Mechanics of Machining
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Lecture - 09
Other Analysis for Force Relationships

Dear students, welcome to the course on Mechanics of Machining, this is 9th lecture. Today I will be talking about various other models of finding out the cutting forces in orthogonal machining. Till now we have discussed about Merchant's force analysis, fused single shear plane model, Ernst and Merchant together they published many papers and Merchant also has published papers. So, that is very popularly known as single shear plane model. Even I discussed about slip line analysis of Lee and Shaffer. Lee and Shaffer slip line solution also assumes type of single shear plane.

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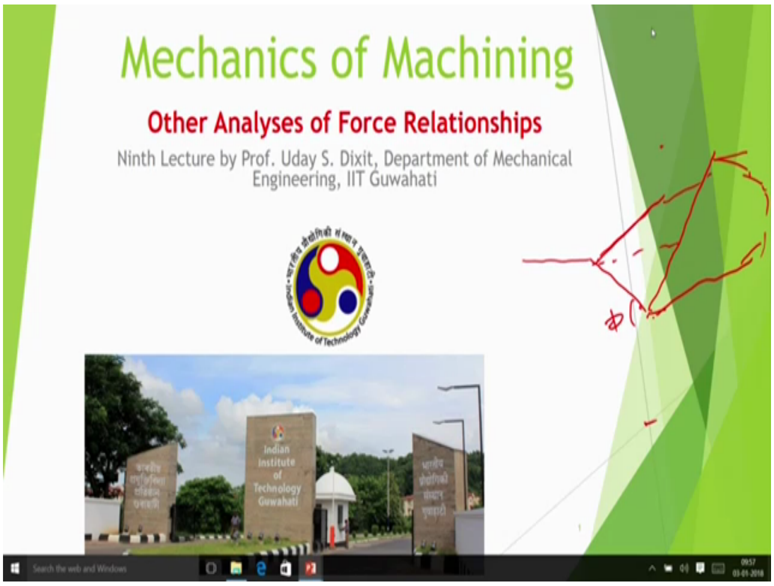

But although it is a slip line, but here we established this type of relation there was phi and then there was slip line. This is considered as a slip line and then there was a free surface, this was the tool like that and it is. And after that he deduced the relation between the shear angle, rake angle and friction angle which is different from merchant's relation and therefore, the power etcetera will also be different.

But nevertheless it is also assuming a existence of a shear plane like this. Then after that there are other models we will discuss those things today in the lecture and also we may give some exposure to the finite element method of finding out the cutting forces.

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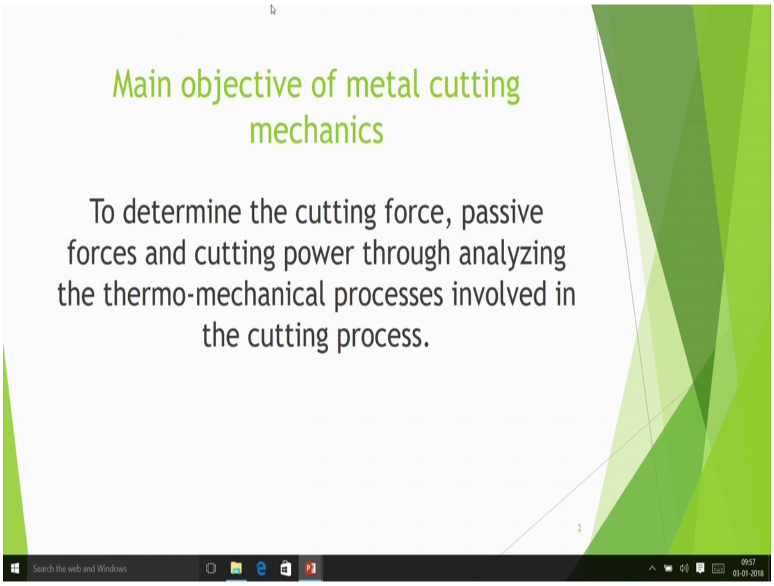

So, if we go to this one that what is the main objective of metal cutting mechanics? The main objective is to determine the cutting force, passive forces means which do not participate like first force which does not produce any power and cutting power through analyzing the thermo mechanical processes involved in the cutting process. So, we have to do temperature analysis also and we have to do stress analysis also. Till now we are only talking about stress analysis, we are yet not talking about the temperature analysis.

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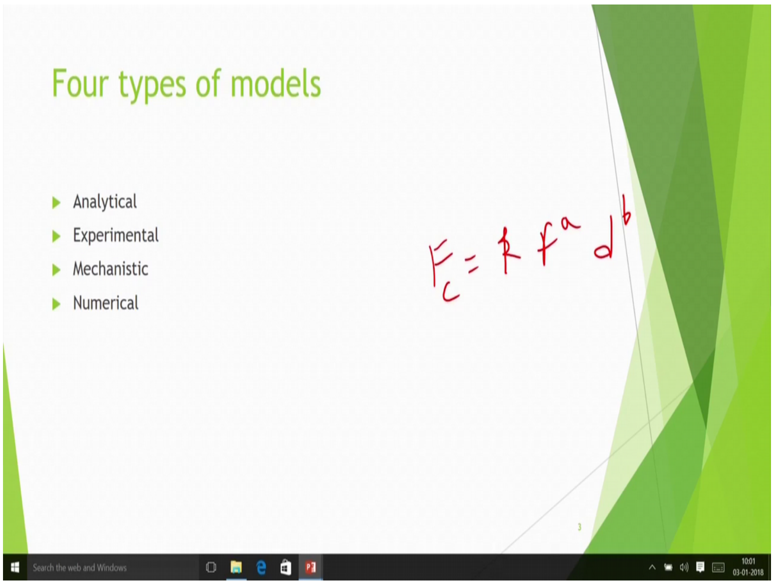

Then, we can say totally there can be four type of models, one is the analytical one like merchants analysis you can say it provides analytical model, it will usually give a closed form solution even, Lee staffers model also gives closed form solution. Then, there can be experimental or empirical model which is based on the number of experiments. We conduct number of experiments and we try to set one relation. In the last lecture also I wrote something like this, cutting force is equal to k times F to the power a and d to the power b here F is the feed and d is the depth of cut. So, such type of relations can be fitted by conducting number of experiments and then finding out some relation.

Then, third type of model is mechanistic model. In the mechanistic model we actually it is some type of we can say semi analytical model, semi experimental model. How do we do that? Suppose we obtain some type of relation that it should be like this, but the coefficients of that relation we can obtain by experimentation. Means, there is some physical basis, but it is not completely only by physics of that. So, that type of model is called mechanistic model ok.

Like I can say with my physical reasoning that the cutting force should be proportional to the chip cross sectional area. So, maybe it is proportional to F into d, but what is that proportionality constant that can come by some other methods. And then, if there is a even number of cutting edges are there are the cutting edges in the helical form, that type of thing also can be done by transformation process we can find out the aggregate effect;

that means, at one location I can make a simple model orthogonal cutting. And then after that I can keep on adding these small contributions and properly transform to account for the angle of the helices.

Then the fourth type of model is the numerical model. In that we use some governing equations which govern the behavior of metal cutting. So, those governing equations we can take like continuity equation, equilibrium equations these equations have to be solved not analytically, but by means of numerical methods. For example, we can use finite difference method or we can use finite element method and other type of techniques also, so such types of methods are called Numerical Models.

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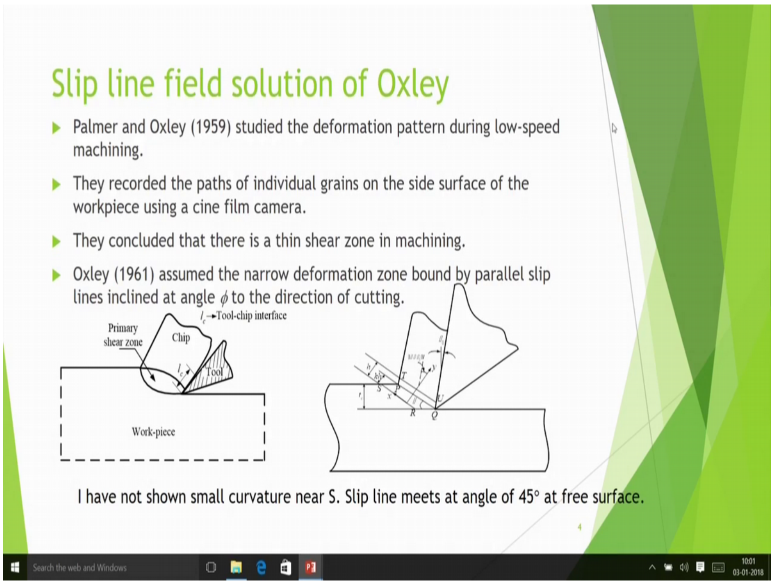

Now, first let me discuss again another slip line feed solution that is given by Oxley earlier that Palmer and Oxley in 1959. They discussed this model and then Oxley further developed it that model I am going to discuss. It is also slip line feed solution, but here they are assuming a shear zone actually not a type of identified shear plane, but there is a complete zone here.

So, they conducted lot of experiments and published paper in 1959 those experiments were of course conducted at low speed machining, and they recorded the paths of individual grains on the side surface of the work piece using a cinema film camera by that they recorded that how the grains are deforming and then they concluded that there is a thin shear zone in machining.

So, Oxley assumed the narrow deformation zone bound by parallel slip lines inclined at an angle phi to the direction of cutting. So, he assumed that although this zone is like this, but to a first approximation I can consider it like zone bounded between two parallel slip lines and these are inclined at angle phi. So, it is just like a shear plane angle. So, here this (Refer Time: 07:39) is one bound and may be T u is another one. In fact, they provided some small curvature near S so that they it ensures that the slip line meets at angle of 45 degree at free surface at this one.

So, they there is a small curvature type of thing. Why it should met meet at angle of 45 degree at the free surface? Because free surface will be divide of any stress and it will be one of the principle stress plane, one of the principle plane. So, the shear plane must make 45 degree angle from this. So, since the slip line is the gives the directions of maximum shear stress. So therefore, it must meet at the free surface at 45 degree angle.

So, that is why they provided some small tilting type of thing, but that is very small and in this figure I have not depicted I have depicted two straight lines. So, may be that this side total zone is h and this may be something like k times h and this may be 1 minus k times h like you that. So, this type of assumption they took.

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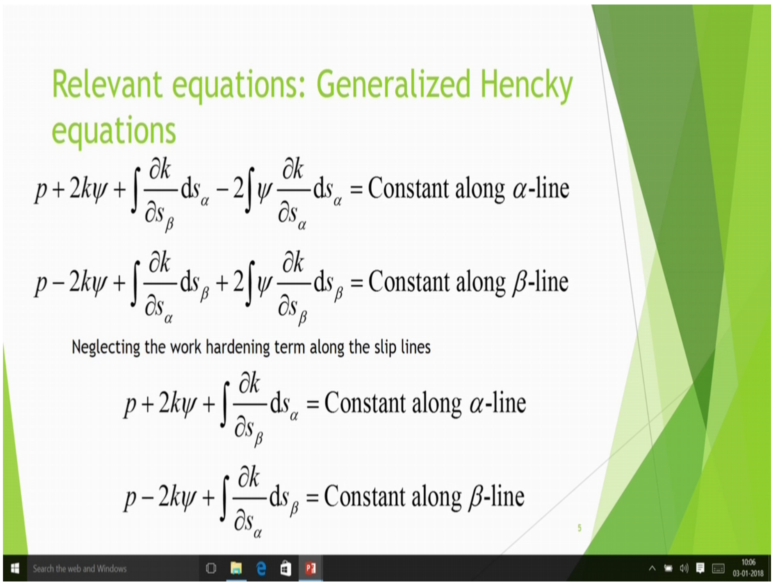

And then after that they I am not going to describe the complete details about that, but they considered the strain hardening also. Usually this slip line method is used for plane strain problems and material is rigid plastic, there is no hardening. But, it can be in calculated, it makes the analysis cumbersome. So, they in calculated the hardening also and relations are like that p plus 2 k psi and this portion. If k is constant throughout then this portion will be 0 and you will get usual Hencky's equation.

So, but these equation are given here. There one equation is constant along alpha line another is constant along beta line. Now if you neglect the work hardening term along the slip line; that means, across this slip line there is work hardening there is no doubt about that. Here, there is a; that means, if you go from S r to T u naturally material will get strain hardened, but assumption is that along S r.

There is no hardening if we assume that then you will get this type of relation p plus 2 k psi is equal to del by because del k by del S alpha here will be 0. So, you will get this type of relation that will be constant along alpha line p plus 2 k psi plus integral del k 2 k by 2 S beta into d S alpha equal to constant and then you will get similarly one relation along beta line.

