Mechanics of Machining
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Lecture - 07
Strains and stresses in orthogonal cutting

Hello students. Welcome to the 7th lecture of the course on Mechanics of Machining. Today we will be discussing about strains and stresses in orthogonal cutting. In the last lecture I developed the force relationships in orthogonal cutting. Forces are important because if you know the how much is the cutting force then you can decide about the power of the motor because, cutting force multiplied by the velocity kind u u the power of the cutting. Then you may get thrust force, thrust force is also needed because if you know that how much force is coming on the machine structure then you can design it accordingly.

But the forces are not the only thing we need to find out the stresses and strains. The stresses give the idea about the relative intensity of the force, force divided by area is the stress. Sometimes the force may be very high, but if the area is too large that means, the stress is not much and sometimes the force may appear to be very small, but the area is so small that the stress will be very high. So, stresses are needed for designing. Now, there are forces acting on the cutting tool in that case there will be stresses generated on the cutting tool these stresses we have to find out. Similarly strains, strains also use some idea about the deformation they give the idea about the relative deformation that is why strains and stresses are important in machining operation.

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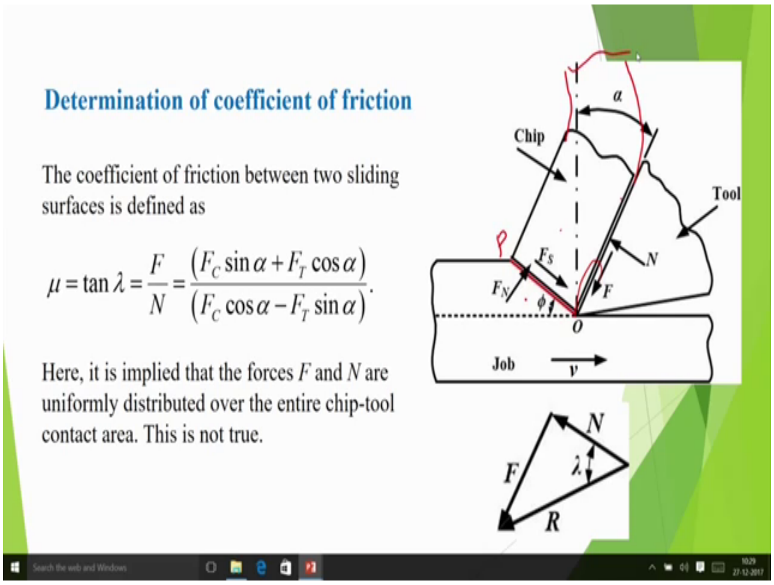

So, we will discuss strains and stresses in orthogonal cutting first based on the single shear plane model where we assume that there is a shear plane that is the this is suppose O P, O P is a shear plane although there may be shear zone in fact, and stresses are strains do not arise (Refer Time: 02:49). But here it in this type of model we have to assume that suppose at this location there is no strain and variation similarly here there is no strain variation, but suddenly across the shear plane there is a huge amount of strain variation that type of thing we have to assume.

So, let us again see the orthogonal cutting in which there is a tool and there is a job here. Now, it is we assume that the job is moving at velocity v and two is stationary. So, there is a relative motion cutting speed is v then the material here it is compressed and because of compression and other components of these stresses you generate shear stress also. We assume that shearing is taking place along that O P. So, in the O P this plane the material is getting sheared and in this situation there is a shear force F S acting here and then there is a normal force F N that is also acting on this that normal force F N is also acting on this shear plane.

This is the plane in which there is maximum shear stress and there is a normal one which kind the also called it is a hydrostatic stress basically it is a pressure. And the phi is the shear lining, phi is the shear lining as usual, (Refer Time: 04:27) on this is you have studied and then this one is the rate I am giving that means, tool F is inclined with the vertical direction. So, this is alpha and machining is taking place here.

Now, coefficient of friction between two sliding surfaces means, sliding surfaces means chip and tool. So, this one is actually nu is equal to tan lambda where lambda is called the friction angle and this is F by N, F is the frictional force and N is the normal force acting on that point resultant is R. This normal makes angle lambda with the R, are R makes angle lambda with N this is called friction angle, so F by N. And we have already told you by Merchant's analysis F kindly expressed as F C sin alpha where F C is the horizontal cutting force that means, in this figure it is horizontal that means, F C is the force N which is in the direction of the cutting velocity.

So, that is what that this is F C is in the direction of cutting velocity and F T is the perpendicular direction in this figure it is vertical. So, F C sin alpha plus F T cos alpha that is F and N will be F C cos alpha minus F T sin alpha, F C cos alpha minus F T sin alpha. You can see that F is the resistance that means, frictional resistance on the tool and that is proportional to means it is the linearly related to F C and also it is related to F T also increases then also this component is going to increase whereas, the normal force acting on the tool in this case if F C is increases then the normal component increases, but if F T increases then the normal component decreases.

So, it is assumed that the forces F and R, N are uniformly distributed over the entire chip tool contact area. Actually in practice this is not true, you may have more amount of the normal force in this particular area near the this side and then after that gradually it may decrease. But we are assuming that in this analysis that it is uniform and it is there is no variation of the normal component or there is no variation of other component here. So, this assumption has to be followed here. So, we can get only stresses in an every sense.

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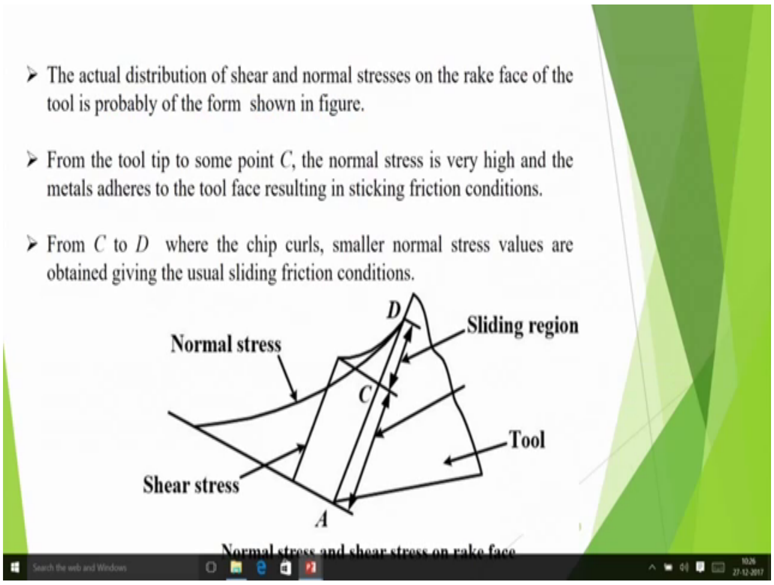

So, let us see that actual distribution of shear and normal stresses on the rake face of the tool is probably of the form shown in this figure. So, this is suppose tool in this you have got normally stress. So, at the point A that means, at the this just at the (Refer Time: 07:54) there is a maximum amount of the normal stress and after that normal stress keeps on decreasing at some point then it becomes 0 that means, the chip as chip rated. If there is no normal force there that means, chip cannot remain in contact chip gets separated at point B up to from A to D chip is in contact, so at D contact has condition is that contact has separated that means, this is the point D and this is normal stress variation more or less it looks like a parabolic type distribution.

As far as this shear stress is concerned in this particular up to point C it is a sticking region that means, chip gets stuck to the tool and in this there is a constant shear stress, in this region no longer the Coulomb's law of friction is valid. But after that there is a sliding region in which this chip is sliding properly and in this region that shear stress keeps on decreasing with the normal stress in this region we can see more or less Coulomb's law of friction may be holding, so that is what we have got shear stress region, constant shear stress region, and then after that we have got sliding region in which the shear stress may be proportional to normally stress.

So, from the tool tip to some point C the normal stress is very high it is because of suppose the normal stress is very high in that case the Coulomb's law may not be applicable and the metal adheres to the tool face resulting in some type of sticking friction condition. Metal may adhere on this face and other layer top layer may you know that is slightly slide on this one it is not that entire metal has adhered then of course, it will not be able to move also, but this layer is of course, may get stuck and other layers on the top may move also.

So, from C to D where the chip curls in this there is chip curling means it adopts this type of shape. You can see that some amount of curling here it is not shown, but we may have that this type of curling. So, chip may curl here and this one is from C to D where the chip curls smaller normally stresses values are obtained giving the usual sliding friction condition. So, this is normal stress and shear stress on rake face that has been shown.

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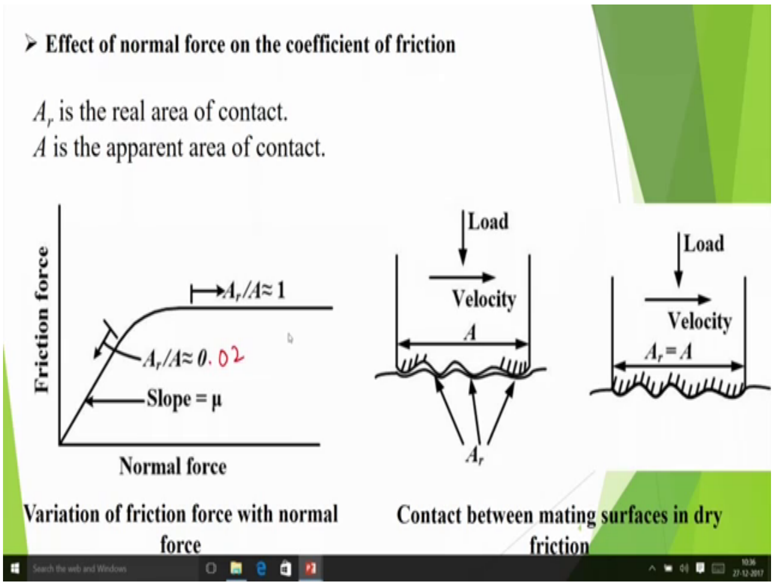

Now, usually it is not only in the case of metal cutting, but even in metal farming also this phenomena will be there that Coulomb's coefficient of friction may not be applicable it may not be constant. So, what happens to Coulomb's law is not valid. Coulomb's law says that the friction force does not depend on the area, but this may no longer the correct it may depend on the area also. So, it is here that if we show variation of friction force with normal force any general normal force is there and then we generate frictional force. So, if the normal force is very small then the frictional force is also small.

