Principle of Hydraulic Machines and System Design
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Lecture – 11
Degrees of reaction: velocity triangle

So, today we will continue our discussion on Principal of Hydraulic Machines and System Design.

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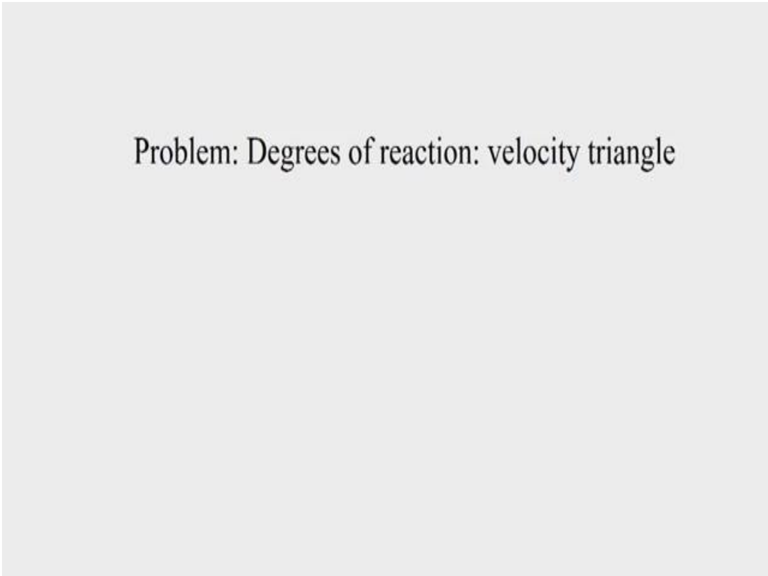

Today, we will discuss about the Degree of reactions: velocity triangles and we will see in detail that depending upon the magnitude of Degree of reaction, how we can construct velocity triangle for the inlet and outlet of the pump. And then, we will briefly discuss upon that what is the importance of degree of reaction; why we need to know about the degree of reaction for a pump?

And before we go to discuss about this, you will discuss one problem which is very important. Probably, I have briefly touched upon that in one of my previous lectures.

So, to start with we will discuss that even if you consider as a pump if it is a radial flow pump or an axial flow pump whatever it is.

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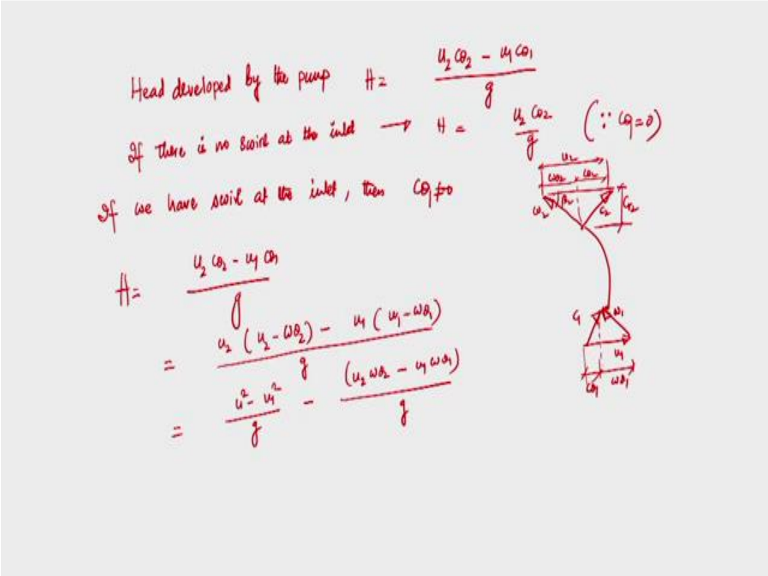

So, the head develop by the pump, Head develop by the pump which is given by Euler's equation for the pump is H is equal to $(u2\ C_{\theta 2} - u1\ C_{\theta 1})/g$. So, this is the Euler equation which produces the head being developed by a pump whatever it is. It is a radial flow pump or it may be a axial flow pump.