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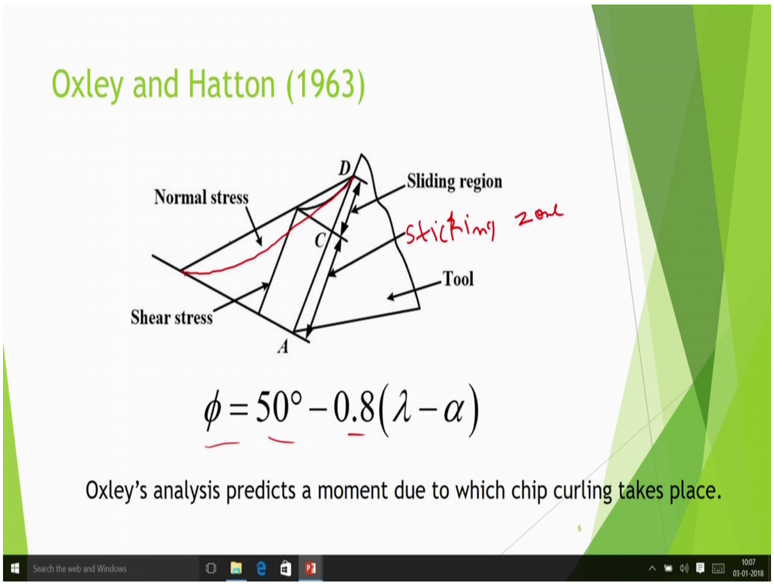

So, these relations they obtained and other procedure they followed in the similar way and then Oxley and Hatton in 1963, they considered this also you know this shear stress is not uniforms on the surface. In fact, there is a sticking zone this is sticking zone from this is the tool cutting edge point A and from A to some distance there is a sticking zone or a sticking region.

So that is, in this portion shear stress remains constant and that is k and after that there is a sliding region in which you can say Coulombs law holds good and then as I have shown in the last class that the normal stress is parabolic type of thing here you get the maximum normal stress and gradually is decreases, but they assume that let us consider that it is not parabolic rather it is just straight line variation is there.

So, if you parabolic means it would have come like this, but some error will be including. So, they said let the normal stress be like that and then based on all these things the analysis was done and you get another type of relation that is phi is equal to 50 degree minus 0.8 lambda minus alpha. And Oxley analysis also predicts a movement due to which chip curling takes place; means he considered in that model that the forces will generally not passed through this one this point rather the force may pass here and this will create a movement.

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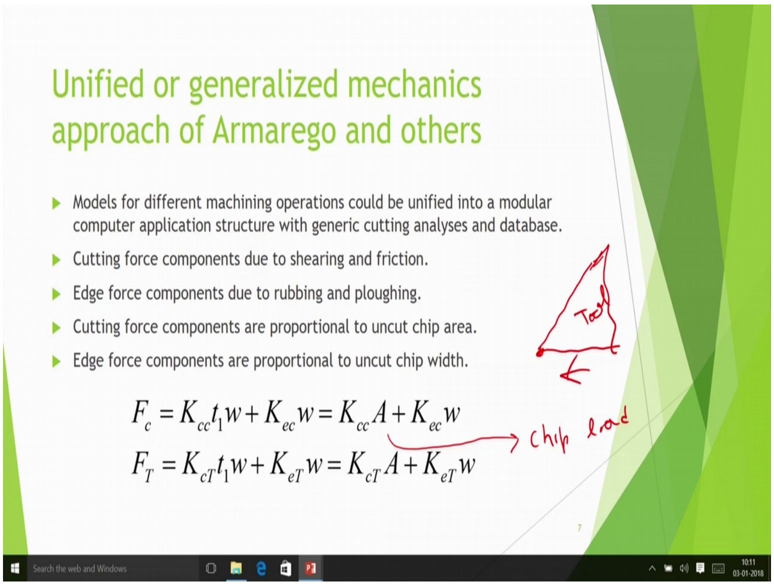

So, that type of thing also has been done those details are available in the textbook, but we are not going to discuss that much or I am just providing you some exposure that such type of analysis is also available in the literature.

Now, I am another one approved that which is. In fact, is still very active approach people are doing it that is, unified or generalized mechanics approach of particularly Armarego and co-workers they propagated that scheme. In this models for different machining operations could be unified into a modular computer application structure with generic cutting analyses and database. Suppose you keep lot of database suppose I keep the database for let us say the turning process, if I have kept the database for turning process how when depth of cut was this and feed was this.

Then this was the force then same type of information can be used for modeling of the milling. If we understand the basic concept only thing that there may be may be two cutters simultaneously cutting. So, you take that effect into account there may be helical type of cutter tooth. So, you take that helix angle into account; that means, you have to integrate small segments helical tooth can be considered as series of the tooth, which are having different types of angles they are arranged little bit staggered that is what.

So, but basic database can be common so that is why we say that generic cutting analysis and database. So, in this we consider cutting force component due to shearing and friction. Then we consider edge force components due to rubbing and ploughing in the basic concept and then cutting force components are proportional to uncut chip area we say cutting force components they are proportional to uncut chip area.

So, and edge force components are proportional to uncut chip width because edge cutting means this is tool and suppose I may make a tool this tool is moving here. So, here this is the cutting edge which is going perpendicular to the screen that edge is there know. So, this edge will do cutting and it is having some width. So, that have other rubbing force may be proportional to chip width ploughing force also.

So, this F c is equal to K cc T 1 w that is the first part, because chip load due to shearing and friction will be proportional to T 1 and W. So, K cc T 1 into W. W is the width of the uncut chip width and T 1 is the uncut chip thickness plus K cc into W we can write it K cc into A, A is the uncut chip cross sectional area and this can be also called chip load people call it as a chip load; that means, this is the total chip load. So, and this one is K ec W edge cutting force. Then first force component can also be written in this similar way F T is equal to K cT 1 W plus K eT W, K cT A plus K eT W ok. This can be written for any general process any machining process you can write.

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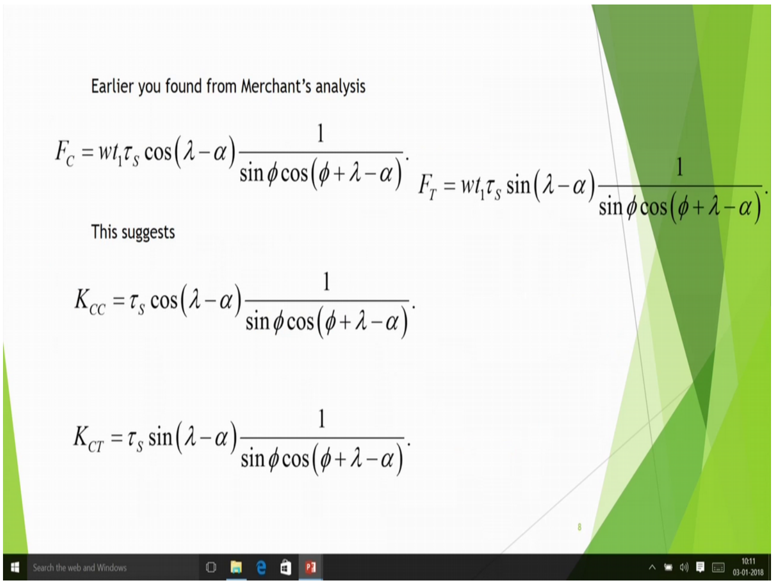

Now, here suppose you want to do that simple orthogonal machining, in that you already have that relation from merchants analysis which was given in this form F c is equal to 2 W T t 1 tau s shear strength and then this was cos lambda minus alpha are that type of things and F T also we derive that expression. If we consider W T 1 as the chip loads then it is clear that this is another portion should be it coefficient. So, that must be K CC. So, like that I have found that K CC is also there K CC is a function of now alpha and it is function of lambda and all that type of thing and K CT is also function of alpha lambda and phi like that.

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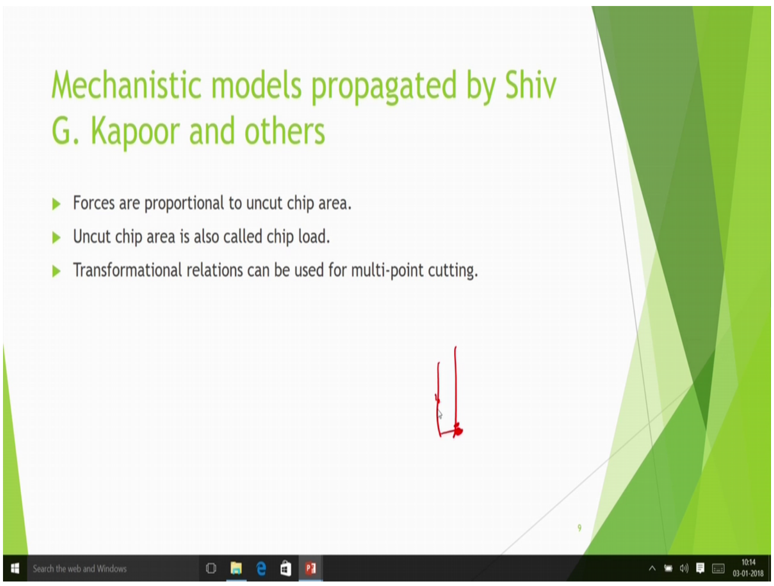

So, this one so like that you can have the data and you can calculate then around the same time Shiv G. Kapoor and that group they developed the mechanistic models. Mechanistic Model and unified generalized mechanics approach more always they are similar, they developed around the same time and lot of work has been done here also they assume that forces are proportional to uncut chip area and uncut chip area is also called chip load. Then transformational relations can be used for multipoint cutting.

So, multi point cutting we can use the transformation type of relations. Suppose my cutting edge is one cutting edge may be here, but suppose this is some type of wind mill. And if you know that at this point how much the force is causing this cutting edge then on the next layer also you can consider that this is also a cutting edge, but angles will be slightly different.

Like that you can find out aggregated effect it is like this that you can have aggregate effect of all the type of forces and then you get mechanistic model approach. So, this lot of papers are there on this topic, each one has developed some mechanistic model for one type of process using the transformational relations these details you can get in other papers. Here, we will not be discussing much about individual papers in detail.

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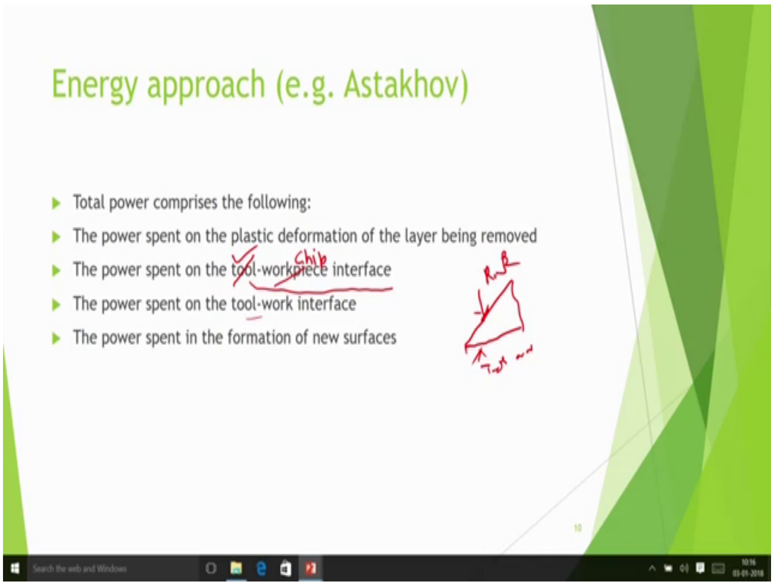

So, we move to another approach; that it energy approach of Astakhov. Astakhov has proposed Astahkov has written lot of papers and books on metal cutting. In fact, earlier also I discussed one paper of Astakhov in which he criticized single shear plane model and merchants analysis.

So, in one of his book he has described the energy approach and he says the forces etcetera can be found by energy approach. Here we consider that total power comprises the following power spent on the plastic deformation of the layer being removed, then power spent on the tool work interface, then power is spent on the tool, this is plastic deformation of the layer removed then power is spent on the chip work. No this is tool chip interface this is tool is and this tool chip interface and another power is the power spent on the tool and work interface because tool is like this. So, on it has got two surfaces with main surfaces that is rake on this the chip is moving.

So, tool and chip are interacting and similarly that here tool and work are interacting. So, that power also as we considered, then power is spent in the formation of new surfaces, that power also he said should be considered like you saw that in Atkins analysis also that type of power was considered.

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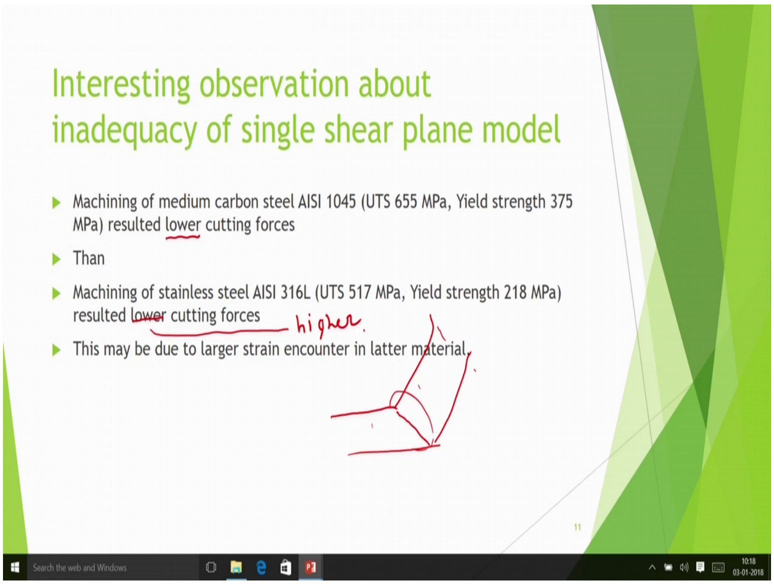

So, that is approach of Astahkov and then, he also pointed out once the interesting observation about inadequacy of single shear plane model. For example, machining of medium carbon steel that is AISI 1045 which has about 0.4 percent carbon in that ultimate tensile strength is 655 mega Pascal and yield strength is 375 mega Pascal.

