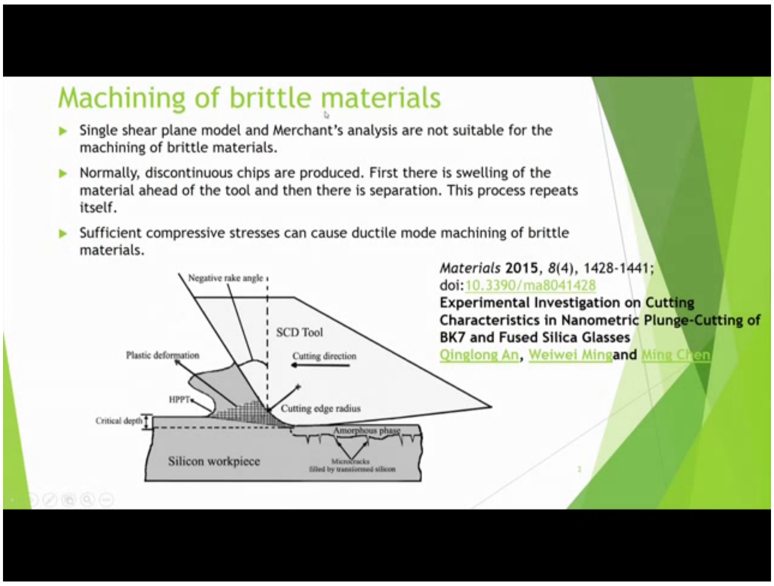
**Mechanics of Machining**
**Prof. Uday S. Dixit**
**Department of Mechanical Engineering**
**Indian Institute of Technology, Guwahati**

**Lecture - 08**
**Measurement of Shear Angle**

Hello students. Welcome to the course on Mechanics of Machining. This is the 8th lecture and today, I am going to describe experimental study of orthogonal cutting and some other models of machining. You have already seen one model of Ernst and Merchant. So, Merchant model is very popularly taught in the undergraduate classes. It is also in most of the books. But there are some other models also we shall discuss about those models and we will tell how people have conducted experiments to find out that whether the chip forms in a particular where we understand or it is something different.

So, lot of efforts were made in the 20th century we will summarize these and we will provide some directions so that if you want to investigate these processes further then, you can make use of the already existing knowledge and work on that.

(Refer Slide Time: 01:37)




So, first let us come to the Machining of brittle materials. Why I am starting my discussion starting from the brittle materials? Because in the last classes you saw Merchant's model and use single shear plane assumption in which there was assumption that the chip is continuously formed.

So, it is coming out of that in most of the ductile materials, this type of behavior is expected and you saw a continuous type of chip like here in this figure. But brittle materials will not deform in that way; they are hardly any model on the brittle materials. Most of the people concentrated on the duct machining of the ductile materials and they assume the continuous chip formation. We know that the chips can be segmented also; they can be discontinuous. But such type of efforts are limited.

So, single shear plane model and Merchant's analysis are not suitable for the machining of brittle materials actually. Normally, discontinuous chips produced in that case. So, first there is a swelling of the material ahead of the tool in the brittle materials. There is a compression material little bit swells and then, there is a separation. Because if it compresses here, then it reduces some shear stress and then it will be very weak in the shear it will separate and this process repeats itself.

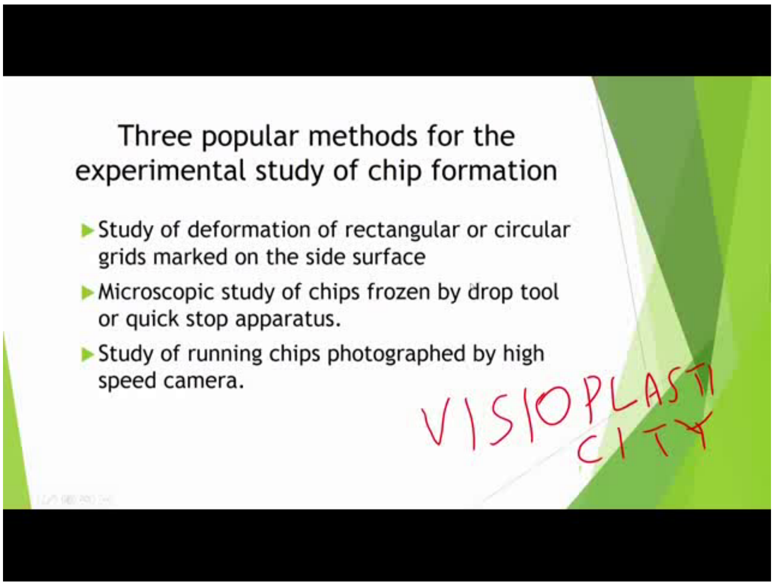
So, there is a compression. Due to that there is a shear and due to shear there is a separation this continues. So, you get this continuous type of chip set, but if we apply sufficient compressive stresses, then sometimes even the brittle materials they behave in a ductile manner. So, they display the ductile model of machining. There is a I have taken one photograph form a recent paper published in materials. It is on experimental investigation on cutting characteristics of Nanometric Plunge-Cutting of BK7 and Fused Silica Glasses.

So, these glasses are actually brittle, but we have done very small depth of cut. Suppose, this is real stone this is a small depth of cut critical depth. It is in nanometer range and we have got here, it is amorphous phase is there and there are some micro cracks and here, we have used single crystal diamond tool. Single crystal diamond tool with highly negative rake angle, you see that here rake angle is negative. Cutting velocity is in this direction, cutting velocity is shown cutting direction is shown.

So, perpendicular to cutting direction is the reference plane; from the reference plane, this rake surface is inclined at an angle here it is inclined. So, here it is inclined this way that is why there is a negative rake angle ok. Earlier I made the figures in which mostly the rake surface, I had shown in this direction. So, it was positive type of rake angle, but now you are having negative rake angle and you can see the negative rake angle has made this portion really quite thicker. So, it is quite strong actually. So, that is advantage of that, but you see that now there is more difficulty for chip to flow because velocity is in this direction chip has tendency to tendency to write like this. But in this case, you have got another tendency; that means, it is turning back.

So, naturally that forces will increase, but here large amount of compressive stress is produced here and there is a lot of so plastic deformation. So, in this, this is called High Pressure Phase Transformation. So, because of the high pressure, there is some case transformation and the behavior becomes from brittle to ductile. So, still you can get continuous type of chip that is what this figure is demonstrating; that means, even the brittle materials behave in the ductile manner. But of course, that machining of the brittle materials is also considered for that there are very few models even till now.

(Refer Slide Time: 06:20)




Now, coming to the experimentally study of the machining process. Earlier effort for mostly concentrated on studying the orthogonal machining because earlier theoretical efforts was also in modeling the orthogonal machining processes. So, therefore, most of the experimentalist, they concentrated on orthogonal machining. Later on some oblique machining studies was also done.

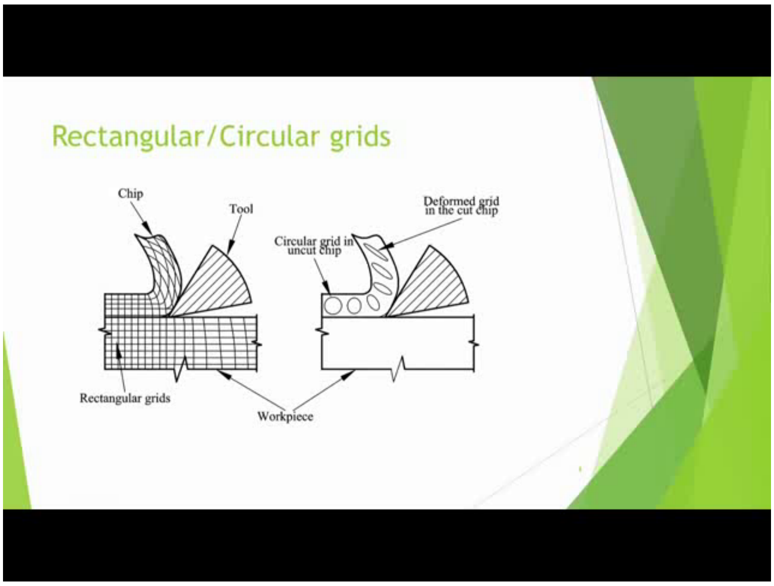
Three popular methods for the experimental study of chip formation are one is that Study of deformation of rectangular or circular grids marked on the side surface. Suppose, you are going to do the machining suppose this is a lock and here you can make this on the side surface, you can make some grids because side surface means external.

This you can observe by your eyes and you make the grids; these grids can be rectangular or circular and when the deformation occurs, then you see the deformed shape. So, that is what this type of study is done. In fact, in metal forming also people have done this type of study that marking the grids on the metal and then, deforming it and see that how the deformation takes place then calculate these strain etcetera.

Such type of study was called Visio-plasticity. Visio-plasticity that means, visual Visio Plasticity. So, this is. So, plasticity. So, Visio-plasticity was done, now here. So, it is same thing visio-plasticity study in machining. So, study of deformation of rectangular or circular grids marked on the side surface then, micro microscopic study of chips frozen by drop tool or quick stop apparatus. How it is like that?

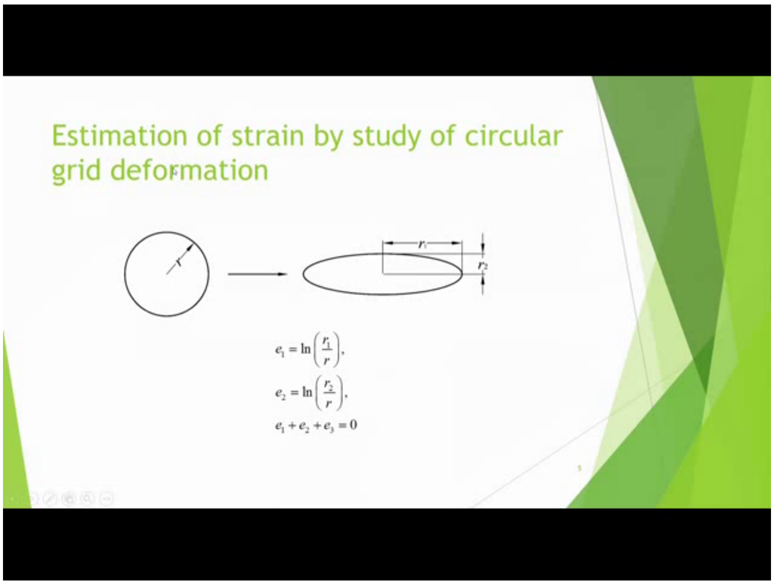
Suppose, you are doing machining and suddenly you stop the process and remove the tool, then in that case chip will remain adhered on the surface. If you can see that, then you can get some insight about how the machining was taking place that type of study was also done. Then, study of running chips photographed by high speed camera, high speed camera was put on and they saw how the chips are flying and how the cutting is taking place that behavior was studied.