So, in this particular region more or less Coulomb’s law is followed that means, friction force is proportional to the normal force and here this slope is basically mu friction force divided by normal force is mu which is called the coefficient of friction. This you know very well in most of the engineering mechanics problems you have being taking transcend coulomb coefficient of friction. In this region basically the real area of contact a is actually very very small compared to this one it should not be said that 0, but it may be very very small may be 0.02 or even smaller than that.

See a real area means suppose surfaces are not smooth. So, if there is some object which is moving with velocity v it may be moving here in this direction horizontal direction as shown and I am applying the normal load on that then there is a area A, that is this nominal area that is apparent area which is appearing towards what we are seeing that, but in practice this surface is not smooth.

So, what is happening that this area is rough, this is you are seeing that this is rough, and it is making contact at certain points mostly expertise where those are those (Refer Time: 13:18) high points and these cumulative are denoted by A r. So, initially A r is very small compared to A. So, really it is very small. So, what is happening? That if high increase the load in that case my A r it is also keep increasing, A r will increase in that proportion and since A r will increase so similarly in that proportion your shear stress will also increase.

That is why normal stress will appear to be proportional to the shear stress. And you say that friction force is not depending on the contact area not depending on the contact area means it is not depending on the area A, which is you are seeing by your eyes, but actually it is depending. What in the beginning there if there is a more contact area suppose A I 0 then there is a no place to shear and therefore, friction force will of course be 0 that is why when the normal force is 0 then the friction force is 0.

But once there is a contact getting stabiles that means, there is A r that means, these surfaces then for the resistance and therefore, you will get friction here, so that will be manifested here. So, this type of thing keeps happening, so that is the region that in the you are getting that friction force is actually increasing with respect to the normal force and if I apply the load. So, there may come one point that when it has made the contact fully and in that case may be A r is equal to A, entire surface is in contact is (Refer Time:

15:16) how they formed. In that case now if I further increase the load in that case because entire area is a already in touch and therefore, that area can offer some resistance for shear deformation or shear depending and there may be shear strength of the material. So, depending you know that how much force is needed to the demand.

So, if A r as already become equal to a there is no scope to increase the friction force and in that case the friction becomes constant. So, therefore, A r by A is equal to A. In between there may be some transition area because this abrupt change will not be there that up to this there is a coulomb coefficient and after that suddenly it becomes a constant shear stress through (Refer Time: 16:09) that is not corrected. In fact, there will be gradual transition, gradual transition will be there.

Actually the things are very complicated people are doing this type of (Refer Time: 16:19) this itself is a research area asperity based model of the friction is usually taken and then people do lot of studies these expertise are deforming in a elastic manner. Also they are deforming in the plastic manner so elasto plastic deformation what is the effect of various things and chemical aspects are these how to be considered in detail, but phenomena logically we are observing this type of behavior that I have shown here.

So, friction force actually is very small and follows the Coulomb's law if the normal force is less and if the normal force is high then it becomes the constant friction condition, so that is why you are getting this type of distribution, but right now I have done Merchant's analysis, right now we are not going to model the stresses in this manner, ok. Later on you know based on some research papers you can actually do very detailed analysis and you can study further.

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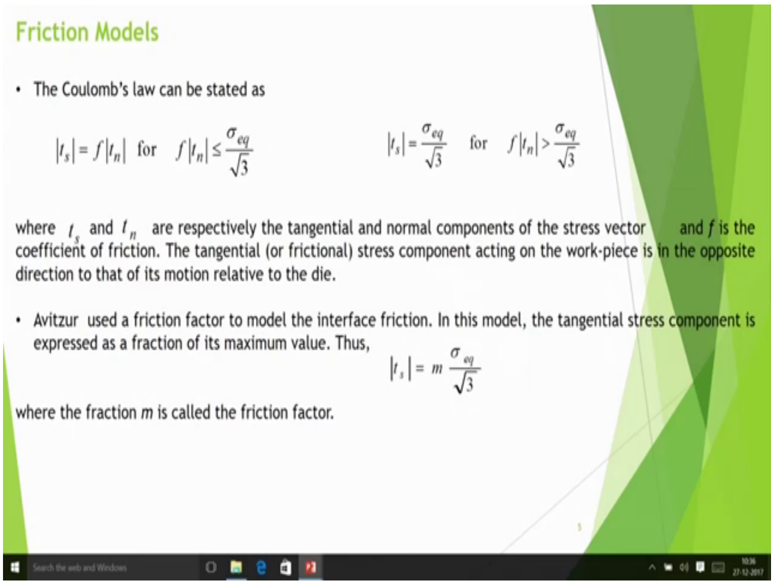

Here, so Coulomb's model can be stated as t s is equal to f times t n, where f is the coefficient of friction, but provided a f times t n that means, the total shear is stress is somewhat is smaller than sigma eq by root 3. What is sigma eq? That means, equivalent stress in terms that means, may be that I can write it sigma Y if I am not considering the temperature and strain effect, so sigma Y by root 3 that means, maximum shear yield stress.

As long as the yielding of the surface is not taking place assumption is that you cannot increase the shear stress on the surface beyond its yield shear stress. So, if t n is smaller or equal to sigma eq by under root 3 this one and t s is equal to sigma e eq by under root 3 for f t n greater than sigma eq by root 3. So, t s and t n are tangential and normal component of the stress vector, we call stress vector that means, unit is Newton per meter square only, but when we talk about the stress on a particular plane getting specified then we can say it as a vector also. And f is the coefficient of friction the tangential or frictional stress components acting on the work piece is in the opposite direction to that of its relative motion this one up tool or whatever type.

Avitzur has (Refer Time: 19:17) lot in metal forming he used a friction factor to model the interface friction in each model the tangential stress component is expressed as a fraction of its maximum value that means, we he does not use the coulomb coefficient of friction he uses the t s is equal to m times sigma eq by root 3. So, you know that maximum possible friction stresses sigma eq by root 3, but here you have t s is equal to m sigma eq by root 3. So, here it is a this fraction m is called friction factor that means, for lubricated surface m may be 0.1 or even may be smaller this type of matter can also be used.

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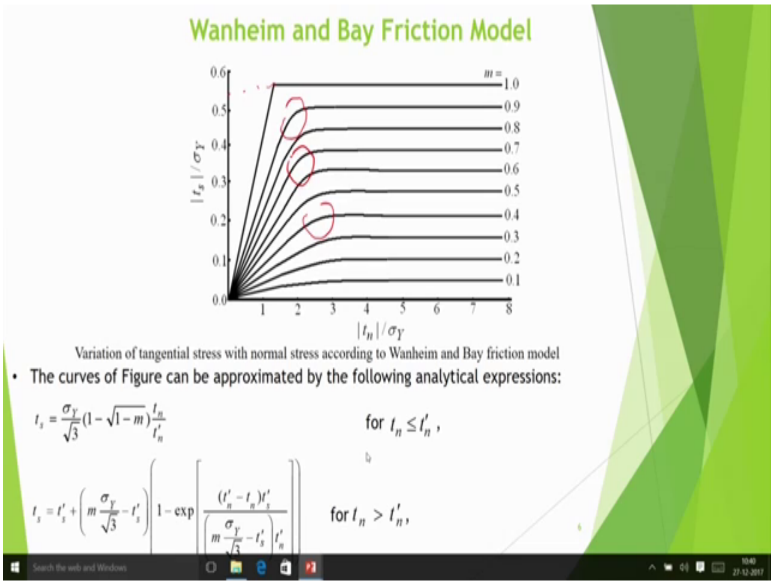

Wanheim and Bay has modeled based on the strip line fluid theory and he had shown this type of arrangement t n by sigma Y, sigma y is the yield strength non dimensionalized way we have plotted the normal stress, and this is the shear stress portion here, and we are getting these type of curves. So, suppose n is equal to 1 in that case constant stress is of course, becomes equal to t s by sigma y by this 1 root 3 is equal to one so that means, 1 by root 3 is actually 0.577. So, this quantity is actually 0.577, that is 0.577.

And here you are having different values of n. So, what he did that the same thing that here he included transition region that means, there is a zone in which the Coulomb's coefficient can be used Coulomb's friction law is valid, and after that there is a zone of constant friction and in between there is a transition zone and this is plotted for different values of n. And these can be approximated by the different this one. Here t n prime is the limit of proportionality that up to which the Coulomb's law hold goods and this is t n is greater than t n prime; t s prime t n prime are the values of the tangential normal stress components at the proportionality limit. He also derived that t s prime by sigma Y is equal to this much and t n prime is also given here.

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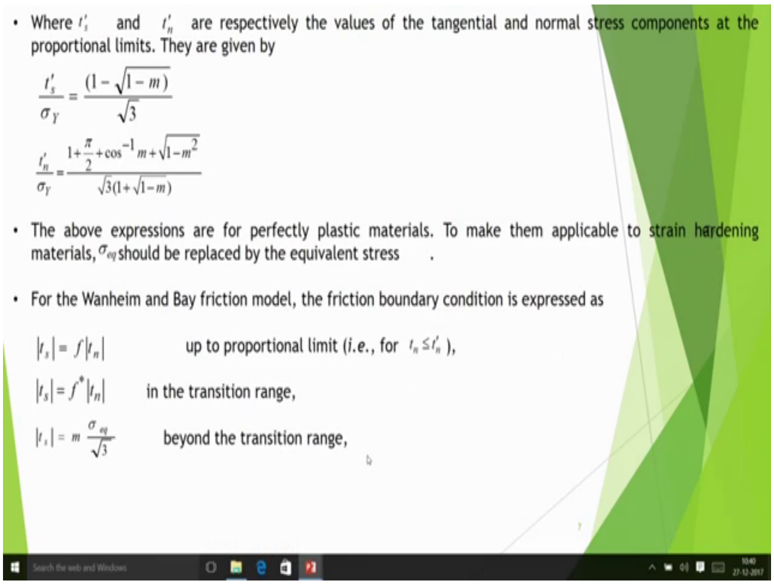

These expressions are for perfectly plastic material that means, we assume that the material means rigid and after certain point it becomes a plastic all of a sudden that is rigid and there is no strain hardening. So, to make them applicable to strain hardening material sigma eq in these equations can be replaced by the equivalent stress that means, sigma Y also I can use that other expression which is dependent on the strain hardening similarly sigma Y may be dependent on the temperature also.

