Mechanics of Machining
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Lecture – 06
Mechanics of orthogonal cutting

Hello a student, this is the 6th lecture on Mechanics of Machining. Today, I am going to talk about Mechanics of orthogonal cutting. Till now we have discussed various types of chips, what types of chips are produced in machining and also we have told you something about the cutting tools. In the cutting tool, there are lot of angles, a cutting tool is designated by seven elements including nose radius and there are six angles in which there are rake angle, relief angle and cutting edge angle.

Now, today first we will discuss only the mechanics of orthogonal cutting in a simple way. Here two angles are important that is one is the rake angle and then we have got some relief angle, but may be the relief angles effect will not be that much visible in the simplified analysis. Now, we are going to tell you about orthogonal cutting, we will now come to know about the forces coming during the machining.

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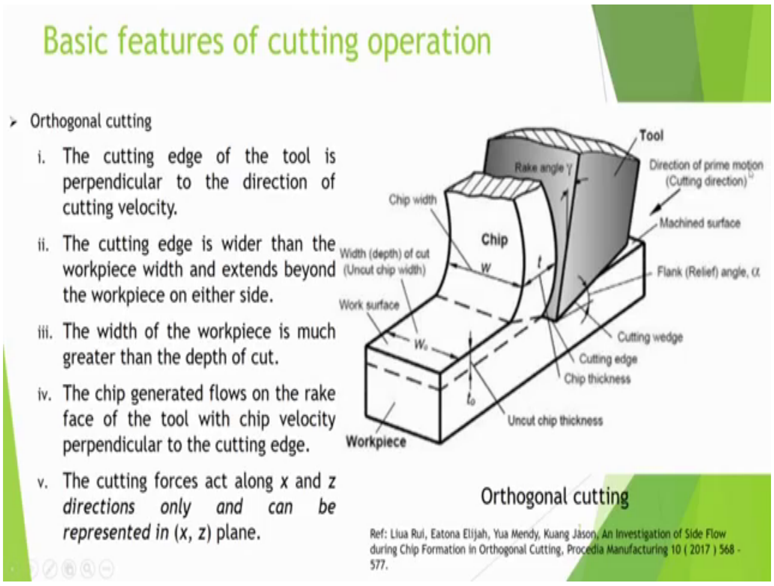

So, here if you go to a orthogonal cutting, this is a picture of orthogonal cutting. Here there is a one piece, then there is a tool. Now, this tool this is having a rake angle gamma. How do we know that this is a rake angle the cutting tool velocity is in this direction that means, horizontal direction perpendicular to the cutting velocity, there is this vertical plane and rake face is inclined with that vertical plane by an angle gamma. So, we have indicated rake angle by gamma in this position, the rake angle is positive so that the chip usually glide solid.

Now, here you also have a flank or relief angle because otherwise this tool a bottom surface that is flank surface, we would rub against the machine surface. So, we have provided some sort of relief and that is why this is called the relief angle, which may be about 5 degree or so. Now, this is the direction of prime motion that is cutting direction we are considering that right now I am not giving any feed, but feed can be given in incremental form also. After one pass is complete you in the transverse direction you both the tool, but that is not needed at the movement because we are going to find out that when the primary motion is taking place then how much force is coming on the cutting tool.

And here in orthogonal cutting, we are assuming that the cutting edge is perpendicular to the cutting velocity. And also the cutting edge is very long; it extends beyond the width of the work piece surface. So, it is extending. So, the complete operation is a two-dimensional one and in that case here what is happening suppose width depth of cut is this one, this is the width of this one, we can call it uncut chip width and may be that during the machining chip width may be somewhat different type it is W 0 and this may be W.

In most of the orthogonal cutting operations, we assume that W 0 is equal to W, but well it can be different also. And the tool has dug this distance, so it is t 0. And t 0 zero is the uncut chip thickness when the chip is coming chip is getting compressed and the chip thickness is basically t. And this is the machine surface behind this is coming like this.

Now, here I summarize that what are the characteristics of orthogonal cutting. In orthogonal cutting, first point is that cutting edge of the tool is perpendicular to the direction of cutting velocity. So, there is a cutting velocity here and this cutting edge is perpendicular to the direction of cutting velocity. Cutting edge is wider than the work piece width and extends beyond the work piece on either side; other side we are not able to see, but the other side also the similar tool is there. This side you are able to see that this cutting edge is actually extending beyond the cutting beyond the work piece width.

And this one the width of the work piece is much greater than the depth of cut so that the plane strain situation occurs that means, width of the work uncut chip thickness is not increasing. The width of the work piece is much greater than the depth of cut. So, what I am calling depth of cut here depth of cut here is the depth of this one; that means, t 0 ok. So, t 0 is much a smaller compare to that. I am not talking about the depth of cut in turning operation. This depth actually is more related with the feed of this one because in the in turning operation that tool is first digging or penetrating inside the feed direction ok. And width is along the depth direction that point we will I will again and again emphasize because it is here where most of these students confuse.