(Refer Slide Time: 08:55)




So, here I am going to show here that first method that visioplasticity type of approach. Here, we have on the side surface, we have marked number of rectangular grids and to each cutting. So, you can really see that here these was some what is called square type of cell that has transformed that got distorted. It has become like a parallelogram or by rectangle and this type of shape has been obtained. Similarly, some people made circular grids, they made suppose there is a circular hole. Now after deformation it has become elliptic so, by that you can calculate this strain etcetera.

(Refer Slide Time: 09:43)



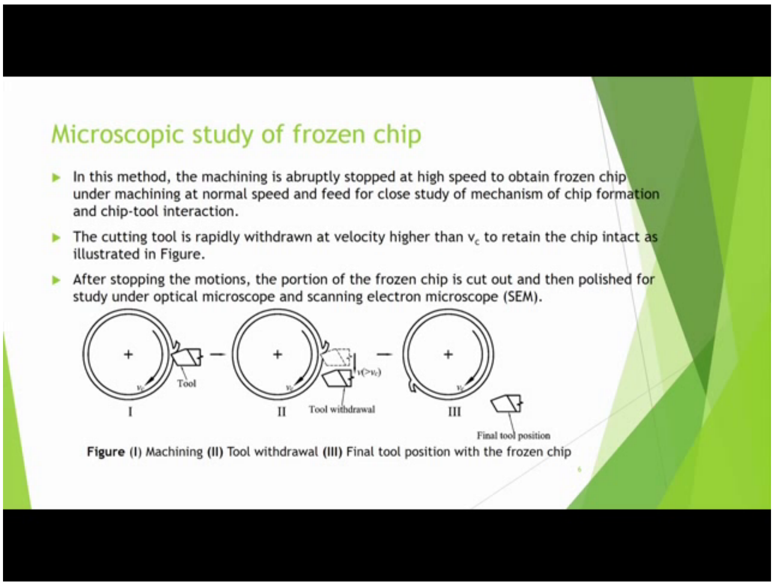

So, this is a simple example that suppose how you can calculate these strain. Now, suppose you have got one circular element of radius r, then it is transforming and it is becoming into ellipse. So, this has become into ellipse and this radius was r, but now here major radius is r 1 and minor radius is r 1; minor radius is r 2; major radius is r 1 and minor radius is r 2.

So, e 1 means strain in this direction in the major axis direction can be calculated e 1 is equal to ln r 1 by r. This is the true strain or (Refer Time: 10:30) strain or algorithmic strain. So, it is r r ln r 1 by r and then, e 2 can be calculated as ln r 2 by r. Then, in that case e 1 and e 2 2 components are known; that means, strain along the major axis and strain along the minor axis, these 2 strains have been calculated and you know that the during machining there is no change in the volume.

So, they are that is plastic in compressibility condition. So, by that volumetric strain will still be 0; that means e 1 plus e 2 plus e 3 equal to 0; e 1 is known, e 2 is known. So, I can find out e 3; that means, perpendicular to the screen what will be the strain that also can be obtained.

So, such type of analysis can be done and you can estimate that, what type of strains you are getting and what is coming by theoretical model these studies have done and now, coming to the other type of study that is Microscopic study of frozen chip.

(Refer Slide Time: 11:31)

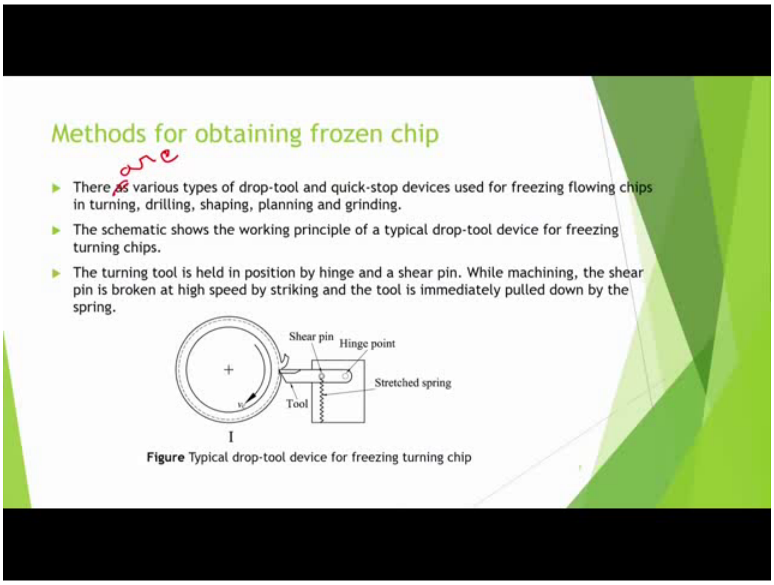



So, here schematically shown here that this is tools and machining is taking place. They certainly we provide very high velocity to this v is much greater than v c cutting velocity and tool withdraws and then, you get some chip which remains adhered in the work piece that type of thing and that chip we can study.

So, in this method the machining is abruptly stopped at high speed to obtain frozen chip under machining at normally speed and feed for close study of mechanism of chip formation and chip tool interaction. The cutting tool is rapidly withdrawn at velocity larger than v c to retain the chip intact as illustrated in figure; that means, your velocity must be much higher, even the chip velocity.

And after stopping the motion the portion of the frozen chip is cut out and then, polished for study under optical microscope and scanning electron microscope. Also you can study you can get more details in scanning electron microscope; how much material is transforming, what must be the conditions you can get some idea.

(Refer Slide Time: 13:02)




So, this study is was also by researchers. Another setup is like this here. You have got machining, suppose you are doing machining on a rake machine, your tool is there and tool is attached to with this base or fixture by means of a shear pin and it is hinged at this point. Now suppose machining is moving on, then suddenly somehow we break the shear pin; either by physically hitting by hammer or automatically by some moving thing. If we break the shear pin, then this will be detached and hinge is there.

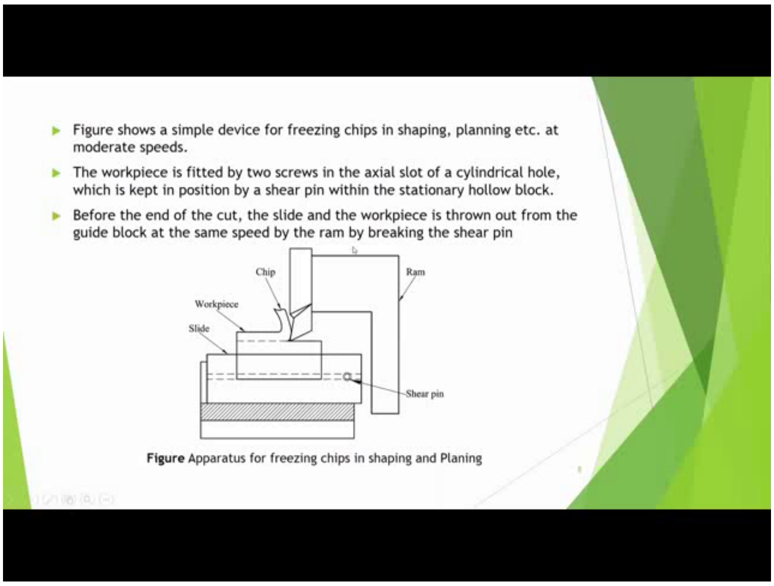
So, it will come down. So, there will no longer be cutting earlier it is tool is like rigid. Once I have broken the shear pin, its connection with fixture gone; that means, it will hang out, it will hang down. And then, you can see the chip will remain attached with this done; that means, half cut type of thing will be shown here.

So, this you can also do that because tool will be basically there is one spring is also put here. So, that spring will itself pull the tool actually once very fast. So, there are various types of there are various types of drop tools and quick stop devices used for freezing flowing chips in turning, drilling, shaping, planning and grinding. This is there are; yes.

Now here, the schematic shows the working principle of a typical drop tool device for freezing turning chips. It is called drop tool type of thing because tool drops down turning tool is held in position by hinge. This is hinge and shear pin; shear pin is here, it goes perpendicular to the spring. While, machining the shear pin is broken at high speed by striking and tool is immediately pulled down by this spring.

So, this type of device also has been used. They have part of making this type of setup. Now, I am giving other ideas of studying.

(Refer Slide Time: 15:14)

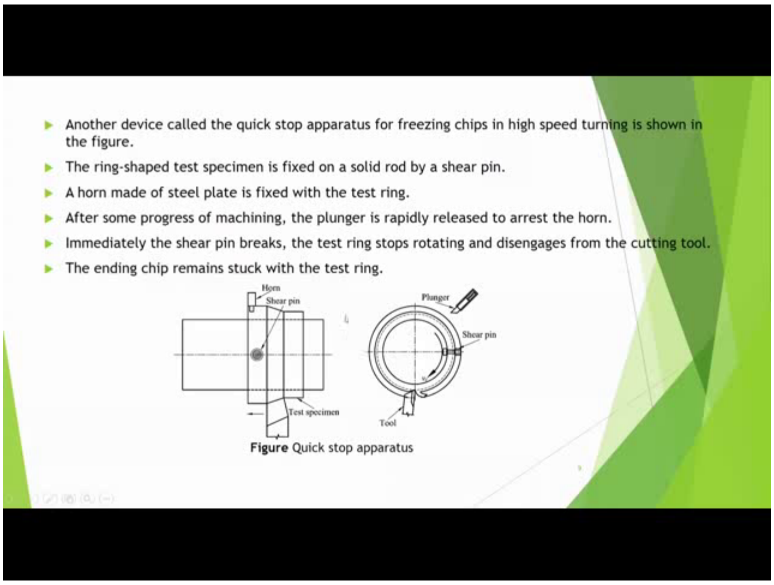



Now, here you see that it is a study on the simple machine. Your tool is there and there is a cutting is going on and then, you are having chip is flowing like this. Now it is this is a simple device for freezing chips in shaping and planning etcetera at moderate speeds. So, in this case work piece is fitted by 2 screws in the axial slot of the cylindrical hole; work piece is fitted here, which is kept in position by a shear pin again.

Again, there is a shear pin here in this slide. Now before the end of cut side and the work piece is thrown out from the guide block at the same speed by the ram by breaking the shear pin. So, suppose you are doing the cutting. So, machining is going on and then, this portion is coming that there is a ram this projected portion here is coming this will go and it will hit this one, this portion.

So, the shear pin will be broken and then the slide and work piece will fly away and then, they will observe that how was the cutting process taking place. So, this type of method also has been used. This is apparatus for freezing chips for shaping and planing such type of study was also done.