So, for the Wanheim and Bay friction model the friction boundary conditions are t s is equal to f t n this is up to the proportionality limit that is if t n less than some threshold value that is t n prime and t s is equal to f star t n for transition region this f star is now, not a constant and t s is equal to m sigma eq by root 3 beyond the transition region.

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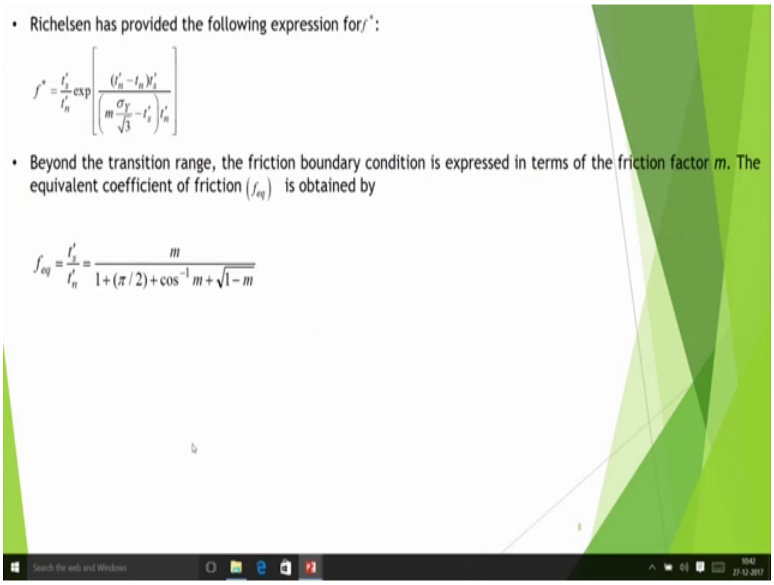

T s star time being expressed in this manner, f star is equal to this one; f star basically so that means, the if we are in this region then the Coulomb's coefficient of friction equivalent Coulomb's coefficient of friction is in fact, dependent on what is the amount of the normal stress T n. So that is what by this expression is dependent like this it is a function of t n.

Beyond the transition range the friction boundary condition is expressed in terms of the friction factor m, the equivalent coefficient of friction can be obtained like that. This is equivalent Coulomb's coefficient of friction if I know some m then I can know that equivalently at low normally stress what would have been Coulomb's coefficient of friction. So, this relation is already there. That is what we have, that is what the Wanheim and Bay model features.

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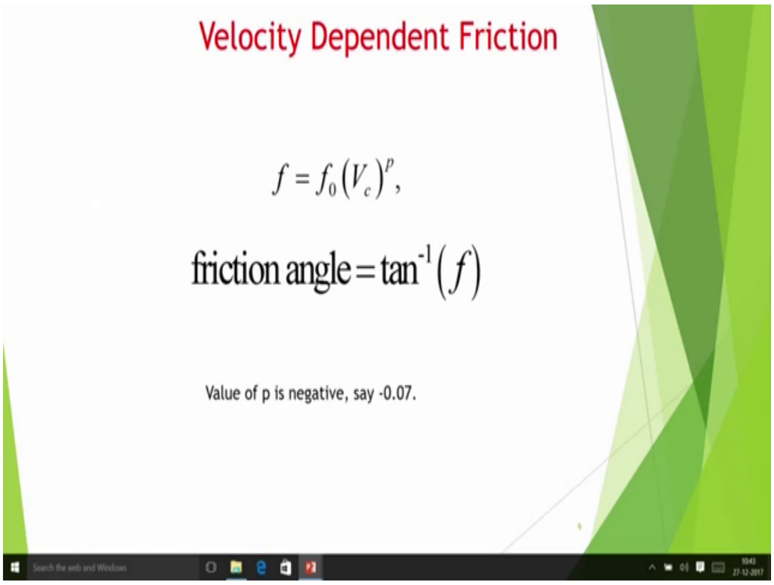

So, has been used option in metal cutting we use velocity dependent friction model. Here we say that the friction force reduces with increasing velocity may with increasing velocity the temperature also becomes high and materials softens a bit. So, f is equal to f 0 v c p, material softening takes place at high velocity at the same time they are may not be enough time for making a contact or welding of the micro welding up to surfaces because in both the surfaces they are all expertise they may not get properly welded.

So, whatever is the reason there may be various factors, but it has been observed that as the velocity increases or cutting speed increases then the coefficient of friction basically decreases. So, if I consider that f as the coefficient of friction then f is equal to f 0 v c p, p is actually negative may be minus 0.07 or whatever is that thing, friction angle is equal to tan inverse f.

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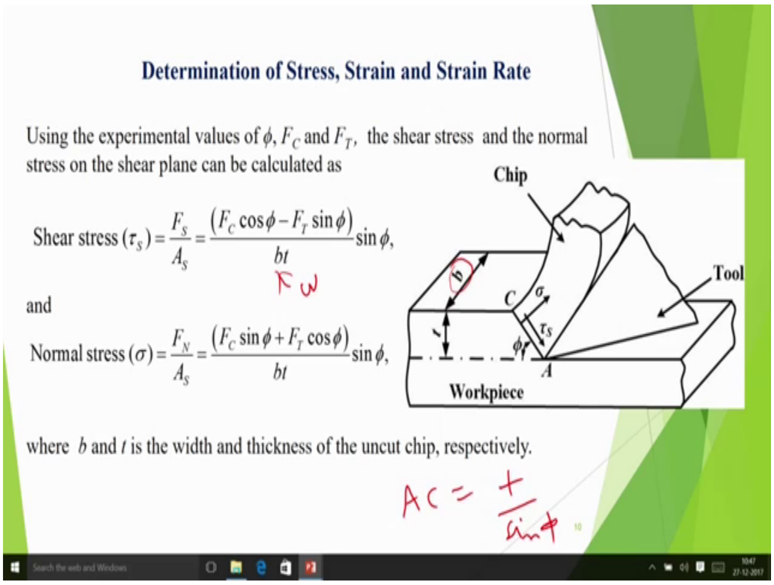

So, having told you about the friction, now let us try to calculate these stresses and strain and strain rate. Using the experimental values of phi F C and F T to know that how I can calculate F C and F T. F C and F T can be measured by dynamometer also, cutting tool dynamometer can be fixed on the machine on which the tool can be mounted and on that you can find out the cutting force and thrust force and then you can also obtain shear angle by photograph by experimental method. Or suppose you know rake angle and friction angle if you have idea then you can find out phi by that relation which combines friction angle and shear angle and rake angle.

So, suppose we know phi and we know F C and F T in that case we can obtain the shear stress on the shear plane. So, shear stress is actually F s divided by A s, this is A s that area of the shear plane A C is the shear plane and F s is equal to F C cos phi minus F T sin phi this comes from the Merchant's relation and divided by this will be shear stress he will be b A s will be b multiplied by T by sin phi. So, this sine phi goes up this A C length is what A C length is basically T by sine phi b is the width of the chip if you say b is the width of the chip.

So, this should be like that and normal some books may use b as a w. So, be careful because you are reading 2 3 books. So, you have to be careful b can be called w also. Here in this figure this is b and this one there. So, here what is it indicating that shear is stress of course, the shear stress increases if F C is more and if F T is more than it is decreasing it is also dependent on phi and it is dependent on b and t. And then the normal stress is sigma that is F N by A s that is F C sine phi plus F T cos phi divided by b t into sin phi b and t, b is the width and t is the thickness of the uncut chip. Actually this length of the A C, A C is equal to t by sine phi, t by sine phi that is how the this coming here that is y t is in the denominator and then phi goes sine phi was up. So, this you have seen and you can find out the shear stress and normal stress on the shear plane.

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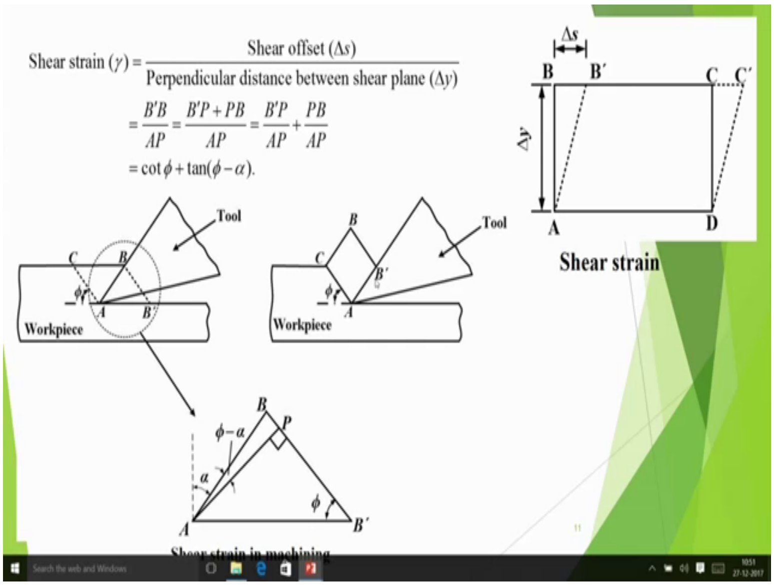

Now, let us talk about the shear strain. You do not know what is a shear strain suppose we have at a point we make very small this type of rectangular A B C D and then slightly deform this rectangular, so that this line A B becomes A B dais and may be C D has become C, C prime, D like that. So, there is only deformation we are not stretching it and there is a change in the angle. So, here this angle was 90 degree. Now, this angle has slightly reduced. So, angle between A D prime and A D is just then 90 degree and since the angle is reducing by convention we call it a positive shear.