Another steel is stainless steel that is AISI 316L its ultimate tensile strength is 547mega Pascal and yield strength is 218 mega Pascal. So, both these strengths are lower it should give that lower cutting force, but actually this give higher cutting force. In fact, this give higher cutting force we aspect that it should give lower, but it give higher and this actually which has more ultimate tensile strength it gives lower.

So, why it is so; that means, Astahkov argued that this may be due to larger strain encountered in that this material that later material. That means, in stainless steel part there may be this one there may be lot of redundant deformation there may be lot of unwanted deformation suppose, I wanted to cut some material and this chip is there this portion has been cut. Now we will require the energy in cutting, but during the process the chip thickens and you know it shortens also

So, this I am doing unnecessary chip thickening is not of any use, but it happens, but it takes energy. So, it is a type of you can say it is redundant energy if another person can do the simple cutting operation same way, but energy is not required; that means, lot of deformation of the chip is not there in that case is energy will be smaller. So, stress is one thing, but the strain is another thing. Stress into strain they give the energy product of stress and strain. So, this may be due to that.

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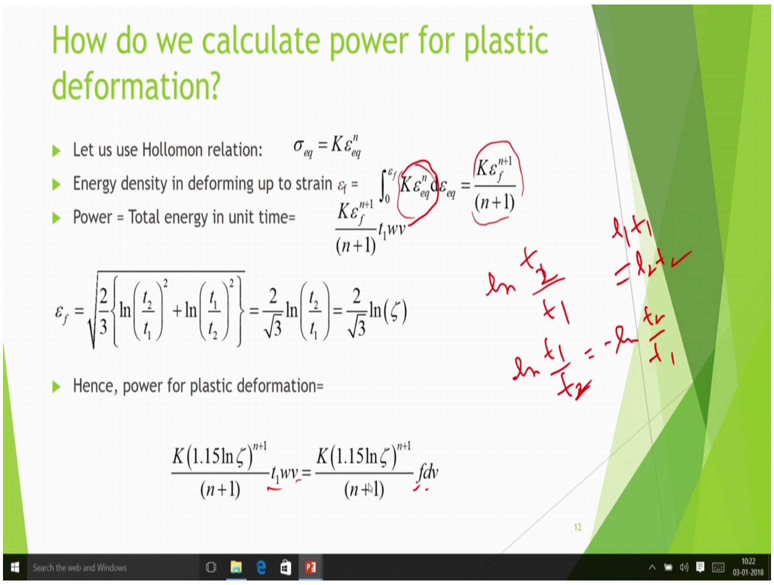

So, you have to consider that strain effect also. Now how do we calculate power for plastic deformation? Why I am will giving one illustration, let us consider Holloman relation of strain hardening.

So, here sigma eq will be K time epsilon eq into n, n is the exponent and K is the strength coefficient. Now energy density in deforming up to strain epsilon if suppose I know that this is my fracture strain; that means, this will be the strain for causing the structure ok. So, then we can integrate 0 to epsilon f K epsilon p e q into d epsilon e q stress into strain give me energy density; that means, energy per unit volume.

So, if I take product of that that stress is this whole quantity; that means, epsilon e q and multiplied by d epsilon e q that will give small energy, but if I integrate between 0 to epsilon f I will get total measure of the energy. So, if I integrate I get this type of expression K epsilon f n plus 1 divided by n plus 1. So, power is equal to total energy in unit time. So, if I multiply this energy per unit volume by volume rate of metal involved that is T 1 W into v v is the cutting speed then I get the total power.

So, we got this type of expression now how do we find out epsilon f if I can measure the chip thickness then I get the idea about that strain; that means, assuming that it is plane strain causes. So, you get the thickness direction strain and the length direction strain of course, the both of these strains will be equal and opposite because the total volume has to be constant.

So, one is that suppose chip has become T 1 to T 2. So, you get that l n T 1 by sorry T T 2 by T 1 and similarly in other case you get in T 1by T 2 because (Refer Time: 25:13) L two by L one, but L two by L one is nothing, but T 1 by T 2 because you know that L 1 L 1 is equal to l 2 T 2. So, these are the logarithmic strain measures and of course, in T 1 T 2 is nothing but minus ln T 2 by T 1. So, I can find out now the equivalent strain equivalent is strain is what square root of 2 by 3 and then I square all the non zero components of these strains and add them.

So, there are only two non-zero components of the strain in this axis system 1 is the chip is thickening and then chip is also getting shortened. So, I took in T 2 by T 1 square plus in T 1 by T 2 square and then I took the square root of this. We get 2 by lambda root 3 In T 2 by T 1 and in T 2 by T 1 is nothing but the chip compression ratio it is more than one and we say 2 by lambda root 3 In zeta.

Hence the power for plastic deformation becomes here two by lambda root three is nothing, but 1.15. So, we substitute in this expression and it becomes k 1.15 ln zeta divided by n plus 1 to the power n plus 1 and T 1 w v and this becomes this I can instead of w I can write f and instead of T 1. I can write d d depth of cut and I get this type of expression.

So, this way I can calculate the power for plastic deformation.

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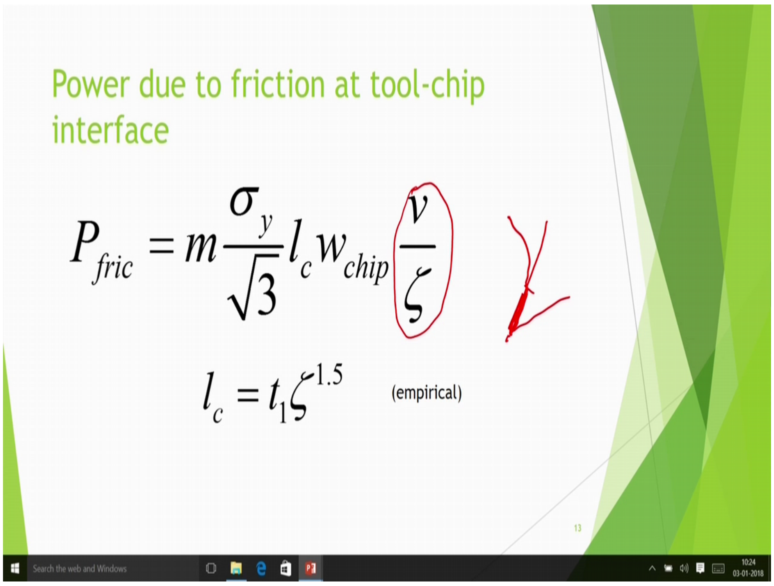

Now, we can find out power due to friction at tool chip tool chip interface tool and chip. Chip is sliding on the tool so here we find out P friction is equal to m, m is a friction factor it is a maximum value can be 1. So, m time's sigma y by root 3 that gives a constant shear stress and l c is the chip length then w chip is the width and then we say that here. So, l c into w chip, chip that will give you that force shear force and then v multiply by v. In fact v c, because the chip is moving with the velocity v c, but v c is equal to nothing, but v by chip compression ratio tau.

So, you get this type of expression m has to be of course, found out by experiments because m is the friction factor if there is some lubrication m will be small if there is no lubrication m will be big like that and then l c is the chip contact length. That means, how much is the length of the contact because chip separates there is a tool and on this tool the chip is moving suppose my chip is going like this. So, contact length is up to this he gave empirical type of expression and this l c is equal to T 1 multiplied by chip compression ratio to the power 1.5. So, this expression can be used and you can get idea about the friction power.

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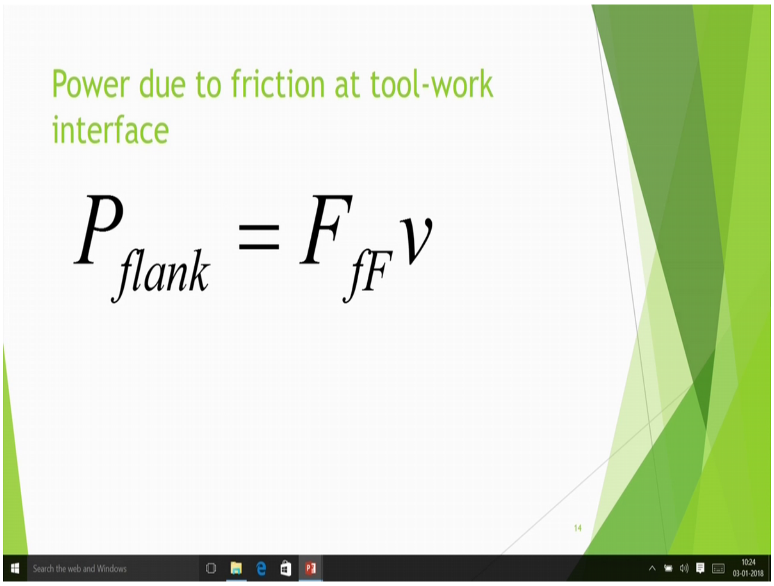

So, two powers we have obtained then; power due to friction at tool work interface can be found P at the flank and be friction force at the flank multiplied by V. So, this portion also you can find out you can consider that this portion is to be proportional to the cutting edge width and it may be fraction function of low radius those types of things can be taken into account and then other things also chip. That power is spent on the separation of new surfaces that can also be considered and all the energies can be combined you get the total energy then divide it by cutting speed V and then you get cutting force.

So, this way you can find out the cutting forces. So, all these analysis are there; there are many other methods also like one is the upper bound method in which we find out up upper bound of the cutting force that how much will be the maximum cutting force like that and there are many slip line feed solutions.

So, all these methods are not able to match the numerical methods like finite element modeling although finite element modeling of machining process is also difficult. But, nowadays finite element modeling has been in calculated in many commercial packages like deform and in abacus etcetera. Also, you can do finite element modeling by doing some work from your own side by writing some small codes etcetera.

So, I will give some exposure about finite element modeling because nowadays you can get even readymade codes and in that you use them as exact box, but you must also know that what is going inside. So, very briefly I will tell about the finite element modeling I at least that what are the questions actually which are being solved by finite element model.

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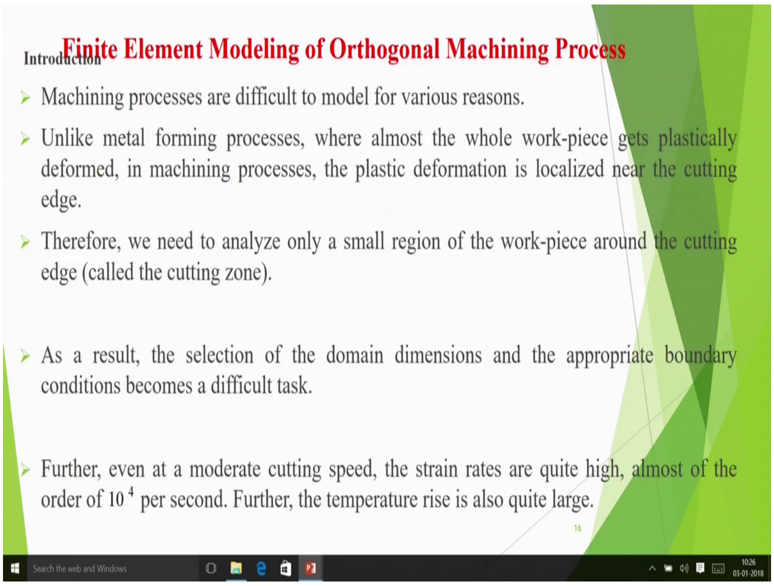

So, first some introduction that machining processes are difficult to model for various reasons one is that unlike metal forming processes, where almost the whole work piece gets plastically deformed, in machining processes the plastic deformation is localized near the cutting edge. So, you have to consider only the localized one. So, that local thing has lot of effect if the local layer is very hard it will have significant effect.

In metal forming whole thing you are analyzing suppose local layer you are not able to model properly or you have taken the average property that will do, but average property type of thing in machining will not work. So, we need to analyze only a small region of the work piece around the cutting edge that is called the cutting zone. As a result the selection of the domain dimensions and the appropriate boundary conditions becomes a difficult task. And further, even at moderate cutting speed the strain rates are quite high although it is also uncertain that how high it is really, but many people believe that it is almost of the order of 10 to the power four per second and temperature rise is also quite large.