(Refer Slide Time: 16:39)



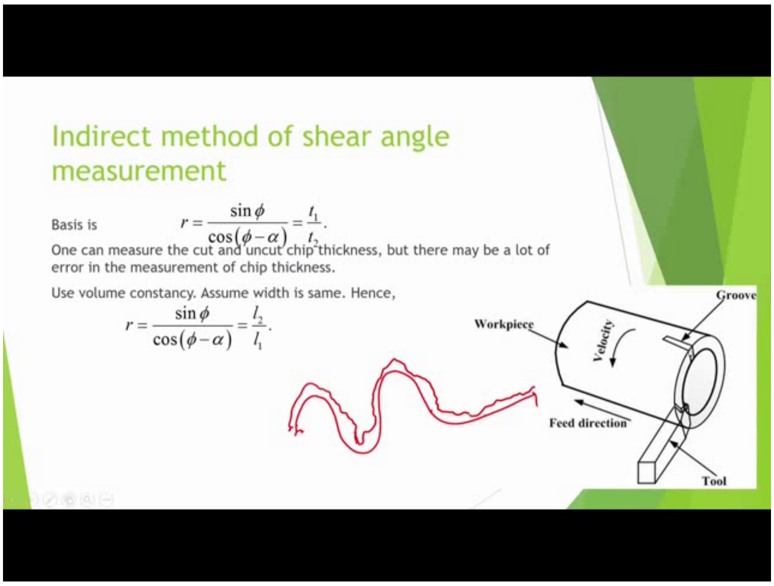

Now, another device called the quick stop apparatus for freezing chips in high speed turning is shown in this figure. In this suppose, 1 ring is rotating; then suddenly from a plunger, there is something attached with the fixed pin and then the plunger comes and enters this hole. As soon as the plunger comes and enters the ring stops rotating and it was attached to by another that job by means of a it was attached with these spindle by means of shear pin. So, shear pin will get broken. Once the shear pin gets broken everything becomes loose and there is no rotation and it is stops.

So, in this ring shaped test specimen is fixed on a solid rod by a shear pin. A Horn made of steel plate is fixed with the test ring. This is the horn here. After some progress of machining, the plunger is rapidly released to arrest the horn. So, plunger goes and arrests the motion of the horn; after some progress of machining the plunger is rapidly released to arrest the horn.

Immediately the shear pin breaks, the test ring stops rotating and disengages from cutting tool. The ending chip remains stuck with the test ring. So, such type of things, were also done ok. I am not showing the very detailed drawings, but you can get the idea and you can think of making the similar type of devices. It is not that when you make a setup, it can be a exactly same.

But you can get the idea that these are the ways that suddenly the cutting process can be stopped so that you can see in abrupt manner how the cutting was taking place. But you have to do at very fast speed, otherwise the chip will detach that is what required here.

(Refer Slide Time: 18:38)




Now, I am coming to this one; some discussion about indirect method of shear angle measurement. In fact, in most of the undergraduate courses on machining, this type of experiment is done also in the lab. Usually, the students try out this type of experiment in their third year by how to measure the shear angle. So, basis is that we have already derived that relation in the orthogonal cutting that r is equal to sin. Sin phi divided by cos phi minus alpha; where, phi is the shear angle and alpha is the rake angle and this is cutting ratio.

Cutting ratio is inverse of the compression chip compression ratio and it is r is equal to t 1 by t 2. You know that t 1 is generally smaller and t 2 is larger than t 1. So, t 1 by t 2 will be less than 1. So, one can measure the cut and uncut chip thickness. Suppose, you can do the machining, you know what is uncut chip thickness? Suppose, you are doing orthogonal machining and you side cutting is angle 0.

In that case speed becomes uncut chip thickness; that means, t 1 becomes f and if even if there is a side cutting is angle is also there, then also you know that you can have t f into cos psi; that means, side cutting edge angle and you can find out that. All these things I have told in the last classes. So, you can measure very easily t 1 and after that you have done the machining, then you can measure the t 2 also.

It may not be given you have to measure that several places, but that surface is also very rough actually. So, there may be lot of difficulties, surface that chip thickness itself may not be much and then there are unevenness because it comes at a high temperature and due to heating there is oxidization, the surface has become totally rough and error in the measurement itself may be very high compared to the thickness of the chip. So, you have to find out some other ways.

Other way is that you can use volume constancy relation; if you assume that width is same, then r is equal to sin phi divided by cos phi minus alpha and I can write is instead of t 1 by t 2, I can write it l 2 by l 1. Here, l 2 is the length; that means length of the chip and l 1 is the uncut chip length; this is l 1. So, here you have got because l 2 times t 2 will be equal to l 1 times t 1.

So, from that it has easily come that t 1 by t 2 is equal to l 2 by l 1. Now as for as l 2 is concerned, how will measure l 2? Because chip may be curved, it may not be straight. Usually you have to find out some ways; you may be having, it may be chip may be suppose chip is like this; how and you have to say somebody has given you this type of chip.

Now, you have to find out which may be suppose you take very small thread and in that thread, you just attach with this means like this parallel way looking tip attach in this side. I am just showing execrated and this thread will go like this and cut it here. You see that here thread is starting here, ending here, then after that straightened thread and you measure. It is a length, like that you can measure; otherwise you have to measure may be segment by segment. Here, you measure up to this, then up to this you have to. Here, it is straight like that whatever it may.

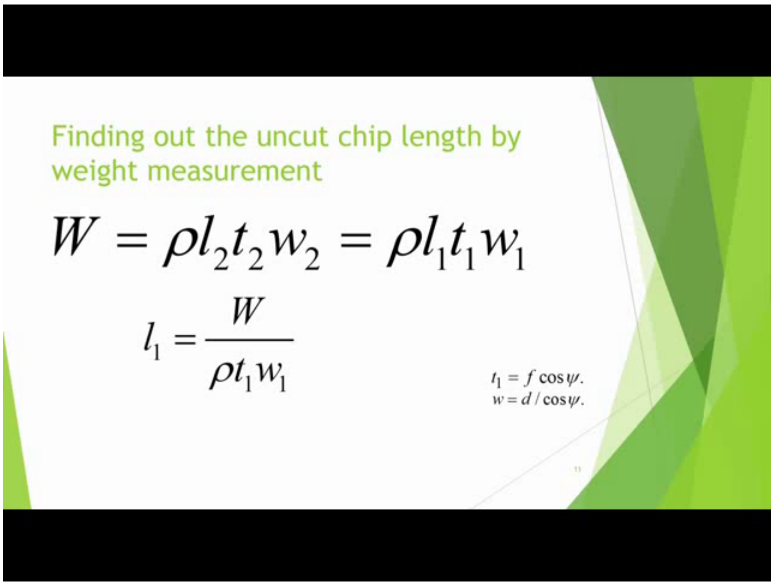
Otherwise, when you take the photograph; if you take magnified photograph; nowadays, it is possible to take by your mobile camera also and then magnified photo, you have taken. Then, you can take it in some editor like photo shop editor etcetera then you can make; any software you can take and then you can measure its length. In that software also all these type of things can be done.

Whatever way you prefer, you can do that and you can measure basically the chip length. But how should I measure the uncut chip length means to which uncut chip length this belongs that becomes the issue. So, that is why people do like this also that suppose, they make a small group type of l so that when I am doing the machining.

So, I will get idea that here that it is basically one length is means this one. So, they say phi d; phi d becomes the length of this thing. If you do not make the group, then 2-3 rotations will take place and in between you will not get any indication and you will think that it may be of one rotation, but it may be result of 2 rotation. So, that is why people make here group, when they are machining here up to this one after that may be the chip may get broken or at least the group mark will be there like that they do.

So, one small group can be made this is another way of doing the things ok; otherwise you can make some sharp grid type of thing here that you can method you can work. So, there are various ways that you can explore, but this is one method by measuring the length. Because thickness measure, you can do by both the methods and then after that you compare that how is the result coming.

(Refer Slide Time: 24:27)




So, this is one method, then I am telling that how you can find out the uncut chip length by weight measurement. Suppose you do not want to make group etcetera here just measure the rate of this one. So, here that you have one formula weight of the chip will be equal to rho is the density l 2 t 2 w 2. W 2 is the width of the chip; t 2 is the thickness of the chip; l 2 is the length of the chip ok. Length will be shorter than the uncut chip length; that means, l 2 is smaller than l 1.

And since, weight remains same; so, same equation is valid here. Rho l 1 t 1 w 1. W 1 is the chip width, but usually w 1 will be w 2 in orthogonal machining. So, you have only this type of thing that l 2 t 2 is equal to l 1 t 1, but let us say at this stage, let me say some even if it may be different; does not matter that this is rho is the density. So, you can obtain l 1 is equal to W divided by rho t 1 w 1. So, W is the weight of the chip.

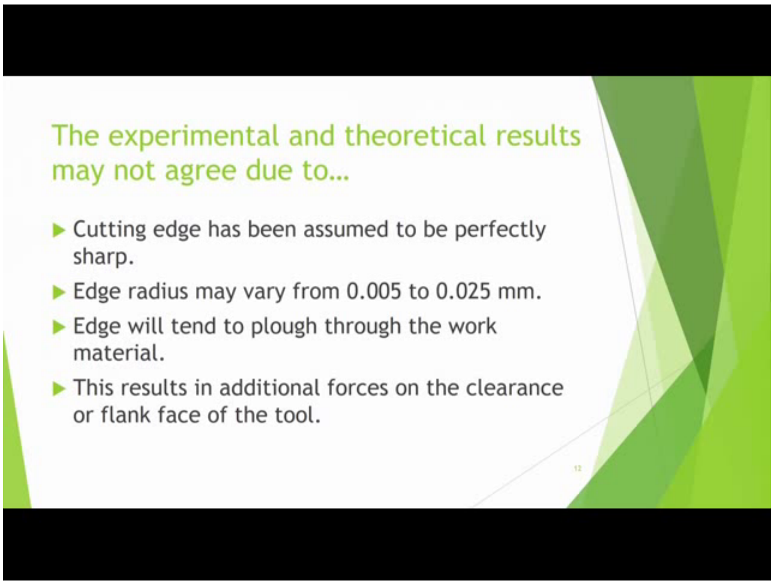
This I have measured by some chemical balance which has very high accuracy also. It is not less than milligram accuracy we can get. We have measured properly and then, so, W is known to me; rho is the density of the material that is also known, we assume that even after the plastic deformation or the chip has formed, chip has the same density as the material. And then, so you get a formula l 1 is equal to W divided by rho t 1 w 1.

w 1 is known because corresponds to depth of cut are if there is side cutting edge angle is there, then w is equal to d divided by cos psi and then, t 1 is equal to f cos psi. So, t 1 is also known to me; f is the feet; d is the depth of cut. So, t 1 is known to me because I am adjusting the machining operation w 1 is also known to me, rho is also known to me and w I have measured by my chemical valence.

So, easily I can find out l 1. So, l 1 has been obtained and this is once I know the l 1, l 2 I have measured by my thread. Then, r is equal to l 2 by l 1. So, I have found basically the cutting ratio and once the cutting ratio is obtained; then, I have found the shear angle by this formula and this using the single shear plane model ok.

So, this method is done students carry out this experiment easily.

