theta direction. And whenever fluid is flowing through you know through the pump rather through the process of the impeller, then the you know stream lines maybe your stream lines are parallel; may be at for downstream and for upstream, but stream lines are not parallel you know adjacent in the blade rather in the near the blade.

So, streamlines are parallel for upstream and for downstream; for upstream and for downstream. While streamlines are not parallel are not parallel near the blades, near the blades and near the blades of axial flow machines. So, why is streamlines are not parallel near the blades maybe for upstream and for downstream, we have seen that the streamlines are highly parallel, but near the blades streamlines are not parallel. That is because of this presence of Cr; that means, the streamlines are not parallel near the blades of an axial flow machines due to presence of Cr.

So, we can say that there is a radical shift of the streamline whenever I mean they are approaching the blades. So, we can see that they are radial shift due to centrifugal action of the blade. So, we can see that we can write or write here that there is a radial shift, there is a radial shift because of the because of the centrifugal action of the blades. So, because of the centrifugal action of the blades, there is a radial shift near the blades and that is why streamlines in the blade near the blades are not parallel ok.

So, this radial shift now this I can write that this radial shift wherever is there near the blades cannot be there if the pressure force balances the centrifugal force. So, force due to pressure defines the balances the centrifugal force, there may not be a radial shift of stream lines whenever they are approaching to the blades. So, so far we have seen that whenever liquid is or fluid is flowing, stream lines are parallel for upstream and downstream, but near the blade there is a radial shift that is stream lines are not parallel. That is only because of the presence of centrifugal force.

Now, I am telling that if the pressure force; so, it is rotating continuously. So, if pressure force balances you know centrifugal force this radial shift can be you know minimized or radial shift can be seized.

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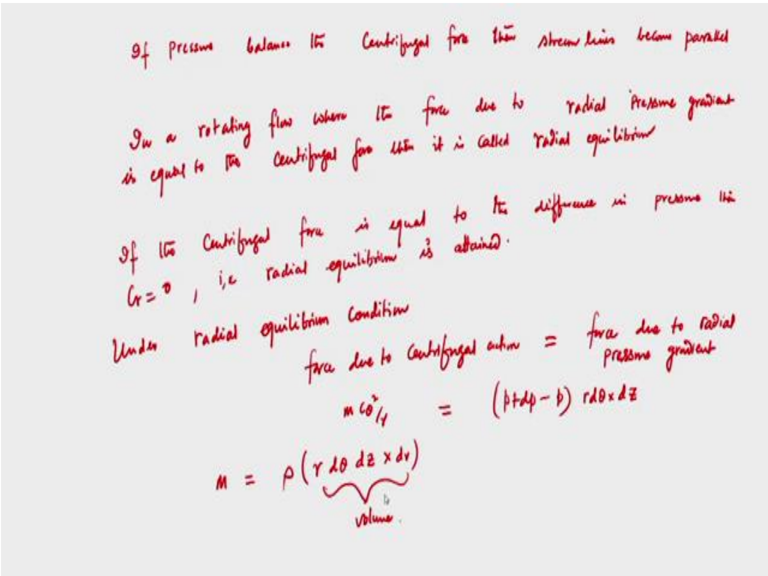

So, I am writing now again that you know if pressure; so, if pressure defined. So, the pressure force I am telling a force due to pressure balances balance the you know a centrifugal force, centrifugal force; then, stream lines becomes parallel, then stream lines become parallel.

So, we have seen there is a radial shift due to pressure difference. But if we if the radial shift is only because of the centrifugal force. We have seen the radial shift it is centrifugal action of the blades and if the pressure force balances the centrifugal force, the stream lines might become parallel. So, that means, we can see that we can that is you can say that in a rotating flow where the force due to radial pressure gradient is equal to the centrifugal force.

So, I can see, I can write that in a rotating flow in a rotating flow where the force due to pressure difference balance or pressure difference I can write force due to radial pressure gradient force due to radial pressure gradient is equal to is equal to this centrifugal force, then it is called so we are having a radial equilibrium. That means, we have seen that because of due to centrifugal action there is a radial shift of the stream line.