So, the width of the work piece is much greater than the depth of the cut. The chip generated flows on the rake face of the tool which chip velocity perpendicular to the cutting edge. So, here the chip velocity is perpendicular to the cutting edge and this plane and the cutting forces act along x and z direction only x direction means along a this direction and z direction in the vertical direction, but there is no force along the width side. This side there is no force and it there so therefore cutting forces can be represented in x z plane and I can carry out two-dimensional analysis.

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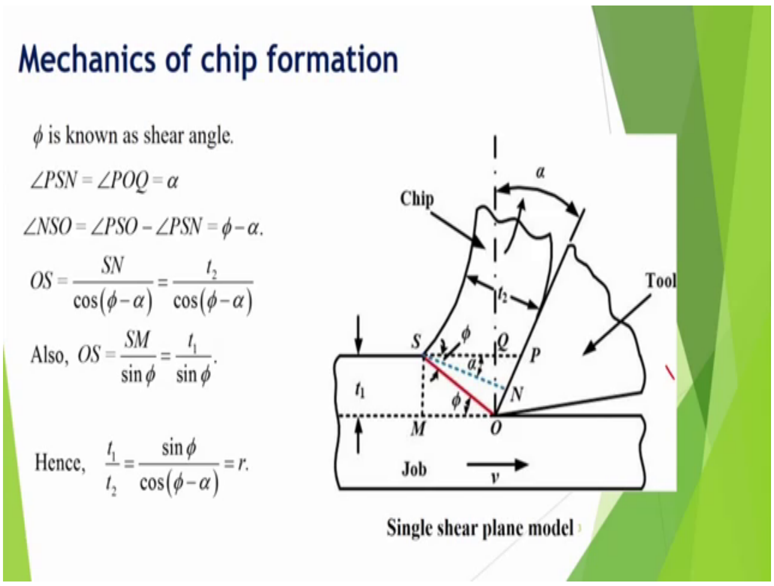

Here let us discuss the mechanics of chip formation ok. The last in this slide I have taken one figure from one open access paper that is a published in Procedia Manufacturing. And in this paper that notation was that see rake angle was shown by gamma and relief angle was shown by alpha. But now onwards I am not going to talk much about the relief angle anyway, so I am indicating the rake angle by alpha because in most of the your text books it is indicated by alpha. And I am going to show you that how the cutting is taking place. And now I will not make any three-dimensional picture instead i represent it by a two-dimensional figure.

So, in the two-dimensional figure this is t 1 and width of this one chip are you can say the width of the work piece is perpendicularly to your screen. You can again see that this is the width here. If I take that side view of that I will be getting 2d picture only, and you can understand that the width is perpendicularly to your screen. And there is a tool then there is a chip. I am now assuming this is called single shear plane model. Many persons have observed this type of behavior that there is a small shear zone based on that this was formulated that there may be shearing taking place along. So, so that means, uncut chip thickness is t 1. And this chip this material actually passes through this shear plane and it goes to in the form of chip with chip thickness t 2.

So, this angle is show is called phi. This angle is called phi. And then this since this so is making phi angle with the horizontal, horizontal means it is the direction of the cutting velocity. Here I am assuming tool is a stationery and job itself is moving with a velocity v. So, the job direction is indicated. Now, if it is so is from horizontal that angle is phi, naturally the same thing here is shown here that this angle is also phi, because sp is a horizontal line.

And then if from S, I drop a perpendicular along this one on the tools face, so this is S N and this line is naturally on the tool surface that P N line. So, S N is perpendicular to that. And this angle will be alpha, because the tool rake surface itself is making from vertical line alpha. So, this will make from horizontal line alpha. And this is this is the picture. Now, here this is t 2 and this is t 1, and this one is the chip.

So, phi is known as shear angle, angle P S N P S N, P S N is alpha that is same as P O Q that is this one and then N S O, N S O, N S O is basically P S O minus P S N; that means, phi minus alpha. So, this angle N S O is basically phi minus alpha. Now, O S is equal to O S is the length of the you can say shear plane here that is S N divided by cos phi minus alpha; that means, t 2 S N is basically t 2 chip thickness. So, it is t 2 divided by cos phi minus alpha. Also O S is equal to from this triangle O S M it is S M divided by sin phi, so this becomes t 1 by sin phi. So, equate both O S, then you get this relation t 1 by t 2 is equal to sin phi divided by cos phi minus alpha. Well, phi is the shear angle and alpha is the rake angle. And this ratio t 1 by t 2 is called the cutting ratio.

As t 2 is more than t 1. So, therefore, cutting ratio is always less than 1. So, cutting ratio you can be say for example, point phi t 1 by t 2 is equal to r. So, this way if you know the cutting ratio, if you experimentally measure the cutting ratio, you can find out shear angle by this relation or if somebody is telling you the shear angle, then you can find out the cutting ratio.