(Refer Slide Time: 27:22)

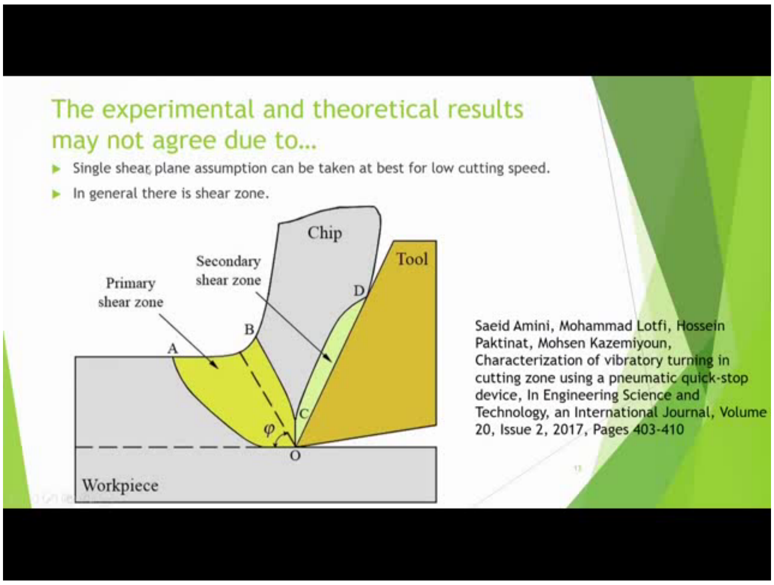



So, they get some idea. Then now, but experimental and theoretical results may not agree; there are reasons why we assumed in the merchant analysis that cutting edge has been assumed to be perfectly sharp. But actually the cutting edge will have some radius that (Refer Time: 27:45). No radius is been formed different; no radius is also there like this. I am talking this cutting edge.

Even here there is some radius. So, this where it will cost some ploughing; this type of thing has not been taken into account till now. So, edge radius may vary from 0.005 to 0.025 mm; up to this much it can be there and (Refer Time: 28:10) radius you know that it can be sometimes even 0.2 mm 0.4 mm like that.

Edge will tend to plough through the work material. So, edge ploughs the work material. This results in a additional forces on the clearance or flank face of the tool. So, if you measure forces by dynamometer, you will see that you will not get proper matching with the theoretical and experimental results. So, additional forces will come.

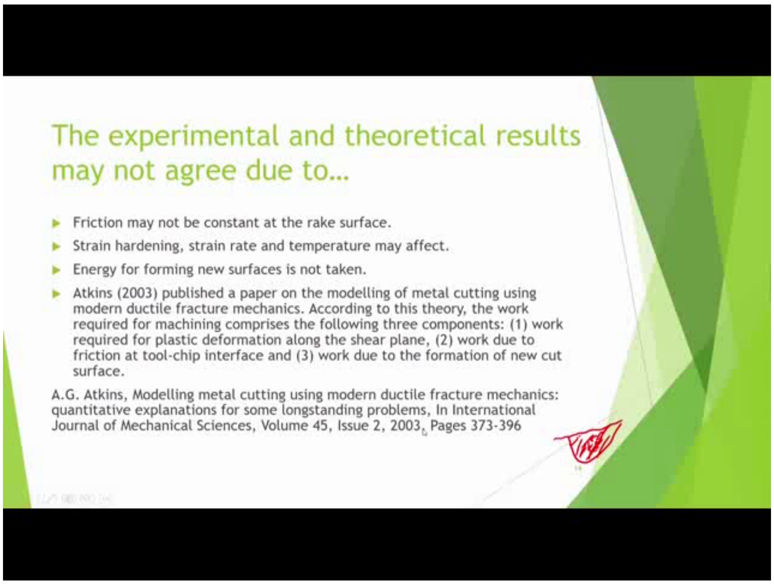
(Refer Slide Time: 28:39)




Now, our other problem is that Single shear plane assumptions can be taken at most at for low cutting speed, but at high cutting speed this type of assumption is not valid. In general you get a shear zone; you really do not get a plane across which the cutting is taking place, you get this type of situation. Here, I am showing that suppose you are doing the machining tool is moving, you have got the primary shear zone. This is primary shear zone. You see the large area is there that from here to here you are doing machining and roughly, we can assume that this is a shear plane and which is at angle phi.

It is just very proved assumption and then, you get a secondary shear zone because the chip is flowing over the tool surface. Here also there is a shearing and lot of distortion is taking place. This is called Secondary shear zone in fact, you have here also some zone that is tertiary shear zone; that means, flank face of the tool and the machine surface all these things are there ok.

(Refer Slide Time: 29:51)




So, now they may also not match; that means, experimental and theoretical results may not also agree due to these reasons that friction may not be constant at the rake surface. At the rake surface friction may keep changing near the tip it may be different, after that it may be different. Moreover that friction will not be constant even at (Refer Time: 30:16) cutting conditions also because temperature another things influence its behavior.

So, these issues are there, but we are taking simple Coulombs coefficient of friction. This is also one severe assumption. Then strain hardening, strain rate and temperature may affect results. Material gets strain hardened; material also gets some hardening because of strain rate and temperatures softens the material. Then, energy for forming new surfaces is not taking place.

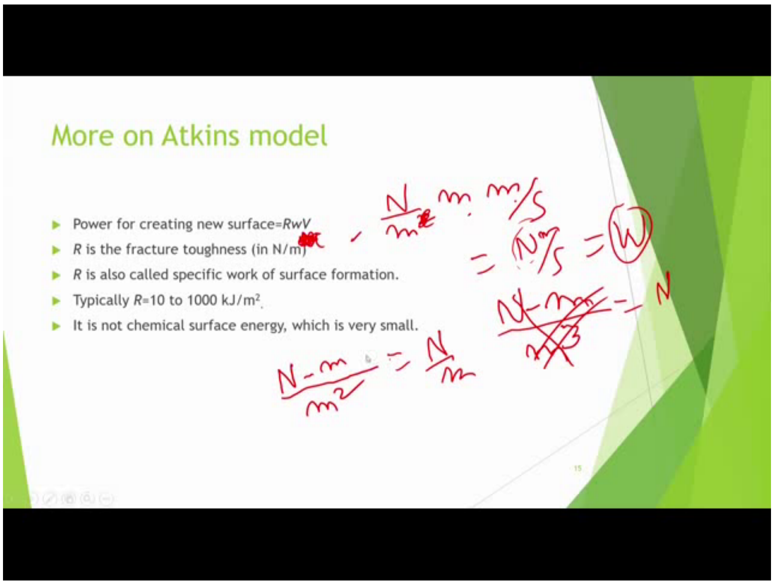
Because when you cut material, new surfaces are created. If you find out the chemical energy formation of the surface energy that is very very small; long back around 50's as saw I have shown that that chemical energy is very small. M C Shaw, he did lot of research and said that energy small, but I am not talking about that energy, chemical surface energy. I am talking different one.

Recently, Atkins in 2003, he published a paper on the Modeling of metal cutting using modern ductile fracture mechanics. Modern ductile fracture mechanics used. And according to that theory, the work required for machining comprises the following 3 components: work required for plastic deformation along the shear plane; that type of thing shear plane or shear zone that portion is very much there; then, work due to friction at tool chip interface that has to be also there otherwise the chip is not moved further and then, work due to the formation of new cut surfaces. That portion he has proposed.

Now, here this is not just the chemical surface, it is something that when you are creating the new surfaces. Now beneath that surface there will be lot of plastic deformation, lot of work will be done because whole thing is ductile. These things will also require some work that cumulative effect has to be taken. So, that type of thing is done. Suppose this is the surface, but beneath it there will be large stresses, those things who will take it into account these types of things.

Here, we are not noticing that plastic deformation. It is within that. Here, some amount of work will be done; that has to be taken. So, Atkins has written this paper Modeling metal cutting using modern ductile fracture mechanics: quantitative explanations for some longstanding problems. This was published in International Journal of Mechanical Sciences, Volume 45, Issue 2, in 2003.

(Refer Slide Time: 33:00)




Now, here he showed that power for creating new surface is represented by R w into V; w is the width and V is the cutting speed. So, multiplied by R; R is basically fracture toughness and this fracture toughness is shown here power; R is the fracture toughness. What should be that? This should be Newton meter square; this is not Newton meter because fracture toughness know; R is in Newton per meter square; that means, it is basically you just see the whether it is dimensional balanced or not. R has got unit of Newton per meter square right, then w has got unit of suppose I system meter and this is meter per second know. So, this meter square comes. So, you get Newton per second; that means watt. So, that is the unit of the power.

So, this is the power. So, it is R basically unit is Newton per meter square; that means, same unit as the stress. R is also called specific work or surface of surface formation. Why it is called specific work? Work has got a unit of Newton meter that is work and if we have suppose it is if I say per unit volume; how much work is done? I remove some volumes. So, this unit is meter cube.

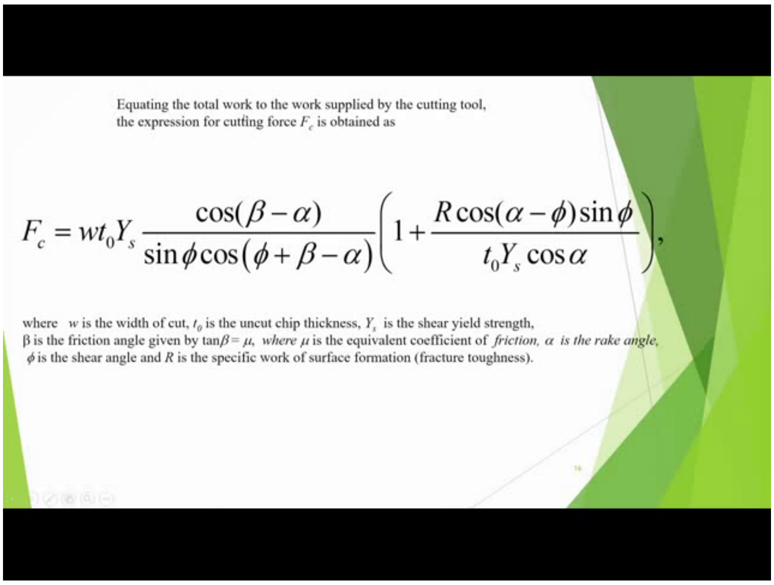
So, this is will be joule per meter cube or it will also come Newton yes per meter ok. So, this is this one. So, this is Newton per meter square meter know there is a mistake here; just let us do that ok. This unit is here, it is called that is Newton meter only. R is the fracture toughness in Newton meter because here power created this is Newton meter; this is Newton per meter; this is meter; this is meter per second.

So, this becomes Newton per second; Newton meter per second. Newton meter is joule, then it will come watt. So, this was correct only. So, this thing is this one, I am removing that; here, this one is correct. But it is good that by this exercise, we have understood critically that what are thing. So, this is correct only and this will be now here.

So, it is here, but here specific work means Newton per meter and this will be Newton per meter specific work of surface formation. So, here it will be meter Newton meter; that means, suppose we have got that something like Newton meter and divided by this one is the suppose you have got surface. So, surface is of area meter square. In the surface, we will take not the volume because there is no concept of the volume here. We have to take only the area per unit area how much is the work done.