But if because of the radial pressure gradient; so, in a rotating flow where the force due to radial pressure gradient balance or force due to radial pressure gradient is equal to the centrifugal force. Then, the radial it is called Radial Equilibrium. It is called Radial Equilibrium or that is called Radial equilibrium is attained. That means, whenever liquid is flowing or whenever fluid is flowing through a in a through the impeller of an axial flow machine, for a upstream for downstream stream lines are parallel, but when they are approaching the blade or near the blade stream lines are not parallel because of the centrifugal action.

Now, if there is a you have seen that there is a radial pressure gradient. So, force due to radial if the force due to radial pressure gradient is equal to the centrifugal force. Then radial equilibrium is attained and it is called Radial equilibrium. So, what we can see that that if centrifugal force equal to difference in pressure, then I mean c r will be 0. I mean that means, so that is what we have you know retain that Cr is 0 near the section of blade.

So, if the centrifugal force, if the centrifugal force is equal to the difference in pressure, then C r = 0. That means, that that means, radial equilibrium is attained; radial equilibrium is attained. So, if the centrifugal force is equal to the difference in pressure in that section then Cr = 0 and radial equilibrium is attained.

So, there will not be any Cr. So, under therefore, we can see that under radial equilibrium condition right, what we have seen? Force due to centrifugal action will be equal to the you know force due to radial pressure gradient. So, that is you know force due to centrifugal action will be equal to force due to radial pressure gradient right.

So, so far whatever we have what you understood that in a rotating flow when the force due to radial pressure gradient is equal to the centrifugal force, then I mean we can switch the radial shift of the stream lines that is it is called radial equilibrium. So, that means, we are having equilibrium in a radial direction and in that case Cr = 0. Cr is the component of velocity in the radial direction. So, on the radial equilibrium condition, we can see rather we can say that the force due to centrifugal force will be equal to the force due to radial pressure gradient.

So, if we now want to quantify this I mean if you would like to give a mathematical form this; form of this you know equilibrium condition. Then, we need to go back to the previous slide where we have drawn you know sectional cross sectional you know view and we had seen that if we consider this particular cross section and then, what it is a centrifugal force acting on that particular element and what is the force due to radial pressure gradient?

So, the centrifugal force acting in that particular element and the force due to radial pressure gradient even acting on that particular element, if we make this come to components equal; then we can obtain the expression of radial equilibrium you can obtain the mathematical expression for the radial equilibrium condition.

$$\frac{m\,{C_\theta}^2}{r} = (p + dp - p) * rd\theta * dz$$

m = $\rho\ r\ d\theta\ dz\ dr$

$$\rho\ r\ d\theta\ dz\ dr\ \frac{{C_\theta}^2}{r}\ = dp * rd\theta * dz$$

$$\frac{dp}{dr} = \frac{\rho \, C_\theta^2}{r}$$

So, this is the volume this is the volume multiplied by density. So, this is the mass and so, if I write this expression again. So, what I can write?

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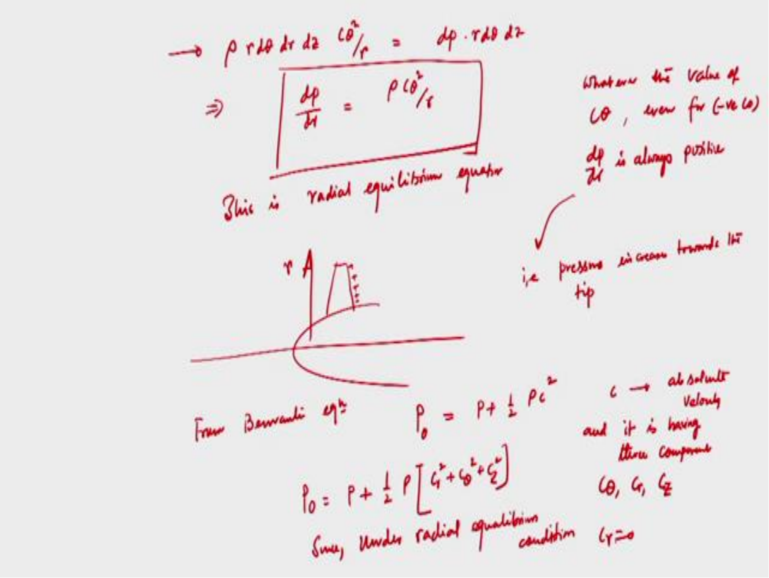