Now, if you consider that there is no swirl at the inlet that we have discussed. If there is no swirl at the inlet, no swirl at the inlet; then, head develop by the pump can be written $u2\ C_{\theta 2}/g$. In that case since $C_{\theta 1} = 0$. So, this is the expression of head being developed by a pump whether it is axial flow pump, radial flow pump or mixed flow pump.

Now, even if there is no swirl at the inlet, I mean then we can express by $u2\ C_{\theta 2}/g$, but if we have swirl at the inlet, then if we have swirl at the inlet. So, if we have swirl. So, if we have swirl at the inlet, then $C_{\theta 1}$ not equal to 0 whether the swirl is having rotation, I mean whether of whether the fluid entering to the pump will have different rotation or will have a same rotation to that which is having the impeller.

$$H = \frac{u2\ C_{\theta 2} - u1\ C_{\theta 1}}{g}$$

$$= \frac{(u2\ (u2 - W_{\theta 2)} - u1(u1 - W_{\theta 1})}{g}$$

$$= \frac{{u_2}^2 - {u_1}^2}{g} - \frac{(u2\ W_{\theta 2} - u1\ W_{\theta 1})}{g}$$

that is from the inlet and outlet velocity triangles, I can express $C_{\theta 2}$ and $C_{\theta 1}$ in terms of tangential velocity of and the relative velocity on the component of a absolute relative velocity on the tangential direction.

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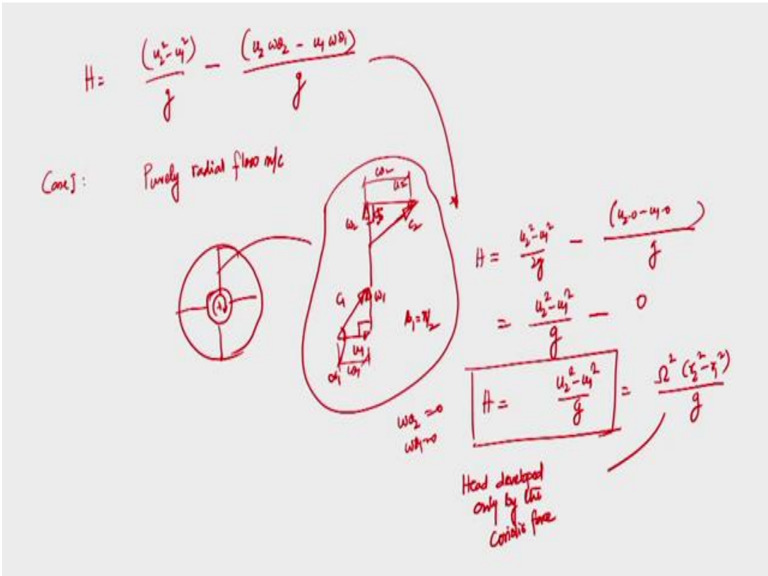

So, this is the head being developed by the pump whether it is radial flow pump or axial flow pump. Now, I am taking a case say let us say case 1, I have solve one problem in one of my previous lecture. If Case I that is Purely radial flow machine, purely radial flow machines; if I draw the impeller, then like this and the blade will be extract and if I draw the velocity triangles at the inlet and outlet. So, this is the blade. So, purely radial flow machine this is w 1. So, this is u 1; this is w 1; this is c 1 and this angle is 90 degree and this is c 1. This angle is $\alpha 1$. So, $\beta_1 = \frac{\pi}{2}$.

Similarly, if I draw the velocity triangles shut the outlet; again I will get velocity triangles like this. This is u 2; this is c 2; this is w 2 and this angle is 90 degree. Now, so, if I try to write this expression for this case, rather if I try to write if I try to obtain the head being develop by a radial purely radial flow machines using Euler equation.

Then, I can obtain H is equal to $\frac{u_2{}^2 - u_1{}^2}{g}$ and note that here for this particular case, I mean for the inlet outlet velocity triangle from the inlet and outlet velocity triangles u 2 itself is equal to $W_{\theta 2}$ and u 1 is equal to $W_{\theta 1}$

$$H = \frac{u_2{}^2 - u_1{}^2}{g} = \frac{(r_2{}^2 - r_1{}^2)\,\omega}{2g}$$

So, in that case that is for the purely radial flow machines head developed only by the Coriolis force. So, in a purely radial flow machines head develop only by the Coriolis force that is what I would like to say from this exercise. Even if it is not a purely radial flow machines, suppose now I will take an example of an axial flow machine.