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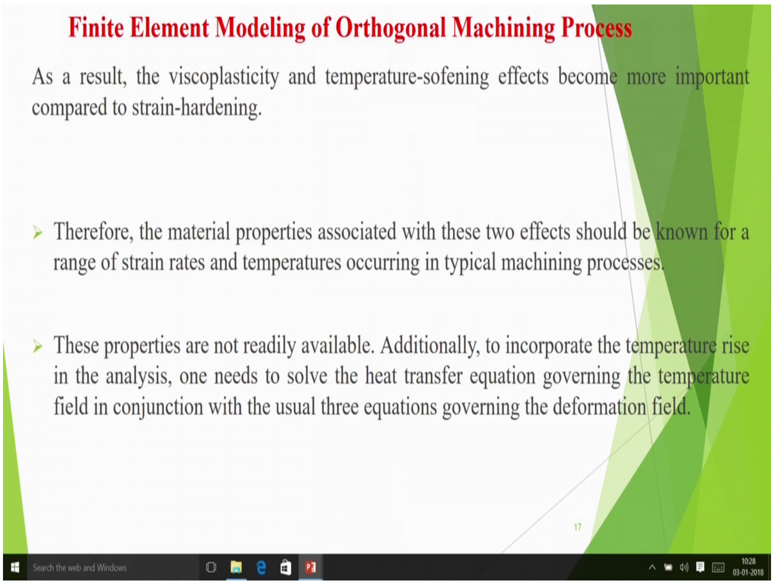

As a result viscoplasticity and temperature softening effects becomes more important compared to strain hardening we consider that flow stress is also a function of strain rate. So, material property is associated with these two effects should be known for a range of strain rates and temperatures occurring in typical machining processes. These properties are not readily available additionally to incorporate the temperature rise in the analysis one is to solve the heat transfer equation governing the temperature filed in conjunction with the usual three equations governing the deformation fields.

So, you get the equilibrium equations those have to be solved in addition you have solve the heat transfer equations, but heat transfer equations you can solve by finite element or you can solve by other methods also analytical methods also.

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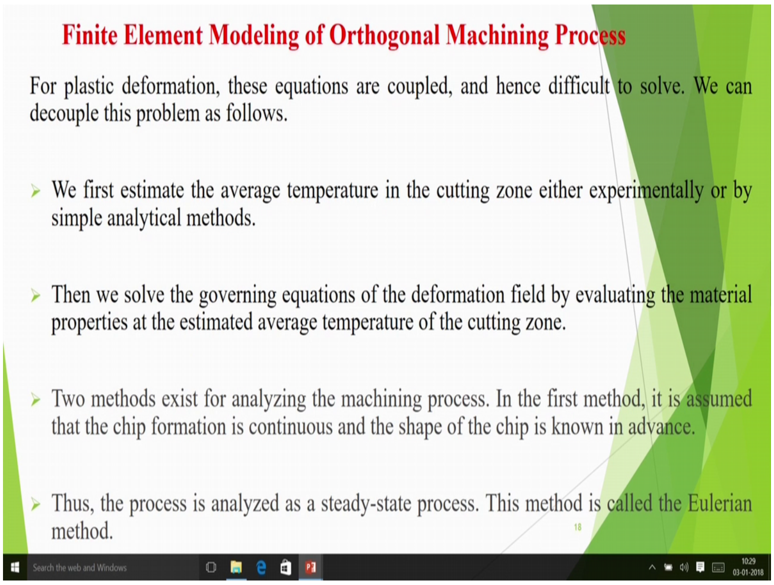

So, for plastic deformation these equations are coupled and hence difficult to solve. So, as such they are coupled, because if temperature effects this one stresses and stresses effect the temperature, but we can often decouple them and we first estimate the average temperature in the cutting zone either experimentally or by some simple analytical methods. And then, we solve the governing equations of the deformation field by evaluating the material properties at the estimated average temperature of the cutting zone. That means, average way I have found and then to methods this process can be repeated after you do that analysis then again you can find out the temperature and again you can do this one. So, you can do in a sequential manner also.

Here two methods exist for analyzing the machining process in the first method it is assumed that the chip formation is continuous and its shape is known a prior. So, chip is known in advance. So, process can be analyzed as a study state process that material is moving through that control volume and this method is called the Eulerian method control volume method.

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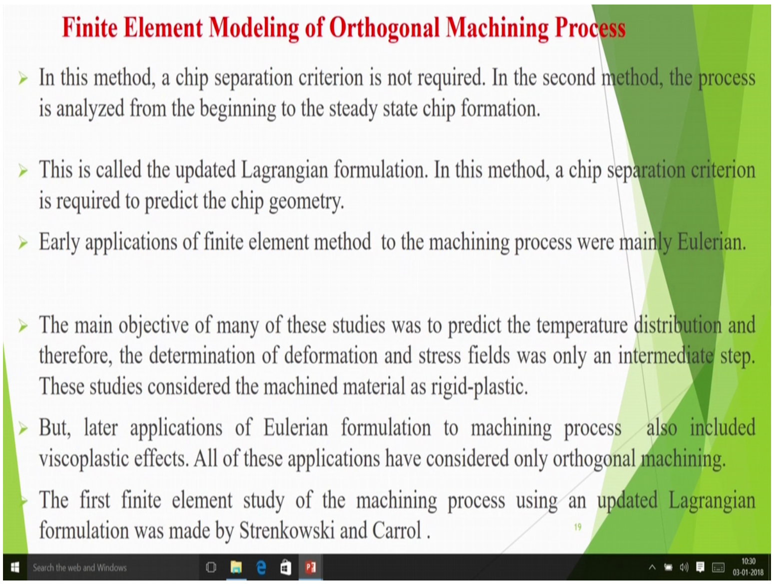

Another so in this method the chip separation criteria is not required. And in the second method the process is analyzed from the beginning to the steady state chip formation in the second method process is analyzed from the beginning to the steady state chip formation. We follow the material though this is called the updated Lagrangian formulation.

In this method a chip separation criteria is required to predict the chip geometry that how frame the chip will separate that type of thing has to be in calculated in this we find out the position etcetera. And stresses of one time step by taking the differences as the previous time step that is why it is called updated Lagrangian formulation.

But earlier people mostly used an Eulerian approach, because Eulerian approach will take comparatively less amount of time or though you get more detailed information if you do Lagrangian approach. So, main objective of many of these studies was to predict the temperature distribution. And therefore, the determination of deformation and stress fields was only an intermediate steps and these studies considered the machining material as rigid plastic.

So, if you want to find out even the temperature distribution still you have to find out stresses or if you want to find out stresses naturally you must know the temperatures. But later application of Lagrangian formulation to machining processing you did Viscoplastic effects and all of these applications have considered only orthogonal machining the first finite element study of the machining process using an updated Lagrangian formulation was made by Strenkowski and Carol.

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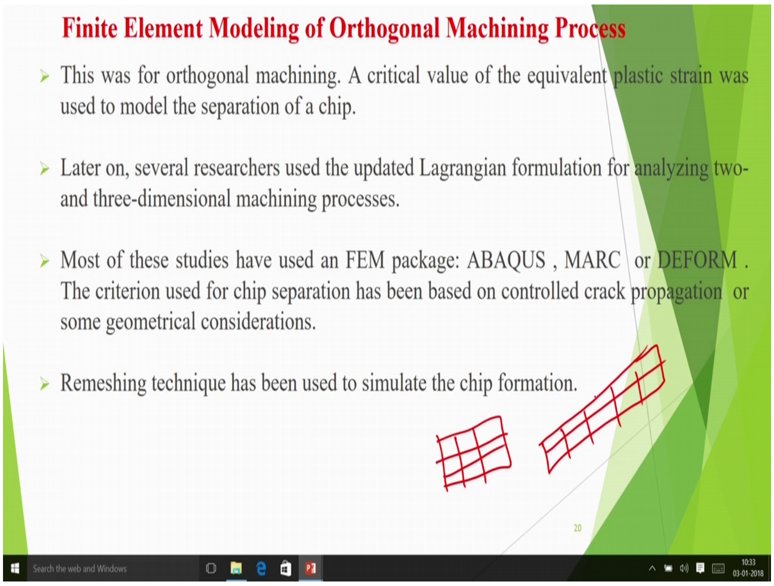

This was for orthogonal machining and they used a critical value of the equivalent plastic strain to model the separation of a chip.

Suppose you keep on analyzing and finding out these strain and when that particular strain becomes more than critical value, then you can say that the chip separation is taking place. Later on several researchers used updated Lagrangian formulation for analyzing 2 and 3 dimensional machining processes most of these studies have used FEM package ABAQUS is one FEM package MARC is another one DEFORM is another one DEFORM is very usual trend it is used for metal forming and metal cutting studies and in other packages you have to work something, but they may have more flexibility.

Criteria used for chip separation has been based on controlled crack propagation or some geometric consideration. And they have used remishing technique to simulate the chip formation suppose when deformation occurs then again you make the miss again. Because, if suppose I make a miss here in this region like this; and then suppose if has deformed then naturally you have to makes a different type of miss that type of thing is also done in those packages.

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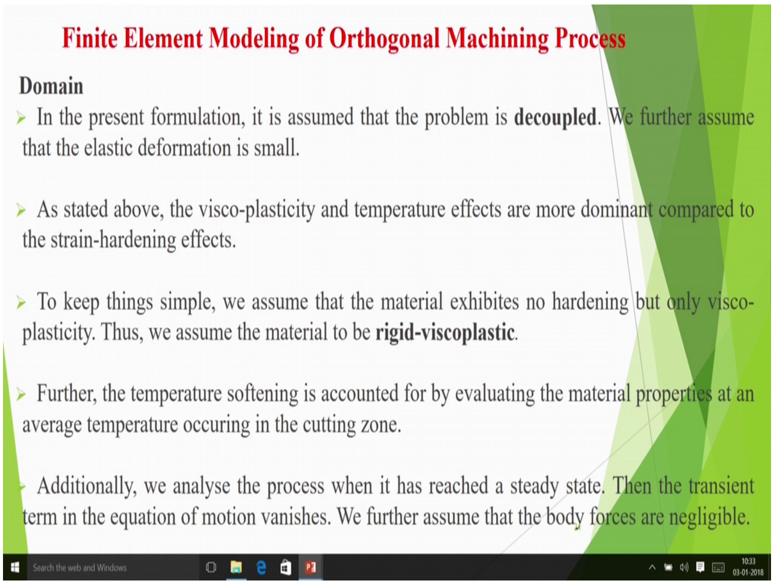

And let us discuss finite element modeling of orthogonal machining process we assume that the problem is deformed; that means, we can estimate some temperature and after that we can do simply the stress analysis and we assume that the elastic deformation is very small. So, neglect any elasticity and viscoplasticity and temperature effects are more dominant compared to these strain hardening effect.

So, we do not consider strain hardening, but rather we consider viscoplasticity and temperature effect and to keep things simple. We assume that the material exhibits no hardening, but only viscoplasticity thus we assume the material to be rigid-viscoplastic. Further, the temperature softening is accounted for the evaluating the material properties at an average temperature occurring in the cutting zone.

Additionally we analyze the process when it has reached a steady state value a steady state condition then the transient term in the equation of motion vanishes. So, transient is not there and we further assume that the body forces are negligible what are the body forces? Like such as the gravity force which is distributed in the body those types of forces are neglected interaction of the forces is only through the surfaces by external agency.

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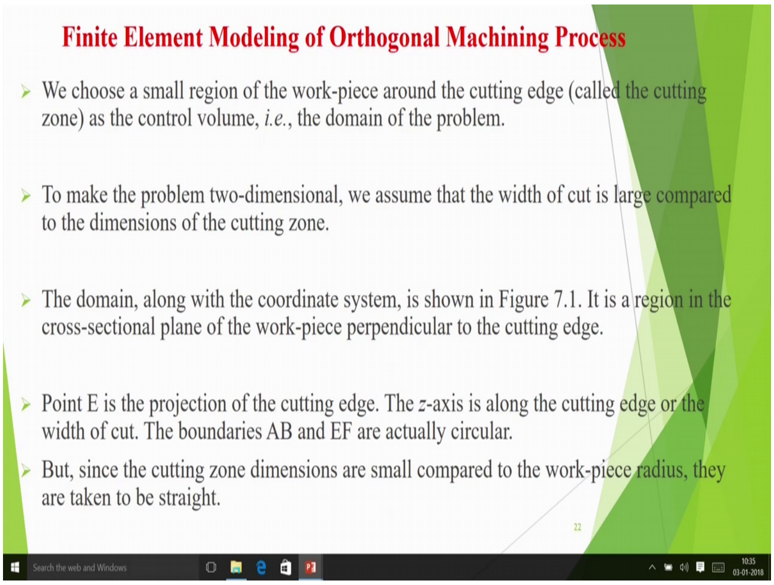

So, here we chose a small region of the work piece around the cutting edge or the cutting zone as the control volume that is the domain of the problem. Then to get the problem two dimensional we assume that the width of the cutting large compared to the dimensions of the cutting zone. So, that we can state is more or less a plane strain type of situation. The domain along with the coordinate system zone in the next figure and it is region in the cross sectional plane of the work piece perpendicular to the cutting edge now here this one is.