So, we will now discuss about that how we can write it in terms of be you know that is dp dr is always positive what it signifies. I mean it signifies that pressure increases towards the tip. So, if I again draw you know if I draw you know in impeller. So, this is the blade. So, dp dr always positive; that means, in that if I take this is the r direction, then pressure is always increased increases towards the top. So, this is plus sign.

So, dp dr is always positive that is pressure increases towards the tips, towards the tips. This is half; this is tip. So, dp dr in the direction of r pressure gradient is positive, that means pressure always increases towards the tips.

Now, if I now apply Bernoulli equation; so, what I can write? The total pressure, I mean if I apply Bernoulli equation now from Bernoulli equation. So, we have seen from the radial equilibrium equation that the pressure gradient is always positive; that means, pressure increases towards the tips.

Now, I would like to see the expression of total pressure and then, I will try to delay that expression in terms of the c theta square or that is a tangential velocity. So, I will write the radial equilibrium equation in terms of the total pressure.

Total pressure Po = $P + \frac{1}{2}\rho C^2$

$$= P + \frac{1}{2}\rho\,[C_r^2 + C_\theta^2 + C_z^2]$$

So, under radial equilibrium condition, as is said in that, I would like to express the radial radial equilibrium equation in terms of the stagnation pressure or total pressure. So, under radial equilibrium condition, since under radial equilibrium condition Cr= 0; then I can write that

$$Po = P + \frac{1}{2}\rho\,[C_\theta^2 + C_z^2]$$

So differentiating wrt r

$$\frac{dPo}{dr} = \frac{dP}{dr} + \frac{1}{2}\rho\frac{d}{dr}\,[C_\theta^2 + C_z^2]$$

$$\frac{1}{\rho}\frac{dPo}{dr} = \frac{1}{\rho}\frac{dP}{dr} + \frac{1}{2}\frac{d}{dr}\,[C_\theta^2 + C_z^2]$$

$$\frac{1}{\rho}\frac{dPo}{dr} = \frac{C_\theta^2}{r} + \frac{1}{2}\frac{d}{dr}\,[C_\theta^2 + C_z^2]$$ {Radial equilibrium equation for incompressible flow}

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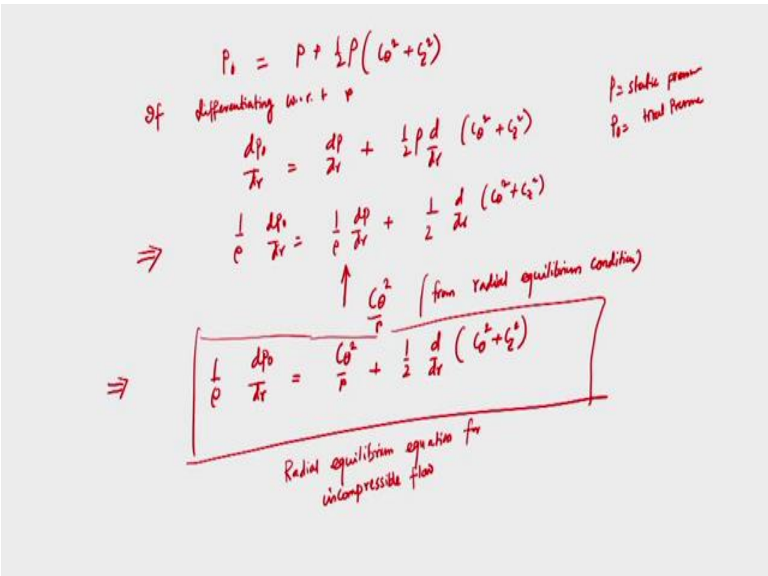

So, this the equation as I said that I would like to express this radial equilibrium equation in terms of the total pressure and after doing some I know steps I mean after having some steps, we have arrived at another equations; another equation which is known as Radial equilibrium equation for the incompressible flow.