So, let us say that B B dais is delta s and A B is delta y then delta s by delta y that will be shear angle. So, shear strain will be shear offset delta s divided by perpendicular distance between shear plane. Same concept we apply here suppose there is a tool I am doing machining may be some material was like this C B, B prime A C this what the part of the workpiece and then tool has penetrated tool has come up to point A. Then this B prime must have gone to B because that is what it is a making this one and from here it is going up, so that is what this one you are seeing the workpiece because this one this portion has gone up prime has gone b and this b itself has moved ahead. So, this C B has become C B here and this is B prime and this one, 4 5 this one. So, how much we are displacing we are displacing the material from B prime to B and from A we can draw perpendicular and we can see what is the perpendicular distance means what is delta y.

So, we can say B prime B divided by AP this is AP that can be written as B prime P, P is the basically put up the perpendicular and this is B prime P plus P B divided by AP this will become equal to B prime P divided by AP plus P, B by AP. Now, from this figure you can very nicely see that B prime P is actually, B prime P by AP is actually cot phi and this one is actually tan phi minus alpha why this because A B rime this A B rime is in fact, making an angle alpha with the vertical and this perpendicular will make phi from the vertical. So, this angle becomes phi minus alpha. So, you get a nice expression shear strain gamma is equal to cot phi plus tan tan phi minus alpha.

You can very well see that if alpha increases then the tan phi minus alpha is going to reduce and therefore, shear strain is going to reduce that is why a large rake angle is beneficial it is giving less amount of shear strain because we do not want that much shear strain also we just want to separate the chip. So, alpha effect is clearly seen as far as the phi is concerned that shear strain increase with phi or decrease. We can see that tan phi part second part is increasing, but the first part is decreasing with increase in phi so that means, there must be some optimum phi at which the shear strain may be minimum, so that is what, but this relation. You can always remember that gamma is equal to cot phi plus tan phi minus alpha.

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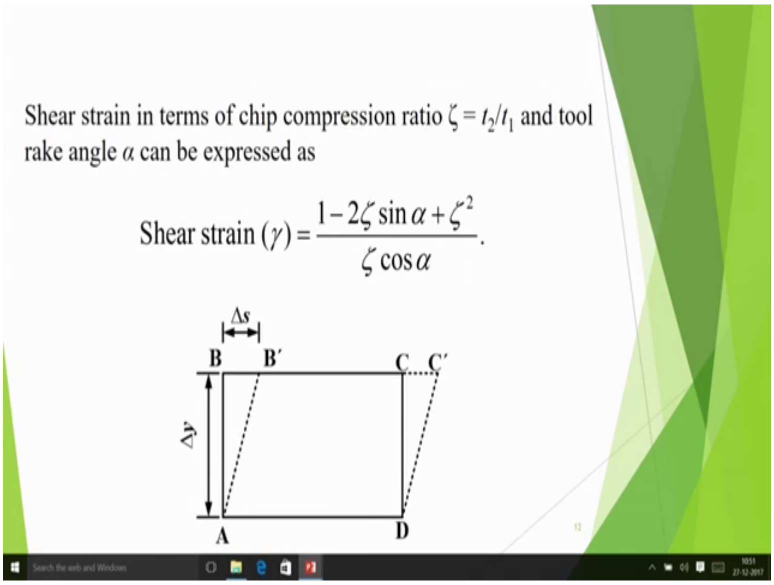

Now, here shear strain can also be expressed in terms of the chip compression ratio. I defined the cutting ratio cutting ratio is t 1 by t 2 chip compression ratio is exactly reverse of that. So, this is the find as t 2 by t 1. Cutting ratio is always less than 1, chip compression ratio is actually more than 1 and shear strain can be expressed as a function of chip compression ratio and rake angle alpha because phi is also dependent on chip compression ratio and rake angle.

So, this we can very easily derive because you have already that relation. You already know that type of cutting ratio relation that we have already shown. So, we will be showing a in the previous like we have already shown. So, shear strain gamma you can express as 1 minus 2 tau sine alpha plus tau square divided by tau cos alpha. This is showing here.

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In this case shear strain is expressed like this derivation you can yourself do here that one outline is given suppose gamma is equal to cot phi plus tan phi minus alpha cot phi (Refer Time: 34:19) is 1 by tan phi and then this you express tan phi portion that shear angle in terms of the chip compression ratio and rake angle like that you express. And if you do some manipulation then after that you get this expression gamma is equal to 1 minus 2 tau sine alpha plus tau square and this is tau square cos alpha this symbol is actually can be called zeta, this is zeta equal to t 2 by t 1. This is and this is 1 minus 2 zeta sine alpha plus zeta square divided by zeta cos alpha that is shear strain gamma.

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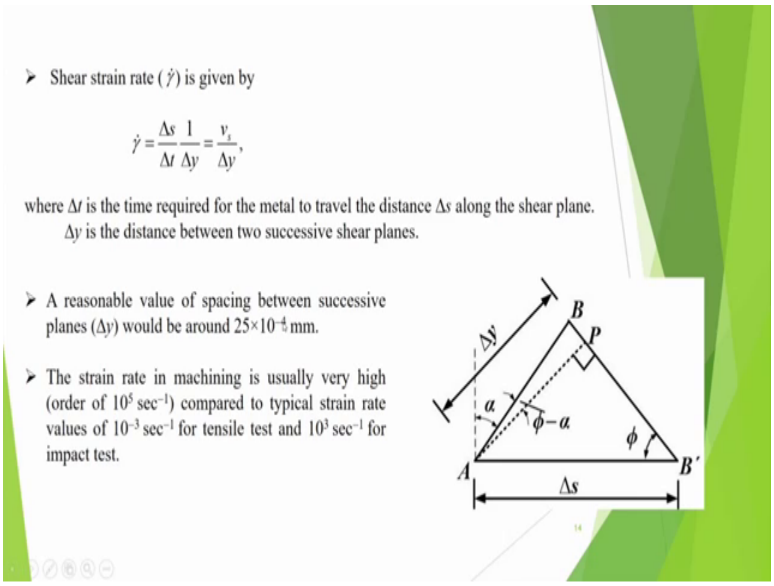

Now, shear strain rate is given by delta s by delta y divided by delta t, that how much is the time required or the metal to travel that particular distance delta s. The metal has moved out come from here to here, but how much time it has taken that point has to be known and to travel the distance delta s along the shear plane delta y is the distance between two successive shear planes that we can take. A reasonable value of spacing between successive plane would be about 25 into 10 to the power minus 4 mm that type of thing can be done, so that you can actually have there may be different type of this one 10 to the power minus 3 mm means 1 micron.

So, we can have about a you may have say 25 micron, there is lot of people have given different different ranges and there is a this one, but you can take any typical value say in this case it is mentioned that 25 into 10 to the power minus 4 mm. Strain rate in machine is usually very high if we calculate like that it is of the order of 10 to the power 5 per second, compared to typical strain rate values of 10 to the power minus 3 per second for tensile test.

And generally in impact test we get the it is of the order of 10 to the power 3 per second for impact test this value really delta y is actually this much or it is different this R is debatable sometimes people have taken some photographs and you can find out from using scanning electron microscopy and transmission electron microscopy you can get some idea and research kind (Refer Time: 37:12) in that direction, but we have just taken some ad hoc value here and then we are estimating that shear strain rate.

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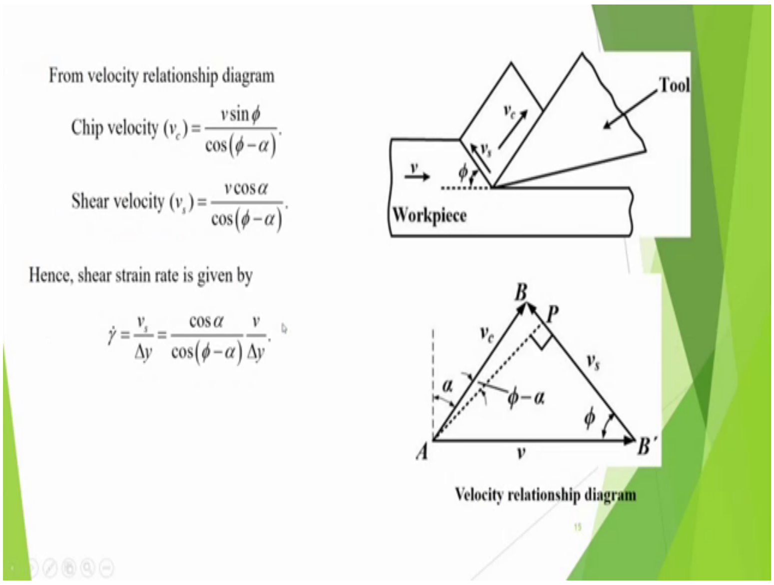

Now, from the velocity relationship diagram suppose we go to the velocity relationship that suppose it is v and then there is a chip is moving here chip may be sliding up. So, v s is shown up job is moving with velocity v and this is v c, v c is the chip velocity we call chip velocity we can make it triangle. Triangle because these are vectors, so suppose v is the workpiece on which there is a chip which is sliding with v s.

So, I have from here I can take this one and this is the absolute velocity of chip that will be called v c, this is the v, v is the velocity of the workpiece on the workpiece itself the chip is moving with relative velocity v s. So, I have drawn it here and therefore, and after that we have drawn A B and this A B is the absolute velocity it is in the direction of where (Refer Time: 38:27), to this angle is alpha, and this angle is no doubt it is phi because v s is in this direction.