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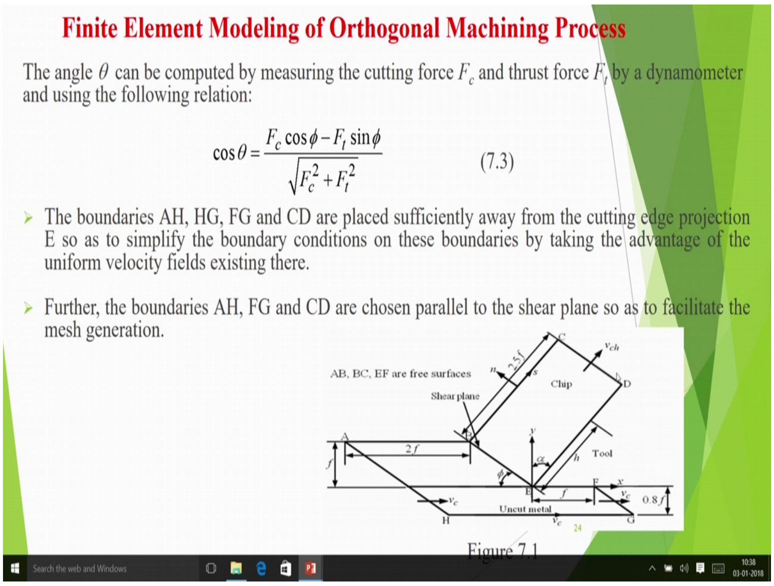

This type of thing we make this type of domain this is uncut material and then this is the domain.

So, here point E is the projection of the cutting edge this point is the cutting edge then z axis is along the cutting edge of the width of the cut. That means, in this screen only you are seeing the x axis and y z is perpendicular to the screen and then boundaries A B and E F are actually circular, but we have assumed A B and E F. This one A B and may be this is E F.

These are free surfaces they may be circular because some bulging will take place here, but we assume that they are straight line and now angle alpha is equal to the rake angle of the cutting tool by now you will very well know it is alpha. That means, surface of the tool is inclined at an angle alpha with the vertical and the distance H is called the tool chip tool chip contact length and we use the following expression for H F sine theta by this one. So, this is the type of expression has been used.

Earlier I told another expression by Astahkov. So, this is this type of things you have to consider. So, suppose they have taken only this type of relation and in orthogonal machining this shear angle can be estimated by measuring the cutting ratio and that is given here. Now here angle theta is actually theta is the angle between shear force and the resultant force and F is the feed.

So, theta is the angle between the Shear force on the shear plane and the resultant force on that plane that can be easily obtained by merchants circuit and it is given in this manner FC cos phi minus FT sine phi divided by this one and FC is the cutting force and this is the thrust force and by a dynamometer. So, we can find out these force boundaries HG FG and C D are placed sufficiently away from the cutting edge projection E.

So, they have been put sufficiently away. So, in this process then we assume that boundaries FG and CD are chosen parallel to the shear plane. So, this way so this and shear plane BE and CD are parallel. So, mesh generation will be easy and here you can assume that at CD you will well this velocity is V CH there is uniformity here.

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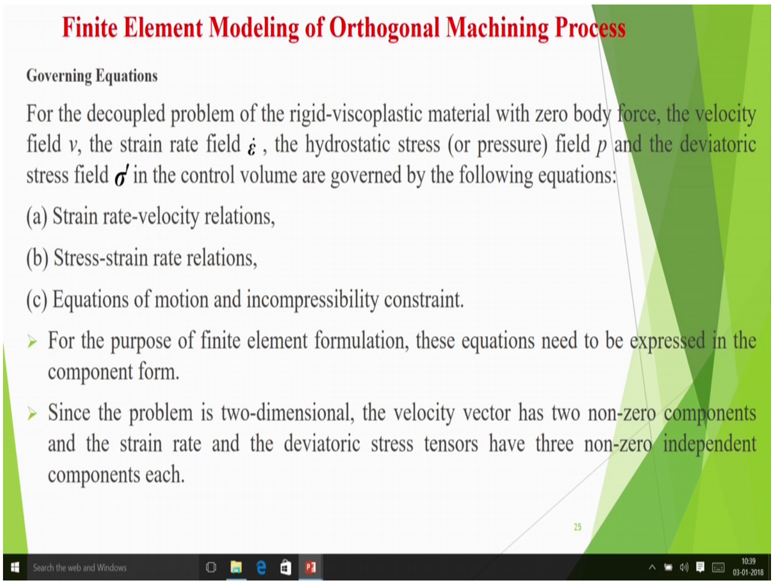

Now, for the decoupled problem of the rigid viscoplastic material with zero body force the velocity filed v, the strain rate field epsilon dot, the hydrostatic stress field p and the deviatoric stress field sigma dash in the control volume are governed by the following equation. We have strain rate velocity relation, then we have stress strain relation, then we have equation of motion and incompressibility constraint.

Now, these equations need to be expressed in the component form velocity vector has two non-zero components and the strain rate and the devoted (Refer Time: 43:37) stress cancel out three non-zero independent components each.

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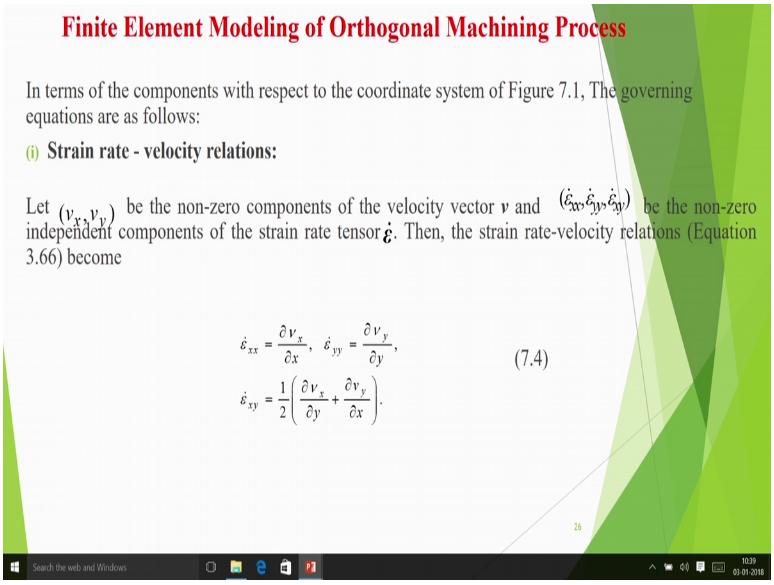

So, what are these equations suppose we write strain rate velocity relation we can write epsilon dot x x is equal to del v x. That means, velocity in the x direction divided by del x epsilon dot y y is strain rate in y direction is this and shear strain rate is given by this that is one type of relation that has to be used, because in a FEM you will be then predicting what are the velocities at different nodes.

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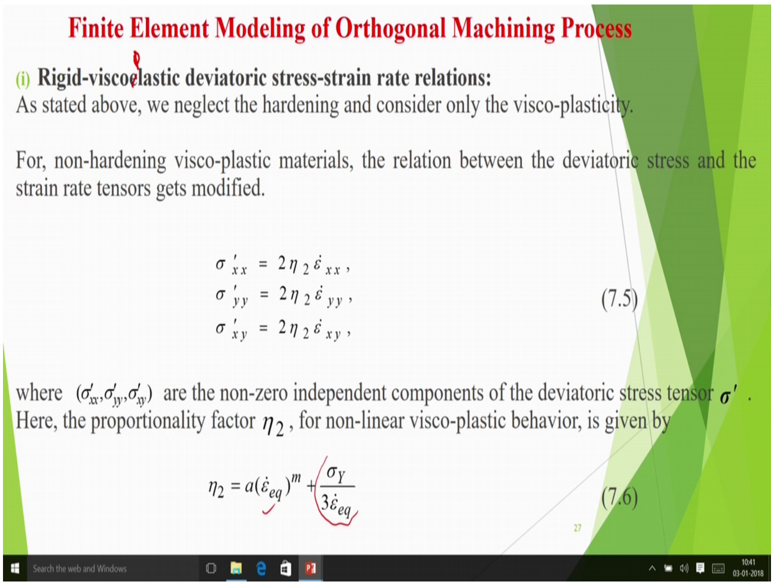

And then there is a rigid viscoelastic (Refer Time: 44:17) deviatoric stress strain relations. So, this is basically viscoplastic; that means, we can say viscoplastic. So, here if it is non hardening then we get this type of relation that deviatoric stress what is deviatoric stress component? That you subtract the hydrostatic part from the stress tensor then you get deviatoric stress tensor and then you have 2 eta 2 into epsilon dot xx and here this is a proportionality factor eta 2 and this is given by eta 2 is equal to a times something strain equivalent strain rate to the power m plus this portion that sigma y by 3 epsilon dot eq that anyway would have come, but this portion has come, because of the viscoplastic effect.

So, that such type of relations of plasticity has being used here.

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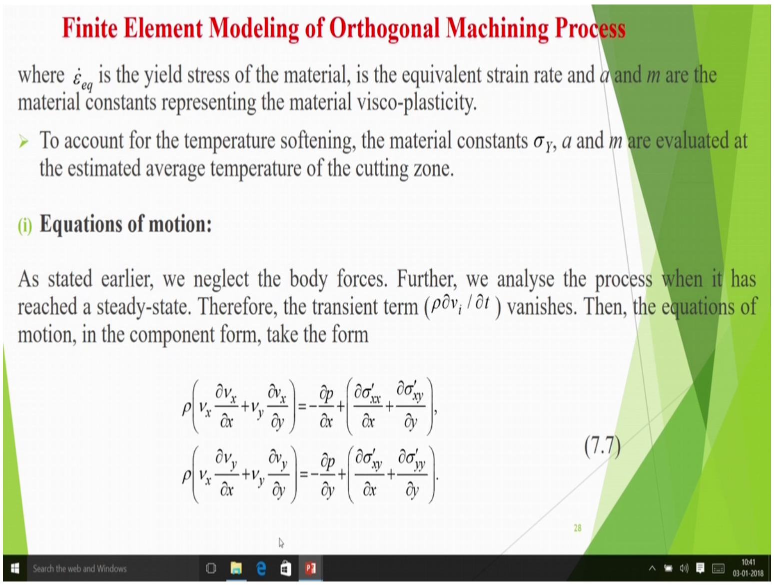

And temperature softening is considered material constants are evaluated at the average temperature of the cutting zone. Then, we write the equation of motion equation of motion are written like this x momentum balance in x direction you used this and momentum balance in y direction use this, but p is the hydrostatic pressure acting on that particular point and this is deviatoric stress. And if we say that inertia effects are neglected then this portion will go here.

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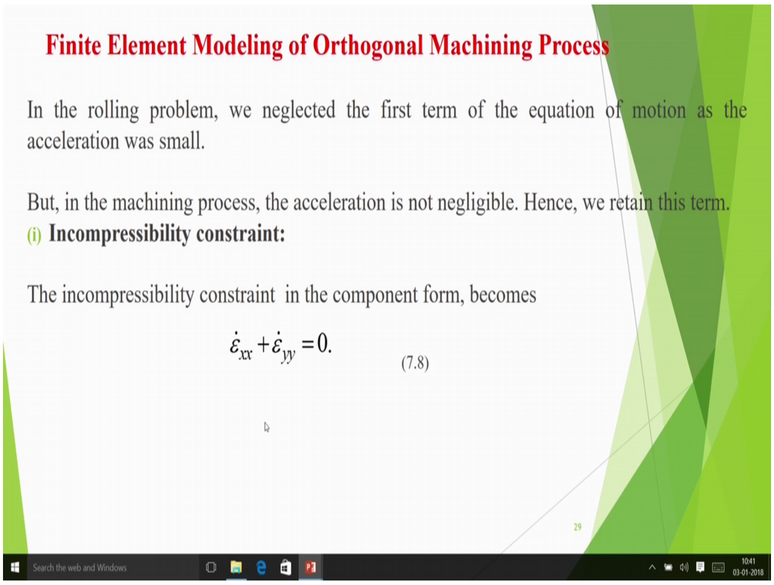

And in that case now in this vv consider incompressibility constraint also. So, incompressibility constraint in the component form becomes epsilon dot x x plus epsilon dot y y equal to 0 that is incompressibility constraint.

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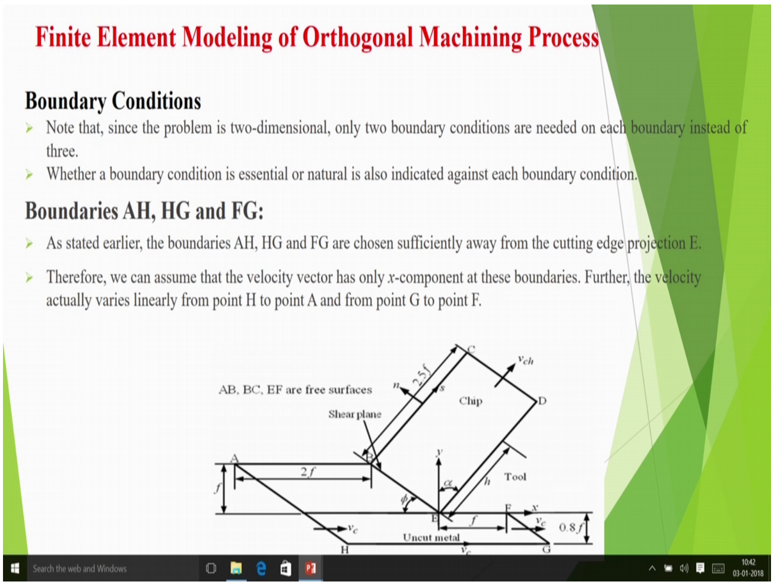

And then we consider boundary conditions note that since the problem is 2 dimensional only 2 boundary conditions are needed on each boundary instead of 3.