Now, it would be nice if would can express the equation for the compressible flow also. Because sometimes in a axial flow compressor, we need to you know handle here. So, what would be the radial equilibrium equation for the compressible flow, now you should look into that?

So, total pressure or stagnation pressure whatever as I should said that you know this P is the static pressure and Po is the total pressure. So, here P is the static pressure that is what you obtain from the relief it will be a equations and P naught is the total pressure or stagnation pressure total pressure.

So, this is what is the radial equilibrium equation for the compressible incompressible flow. Now, we should try to express this equation when the flow is compressible.

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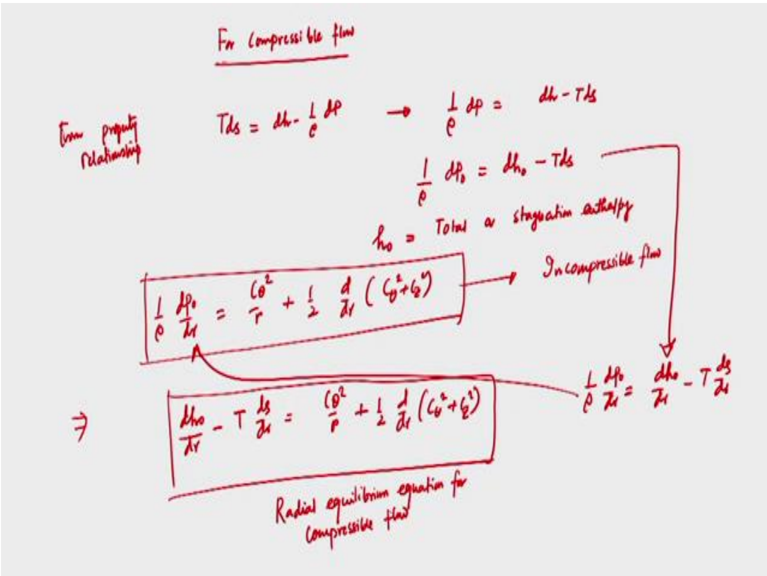

So, for compressible flow; so, for compressible flow; so, for compressible flow, what would be the equations? We know that

$$T\,ds = dh - \frac{1}{\rho}dP$$

$$\frac{1}{\rho} dP = dh - Tds$$

$$\frac{1}{\rho} dPo = dho - Tds$$

$$\frac{1}{\rho}\frac{dPo}{dr} = \frac{C_\theta{}^2}{r} + \frac{1}{2}\frac{d}{dr}\left[C_\theta{}^2 + C_z{}^2\right]$$ for incompressible flow

$$\frac{dho}{dr} - T\frac{ds}{dr} = \frac{C_\theta{}^2}{r} + \frac{1}{2}\frac{d}{dr}\left[C_\theta{}^2 + C_z{}^2\right]$$ for compressible flow

So, this is the equation for compressible flow. So, this is radial equilibrium equation for the compressible flow. So, this is Radial equilibrium equation for compressible flow. Now, here you know as I said that Po a total pressure stagnation pressure is equal to static pressure plus half rho c square; if Po is not varying with radius, then suppose this is the equation for the incompressible flow.

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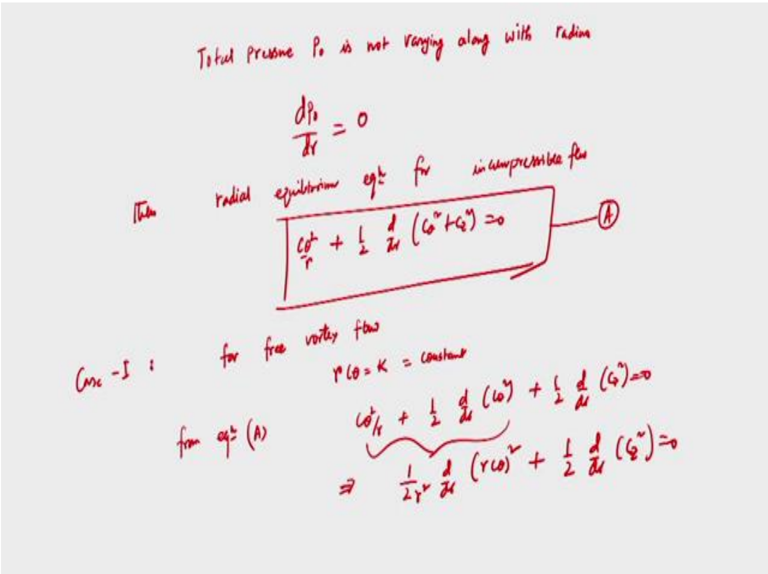