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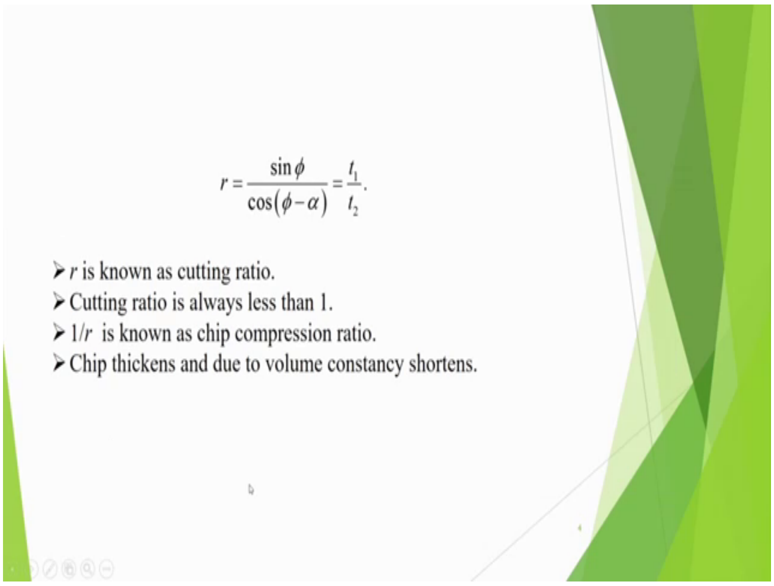

Now, here so you get a nice relation r is equal to sin phi divided by cos phi minus alpha is equal to t 1 by t 2. And r is known as cutting ratio, cutting ratio is always less than 1. And 1 by r is known as chip compression ratio; 1 by r is always more than 1. So, it will show how much compression of the chip is taking place because the chip is getting compressed t 1 is becoming t 2. And chip is also getting short end also volume remains same. And suppose we assume that the width of the chip is also remaining same. So, if the, its thickness is increasing chip thickness, then the chip length will decrease. So, 1 by r is known as chip compression ratio and chip thickness and increases. And due to chip length in decreases due to volume constancy is shortens. So, chip thickness increases and due to volume constancy that it presence shortens.

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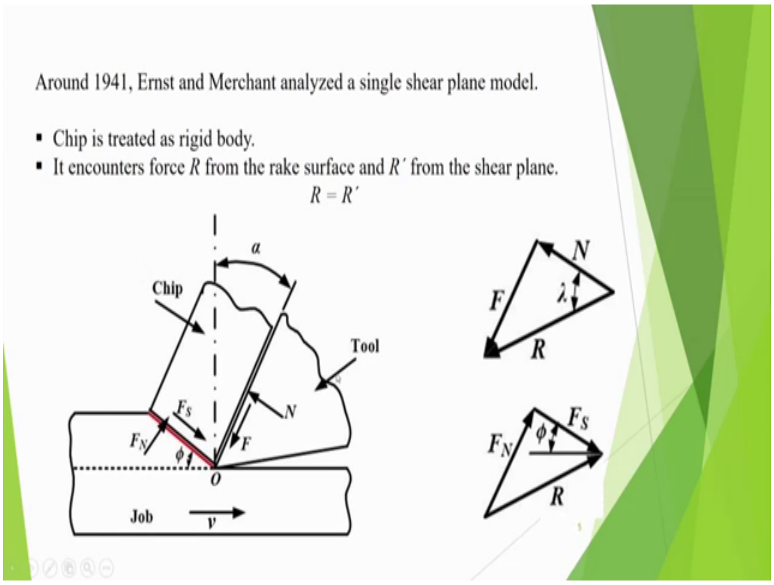

So, around 1941, Ernst and Merchant analyzed a single shear plane model. In fact, we plane etcetera they also have given one cord model that material is getting removed by shearing action just like a dig of cord, one cord is sliding was the other. So, based on that model, this has been indicated. Here chip is treated as rigid body. It encounters force R from the rake surface and R lies from the shear plane. It is if you make a free body of diagram of the chip, you will see that there will be two contexts one is chip is touching this tool surface. So, resultant force is R here and another is that actually on the shear plane here also the resultant force is basically R. So, this is F n and F s. F S means the shear force on the shear plane and F n is the normal force.

If we draw this diagram here, and this side you have got F, F is the friction force on the rake surface and N is the normal reaction of the rake face, in that case this resultant will make angle lambda from the normal and that is called friction angle. You know very well that resultant of the normal force and friction force that makes angle in the with the normal one and this is lambda here and on the other side you have got F N and F S here.

So, this is F N and F N, so they will write to give you resultant are same way because now if you see that on the two sides one side there is force all other side also there is a R. And it is a two force system, so both the forces should be colonial and they should be equal. So, this is also R, F N into F S. However, this F S is now making the angle phi from the horizontal because this is shown here F S is making from the horizontal this one. So, this is the situation job is moving in the from left to right.