So, whether a boundary condition is essential or natural is also indicated against each boundary condition; that means, in a FEM we have two type of boundary conditions one is in terms of the primary variables like our velocity and hydrostatic pressure or the primary variable. So, we say suppose velocity is specified that is called essential boundary condition and if the derivative terms are used in the boundary like suppose strain or stress is the derivative of the velocity then we say that it is natural boundary condition.

So, at these boundaries HG and FG here we assume that the velocity vector has only x component at these boundaries HG here only x component and then you have got HG then you have got FG. So, here FG also only this one and HG also if we assume that it is only cutting like this. So, we assume that it is only in x direction

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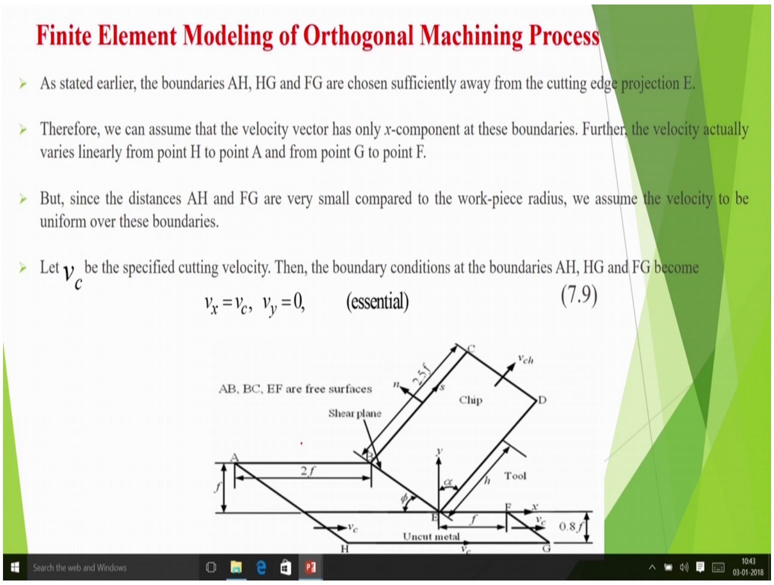

That is V c that and then as we can assume that the velocity vector has x components. Now I am actually the velocity values linearly from H to point A; that means, from here to here and from point G to point F, but since the distances are very small we assume the velocity to be uniform ok.

Let now, V c be the specified cutting velocity then the boundary conditions become Vx is equal to Vc and Vy is equal to zero. So, that type of boundary condition

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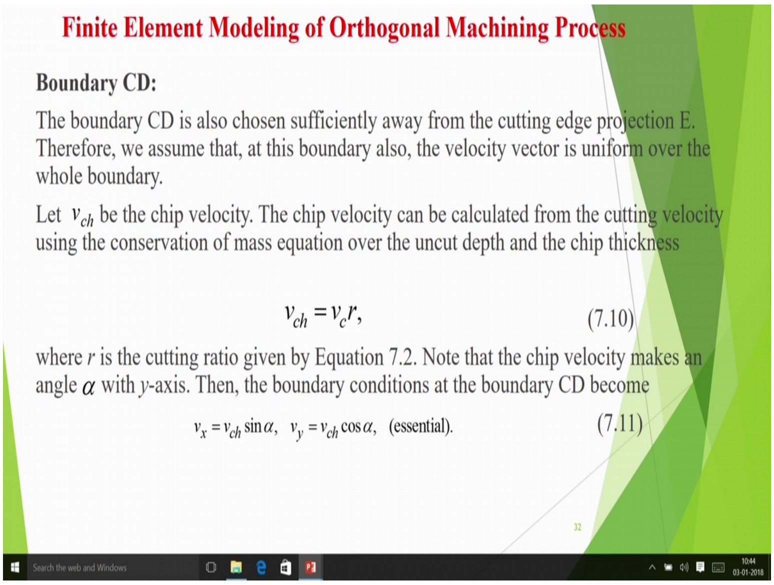

We put at boundary CD; that means, this boundary CD this is that three boundary here at this boundary from the cutting edge we assume that this boundary velocity vector is uniform over the whole boundary. So, V c h is the chip velocity and this is equal to V c times r and in this case boundary conditions at the boundary C D becomes V x is equal to V c h sine alpha V y is equal to V c h cos alpha. That means, we have expressed that in terms of V x and V y because ultimately at each node we will be having unknown valuable like V x V y and pressure.

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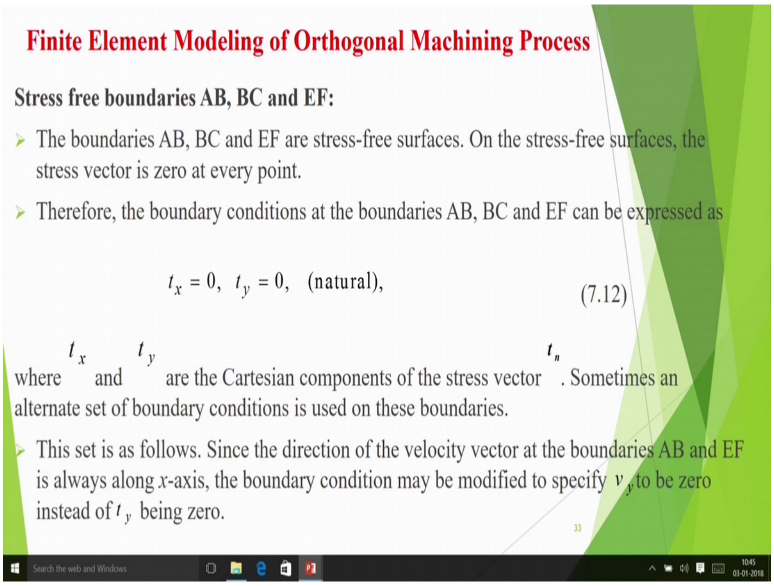

So, we express in that way that portion has to be done and stress free boundaries AB BC and EF on these because these are stress free surfaces AB. So, we simply say that t x is equal to 0 direction in x direction is equal to 0 on t y equal to 0.

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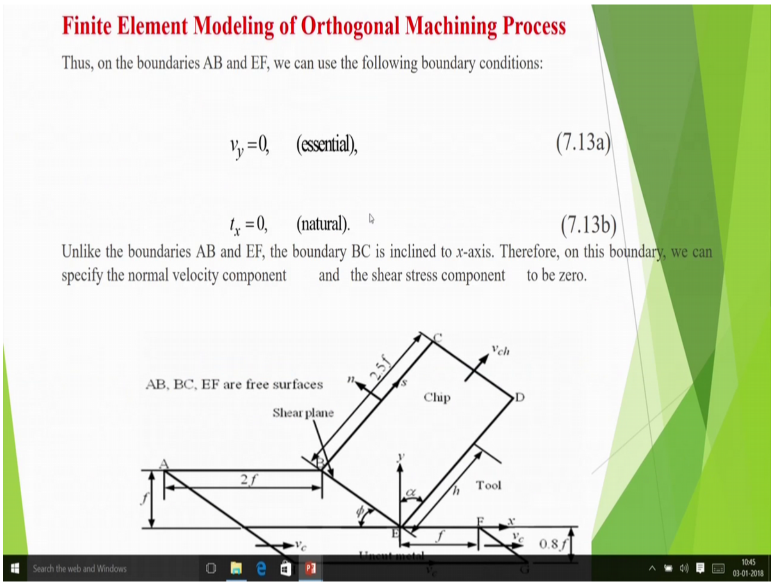

And now this one is now we can of course, take any other type of boundary conditions also, but we are generally taking 3 surface 1 and now what about that boundary BC BC yes BC is inclined here, since the inclination of boundary BC with yx is alpha. So, we can write V n. That means normal component across V c 0. So, n equal to 0, but by trigonometry you have to transform V n in terms of x and Vy and you get V n is equal to minus V x cos alpha plus V y sine alpha equal to 0.

That is essential boundary condition and then you get a natural boundary condition ts equal to 0 because it is free surface t s means tangential stress is not there here, but that is expressed in this form and they are expressed like this then at tool chip interface velocity along the tool chip interface can be approximated by the following relation.

This is a sort of implical relation this is given here and then velocity keeps on increasing first and then after that it becomes constant.

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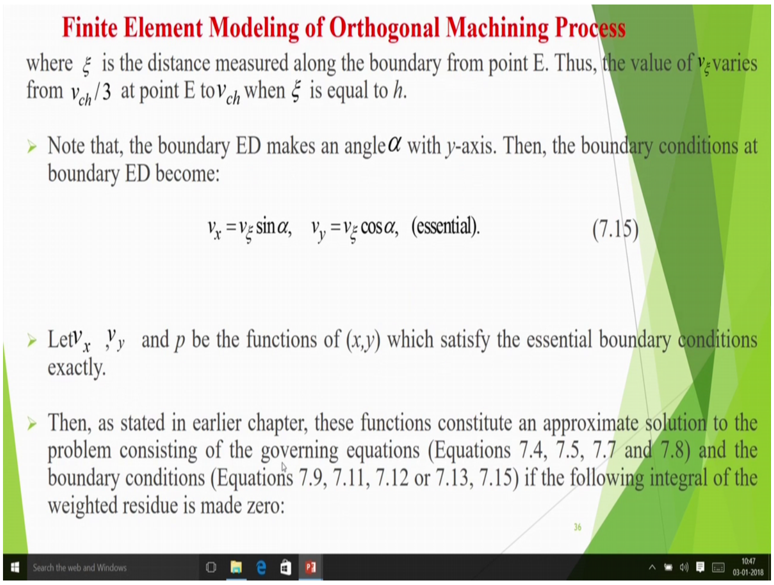

This one now node that this makes angle alpha with y axis; so we can again transform in V x and V y like that we do all these type of things and then.

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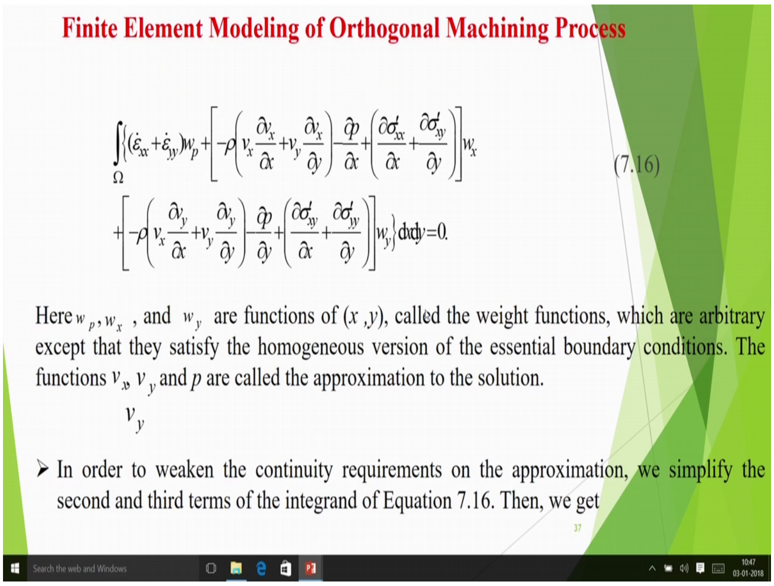

What do we do that we take the easy questions suppose you have this equation epsilon dot xx we have this one we have this equation. Now we know that epsilon dot xx is what in terms of the velocity derivatives and we integrate this equation we multiply each equation by weight some weight function W p W x W y, and we say integrated thing 0.

That is the procedure which is followed in (Refer Time: 51:39) finite element approximation procedure ok.

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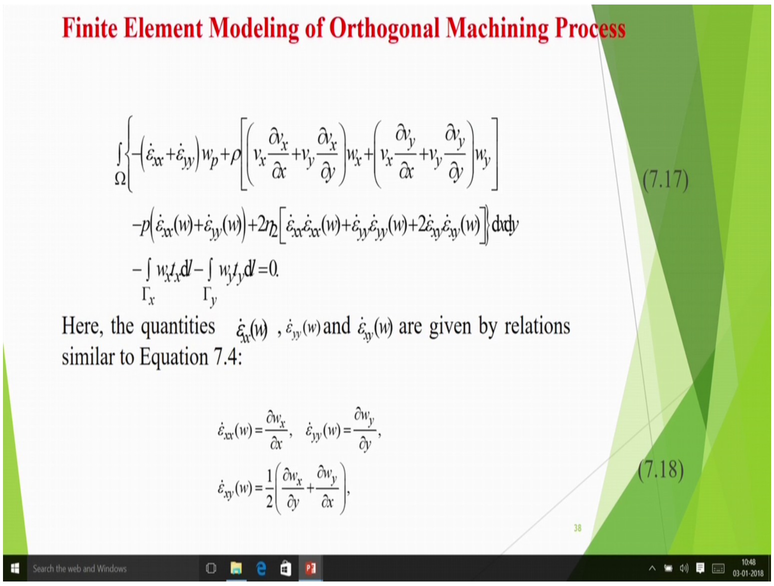

So, I am just telling that this one is done and then after that we obtain the derivative of these equations. That means we integrate these equations by part, because we observed that sometimes there is second derivative of velocity some place is there is a first derivative of the velocity. So, if we integrate by part then we can reduce sometimes the order of derivative that procedure is done, and as a result you get this type of equation I am not going to discuss that in detail.