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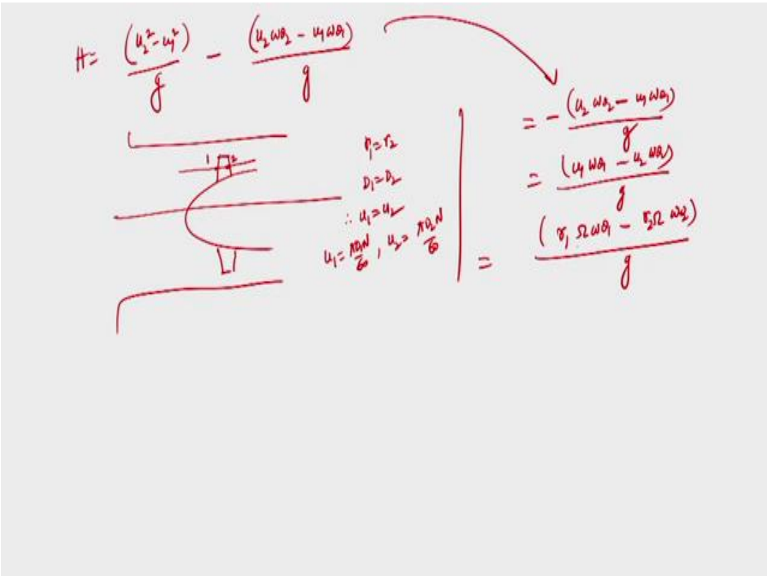

$$H = \frac{u_2{}^2 - u_1{}^2}{g} - \frac{(u2\,W_{\theta 2} - u1\,W_{\theta 1})}{g}$$

Now, if it is a purely axial flow machine, then no swirl at the inlet. Suppose, if it a purely axial flow machine. So, if I draw a purely axial flow machines and if it a purely axial flow machine; so, this is point 2; this is point 1. So, if it is a purely axial flow machine, then mind it r 1 = r 2; then d1= d 2; that means, u 1 = u 2.

$$H = -\,\frac{(u2\,W_{\theta 2} - u1\,W_{\theta 1})}{g} = \frac{\omega(r1\,W_{\theta 1} - r2\,W_{\theta 2})}{g}$$

So, here the head develop by only this component this one and again, we have been able to show that we have shown in day that if the blades are design following a logarithmic spiral; then logarithmic spiral if we follow to design the blade, then for of it can be shown that this first term is equal to this term from, it will go way. So, that is the case.

So, after having this discussion, now we look we will simply go to discuss about what is Degree of reaction? So, if I go to my next slide, then we will discuss about the Degree of reaction.

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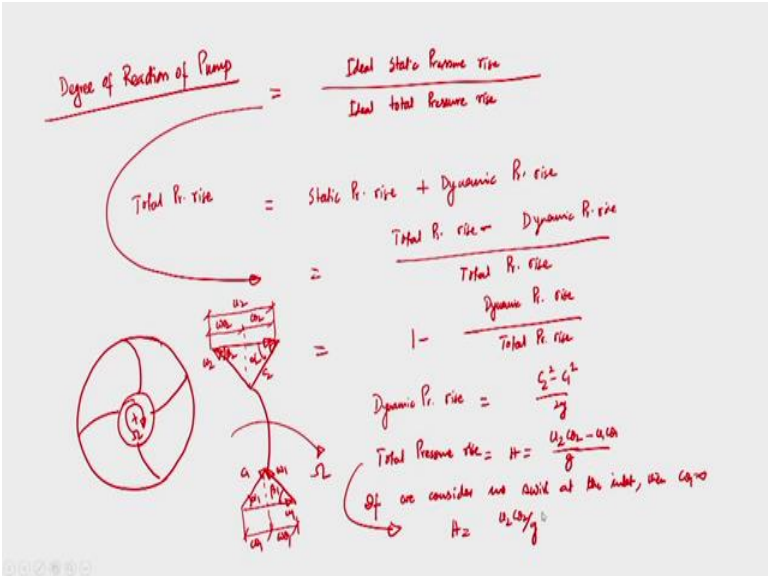

So, Degree of reaction; degree of reaction of pump, so, what is degree of reaction of the pump and why it is important and particularly from a degree of reaction we can design the inlet outlet velocity triangle. You can construct the inlet and outlet velocity triangles and it may help to design also pump.