Now, what Merchant did one thing that he is observed that see there are two forces F and N both are orthogonal, they are giving resultant R. Then there is a force on the shear plane coming normal and on the shear plane in the shear plane direction tangential that is F S that is also at 90 degree and if this will also give resultant. Similarly, my cutting tool is moving in the velocity direction or job is moving with a velocity v, so there will be one force which is along the direction of the velocity that is the main cutting force that is the active cutting force, because it gives you power F into V.

And then there will be a vertical force that is first force that is perpendicular to the cutting force, but these two force is are also 90 degree and these are the things which machine is supplying. See machine is not giving you F N and F S. F N and F S are generated inside the material machine is giving you one cutting force and one first force will come on the machine is structural. So, those are also 90 degree, there resultant will also be basically same R. So, Merchant thought to represent these force components by means of a circle.

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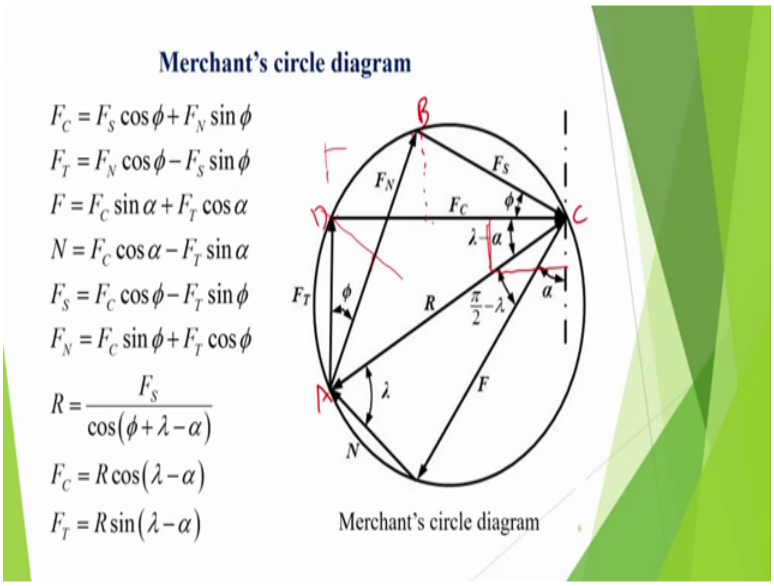

And this is called Merchant's circle diagram. This has got advantage that there is a theorem that in a semi circle if you make a triangle with one side as the diameter, then the angle between two lines is 90 degree, this is theorem. So, taking advantage of this fact that he decided to represent it in the form of a circle and that is why it is Merchant circle. So, in the Merchant circle if you see what has been same thing has been done here diameter is represented as the resultant force. So, there is a friction force on the tool that is F that is making F itself is making an angle alpha from the vertical, because this is clearly from here F is making a angle alpha from the vertical. So, it is making it here and perpendicular to that of course is N that another component must be N.

Suppose, suppose I have made this diameter like this and then perpendicular to that in a or what I can do that I can draw a this one line from here at alpha. So, it will cut it here. Once I have drawn it at alpha from here and it is cutting this way then naturally we can take perpendicular to this that is N. And one can understand that in this case this should be this angle is lambda, because the normal is making with the resultant direction. So, this one is lambda and then this becomes pi by 2 minus lambda that is one part is complete.

Then on the shear plane what is happening in the shear plane we have got this force F S that is from here we draw in the phi direction, because this is shearing is taking place in the phi direction, this is like this show F S. And perpendicular to that I draw that means; I join this point to this point that is F N. This angle is also 90 degree because it is in the semi circle. And then similarly if I draw a horizontal line here that is cutting force F C and then we have got first force F T. So, if this one if we have got F S that is making from horizontal phi, then F N will make the phi angle from F T that means vertical direction.

Now, you note very well that this is alpha and this is phi by 2 minus lambda. And these whole angles should be 90 degree that means, this whole angle from here to here this is 90 degree. So, therefore, you can calculate it very well, this small angle is coming lambda minus alpha. So, in the triangles, now I am knowing all the angles, then it is very easy to get the two components in one system if the components in the other system are known.

For example, if I know F S and F N, F S and F N and I want to find out F C. So, from the diagram, we can see we can resolve this in along the F C direction horizontal direction, so we get F C is equal to F S cos phi and F N sin phi because if I take the projection this will be in this direction F N projected along this direction like this and so from here this point projected here, so this is one component and F S cos phi. Similarly, F T that means, in the first direction F N cos phi, because this will be projected vertically F T, but F S this is opposite because this arrow is down, so F N cos phi minus F S sin phi. So, we got one variation.