So, I can consider one case that total pressure total pressure P naught is not varying along the radius, along with radius.

So, if total pressure Po is not varying along with the radius, then $\frac{dPo}{dr}$ = 0. Then, the radial equilibrium equation even for the incompressible flow becomes, then radial equilibrium equations for compressible for incompressible flow for incompressible flow becomes what?

$$\frac{{C_\theta}^2}{r} + \frac{1}{2}\frac{d}{dr}\left[{C_\theta}^2 + {C_z}^2\right] = 0$$

So, this is the radial equilibrium equation for the incompressible flow, when pressure total pressure is not varying along with the radius right. So, now, we will discuss about a few cases; so, whatever we have derived, we have derived the radial equilibrium equation. What is the radial equilibrium? Equilibrium for the axial flow machines; that means, in a rotating flow I mean in a flow through axial flow machines, there is a radial shift of the stream lines near the blade for upstream and found for downstream blade for highly for upstream and for downstream, you have stream lines are highly parallel, but when there approaching the blade there highly there is a radial shift.

Now, this radial shift is only because of the centrifugal force and we have seen that it force due to radial pressure gradient balance the or is equal to the centrifugal force, then this radial equilibrium or the radial shift can be ceased rather radial equilibrium is attained.

We have tried to quantify rather we have tried to mathematic give a mathematical form of the radial equilibrium equation where force due to radial pressure gradient is equal to the centrifugal force and from there, we have derived the radial equilibrium equation for the incompressible flow and also we have try to express this equation when the flow is in flow is compressible.

We have seen that when total pressure or stagnation pressure is not changing rather is not varying along with the radius, then I mean what will be the form of this radial equilibrium equation for the incompressible flow that that is what you have also derived. And now, we will discuss a few cases; that means, first of all I will discuss case for the free vertex flow, what will be the equation? So, case - I, I will discuss Case - I that is for free vertex flow, for free vertex flow.

Case 1: for free vortex flow

$$r\, C_\theta = constant$$

For free vertex flow, we know r c theta is equal to constant that is a K is equal to constant. That means, we are now trying to see if the for a free vertex flow is no vertex in the flow, then if we I mean if we from the radial equilibrium condition; then what would be the velocity distribution at the inlet and the for upstream for downstream?

So, the for the free vertex flow r c theta is equal to you know constant and your all your also has being that the total pressure is not varying along with the radius. Then, from this equation let say this equation is A and from this equation. So, from equation A, we can write,

$$\frac{C_\theta{}^2}{r} + \frac{1}{2}\frac{d}{dr}\,C_\theta{}^2 + \frac{1}{2}\frac{d}{dr}\,C_z{}^2 = 0$$

$$\frac{1}{2\,r^2}\,\frac{d}{dr}\left(r\,C_\theta{}^2\right) + \frac{1}{2}\frac{d}{dr}\,C_z{}^2 = 0$$

Now, we would like to apply if the flow is force vertex that is for a force vertex flow since r c theta, r c theta will be constant. So, we will go to the next slide and we will write again.