So, Newton meter by meter square and then, it comes out to be Newton per meter that is and otherwise the same thing is also written as Newton meter is joule. So, we can say joule per meter square or it can be expressed in kilo joule meter square. So, typically this R is going from 10 to 1000 kilo joule per meter ok so, that type of thing is there. So, we have to remember that it is not chemical surface energy which is actually very small.

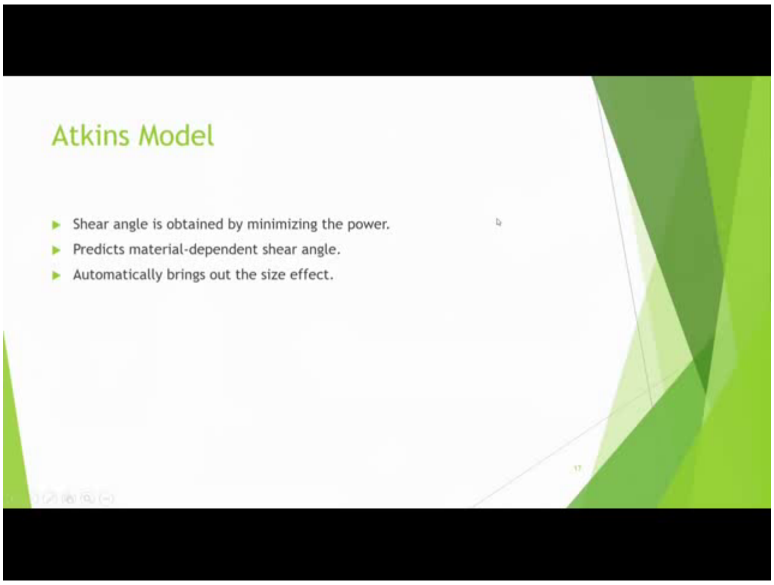
(Refer Slide Time: 37:05)




So, equating the total work on the work supplied to the work supplied by the cutting tool; that means, F c into V is equal to whatever 3 components of the forces power, he has described. Atkins equated that and he got this type of expressions. F c is equal to w c t 0 Y s. Y s is the, w is the width of the cut; t 0 is the uncut chip thickness. Y s is the shear yield strength. Beta is your friction angle; tan beta is equal to mu; mu is the equivalent coefficient of the friction and alpha is the rake angle; pi is the shear angle and R is the specific work of surface formation; that means, it is called structure toughness. If R is equal to 0, your relation is same as the we got in the Merchant's relation.

But here R term is coming here. So, you can see that this also affects the cutting force. So, cutting force is there.

(Refer Slide Time: 38:04)



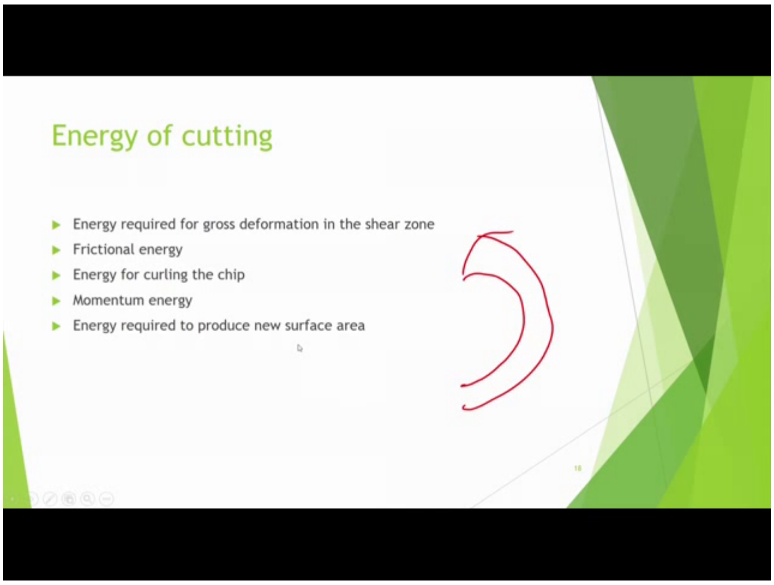

Now, what are the features of this? You can note down this type of expression by at equation, you can calculate, you can take some value of R, you will notice that shear angle in this case here also obtain the shear angle by minimizing the power. But now the whole thing is has to be differentiated. So, in the expression that R term is also coming.

So, therefore, it predicts material depend shear angle. Earlier you got the expression for the shear angle which was dependent on the rake angle and it was dependent on friction angle, but material con was not coming in that first solution of merchant. In the second solution, some material thing was coming because you use Bridgman relation.

But here, even if you take the first solution, then you will get material dependent shear angle because R term is also be there and it automatically brings how the size effectives. You see that if the size becomes very small, then R is constant type of thing. Therefore, that force will be even higher proportionately that force will be higher that type of effect will come. Here, you can see in the bracket also if t 0 is very small then this term will be increasing.

Here, this term is proportional to w into t 0, but this portion is not proportional to w into t 0 because t 0; t 0 will cancels. So, that is why this effect will be there. So, size effect will automatically come, we do not have to assume that any other thing about the size effect here.

(Refer Slide Time: 39:46)

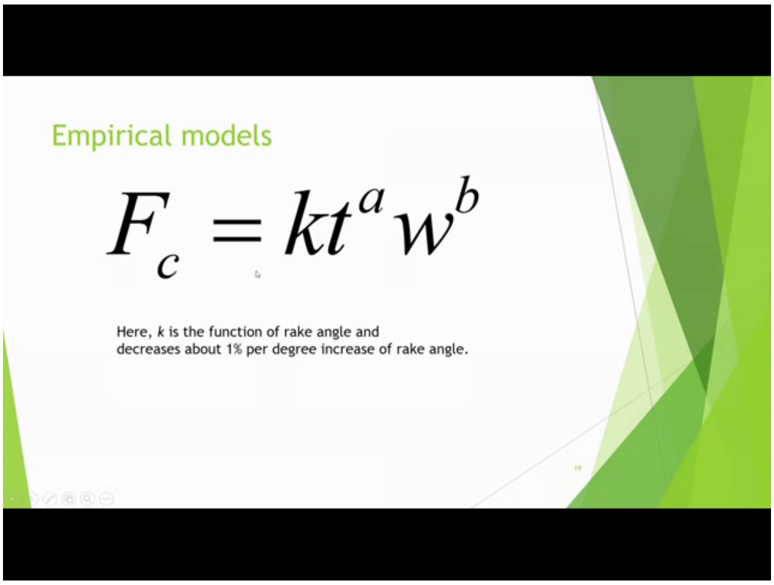



So, he has a done that type of model and that interesting results have been obtained. Sometimes it works well and then, let us now discuss about what are the other energies of cutting. So, one is that energy required for gross deformation in the shear zone; Atkins pointed out about that, but he calculated the energy by single shear plane model, only in that paper he said that it can be applied in the other for other models also.

Just he presented the concept because he wanted to highlight that what is the effect of the surface energy. Now, any other is the Frictional energy then, Energy for curling the chip. So, curling the chip means chip comes and then, you see that the chip gets curved like this like this type of thing.

So, that also has to be taken. Then, Momentum energy because tool is coming with momentum; at high speed machining it may get important and then, Energy required to produce new surface area that I have told. Remember that this is not the chemical surface energy, it is because you have to do lot of work, due to ductile behavior of the material in pulling out that thing that is why you have got this thing.

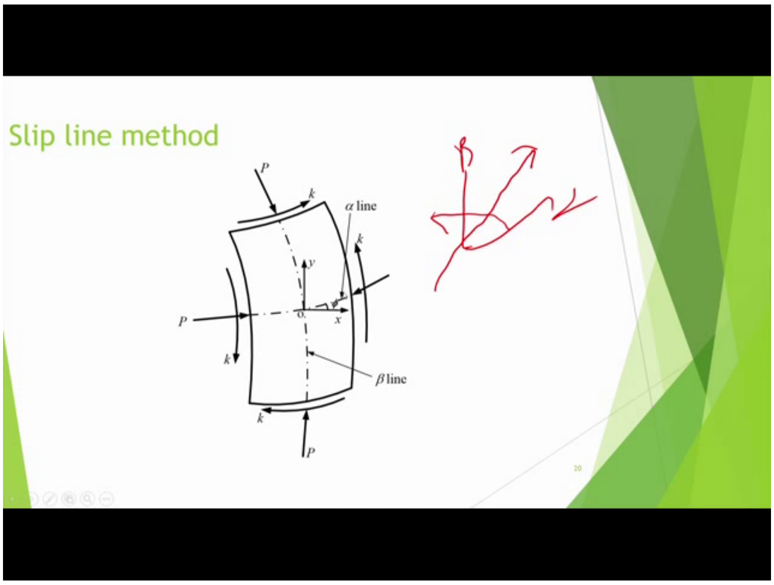
(Refer Slide Time: 40:59)




Then, there were some empirical models like that one model is like this F c is equal to k t to the power a and w b; exponents a and b; k is the function of rake angle and it decreases about 1 percent per degree increase of rake angle.

So, we see there is some dependency, but may not be that severe. But t of course, is affecting the F c chip thickness and over here it is uncut chip thickness because and w is the uncut chip width. Now here, these a and b are the exponents which may depend on so many things on material and tool combinations. So, these type of empirical models also have been used actually.

(Refer Slide Time: 41:48)




Now, I am going to tell you about something about the slip line method. Slip line method is the another method of practically understanding the metal forming and metal cutting process. But it is applicable only in the plane strain case, other cases sometimes people have tried to do also, but it is not that effective. Hardly, there may be 1 or 2 real papers. Usually it is a method for understanding the plane strain deformation and people observed that in the suppose in cutting process material is felling due to shear.

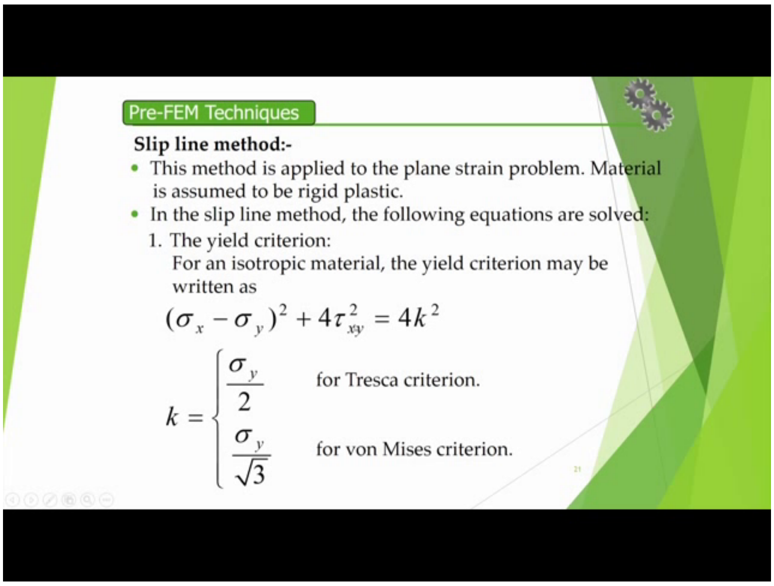
So, it is hypothesized here that the material deforms along this maximum shear lines. This is the in this direction here is maximum shear. So, this is k and here orthogonal you know that shears are so come in combination, if there is a maximum shear in this line; then in the orthogonal line also there will be maximum shear and on this plane there may be hydrostatic stress also. That means, some normal stress may also be there. It is not necessary that when the shear is maximum; then the normal stress is 0 that is not the situation.