So, in that case this angle comes out to be phi minus alpha if you draw perpendicular here and now, we can find out from this triangle that we can apply the sine law, and we can say chip velocity v c is basically v c by sine phi this divided by this and v s this is v divided by that angle. So, this relation comes out to be v sine phi divided by cos phi minus alpha, right. So, v sine phi divided by cos phi minus alpha. If v increases v c also increases if phi increases then also v c increases, and shear velocity v s from this diagram itself I can find out v s is equal to v cos alpha v divided by v s divided by this 90 minus alpha sine 90 minus alpha sine law will give me that. So, you get v cos alpha v divided by cos phi minus alpha.

So, we have got chip velocity, we have got shear velocity then the shear strain rate can be given as gamma dot is equal to v s divided by delta y and this can be written as cos alpha divided by cos phi minus alpha v divided by delta y. But this delta y we have to make some assumption delta y is not known very precisely, so that is what we have to make some assumption and this is what we have written here like this.

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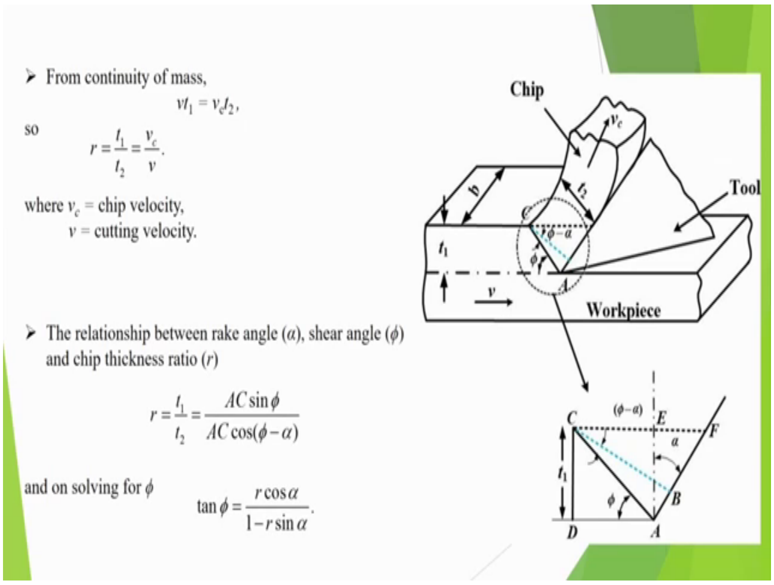

From the continuity of mass v t 1 is equal to v c t 2 that also we can do in the width direction the chip is not spreading. So, we get v t 1 is equal to v c t 2 like that. And then r is equal to that is cutting ratios t 1 by t 2. So, cutting ratio is basically nothing but v v c by v, v c is the chip velocity and v is the cutting velocity at most the cutting velocity can be equal to 1. So, v c can at most be equal to v.

Relationship between rake angle for shear angle phi and chip thickness ratio R that is already known to you r is equal to t 1 by t 2 and this is A C sine phi divided by A C cos phi minus alpha from this figure. So, phi solving we had got tan phi is equal to R cos alpha divided by 1 minus R sine alpha. So, if you know relationship between rake angle and shear angle and suppose you have measured of course, if you have measured the chip thickness ratio you can estimate phi and you can also estimate v c.

And then this is about this one that how do you find out a this one, and if you want to find out the stresses on the workpiece are rather on the tool then in that case you have to take that force is coming on the tool that we know F and N and divide by that particular contact area which can also be easily obtained you can have. But you have to consider that what is the chip contact length how much will be up to which point the chip will be in contact. It need not be in the contact up to this point it may go beyond and you can estimate in the average way.

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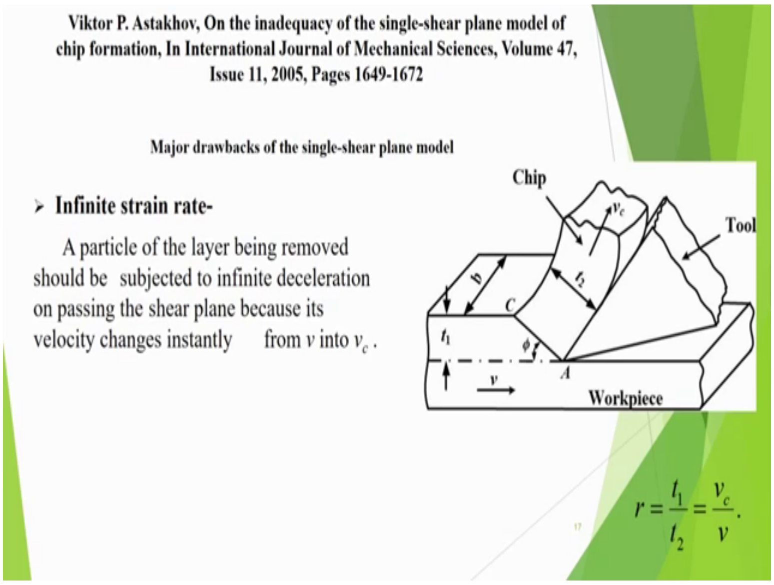

Now, I am going to discuss you one interesting paper by Viktor P Astakhov that was published, In International Journal of Mechanical Sciences in Volume 47, Issue 11, in 2005. And it is about 23 page paper in that he had discussed in detail that shear single shear plane model is not really required in the modeling of the metal cutting process. So, major drawbacks of single shear plane model have been discussed by Astakhov in these manners in finite strain rate that means, a particle of the layer being removed should be subjected to infinite deceleration on passing the shear plane because its velocity changes instantly from v into v c.

You know that v c is smaller than v because v into t is t 1 is equal to v c into t 2 and t 2 is always greater than t 1. So, v c must be always less than v so that means, the velocity changing, but it is changing all of a suddenly this model assumption. That is why from v to v c you have come suddenly. So, naturally if you start calculating the deceleration that will come out to be in finite that is one point r is equal to t 1 by t 2 v c by v that is this one.

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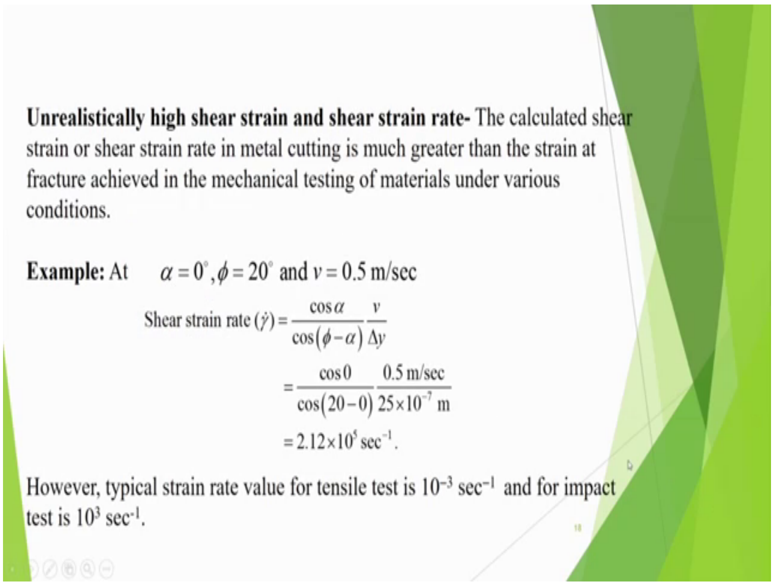

And then unrealistically high shear strain and shear strain rates are predicted by this model Astakhov feels that the strain rate may not be actually that high it may be only of the order of 10 per second. But these flow (Refer Time: 44:03) predict 10 to the power 5 etcetera. Calculated shear strain are shear strain rate in metal cutting is much greater than these strain at fracture achieved in the mechanical testing of materials under various conditions. I give one example.

Suppose I take alpha is equal to 0 degree phi I take 20 degree and v is equal to 0.5 meter per second that means, 30 meter per minute. Then shear strain rate is cos alpha v divided by cos phi minus alpha into delta y put these values alpha rake angle is 0, after putting these values and delta y I have put 25 into 10 to the power minus 7 meter then I am getting 2 into 10 to the power 5 per second, 2.12 into 10 to the power 5 per second. How you are typically strain rate values for tensile test is 10 to the power minus 3 per second and for impact test it is 10 to the power 3 per second. So, these values are much higher.

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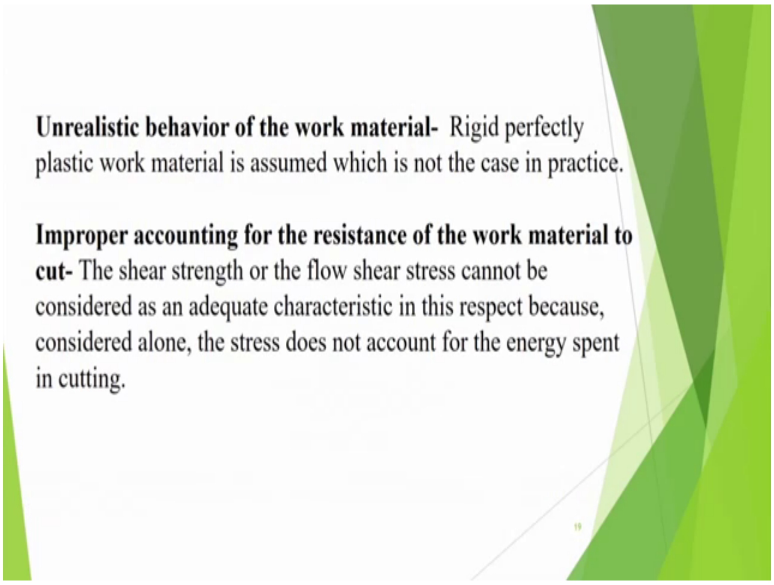

And unrealistic behavior of the work material: Rigid perfectly plastic work material is assumed which is not the case in practice because we assume that it is rigid perfectly plastic, but that may not be true. Improper accounting for the resistance of the work material to cut: The shear strength or the flow shear stress cannot be considered as an adequate characteristic in this respect we got stress does not account for the energy spent in cutting, ok. So, it is a there may be lot of other factors that is another drawback here.