Similarly, we can say F is equal to this is F is equal to F C now these angle is what F C this is making total angle pi by 2 minus alpha. So, this becomes F is equal to F C sin alpha plus F T cos alpha F T F is equal to F C sin alpha and F T also we will projected here F T cos alpha. And similarly n is equal to F C cos alpha minus F T sin alpha. And F S will be F C cos phi because this can to be projected and F T if I project it along this direction this projection will be in opposite direction with the (Refer Time: 21:58) here. So, this is F C cos phi minus F T sin phi; and F N is equal to F C sin phi plus F T cos phi.

So, at the same time, from this triangle, we get R is equal to F S divided by cos phi plus lambda minus alpha, if I call it A, this is A, B, C. So, from ABC triangle if I write BC divided by AC will be this is phi plus lambda minus alpha. So, cos phi plus lambda minus alpha that means, R is equal to F S divided by cos phi plus lambda minus alpha that we have got R.

And F C is equal to very easily we can say is nothing but R cos lambda minus alpha, and F T is equal to R sin lambda minus alpha that is appear from figure you see; and if I name it D, ACD triangle. So, by if we make just a simple Merchant’s circle diagram, it is very easily for me to write the relation between the force components, you need not memorize it that is why this is very important.

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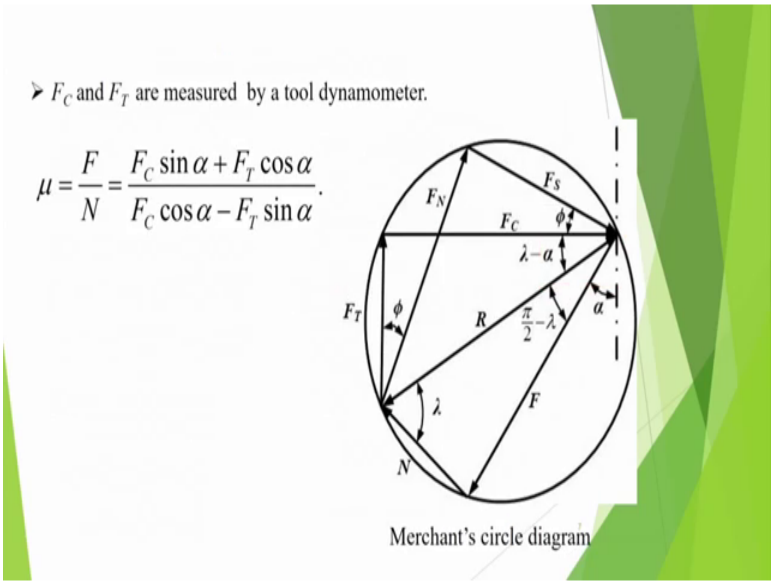

Now, F C and F T are measured by two dynamometer. Forces these are the force components dynamometer can be fitted on the machine; it can measure the cutting force horizontal force coming on the machine and also the vertical force. So, mu is equal to F by n because mu is the friction coefficient here on the two surface. So, mu means coefficient of friction on the surface that means, Coulomb coefficient basically we can say that is null a tangential force divided by normal force so F by N. And that is F C sin alpha because F is equal to F C sin alpha plus F T cos alpha and divided by N is F C cos alpha minus F T sin alpha.

So, if I can measure these forces F C and F T by dynamometer way angle by geometry I am knowing, then I can estimate that what will be the coefficient of friction on the weak surface. So, this is what this one this diagram is again showed to you for this side, so that you can know that F is really F C sin alpha plus F T cos alpha and n is F C cos alpha minus F T sin alpha. Now, how do you find out F S, F S is the shear force coming on the material on that shear plane, this is the shear force.

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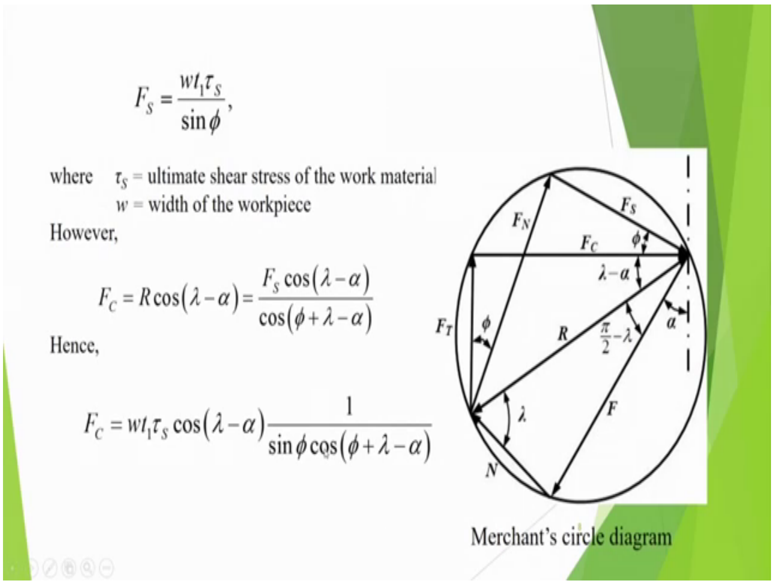

If we know the ultimate shear strength of the material, suppose tau s is the ultimate shear strength. So, if I multiply tau s by the area on which this force is acting, area is what this length of the shear plane, the length of the shear plane multiplied by the width. So, area will be w into t 1 by sin phi; t 1 by sin phi is the length of the shear plane. So, w t 1 by sin phi is the area on the shear plane; and tau s is in Newton per meter square it is the ultimate shear stress of the work material and you get F S.