So, these shear lines they are called alpha line and beta line. So, one line is alpha, the other is beta. Which one is called alpha and which one is called beta? Well, they have adopted a sign convention that if we make that alpha and beta line in this fashion; that means, suppose I say that this is alpha I, can say this is alpha and I can say this is beta line orthogonal way. Then maximum principle is principle is stress bisects them that means, maximum principle is stress is in this direction that is the convention.

So, that means, if you take in this counter clockwise sense alpha to beta, then maximum principle stress is here. That convention most of the people have adopted or you have to follow one convention. So, suppose here this is alpha line and this is beta line and we are showing that at this move movement, alpha line is making an angle phi with this some axis direction and after that it will be terminal also.

So, you know that alpha line can be straight or it may turn.

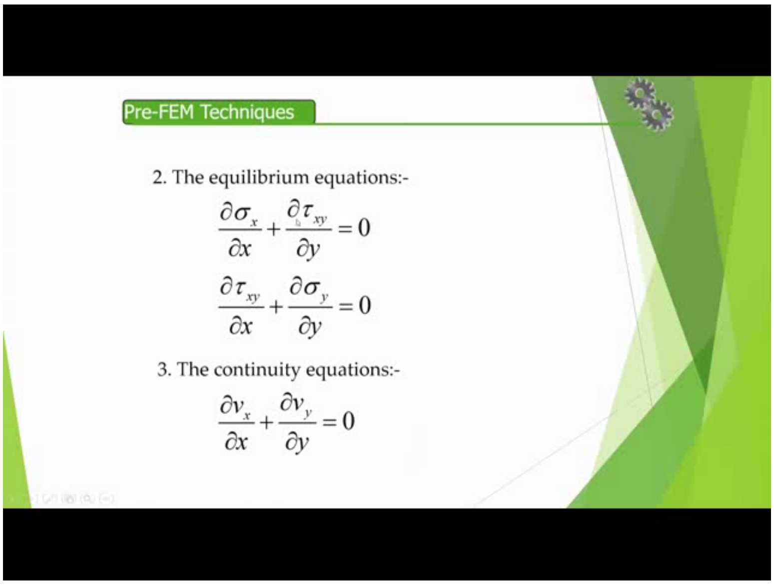
(Refer Slide Time: 44:17)




So, what is the basis of this slip line method? Actually this method is applied to the plane strain problem and in this, we assume that material is rigid plastic; that means elastic deformation is neglected. So, material in a is rigid and suddenly, it becomes plastic. Now in this slip line method, the following equations are solved.

We solve the yield criteria; that means, for an isotropic material in the plane strain case, the yield criteria may be written as sigma x minus sigma y square plus 4 tau x y square sigma x and sigma y are directly stresses along x and y direction and tau x y is this here is stress in that plane and this is equal to 4 k square; k is can be called as the shear yield stress. But, k is equal to sigma y by 2 for Tresca criterion. Sigma y is the yield stress and for von Mises criterion, if we use, then it will come out to sigma y by root 3. So, these are the criterion.

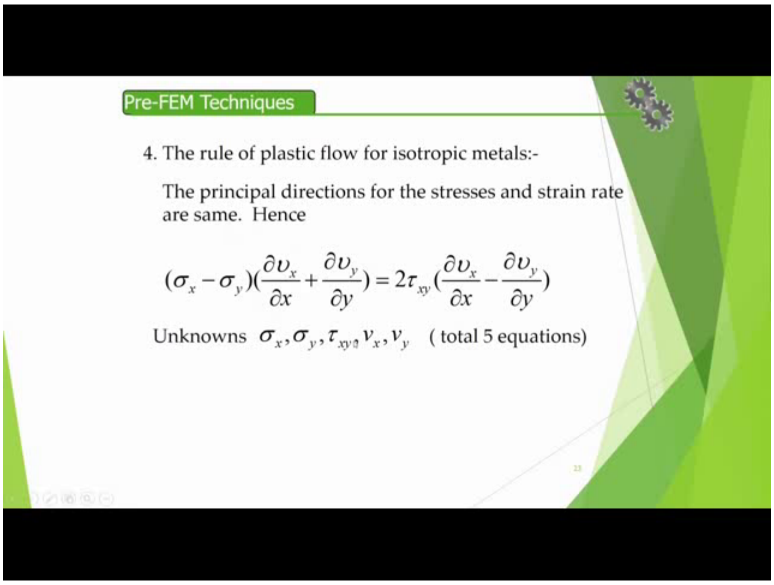
(Refer Slide Time: 45:30)




Now, go to next one; next set of equation equilibrium equations. Newton's laws have to be satisfied (Refer Time: 45:36). So, internally there are stresses and when you take some small tetrahedron and you take make the force (Refer Time: 45:45) all these things in 2 D case, then you will get this type of equation. For 2 dimensional situation you may get this type of situation. There is sigma and there are no other body forces etcetera.

Then we get this type of equation. This is x direction force variance, this is y direction force variance. So, these equations have to be satisfied; then you have to satisfy continuity equations that del v x by del x plus del v by y equal to del y equal to 0 that is continuity equation or you can say del v x by x basically small, this is the strain rate epsilon dot x x and this can be epsilon dot y y. So, epsilon dot x x plus epsilon dot y y 0 because epsilon dot z z is anyway 0; it is a plane strain situation.

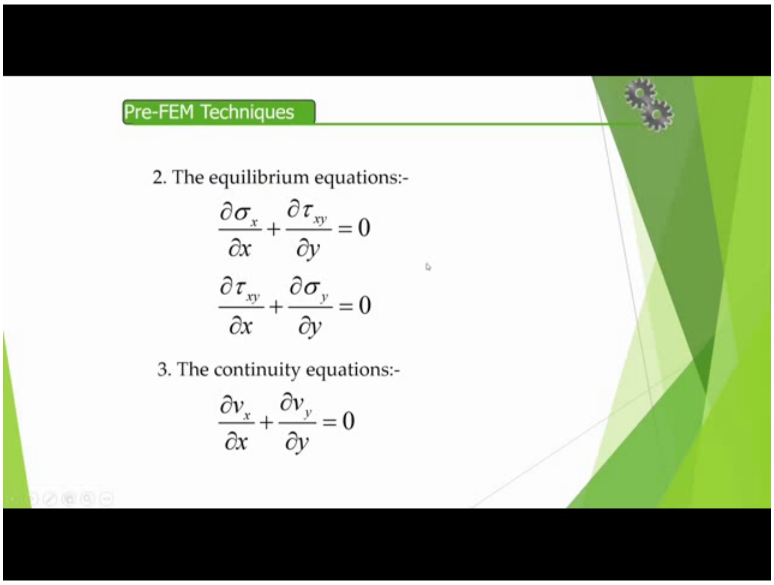
(Refer Slide Time: 46:40)




And then, the rule of plastic flow for isotropic material is this, principle direction for the stresses and strain rates are basically same. So, if you put that condition, you get this type of equation. You can say sigma x minus sigma y, this side you have 2 tau x y. If you remember you strength of material of course, you know that something 2 tau x y divided by sigma x minus sigma y that was the direction of that stress that was this one and here in the similar way, you have this is basically this one del v x by del x minus del y and this one will be like one is portion of the shear and other is the direct strains.

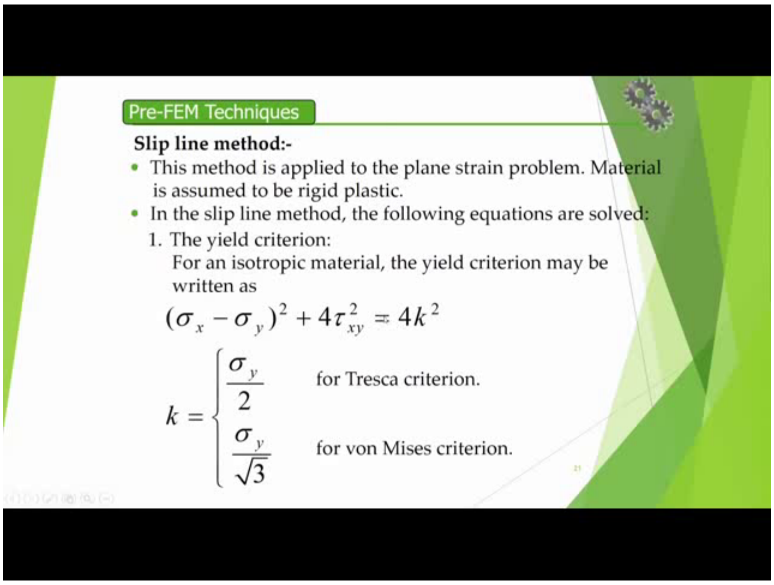
So, you get this type of variation, you can this means this relation I have just written, but I am not going to derive etcetera. So, therefore, do not bother about that just I am telling that the fourth assumption they have made that the principle direction for these stresses and strain rate, they are basically same.

(Refer Slide Time: 47:51)



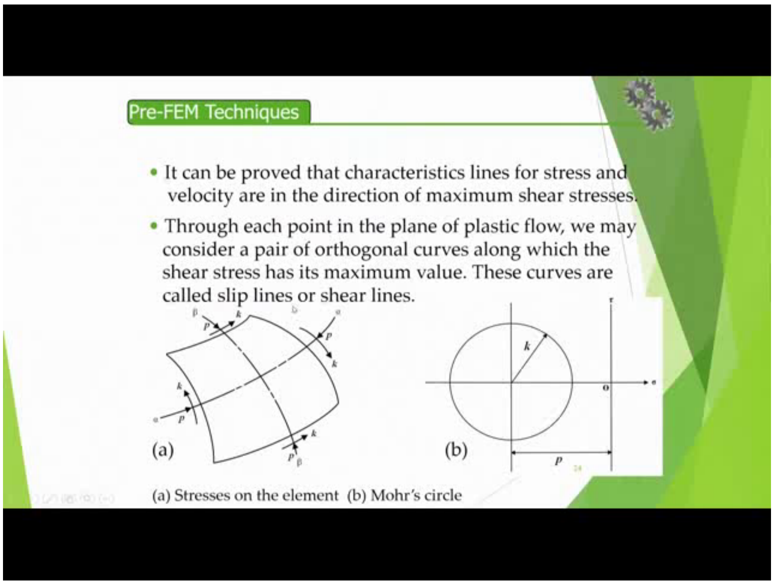

So, this set of there are 5 equations basically.