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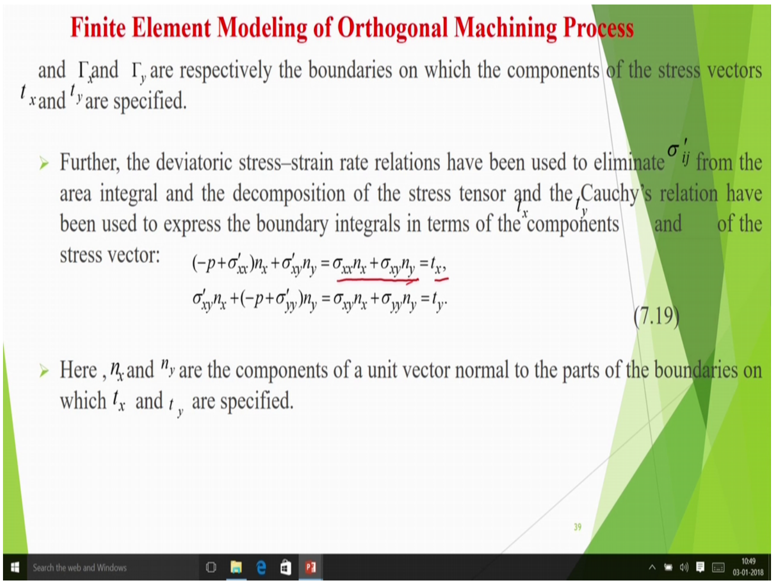

And, then we have this type of relations that of course, you get these equations at the boundary sigma x x n x that is actually a relation of Cauchy by which you get t x direction t x is equal to sigma x x n x, n x is the x component of the normal direction and in y this one. So, these equations are there if you have to just revise the basics of this solid mechanics otherwise time being I am just telling you the basic concepts.

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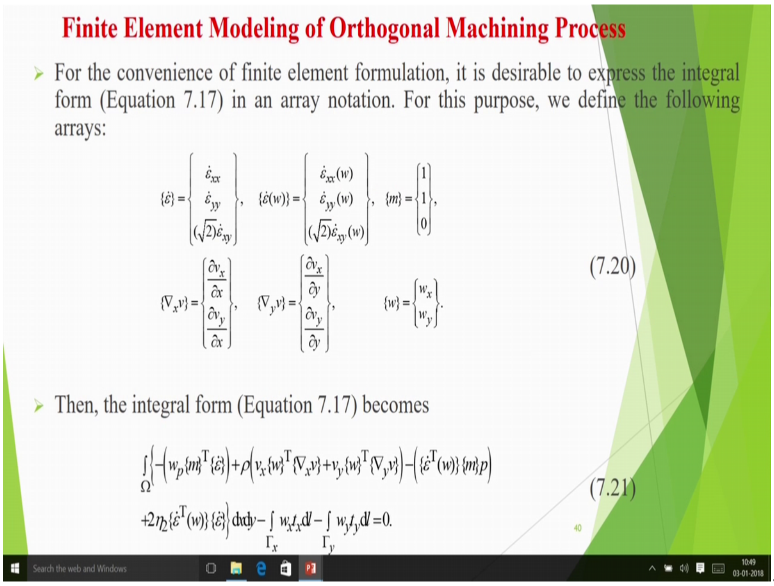

.

So, you can ignore if you do not really know, but here after that we get this type of v represent everything in the matrix and vector forms. So, that it becomes easy to manipulate. So this is a strain rate, and this is this one, and after that we put all that type of data and we take.

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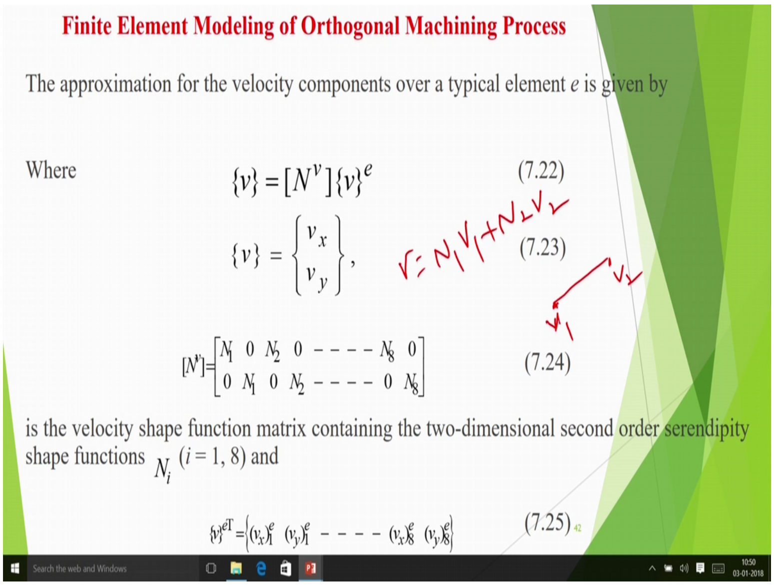

There is a some concepts of that suppose velocity at each node as two components and it is inter projected type of thing that we assume that velocity inside the element will be inter correlation of the model velocities and these inter correlation functions are basically called shape functions. Suppose I have the velocity V 1 here suppose some velocity V 1 and some velocity V 2. So, we can say V in the domain V is equal to n 1 v 1 plus n 2 v 2 where n 1 and n 2 are called shape function or inter correlation function and n1 may be the function of xn2 may be function of x. So, like that so it is nothing but the interpolation function I am mathematics you can study later on in detail I am just telling you basic concepts.

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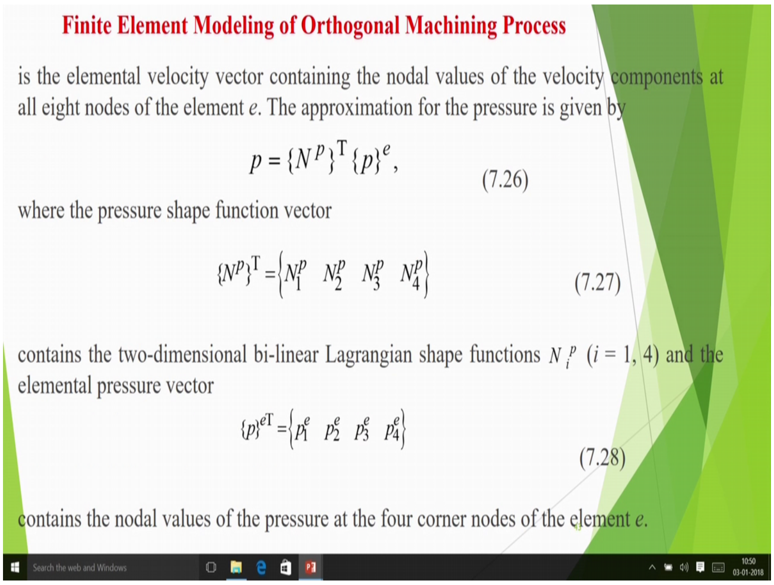

So, that it when you study mathematics then it will be easy for you to follow.

So, this shape functions everything is expressed in terms of the shape functions in interpolation function, because infinite element we find out only the nodal velocities, but of course we can say that we know everything because we can interpolate them. So, interpolation is applied and then we put

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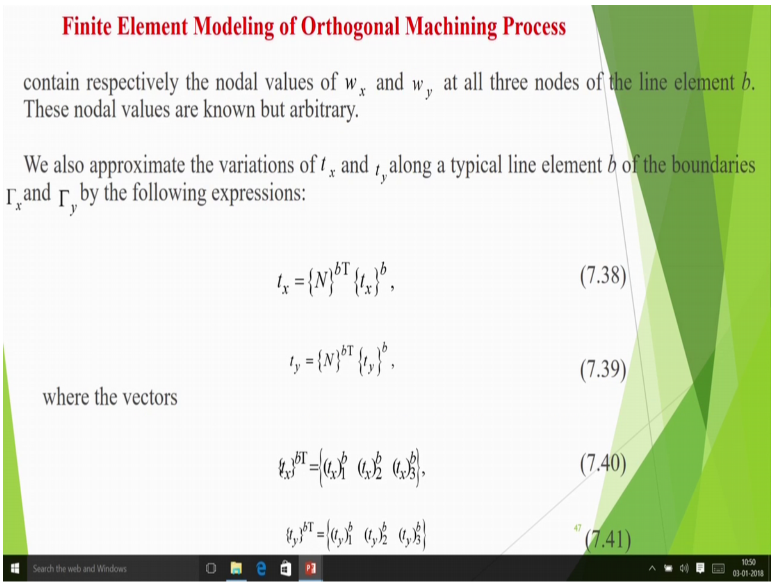

All these thing here we obtain these type of relations B times v e.

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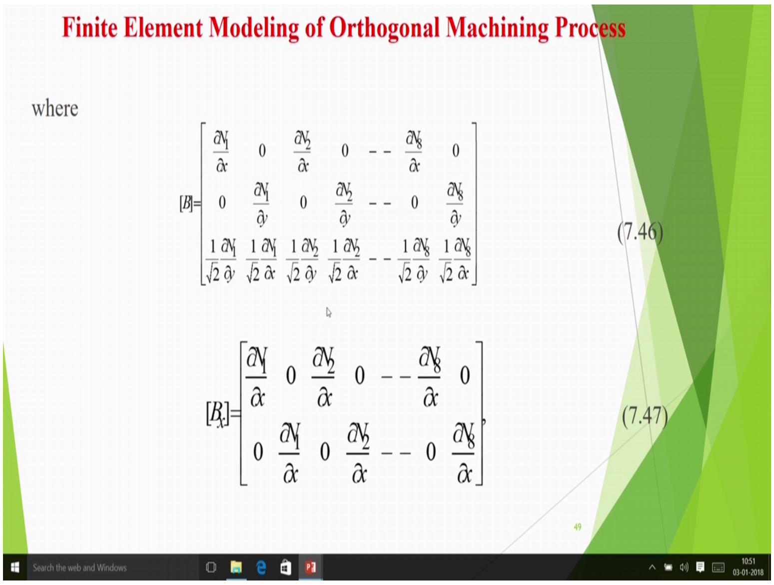

And then this is B matrix derivative matrix means inter correlation functions have to be derive the means differentiated because you have to get a strain rate from velocity. And then you substitute all these type of things and then you get in the matrix form you get nice type of expression.

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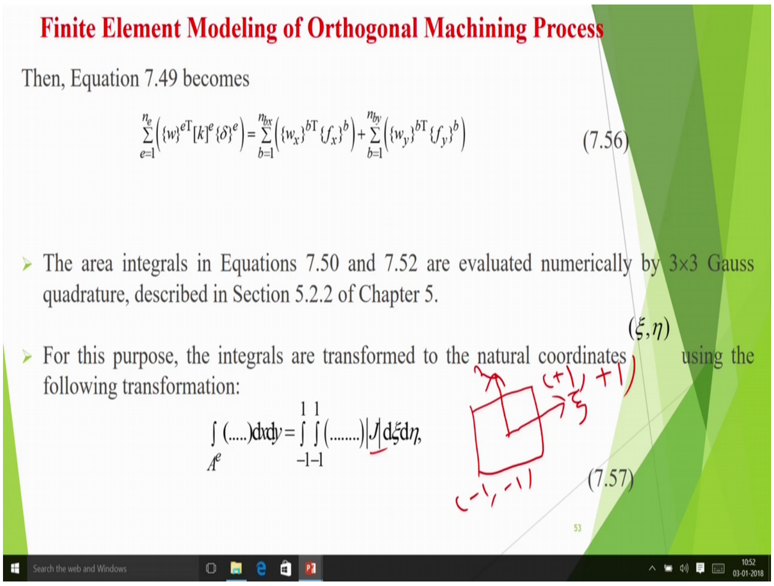

Finally you get that this type of thing that k e delta e and this is this one, but if this requires lot of integration in side because you have to integrate. So, integration is usually done by numerical method like Gauss Quadrature etcetera. And here you transform that transformation is done how that you transform from physical coordinate system to natural coordinate system in natural coordinate system any domain is getting transformed to minus 1 to 1 and minus 1 to 1. That means, square domain here; that means, transformation you can do, but in applying the Gauss Quadrature. We generally try to make this type of domain jai and eta and this is minus 1 minus 1 and this is plus 1 plus 1 and you get Jacobein determinant of Jacobin that is basically transformational relation.