However, now F C is equal to R times cos lambda minus alpha. So, we can say r instead of all we can write F S divided by cos phi plus lambda minus alpha because these angle is phi plus lambda minus alpha. So, F S divided by R is equal to cos phi plus lambda minus alpha. So, instead of R, I can write F S into cos phi plus lambda minus alpha. And this will be cos lambda minus alpha multiplied here. Hence F C will be equal to I put the value of F S here. So, we get w t 1 into tau s into cos lambda minus alpha divided by a sin phi into cos bracket phi plus lambda minus alpha bracket close.

Now, you can see that here alpha is known from the geometry cutting tool is having some geometry, where we have mounted the cutting tool. Now, alpha is fixed we cannot do anything for alpha. Lambda is from the physics of the process because it is the coefficient of friction, it depends suppose you are putting some coolant etcetera some lubricant then the friction will be different lambda is a friction angle basically you know tan lambda is equal to mu. So, here that is also fixed only thing about phi I am not moving. So, phi is internal thing that is called as a shear angle.

So, what do we do that here if we apply one basic principle that actual cutting process, we will try to minimize the energy consumption so that means, that F C should be minimum so that means, I can minimize this expression with respect to phi and then I can find out that the what is the value of phi.

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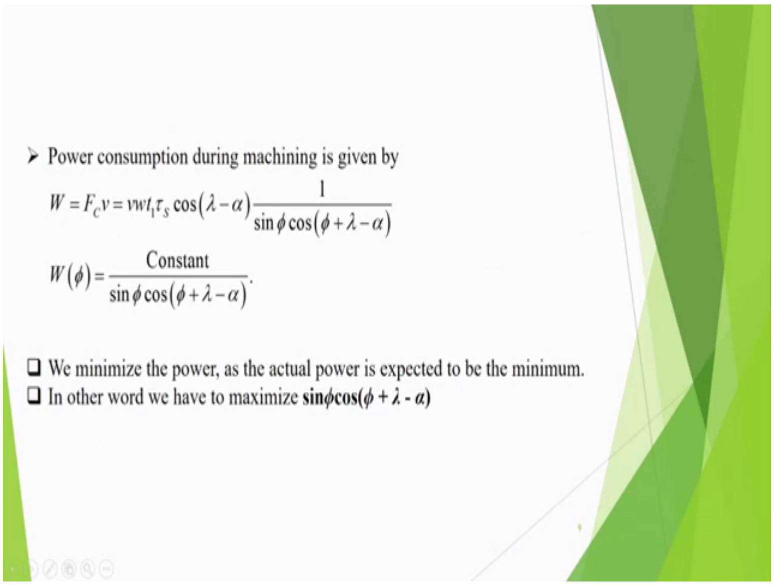

So, power consumption during machining is given by w is equal to F C into v that means, this quantity and all other things are constant. So, w becomes a function of phi that is some constant divided by sin phi into cos phi plus lambda minus alpha. So, you are getting this one. Now, we minimize the power as the actual power is expected to with the minimum. In other words we have to maximize sin phi into this quantity cos phi plus lambda minus alpha. If we want to minimize w phi; that means, we have to maximize this quantity. So, for maximization this is a continuous function. So, this is condition is that if you differentiate this function with respect to phi and make it equal to 0.

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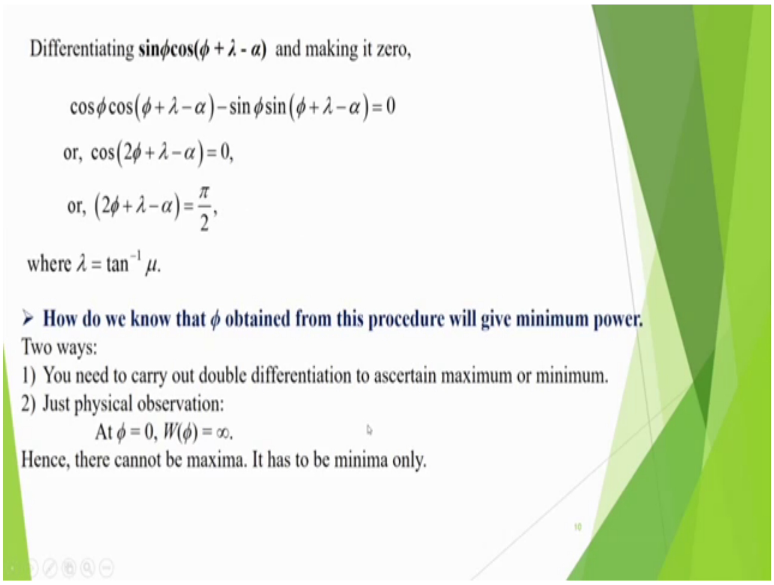