We know that whenever fluid is flowing through the impeller, I mean then there is a rise in pressure. Of course, as I said with that pump absorbs energy. So, whenever we run pumps either using an electric motor or diesel engine, we are giving some energy input to the system; that means, here rotating the impeller. So, that mechanical energy rotational that whenever impeller is rotating that mechanical energy is converted or transferred to the stored energy of the fluid to increase a pressure or velocity whatever it is.

In a pump our prime objective or inter purpose is to develop it that is to develop pressure by at the cost of some energy that we are going to input either through an electric motor or diesel engine. So, if our target is to obtain pressure and this how much energy we are getting. So, whenever liquid is flowing through the impeller, impeller is rotating and there is there we are having some in input of mechanical energy. So, out of that in mechanical energy, how much energy you are extracting rather how much energy you are getting in terms of pressure, pressure head that is very important.

So, whenever it is going whenever water is flowing through the impeller, there are 2 different kind of you know energy if you talk about head, pressure head development of pressure head; there are 2 different type. One is Static head, another is Dynamic head. So, degree of reaction which gives an indication about that you know fraction of static pressure rise out of the total pressure rise in a pump that is whenever a liquid or fluid is flowing through the impeller of a pump. Then, what fraction of static pressure rise out of the total pressure rise is there and that information is obtained from the degree of reaction.

So, I can we can define degree of reaction like this that this is defined as the ratio of ideal static pressure rise, ideal static pressure rise to the ideal total pressure rise right. So, wherever you are defining degree of reaction which is important which gives us information about the fraction of static pressure rise is there out of the total pressure rise. Because whenever we are handling or a whenever a pump handling water, then of course, our target is to develop phase or have to increase the pressure head of the water that is total pressure rise. So, out of this total pressure rise, what fraction is the static pressure is that is obtained from this quantity that is degrees of reaction.

Degree of reaction = $\frac{ideal\ static\ pressure\ rise}{ideal\ total\ pressure\ rise}$

Total pressure rise = static pressure rise+ dynamic pressure rise

Degree of reaction = 1- $\frac{dynamic\ pressure\ rise}{ideal\ total\ pressure\ rise}$

Now, if I draw let us say radial flow pump and suppose, you are drawing an impeller of a radial flow pump and which is rotating at an angular velocity omega in the clockwise direction. So, say the impeller is having a few backward card veins and if I draw the velocity triangles of the inlet and a outlet; then, this is outlet velocity triangles. This is c 2; this is u 2; this is w 2; this triangle is beta 2; this angle is a alpha 2; this is u 2; this is $W_{\theta 2}$ and this is $C_{\theta 2}$.

Similarly, I can draw velocity triangles at the inlet. This is c 1; this is u 1; this is w 1; this is beta 1; this is alpha 1. So, this is $C_{\theta 1}$; this is $W_{\theta 1}$ and it is rotating at an angular velocity omega in the clockwise direction. So, we all know about the component of velocities, I mean absolute velocity, relative velocity, tangential velocity and different angles flow angles and blade angles. So, can you tell me from this expression what could be the dynamic pressure rise and the total pressure rise?

So, dynamic pressure rise dynamic pressure rise across the impeller when water is flowing through the impeller, then dynamic pressure rise will be equal to $\frac{c_2{}^2 - c_1{}^2}{2g}$ and total pressure rise across the impeller, I am talking about. Total pressure rise should be equal to u2 $C_{\theta 2}/g$ that is equal to H; if you do not have swirl at the inlet.

So, but for this case, it should be minus u1 $C_{\theta 1}$. If we assume, if we consider, if we consider no swirl at the inlet; no swirl at the inlet, then $C_{\theta 1} = 0$. Then, the total pressure rise will be equal to H is equal to u2 $C_{\theta 2}/g$.