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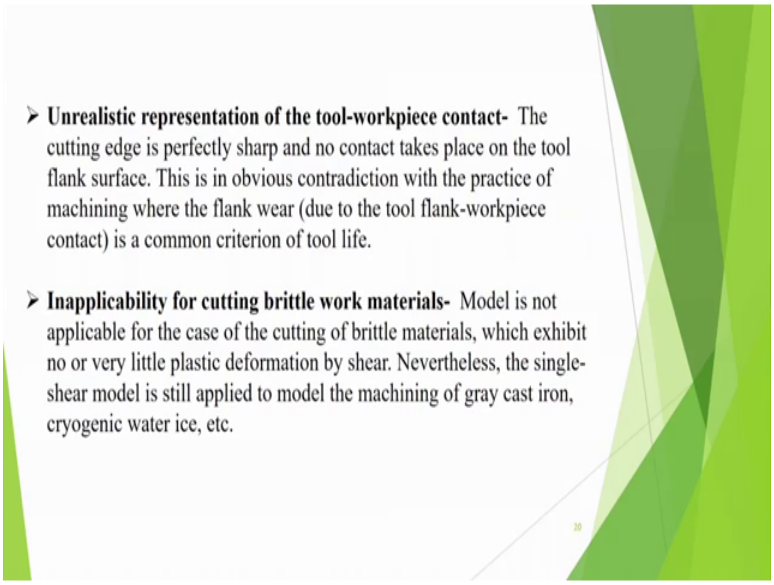

Unrealistic representation of the tool workpiece contact: The cutting edge is perfectly sharp we assume and no contact takes place on the tool flank surface. This is this is in obvious contradiction with the practice of machining where the flank wear due to the tool flank workpiece contact is a common criterion of tool life. That means, there is some amount of the flank wear on the flank surface there will be contact. This is basically this surface is the flank surface here some contact is there, but you know Merchant's model are single shear model is not taking into account that thing.

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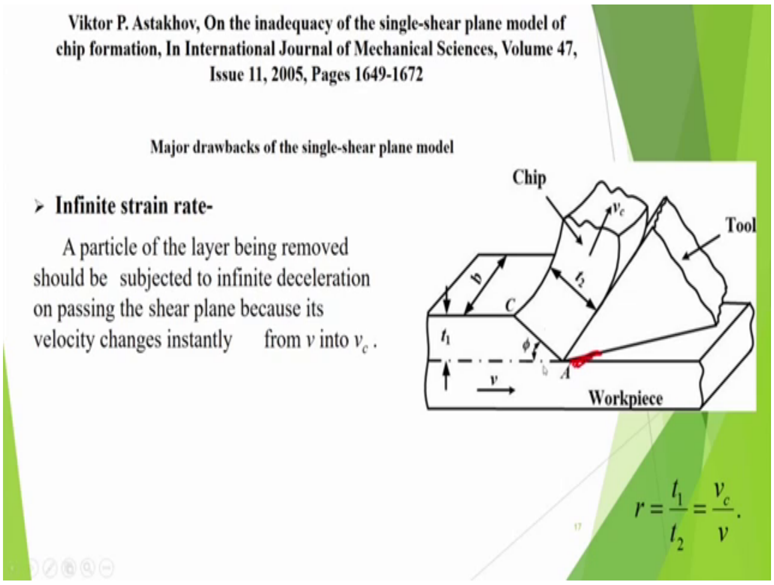

And then inapplicability of for cutting brittle work materials: Model is not applicable for the case of cutting of brittle materials; this exhibits no or very little plastic deformation by shear. A brittle material does not exhibit plastic deformation by shear, but here whole basis is that there is shear taking place. So, therefore, this model may not be adequate for the cutting of the brittle material. Although at certain conditions brittle materials may be (Refer Time: 47:09) ductile fashion also in some cases we call that thing a ductile is a machining but that we are not going to discuss here.

Nevertheless, the single shear plane model is still of light to model the machining of gray cast iron that also then cryogenic tool type of supported in which we use cryogenic cutting that means, we may put some liquid nitrogen in the cutting zone. So that the temperature goes in the negative, that time the behavior of the material may become brittle, but even then we use this model in such type of circumstances also.

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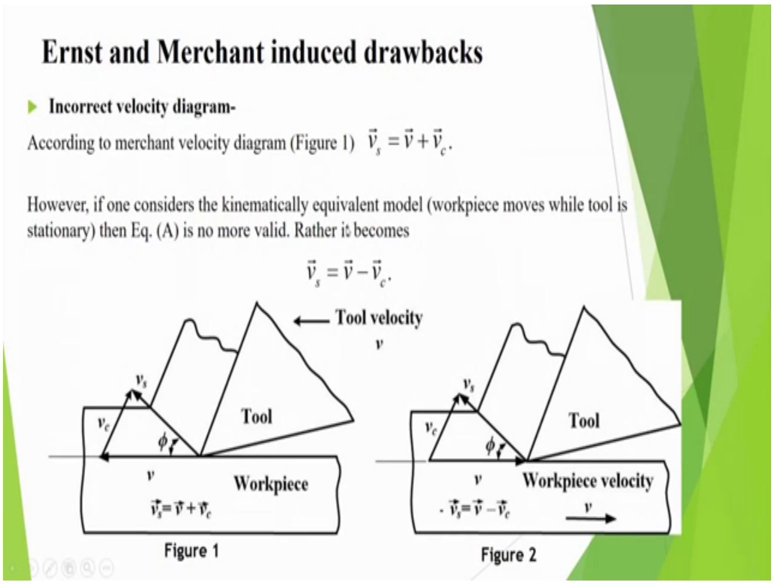

And then Astakhov pointed out that Ernst and Merchant including some drawbacks in the velocity diagram because merchant model the velocity diagram like this. In this case he assumed that the tool is moving with velocity v and in that case v s is equal to v plus v s, but if we take the equivalent model in that case this minus v s will be equal to v minus v s.

So, it is depending on that type of thing whether this one has to consider that: what is the velocity relation. So, there are some problems with the velocity diagram also. Means if I have taken in the earlier case mostly in the analysis I always talk about tool at the stationary and the chip velocity becomes the absolute velocity. But if the tool is moving then what I will be defining at the chip velocity will it be the relative velocity of the chip on the tool surface or will it would be the absolute velocity of the chip or these type of things nobody discussed much that point is there.

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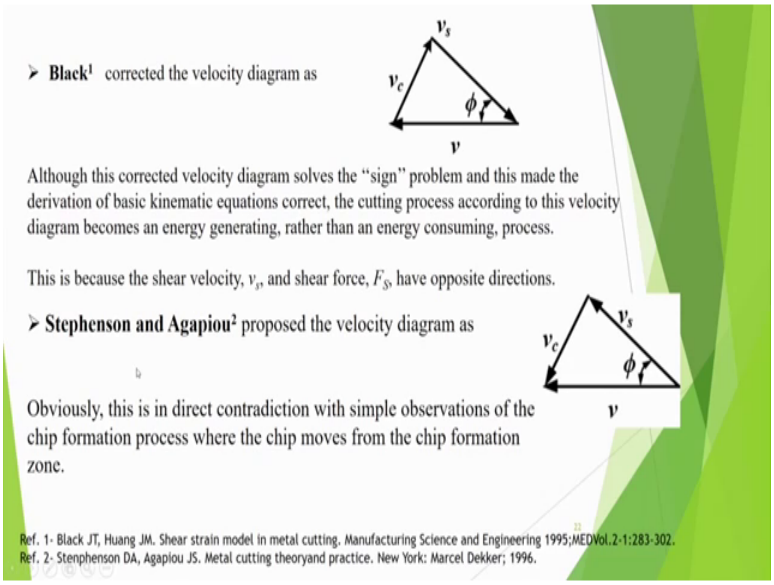

So, Black, corrected the diagram like this Black as need this type of diagram blacks peoples also there although this corrective velocity diagrams are the sign problem. But in this diagram also there is a some problem because shear force and v s both are in opposite direction, then there are some other authors they propose another type of velocity diagram.

Now, this is in direct contradiction with simple observations of the chip formation because chip is going up where this chip moves from the chip formation zone. So, these are some problems in the velocity diagram also, but in most of your analysis and in your books we are just taking the case of two is a stationary and we are making one type of velocity diagram, but you must be aware about the counter points.

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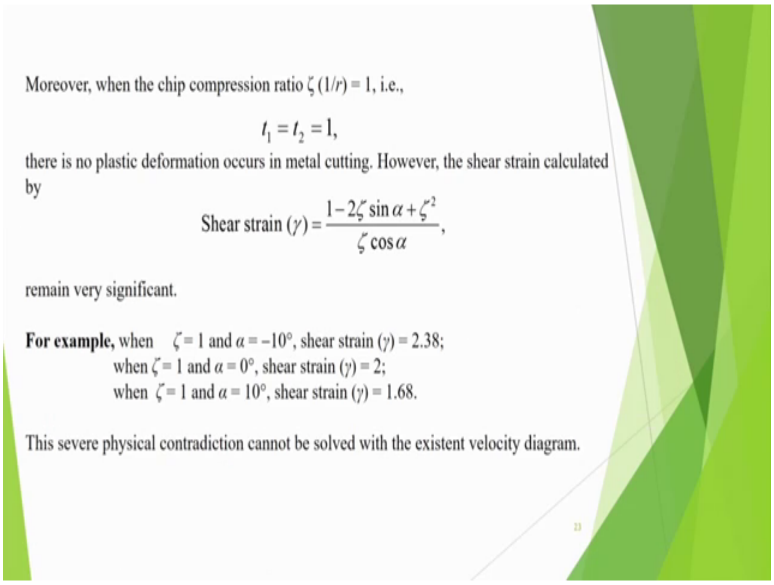

Moreover when the chip compression ratio is one suppose chip compression ratio is one that is t 1 is equal to t 2 actually there should not be any plastic deformation in metal cutting it is just rotation type of thing. But you are calculating the shear strain using this formula then you will still be getting some shear strain because this formula is 1 minus 2 tau sine alpha plus tau square by tau cos alpha. So, put tau sorry it is it should be called zeta and this is what zeta is this one.