(Refer Slide Time: 47:53)




One is the you have seen the yield criteria; 1 equation, 2 equations are here; 3rd equation is here. So, total 4 and this is the 5th equation. So, now, there are 5 equations, but there are 5 for known’s also; sigma x, sigma y, tau x y, v x, v y. So that means, this system can be solved and in fact, this method is type of graphical method of solving these type of equations only.

(Refer Slide Time: 48:20)

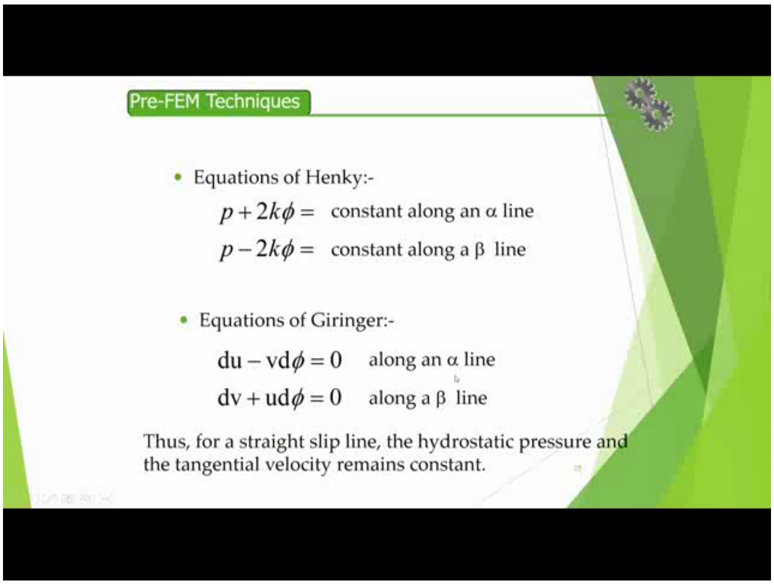



So, it can be proved that the characteristic lines for stresses and velocity are in the direction of maximum shear stresses. So, these are basically characteristic lines you come when you solve partial differential equations, then you get characteristic lines ok; but if you do not know, then do not bother about this. I just explained that slip line method as or can be mathematically related with the characteristic lines.

Through each point in the plane of plastic flow we may consider a pair of orthogonal curves along which the shear stress has its maximum value. These curves are called slip lines or shear lines. So, these are called slip lines or shear lines and you are having this type of situation. If I draw a more circle here, I will show like this because here compressive stress direct stress is P. So, P has come here and k is the maximum shear stress. This is here and also the yield criteria is satisfied.

So, therefore, this always there will be radius k and you will draw like this. So, that means, this is the maximum shear stress k and here it is like this. So, here you have got one principle stress as P minus k and other is P plus k type of thing. So, this can be shown like this.

(Refer Slide Time: 49:44)

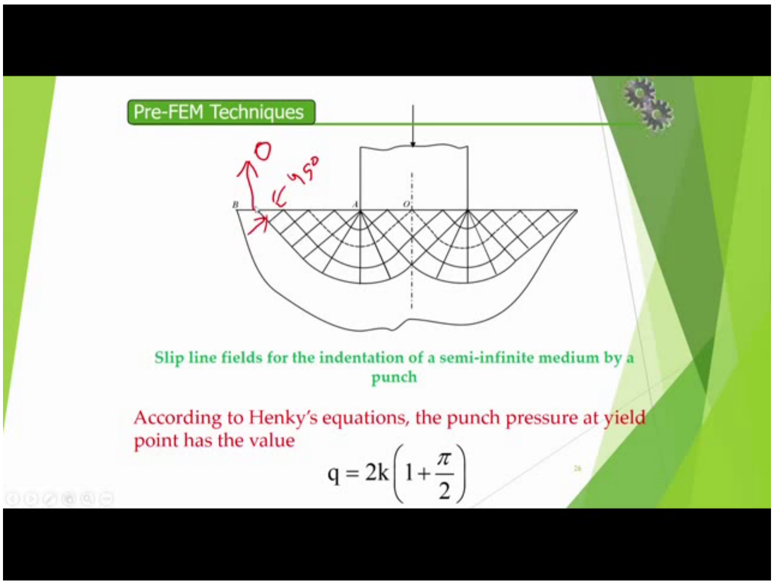



Now, if we make equal (Refer Time: 49:48) equations along alpha line, then we can derive this type of equation. Equilibrium equations you know we have represented in this way Cartesian system, but if we are also try to do along the alpha line, you get P plus 2 k phi equal to 0 and phi is what? Phi is basically the term in of that slip line that I have shown; phi means the terms; so, this is this one.

So, P plus 2 k phi; phi can be measured from any line. Important is not the phi, it is change in phi later on you will see that and this is P minus 2 k phi is constant along beta line and Giringer derived this type of equation for velocity relation that d u minus v d phi equal to 0 along alpha line and d v plus u d phi equal to 0 along beta line.

For straight slip line, if the slip line is just straight line, there is no turning; it is just straight line. Suppose a straight line is like this, like this. In that case, hydrostatic pressure will remain same because this relation is showing that P remains same and here d u will also become 0. So, tangential velocity is also remain constant ok; that means, just flow like that.

(Refer Slide Time: 51:08)




So, this is important thing. Now, if this slip line method has been applied in number of places say for example, I am now going to show slip line field for the indentation of semi infinite medium by a punch. This finds application at many place in geo technical field also suppose you have some here soil and then you are having some figure type of thing which is resting on that on the foundation, then you can see load is coming and this is a semi infinite medium. That means, in this direction there is one line, but the other direction, it is going up to infinity ok.

So, in this case I put a rigid punch, then it will deform and in order to find the punch load etcetera, they assume some slip line that deformation may be taking place like this that here, they will make this type of slip line. They made straight for simplicity. Suppose, they say how they make straight like this, this angle is 45 degree. This angle is actually 45, why? Because they say that this is a free surface. So, there is no vertical stress; that means, this stress is 0, this is 0.

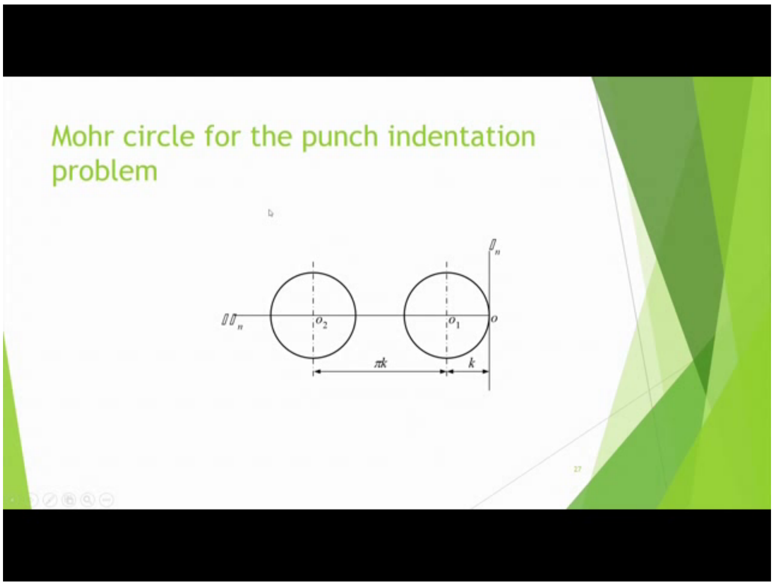
So, it is a free surface, smooth free surface. So, this is one principle stress and there will be maximum here in this direction there will be some stress. So, in this case, this slip line must meet at 45 degree free surface because that maximum shear stress you know bisects the principle stresses. You know that maximum shear stress is at 45 degree to principle stress. So, they make like this and they say up to this it is coming and from here to here, they have made and then here also the punch friction they have neglected.

So, it is a smooth one, since it is a smooth surface. So, tangentially there is no shear here. So, it cannot be maximum. So, if there is no shear on a plane that is principle plane. So, this is one principle plane is A o and therefore, again the shear stress should meet at 45 degree. So, they make like this and after that they may join it here by a centered fan type of structure, here they join like this.

So, this has come like this. So, this type of slip line field has been constructed and they you they can analyze also by the velocity diagram etcetera, photograph they can make that how it is going. So, people have done this type of work and they assuming this one and then they apply the Henky's equation; Henky's equation means these p plus 2 k phi p minus 2 k phi and here.

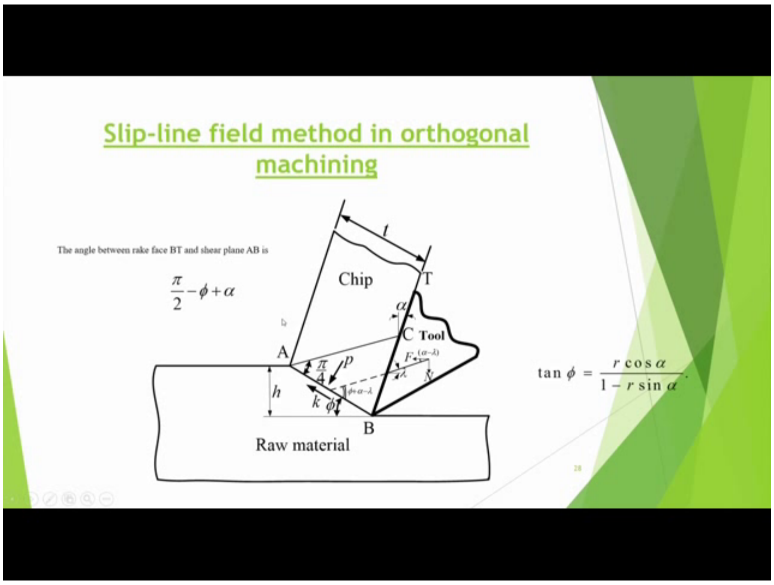
Here they say suppose here to here the stress is constant and then, suddenly there is jump and this angle if it is phi here they find out this angle is pi by 2. So, they put that type of thing. So, change in this one they find out p plus 2 k phi. So, they find out another p and they find out the hydrostatic pressure in this area. Then they find out punch pressure at yield point as the value of this. I am not telling in detail I am just giving you exposure about this procedure and 2 k 1 plus phi by 2.

(Refer Slide Time: 54:37)




So, they can make more circuit for different regions. Suppose this starts from this region; in this region this one principle is stress 0 and they draw it like this and in another one that because it is 2 by it is changing by phi by 2. So, as you know here 2 k phi. So, the pressure must change by 2 k phi factor. So, this is pi k. So, hydrostatic pressure has come here. Actually this central portion this is only the hydrostatic pressure. So, this is pi k and then again they draw the more circuit here and they find out this one is the maximum vertical direction that is t n and then they find out.