Now, if I try to put the values of dynamic pressure rise and total pressure rise in the expression of you know reaction of pump; then what can I write.

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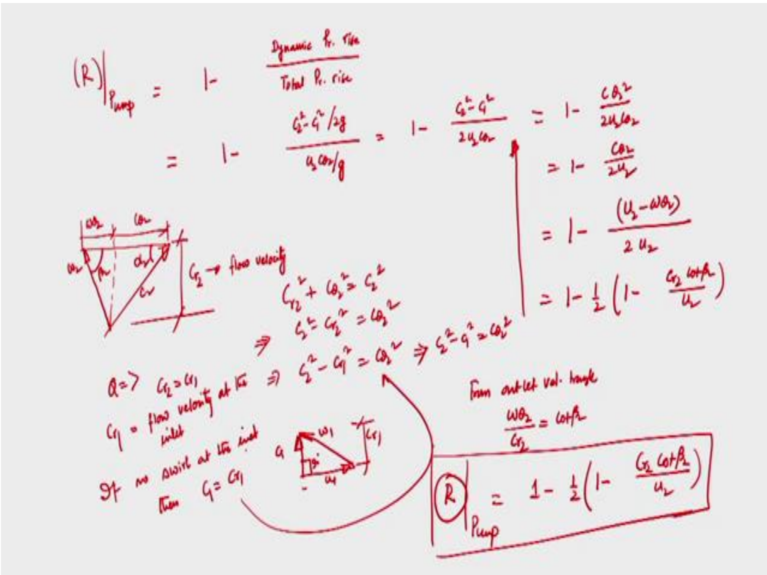

R = $1 - \frac{dynamic\ pressure\ rise}{total\ pressure\ rise}$

$$= 1 - \frac{\frac{c_2{}^2 - c_1{}^2}{2g}}{\text{u2 } C_{\theta 2}/g}$$

$$= 1 - \frac{c_2{}^2 - c_1{}^2}{2 \text{ u2 } C_{\theta 2}}$$

Since, Q though the impeller is same; it the flow through the inflow through the rotating passes the rotating impeller, it is as if flow through the convergent divergent part. So, flow rate has to be same. In some place velocity is higher area is less, velocity is less area is higher; in some places area is less, velocity is higher. So, this is flow velocity at the inlet at the inlet. So, Cr2 = Cr1 and from this velocity triangle, what can I write that what can I write? c 2 square, this is c 2.

$${C_{r2}}^2 + {C_{\theta 2}}^2 = {C_2}^2$$

$${C_2}^2 - {C_{r2}}^2 = {C_{\theta 2}}^2$$

If no swirl at inlet C1 = Cr1, Cr1= Cr2

$${C_2}^2 - {C_1}^2 = {C_{\theta 2}}^2$$

$$R = 1 - \frac{{c_2}^2 - {c_1}^2}{2\ \mathrm{u2}\ C_{\theta 2}}$$

$$= 1 - \frac{C_{\theta 2}}{2\ \mathrm{u2}}$$

$$= 1 - \frac{u2 - W_{\theta 2}}{2\ \mathrm{u2}}$$

$$= 1 - \frac{1}{2}(1 - \frac{C_{r2} \cot \beta 2}{u2})$$

So, this is the expression of the reaction of a radial flow pump. Now, we can discuss a few different cases. So, let us say if I take different values of R. So, first case I will discuss that when R is equal to 5.

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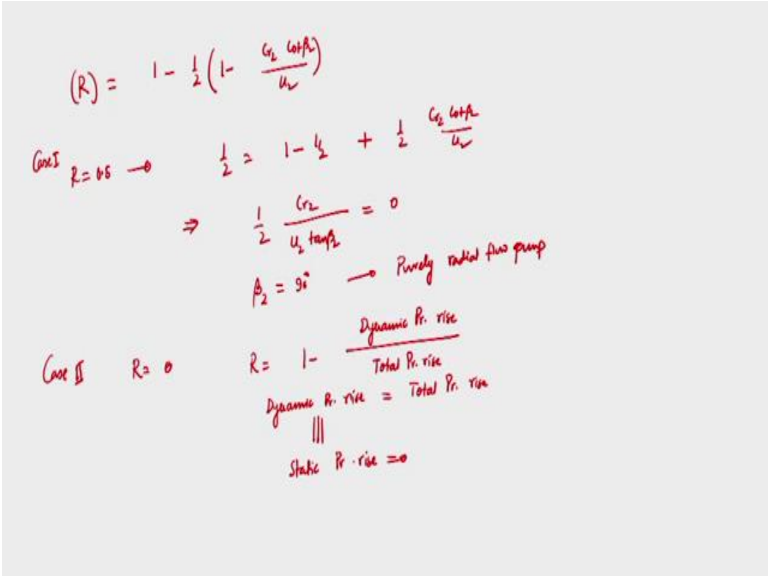