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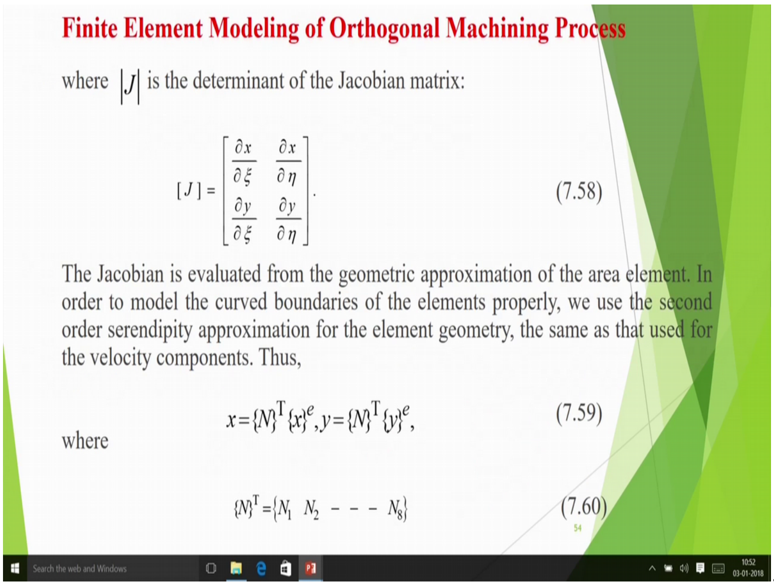

That means, Jacobian can be defined be like this. These type of thing we can put even the coordinates also x position at any point is also inter correlation of values nodal coordinates and we put all that type of thing and then after that you will get. Finally, this type of equation K delta is equal to f and this will be algebraic system of equations. So, we started with the differential equations. But finally, we ended up with algebraic equations, but of course, this K may be non-linear; that means, k may be function of delta. In fact, in metal cutting glasses it is non-linear.

So, we are getting simultaneous not the linear system of equations, but simultaneous non-linear system of equation K is called the coefficient matrix sometimes since packages they just call it is stiffness matrix and delta is basically unknown primary values vector which can be called displacement vector and f is the right hand side force vector or it is just called right hand side vector.

So, we do that and then after that we apply the boundary conditions. So, boundary conditions properly can be applied and there are some terms like eta 2 etcetera. In that relation we told you about eta 2 in that relation it was there just you saw that here this was relation of eta 2 say something when we discussed about this one about relations that yes it in that we discussed what are the type of equations. And we discussed that there is a constitute relation like this.

So, you are getting some eta 2 now suppose epsilon dot eq is very small. Then eta 2 will become very high is not if epsilon dot e q 0 it becomes infinite we do not want that type of situation. So, we limit that in numerical methods you have to do such type of things that all though theoretically if siren dot eq is very small ets 2 must be very large, but you say no I will not allow it to go beyond 1400. And then, you solve these non-linear equations in iterative way till it converges within 1 percent between two successive iterations.

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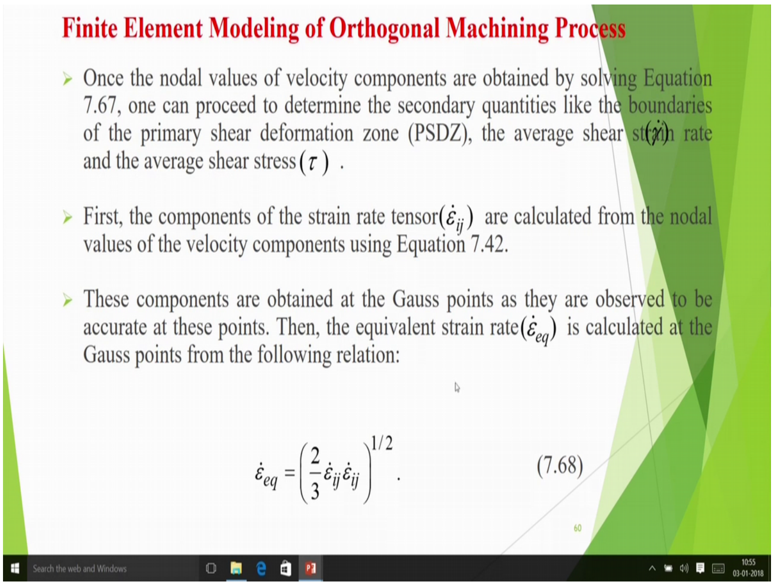

And then after you have find out everything then you find out the derivative components, and then you find out epsilon dot equivalent q is equal to 2 by 3 here I used index notation that epsilon dot i j epsilon dot i j means i varies from 1 to 3 in 3 dimensional problem j changing from 1 2 3, but each components since i and j are occurring twice.

So, all the terms are summed; that means, epsilon 1 1 is square epsilon 2 2 square epsilon 1 2 square like that we add. So, that is summation convention we have used and we find out equivalent strain. And then after that we put some criteria that if a equivalent strain is very small say it is one percent top the maximum strain rate then that portion can be considered as the boundary of the plastic and rigid zone. So, like that we estimate the primary shear deformation zone and its width we can find out ok. So, we do that this way and now what happens is that here now in this one here, if we find out gamma dot shear strain component engineering strain rate is given like this two times epsilon dot 1 2 and average of primary shear deformation zone that shear stress and then after that equivalent strain rate is also defined like this in this fashion three by two sigma dash i j sigma dash i j and this is square root of that.

Then we put all these things and then after that we can estimate the shear zone etcetera. That means, that mathematics I am skipping, but you can see that here some results that we took this data next you know is 7860 sigma y is given m is this is a and this is some exponent used in that model. This is not the aim of that friction factor this is something that in this constitutive relation and we meshed the region like this. And then after that we use the cutting velocity feed and rake angle we have taken in one way like this. And here we use three portion cutoff criteria that primary shear deformation zone boundary is taken at that point in which the shear stress is about not the shear stress, but the strain is three percent of the maximum strain.

So, by that we estimate the shear deformation zone. That means, DS what is the length of the DS and DS came out to be like this and experimental results from the literature provided this. And these are the predictions from the present model and experiment

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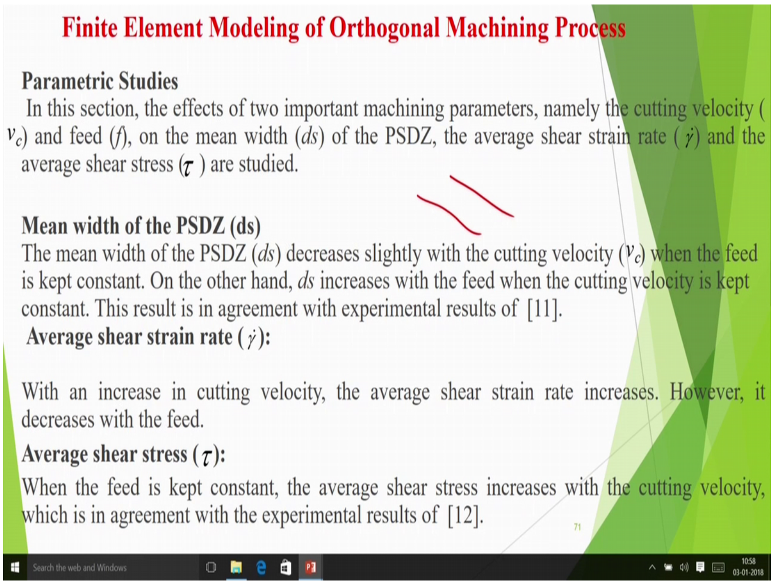

So, matching is quite good and then we did some parametric study we obtained that mean width of the primary shear deformation zone. That means, the width of the shear zone this one decreases with the cutting velocity and it increases with feed and then every shear rate increase in cutting velocity every shear strain rate increases expected and it decreases with the feed.

And every shear stress increases with the cutting velocity which is in a agreement with the experimental result this is because of the shear; that means, viscoplasticity effect

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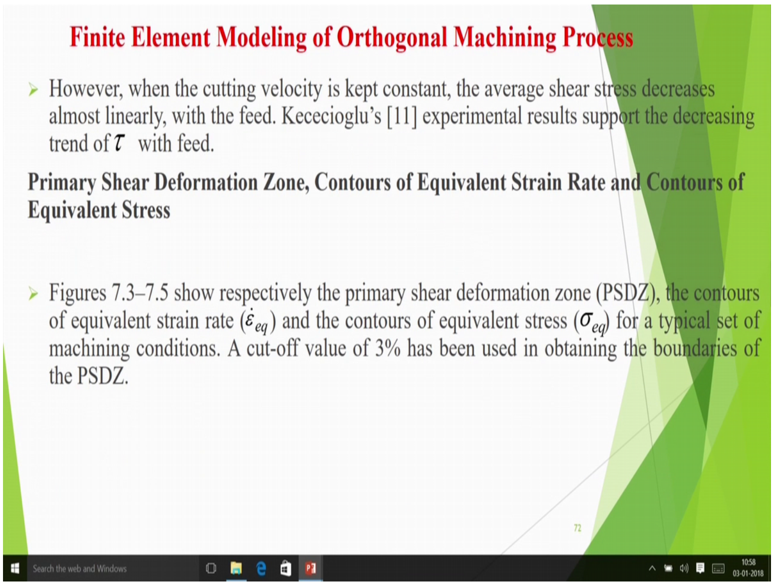

And when the cutting velocity is kept constant then the every shear stress decreases linearly with the feed and then we obtain the contours of the equivalent strain rate these contours have been planted here.

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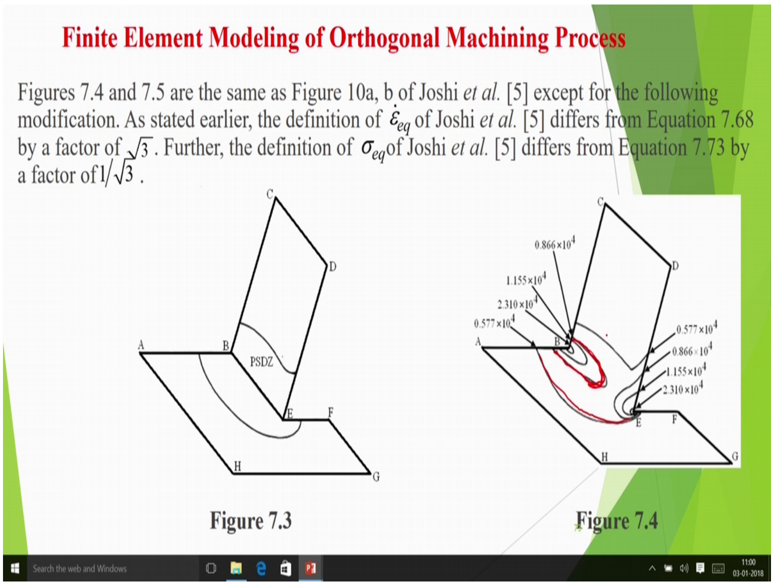

So, these contours are presented here this is primary shear deformation zone suppose this one we have taken this was I think experimental one here that experimental one. And this we have got the contour shear that this one is say here you are getting equivalent strain rate point 0.77 into 10 to the power 4 here. It is this much it is very high here it is somewhat here also it is high then it is somewhat it is here two point three into 10 to the power four like that. So, we have these values and we get the contour; that means, contour meaning is that from here in this region from here to here one particular equivalent strain is there here one particular strain is there like that these contours are plotted.

So, this way that we can get some information about the cutting mechanics, but even if you do this analysis usually it will take lot of time it is not that it is very fast, but it can give you all the insight about that that where these strain rate may be high where the stresses may be high such type of things are done ok.

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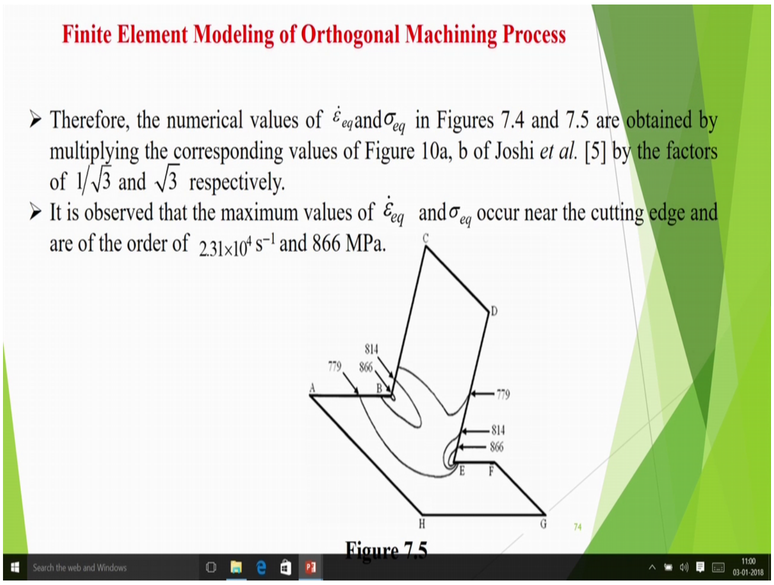

So, now these are the equivalent stresses and there are it is observed that the maximum value of equivalent strain is 866 800 66 mega Pascal 866 mega Pascal is this 1 and then this is equivalent stress contour. That means, here closed type of thing here you are getting less, but here you are getting more and here also you are getting more this one.

So, this is done. So, this much I have told about that I have given some exposure about the finite element model of metal cutting also in more detail you can see from the published papers.

Thank you very much.