So, this should be doing giving you the result. So, I differentiate this with this respect to phi one by one this is a product rule I apply, first sin phi differentiated use cos phi and this portion is left as it is, then I differentiate cos phi plus lambda minus alpha with respect to phi sin phi is left as it is, so this differentiation becomes sin phi plus lambda minus alpha and this is sin phi. So, this is equal to 0. Or this can be seen I apply the formula cos A cos B minus sin A sin B then you get cos 2 phi plus lambda minus alpha, this gets added, and this is equal to 0.

So, this implies that 2 phi plus lambda minus alpha must be equal to pi by 2. And lambda is of course, tan inverse mu. So, lambda is also known to be. So, by this formula I can find out the value of phi. How do we know that phi obtained from this procedure will give minimum power I will just made the phi because it can be minima and it can be maxima also. So, two ways in order to visual you differentiate this expression one more time double derivative and put the value of phi which has been obtained here.

And see whether the double derivative thing is positive. If it is positive, then it is minima; if it is negative, then it is maxima that is one way. Otherwise just by physical observation you see that anyway w phi will be infinite if phi is equal to 0. So, it cannot be maxima. So, phi must provide some minimum energy because at phi is equal to 0, you are getting w phi equal to infinity. Hence there cannot be any maxima it has it is at in it will be at phi equal to 0, so it has to be minima only.

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$$F_C = \frac{2wt_1\tau_s \cos(\lambda - \alpha)}{1 - \sin(\lambda - \alpha)}$$

Now, what happens that after I have obtained the value of phi this value you obtained, and I have expression for F C i put that value of phi here in this expression of F C, then I getting very nice relation F C is equal to 2 w t 1 tau s cos lambda minus alpha 1 minus sin lambda minus alpha. Notice that now phi has vanished because phi now I who obtained from this condition which phi will be the minimum energy ok. So, we got very nice expression like this 2 w t 1 tau s which is indicating that the cutting force is linearly proportional to chip width and it is also proportional to uncut chip thickness.

It is also linearly proportional to tau s ultimate shear strength. And it is of course, dependent on friction also cos lambda that point is there and here also this point is there if friction will increase then sin lambda this portion will increase. And therefore, 1 minus sin lambda minus alpha will decrease, and therefore, effect will be to increase F C cutting force or if I am increasing the alpha then cutting force will reduce, because if I am increasing alpha that means, this portion is going to be bigger. So, it will reduce and similarly here you can argue.

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Now, how does Merchant's original model agree with experiments? This is Merchants original model. It agrees when cutting synthetic plastics it was observed like that, but it is having poor agreement with metals. If the metals this model is not predicting the correct thing. In fact, 2 phi plus lambda minus alpha should be equal to phi by 2 that type of thing may not come properly.

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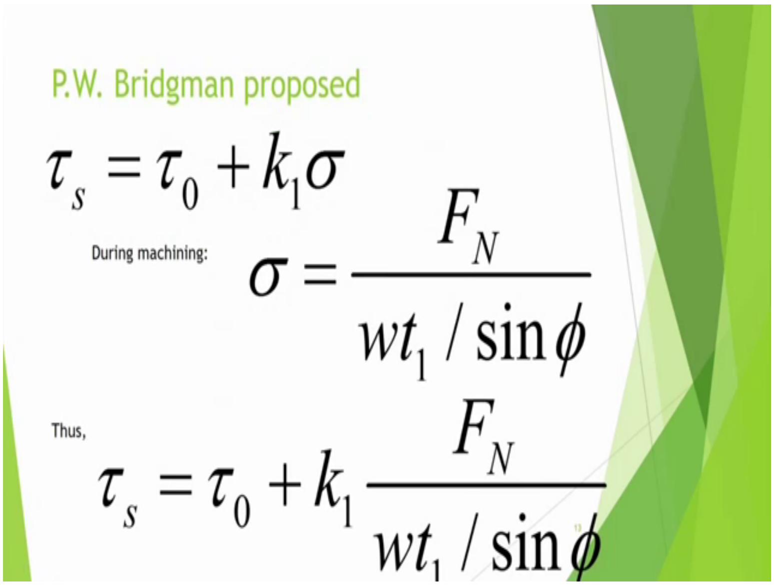

So, here what happens that P. W. Bridgman has earlier proposed that tau s is equal to tau 0 plus k 1 sigma; that means, sigma is compressive stress so that means, if on a surface there is a compressive stress coming then it is a ultimate shear strength is also increasing in a linear way. Tau 0 is some base strength. So, tau 0 plus k 1 sigma, so by the Merchant took this relation of P. W. Bridgman and he try to put it in the his expression.