If R= 0.5

$0.5 = 1 - \frac{1}{2}\left(1 - \frac{C_{r2}\cot\beta 2}{u2}\right)$

$\frac{C_{r2}\cot\beta 2}{u2} = 0$

$\beta 2 = 90$

If R = 0 , $\quad$ R = $1 - \frac{dynamic\ pressure\ rise}{total\ pressure\ rise}$

Dynamic pressure rise = total pressure rise

If R =1

$1 = 1 - \frac{1}{2}\left(1 - \frac{C_{r2}\cot\beta 2}{u2}\right)$

$\frac{C_{r2}\cot\beta 2}{u2} = 1$

$\frac{C_{r2}}{u2} = \tan\beta 2$

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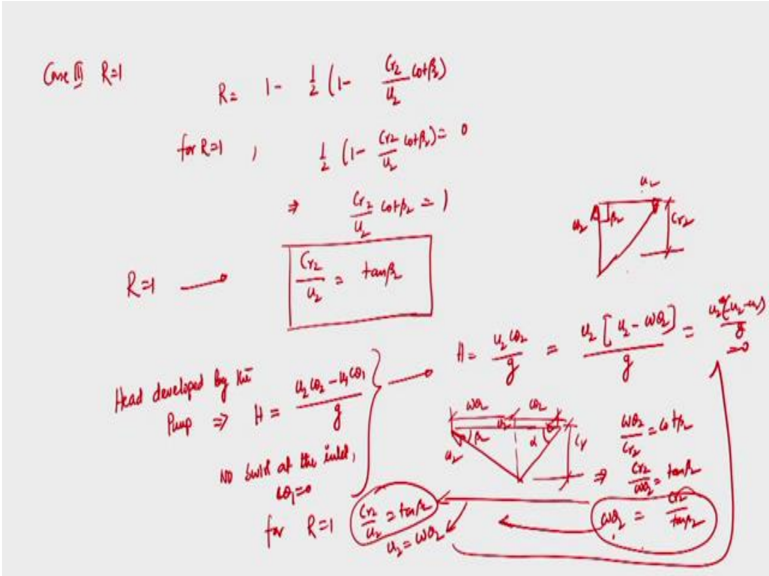

H = $(u2\ C_{\theta2} - u1\ C_{\theta1})/g$

For no swirl at inlet

H = $u2\ C_{\theta2}/g$ = $\frac{u2(u2-W_{\theta2})}{\mathrm{g}}$

$\frac{C_{r2}}{W_{\theta2}} = \tan\beta 2$

For R =1 , $\frac{C_{r2}}{u2} = \tan\beta 2$

So u2 = $W_{\theta2}$

For R= 1, H =0

or even if we apply that if I apply degrees of reaction its degrees of reaction 1 and if you draw the velocity triangle theta 2 and alpha 2 and from there we have been able to show that for this particular case for if we consider R is equal to 1, then method develop by the pump will be equal to 0 ok.

So, now I will draw a figure that we have seen that depending upon the magnitude of a R and of course, the blade this and the blade angle. These 2 are very important to obtain the head develop by the pump.