It remains very significant. So, when for example, when zeta equal to 1 and alpha is equal to minus 10 degree if I put this value then I will be getting shear strain as 2.38 and when I put zeta equal to 1 and alpha is equal to 0 degree then I am getting shear strain gamma is equal to 2 and when zeta equal to 1 and alpha is equal to 10 degree then I am getting shear strain equal to 1.68.

So, for 0 rake angle you are getting shear strain equal to 2 when this zeta is equal to 1 so that means, this is the minimum amount of shear strain you are expecting from this model. Although actually that may not be the shear strain it may be due to rotation, chip may rotate, but rotation does not mean the shear know. So, these this severe physical contradiction cannot be solved with existing velocity diagram. One thing we can of course, observe that here if alpha is increasing from minus 10 degree to 10 degree then the shear strain is decreasing, ok.

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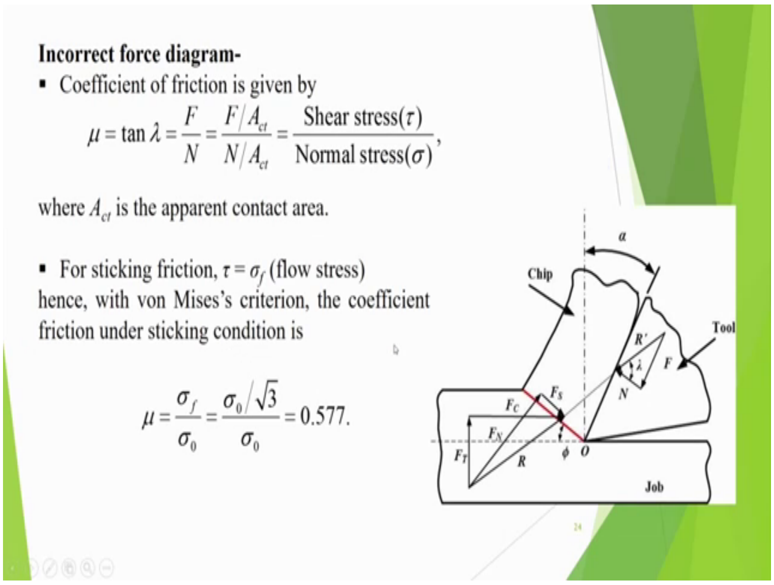

Now, Astakhov pointed out that the diagram is also incorrect force diagram because coefficient of friction is actually given as mu is equal to 10 lambda that is F by N that is F by ct which is appear in contact area, N by this one act shear, stress by normal stress. But you get basically this is coulomb coefficient is assumed as a constant, but for sticking friction tau is equal to sigma F that means, flow stress is sigma F with von Mises's criterion the coefficient of friction under sticking condition is mu is equal to sigma F by sigma 0, sigma 0 may be the yield stress and sigma F is the shear stress that is sigma 0 by root 3. So, it comes out to be 0.577.

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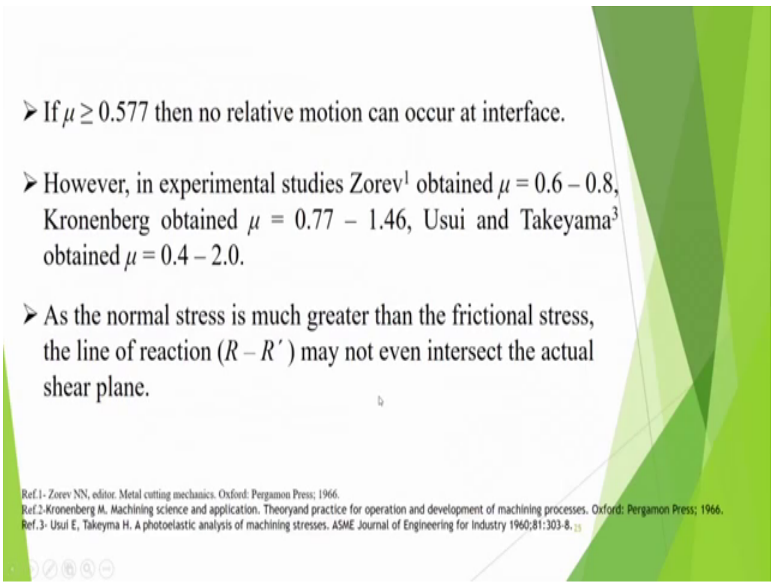

So, this becomes, this one if mu is greater than 0.577 then no relative motion can occur at interface that is the point and however, in experimental studies Zorev obtained Zorev obtained mu is in the range of 0.6 to 0.8, means this is the average coefficient came out to be 0.6 to 0.8 because of these type of problems and because we are not able to monitor proper variation of the coefficient of friction and Kronenberg obtained mu in the range of 0.77 to 1.46 also. So, normally we say that mu should not exceed 0.577, but here they are obtaining very high value, Usui and Takeyama obtained mu in the range of 0.4 to 2.

As the normal stress is much greater than the frictional stress the line of reaction may not even intersect the actual shear plane, that is also another effect and these and these forces are acting far away from these they may produce chip curling. Chip curling is not pointed is taken into account here.

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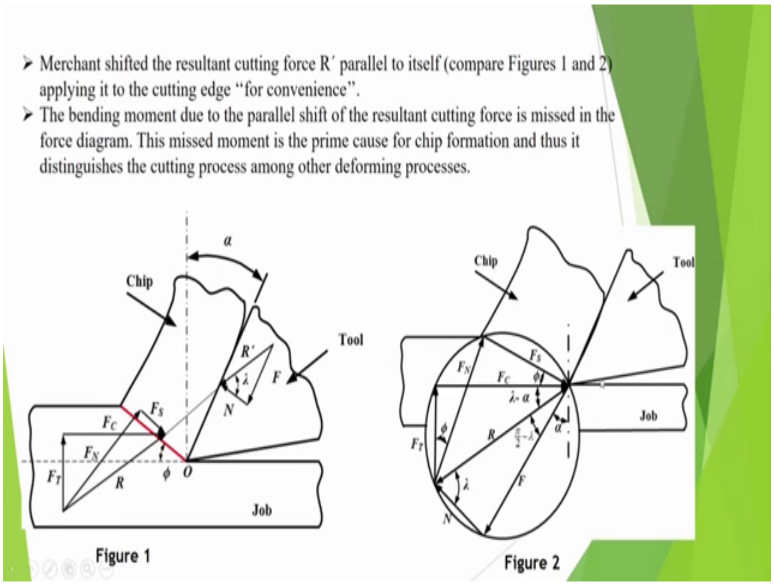

Merchant shifted the resultant cutting force R prime parallel to itself just see if you compare the figure that here he is making that actually the forces are acting here, but in the merchant circuit diagram they are shown here in this one at the tool point is considering this as tool tip.

So, here it is bending movement due to the parallel shift of the resultant cutting force as not been taken this missed move movement component is the prime cause for chip formation, and thus it distinguishes the cutting process among other deforming processes that has been taken, this one.

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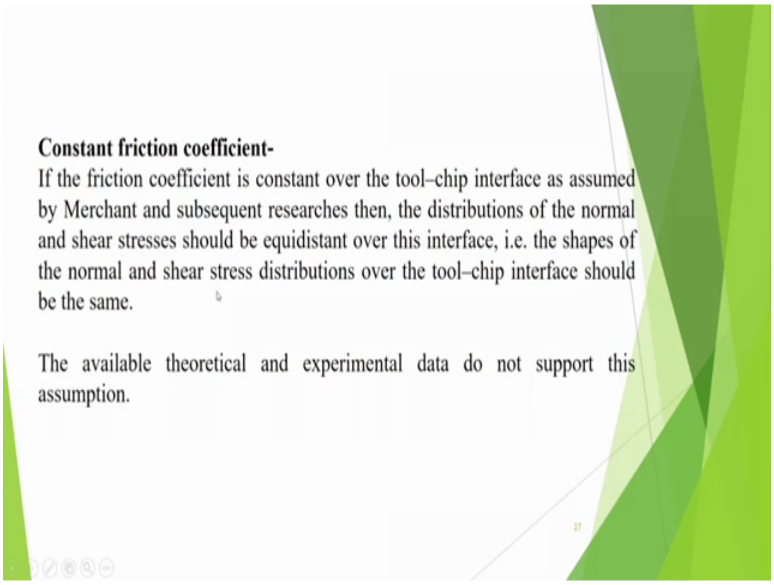

And naturally the constant friction coefficient has been taken if the friction coefficient is constant over the tool chip interface as assumed by Merchant and subsequently researchers. Then the distribution of the normal and shear stress should be equidistant over this interface that is the shapes of the normal and shear stress distribution over the tool chip interface should be the same, but that is not true. I already have shown that how these shapes look.

You have already seen here, this I can show you again that you are seeing that shape of normal stress distribution is this and shape of shear distribution is this. But naturally this cannot be taken into account by simple merchant analysis. These are the problems associated with this model. So, then it is yes, in this case we have taken the available theoretical and experimental data do not support this assumption.