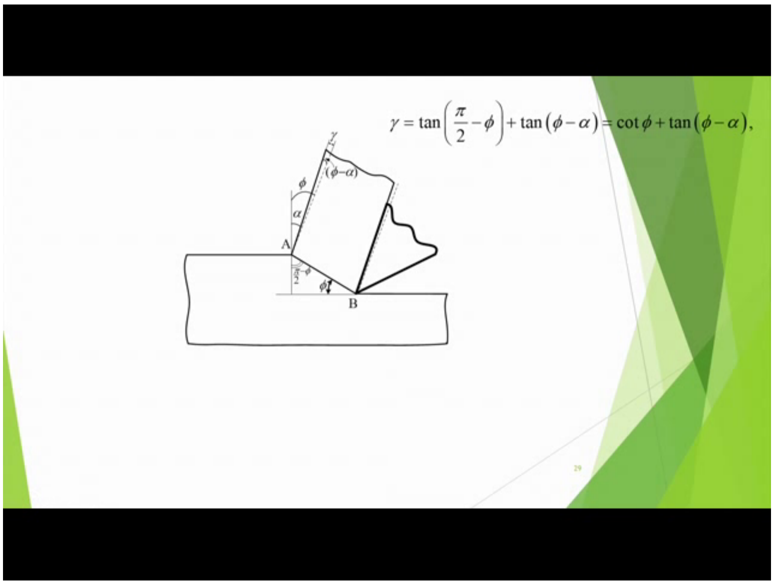
(Refer Slide Time: 55:17)




So, this has been done here and now this method has been obtained. Now I am showing the same thing that slip line field for are orthogonal machining also has been proposed. Here they have obtained this type of relation; there is a raw material here and they are making this chip line. So, they say maximum shear is along A B.

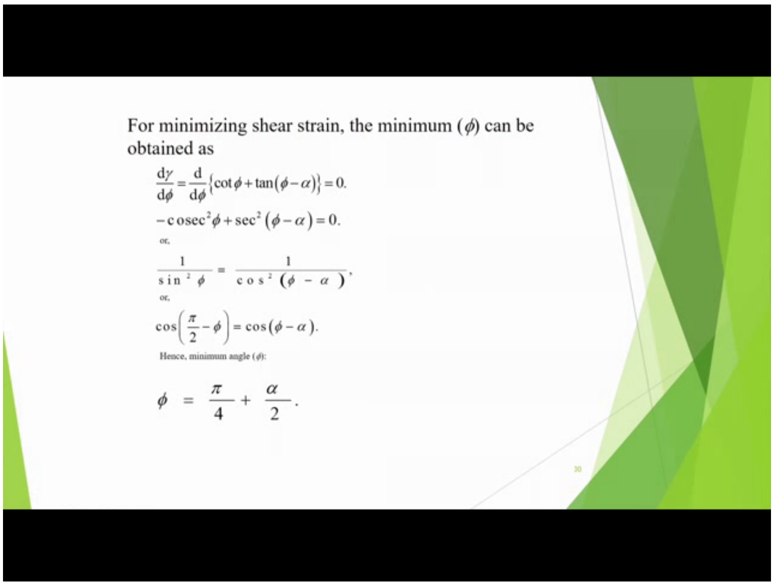
So, one slip line is this one and another they say that A C is the free surface type of thing. They assume that here A C is the free surface and therefore, this slip line should make angle pi by 4 from here and then they show this is k and this is p hydrostatic pressure acting on there and these angles have been shown pi; you can see angle between the phase BC and AB will be pi by 2 minus phi plus alpha and this relation is already you know.

(Refer Slide Time: 56:13)



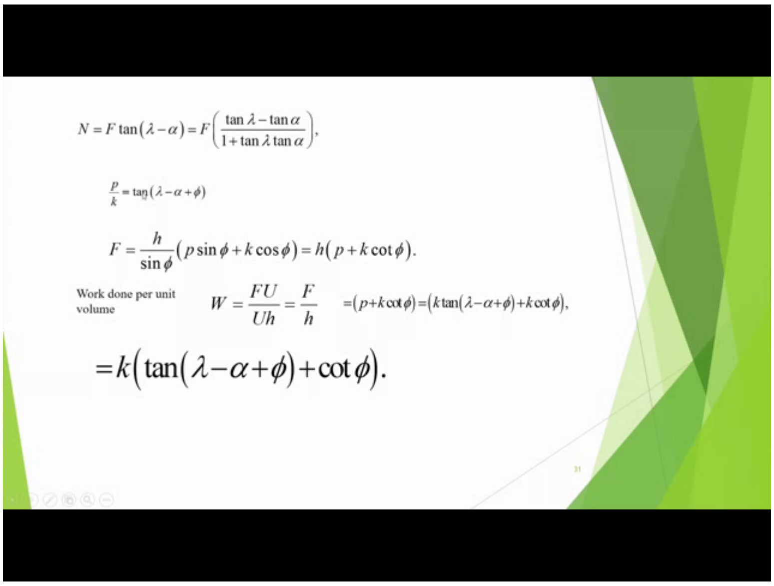

And then, this shear strain gamma shear; strain gamma I have already derived that it was cot phi plus tan phi minus alpha that type of thing is always there and for minimizing the shear strain, we have to differentiate with respect to phi. You get this type of expression; this also I think once more I have discussed also earlier.

(Refer Slide Time: 57:26)




So, 1 sin square pi and this is this one and they equate both the portion. So, you get the minimum angle phi equal to pi by 4 plus alpha by 2. So, that means, it bisects that this included portion. I think it would bisect this one ok. So, that you can you can check that pi by 4 plus alpha by 2.

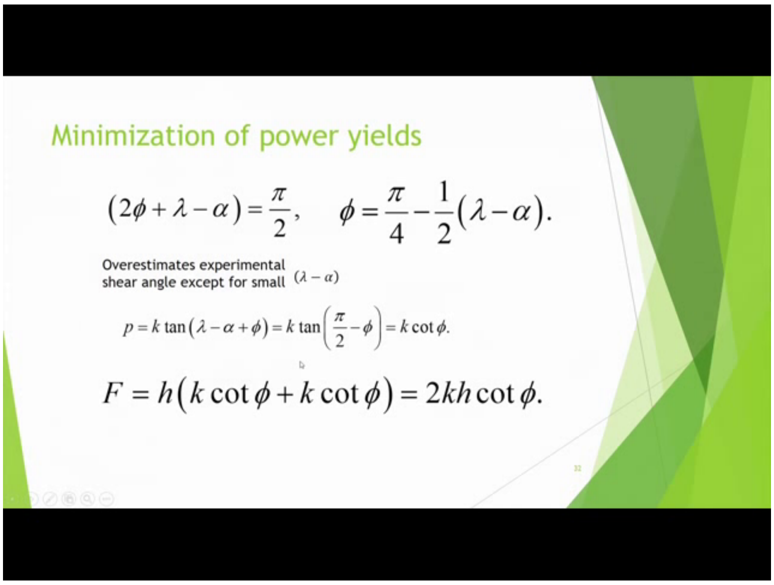
(Refer Slide Time: 57:01)




And then, there was reservation between N and F on the rake phase that was N is equal to F time lambda minus alpha this was also done. And if we have done this type of relation and here we also have obtain P by k is equal to this one because P is the force coming on this force per unit area. Of course, whether you say force are force per unit area. So, P by k was by Merchant's (Refer Time: 57:31).

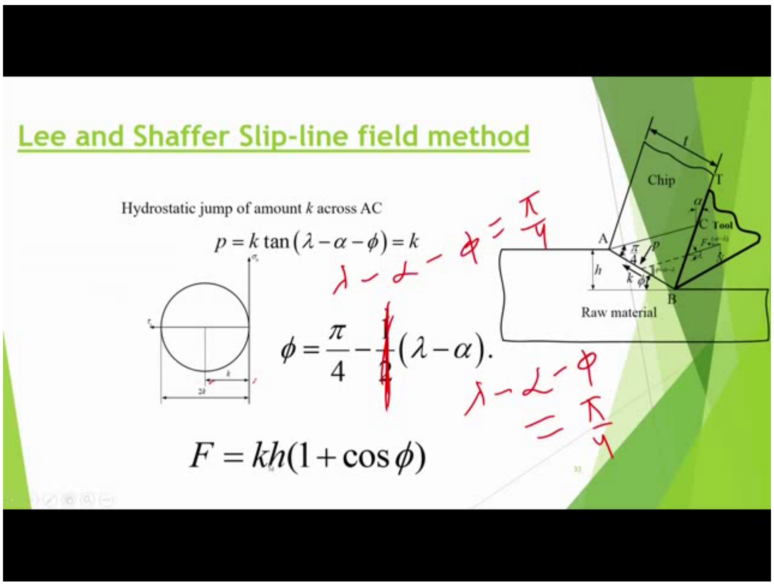
This was relation was there that is tan lambda minus alpha plus phi. So, then we say F is equal to this much that h times this one they put that that h is the cutting force horizontal force that was done and then, work done per unit area FU divided by U h per unit volume. So, FU by U h and then, it comes out to be like this. So, you get a basic relation like this work done per unit volume has come like this.

(Refer Slide Time: 58:02)




But you put a minimization of power if you minimize the power, then you get phi is equal to pi by 4 minus half lambda minus alpha; but this we have already done. I think I am just repeating this portion, but this over estimate experiential angle expect for small lambda minus alpha ok; that is why merchant proposed the second solution. But then, if I put p is equal to k tan lambda minus alpha plus pi, that becomes this much k tan pi by 2 minus phi is equal to k cot phi and then you will get F is equal to simply 2 kh cot phi.

(Refer Slide Time: 58:45)



Then, Lee and Shaffer proposed this solution that he said suppose this is a slip-line. So, it is a k and this is a free surface. So, simply draw more circles here and say that this in that case it is observed that p, which is nothing but the this portion; that means, this distance from here to here, here to here that is k. So, p is equal to also k, k tan and this relation is fundamental from merchant circuit. So, k tan lambda minus alpha minus phi equal to k and therefore, you get phi equal to pi by 4 minus half lambda minus alpha. So, you see from earlier thing that this one; that means, lambda minus alpha; so that means, this will be ok, you can see from here that this will be tan lambda rather I would say that we will get lambda. Lee and Shaffer will give lambda minus alpha minus phi that is equal to pi by 4 ok.

So, phi is equal to this one. So, this half portion is not there ok. I did cut paste from the previous one; in the previous one, it was pi by 4 minus half lambda minus alpha. In this case, you are having pi by 4 minus lambda minus alpha because here it is obvious this is giving me lambda minus alpha minus phi is equal to pi by 4 or phi is equal to 2 pi by 4 minus lambda minus alpha and if we put this type of relation here, then your F is coming kh 1 plus cos phi. Earlier your F was coming 2 kh cot phi by Merchant’s this one.

So, there is some difference between the: Lee and Shaffer’s slip-line method and Merchant’s method. So, this I have demonstrated to you; just I have provided the exposure. All though in most of the undergraduate classes, slip line filed method is not taught ok.

Thank you very much.