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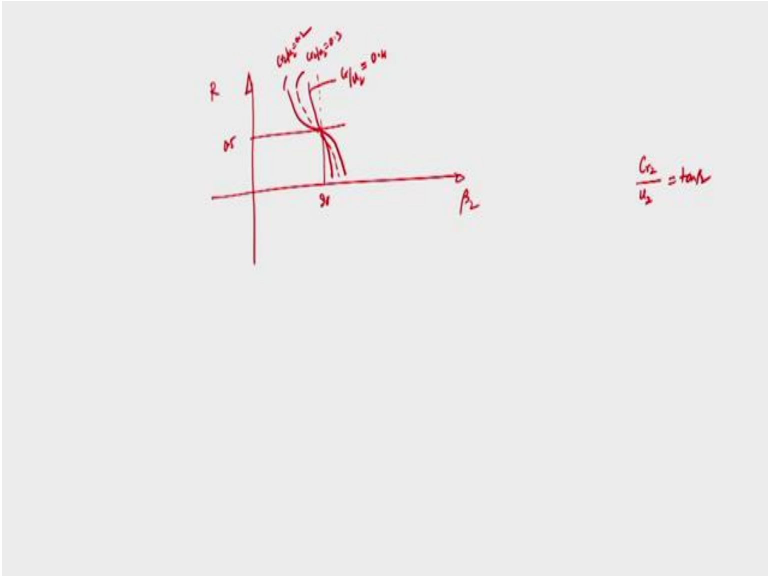

So, now, if I draw R versus beta that is this is blade angle beta and this is R. So, we have seen that for R is equal to 0.5 this beta is equal to 90 degree right. So, this is 90 degree when R is equal to 0.5. Now depending upon the magnitude of $\frac{C_{r2}}{u2} = \tan \beta 2$.

So, depending upon the magnitude ofCr2, flow velocity divided by tangential velocity flow coefficient is called flow coefficient at the outlet, we can obtain a several values of the and degree of reaction like this.

So, depending upon the magnitude of Cr2/u2 that is flow coefficient flow velocity to the you know, tangential velocity, we can obtain different degrees of reaction for different values of beta that is what this plot is indicating. So, an orderly that depending upon the magnitude of degree of reaction, we can again construct you know depending upon the magnitude of degree of reaction, we can again construct we can design the axial flow pump and that we will discuss in the next class. And I also will discuss of few problems which are very important that are based on the lectures whatever we have discussed with other based on the previous lectures, we will walk out of few numerical problems and probably that problem we will discuss in our subsequent lectures.

Ah And so if I try to now summarize whatever I have discussed today is that we have seen that for a purely radial flow machines, the head develop by the pump is only because of the Coriolis force. On the other hand, if you have seen that even if it is a purely radial flow machines and not only purely radial flow machines, if the blades of a radial flow pump are designed following a logarithmic spoiler logarithmic spiral, I mean sometimes blades are designed the following a particular you know step. So, if we use logarithmic spiral to design the blades, then it can be shown that $r_1\ \omega_{\theta 1} = \ r_2\ \omega_{\theta 2}$.

we have discuss about the degree of reaction that is very important that that degree of reaction is very important in the sense that whenever we are using a pump that is a objective is to develop a pressure. So, whenever fluid is flowing through the impeller, there is a rise in static pressure as well as a dynamic pressure. So, what pressure that is to that is a total pressure that is static plus dynamics.

So, the fraction of static pressure, how much fraction of static pressure is developed out of the total pressure in a pump that information is obtained from the degrees of reaction and depending upon the degrees of reaction, we can construct velocity triangle and that what we have discussed today are for different value of R 0.5 and 1. R is equal to 0 is a case when I know beta 2 is 90 degree that is a purely radial flow machine and for R is equal to sorry R is equal to 0.5 that we have discussed a purely radial flow machines and R is equal to 0, where we have seen that the you know static pressure rise I mean is of is equal to 0, there is no static pressure rise.

And for from a value from a value of R is equal to 1 that a for a value of R is equal to 1, we have been able to show that the head rise I mean a head develop by the pump is equal to 0. I mean and then, we have discuss about the at it depends upon the you know Cr2/u2 that is $\tan\beta 2$ and then, we have tried to plot a value a plot R versus beta and we have seen that when R is equal to when beta is equal to 90 degree R is equal to 0.5. Now, depending upon the magnitude of Cr2/u2 are flow coefficient, we have we if we change beta; then, what will be the degree of reaction that is obtained from this plots ok.

We stop here today and I will discuss, I will continue in the next class.

Thank you.