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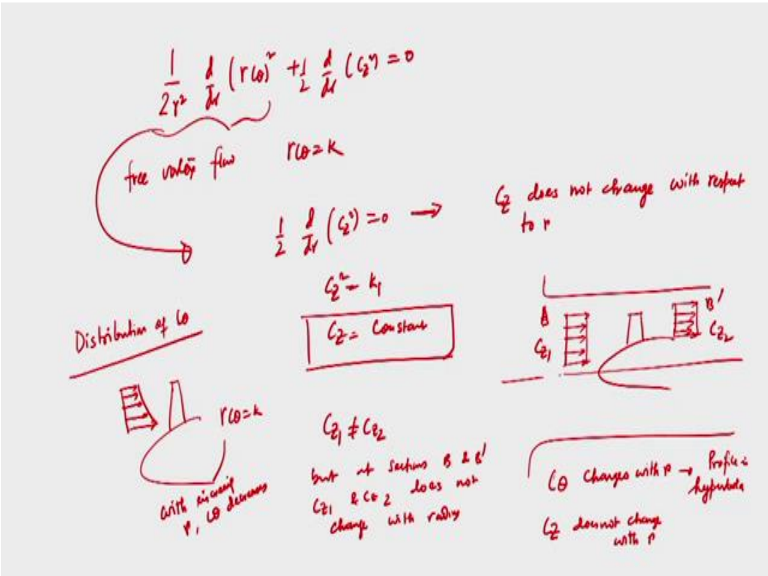

So, we have seen that the radial equilibrium equation when total pressure is not changing along with the radius, the radial equilibrium equation for incompressible flow can be written

$$\frac{1}{2\,r^2}\,\frac{d}{dr}\left(r\,C_\theta{}^2\right) + \frac{1}{2}\frac{d}{dr}\,C_z{}^2 = 0$$

So, this is the equation for the radial this is the radial equilibrium equation for incompressible flow when total pressure is not changing along with the radius. Now, as I said special case free vertex flow. So, for free vertex flow r c theta k is equal to constant; that means, this first term will be 0.

So, if I apply this equation for a free vertex flow, then you obtain half d dr of c z square equal to 0.

$$\frac{1}{2}\frac{d}{dr}\,{C_z}^2 = 0$$

Cz = constant

That means, if I plot the velocity distribution, suppose if I am drawing the axial flow machines and if I draw the impeller and this is the blade. So, pressure total pressure or stagnation pressure is not changing along with the blade and if I draw for upstream here c z is not changing. Here also c z not changing. From this, we conclude that the component of velocity in the axial direction is not changing with r; that is not a function of r. But 2 different locations you know if may change.

So, but at section A and section B B c z and c z change. So, suppose c z 1 is not equal to c z 2, but at section B and B prime c z 1 and c z 2 does not change with radius. So, this is what is about the conclusion. If I radial equilibrium equation for a free vertex flow and then, we can see that that c z 1 naught equal to c z 2; but at a section B or section B prime, c z 1 and c z 2 does not change with radius.

So, that is what is conclusion and what will be the distribution of C_θ. Say if I write here the distribution of c theta because we have considered free vertex flow; so, distribution of C_θ. So, if I draw again and impeller and blade.

So, r $C_\theta = k$; so that means, that means, we will have a velocity distance like this. that if with increasing r that is r C_θ decreases.

So, we have obtained distribution of c theta, distribution of c theta and distribution of c z at for upstream and downstream we have seen that the c z is not a function of r. So, at any given section at any given you know radial location c z is not changing is remaining constant is not function of r rather. But c z and c z may vary in the axial direction.

So, c z 1 is not equal to c z 2 and you have you have also I know seeing the distribution of c theta because it is a free vertex flow. So, of course, with increasing r, c theta will decrease and that is c theta changes. From here we can change the c theta changes with r with r and profile is profile is hyperbola. But c z does not change with r.

So, this is a conclusion that we apply a radial equilibrium equation for an incompressible flow where total pressure is not changing along with a radius. If our special case is that the flow is free vertex flow, C_θ Cr radial equilibrium is attained. So, Cr has to be 0; C_θ is changing with r. I mean you know profile is hyperbola that is with increasing r, C_θ decreases and Cz is not a function of r.

So, c z is remaining uniform at any section that is for upstream and downstream; but their magnitude may differ if you go from downstream to upstream ok. And another case will be the force vortex flow another case will be the force vortex flow and we will discuss this case in the next lecture. So, I will stop here today.

Thank you.