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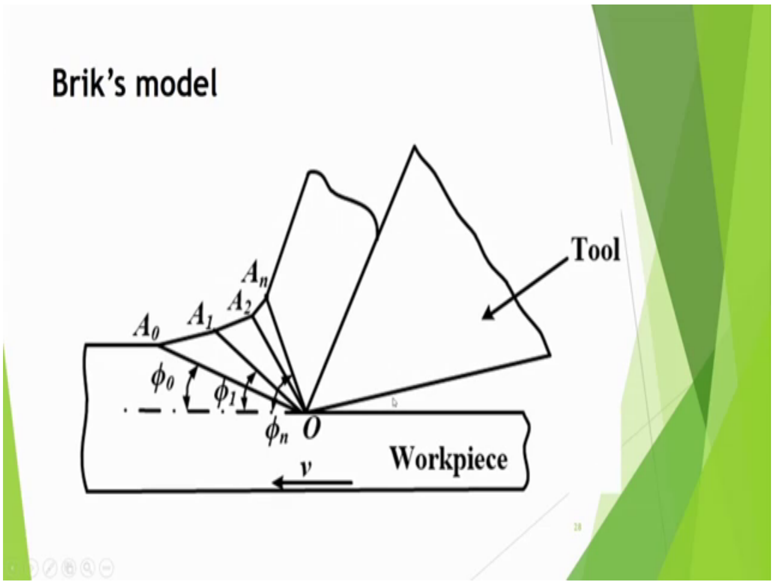

So, Astakhov pointed out that there were other people much before the merchant also they had proposed other models also like Brik's model, around 1896 it was proposed in which he proposed this type of model that there is a shear taking place along A 0, then A 1, this is A 2 and A n. So, here gradually the shear angle is increasing and there is no abrupt increase in the stress and strain this model was taken, but people have not believed that model.

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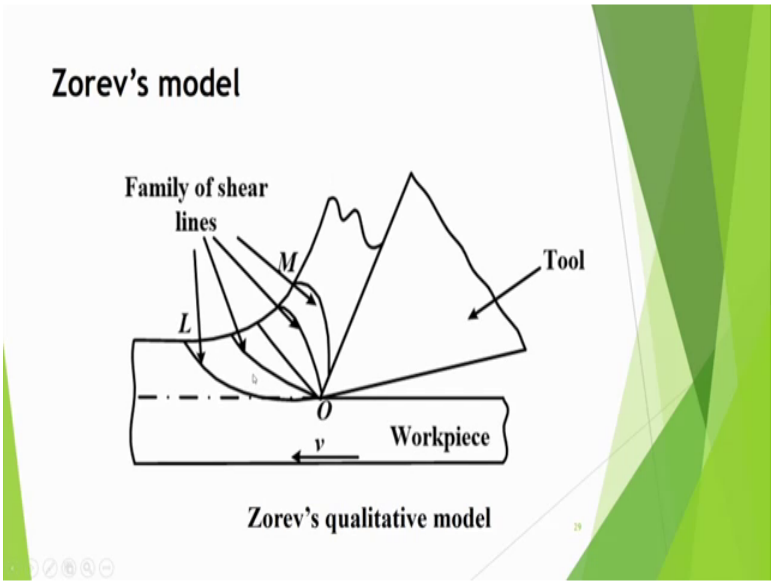

In fact, Zorev presented another type of model in which he assumed the curve line these are the shear lines they are having some curvature. He qualitatively he told about this model.

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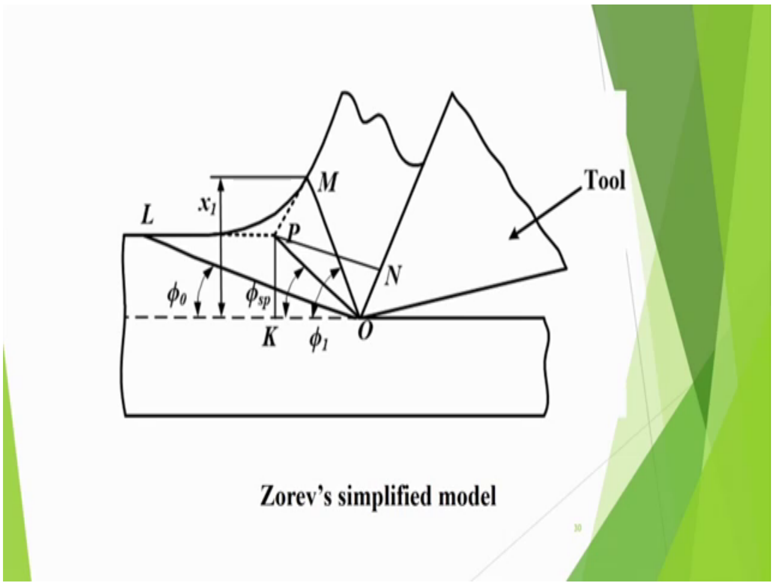

He simplified this ultimately again and he said let this curve portion should be taken as a straight line, then he is getting this type of model here and this and this may be that he has taken some representative value of this one, that shear angle ultimately it boils down to single shear plane model and this one.

So, these are the problems that you know earlier time as discussed about this one in 18-19th centuries something about single shear plane model then other researchers like brick and Zorev Brick and Zorev they also had discussed. Then after that, but ultimately again we took the single shear plane model later on some researchers have told about the shear zone model also, but Merchant's model is still actually being taught in most of the colleges and in most of the text book this model is included because it may be easy to teach properly. So, this I have shown you that paper.

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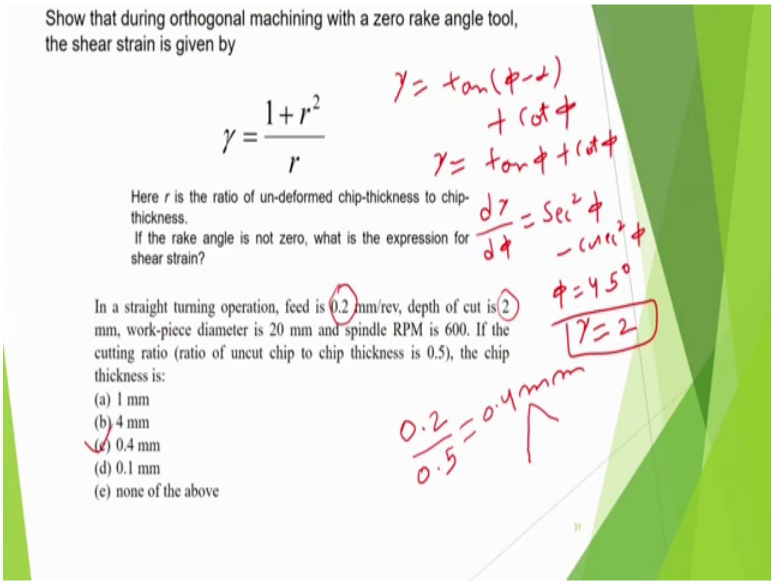

And now, let us see some questions related to this show that during orthogonal machining with zero rake angle this shear strain is given by gamma is equal to 1 plus r square by r this you can derive just you have to put the expression r is the ratio of un deformed chip thickness to chip thickness, if the rake angle is not 0 what is the expression for shear strain, this you can practice.

Another question is this in a straight turning operation feed is 0.2 mm per evolution, depth of cut is 2 mm, work piece diameter is 20 mm and a spindle rpm is 600. If the cutting ratio that ratio of uncut chip to chip thickness is 0.5 the chip thickness is how much. So, straight turning means there is side cutting edge angle 0. So, it is cutting like this in that case as we have already understood that the chip thickness in this case is 0.5 times the uncut chip thickness and uncut chip thickness uncut chip thickness will be equal to feed 0.2.

So, therefore, 0.2 and divided by 0.5 0.5 divided by 0.5 this will be how much? It will be 0.4, so therefore, it will be 0.4 mm that is this one. So, this option will be correct 0.4 mm this one. Do not confuse it with the depth of cut. Depth of cut will give the width of the chip.

Now, here depth of cut will give the width of the chip. So, this in this actually I have discussed, let us just summarize that what we have discussed we have discussed about the strains and stresses in orthogonal cutting, but we discussed in the (Refer Time: 60:18) sense. So, in this case we have considered Merchant's forces then divided by the area. I discussed the difficulties means we have not obtained much quantitative expressions, but we discussed difficulties in the simple Merchant's model like friction behavior etcetera in Merchant's model friction is not taken as a function of the velocity, but you can that that portion can be easily taken you can always do that, but of course, he assumes that constant coefficient of friction. So, here I am calculating the shear stress and normal stress on these planes and similarly we can find out the stresses average stresses on the tool surface also, if you get idea about the tool chip contact length.

And then these expressions we have derived and mostly then we have discussed that: what are the difficulties in the single shear plane model of the merchant. This paper is very interesting this you can read In International Journal of Mechanical Sciences, volume 47, this paper of Viktor P Astakhov. Here he has discussed the drawbacks of the single shear plane model which will be which provides hindrance. Now, what is we can model the machining process by finite element method also. In this course we will not be discussing about the finite element model, but in the finite element model we can find out the stress and strain distribution.

We can obtain the stress contours also and we can do much detailed analysis in that case many things we can reveal, but come to stationary it will take huge amount of time. So, we have done we if we do the finite element analysis then may be by analysis itself we can get somewhat this type of zone, we can observe that in this zone there will be plastic deformation. If we do photographic techniques etcetera and if we make a grid pattern and we find out experimentally then also we can observe this type of zone we will discuss some of these experimental techniques in the next lecture.

So, Brick's model, then similarly Zorev's model is there, and then after that Zorev's simplified model is there and then you can solve some problems of this one and finally, last point just I am trying to show you that suppose we take expression for shear strain as tan phi minus alpha plus cot phi. Now, if alpha is 0, rake angle 0 then expression becomes very simple you can remember also that it becomes tan phi plus cot phi, right.

Now, we want to find out that which shear angle will give me minimum strain. So, we just give d gamma by d d phi which will give me minimum I am interested. If you talk which will give me maximum I can do it by observation also at phi equal to 0 cot phi will become infinite, at phi equal to 90 also gamma will become infinite because tan phi. So, if we do like that so then what do we get we get? Sec square phi minus cosec square phi that means, 1 by cos square phi minus sine square phi. So, cos phi becomes sine phi so that means, phi is equal to 45 degree. So, at phi is equal to 45 degree tan phi becomes one and cot phi becomes 1. So, gamma becomes 1. So, these type of simple exercises you can do in order to become familiar.

So, in the next lecture we will discuss more about the stresses and strain and also some experimental methods.

Thank you.