So, what he did that during machining sigma is equal to normal stress coming on the shear plane is F N that means, normal force on that you are seeing that F S and F N, I can show you again. This is F S and this is F N, normally it is coming like this. And this F N divided by F N divided by w t 1 sin phi, w is what width of the shear plane and t 1 by sin phi is the length, so this is area. So, sigma you got. And put it in the expression, so tau s is equal to t 0 plus k 1 F N divided by w t 1 by sin phi.

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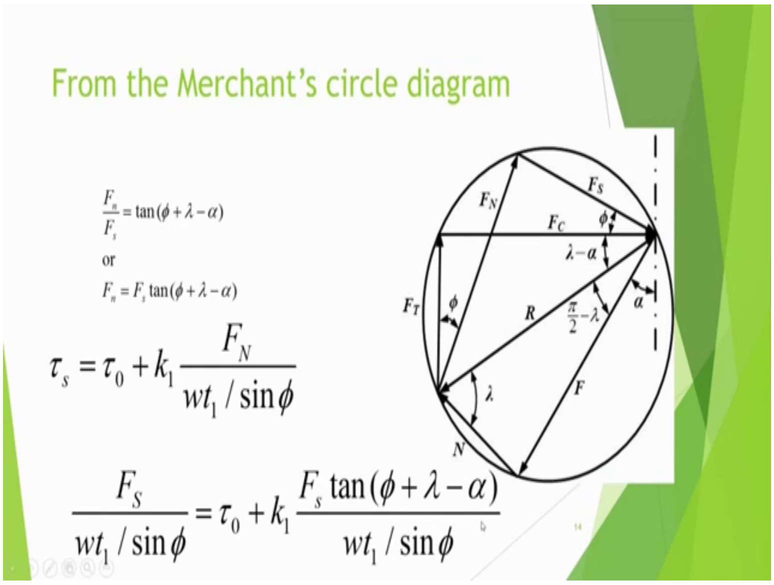

Now, from the Merchant's circle diagram, we see that F N by F S; that means, normal force shear F N by F S this is tan phi plus lambda minus alpha or F N is equal to F N is F S tan phi plus lambda minus alpha. So, what we do that tau s is equal to tau 0 plus k 1 F N by w t 1 by sin phi, and here tau s, I am putting like this F S divided by w t 1 by sin phi because that is tau is F S divided by the area of this shear plane.

And this is equal to this is equal to tau 0 plus k 1, k 1 is what k 1 is the constant in the Bridgman variation, k 1 is some material constant k 1 it may indicate dependency of the ultimate shear strength and the normal stress. If k 1 is equal to 0; that means, ultimate shear strength is basically constant, it is not dependent on sigma. So, k 1 is that one. And

F N is equal to basically F S tan phi plus lambda minus alpha and divided by w t 1 sin phi as it is it comes. So, you are getting this type of relation.

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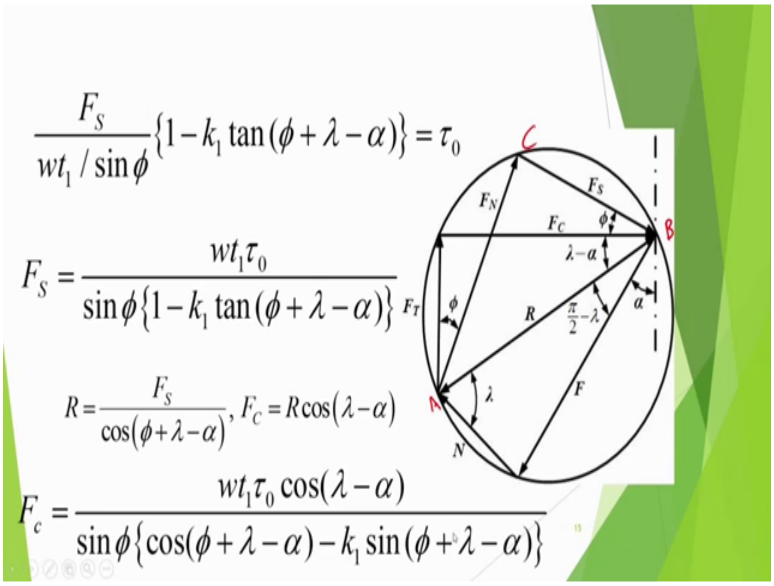

So, F S is a both side, what you can do you can take F S one side, you get this nice expression F S w t 1 sin phi and bracket 1 minus k 1 tan phi plus lambda minus alpha is equal to tau 0. So, F S is equal to now this will be equal to w t 1 tau 0 sin phi 1 minus that much F S as come this way, and a once we have known the F S then we can find out r is equal to F S divided by cos phi plus lambda minus alpha F S divided by cos phi plus lambda minus alpha from this triangle that means, if I name it triangle ABC in from the triangle ABC, you can see this one that F S divided by R is equal to cos phi plus lambda minus alpha.

So, this relation we need use of that. So, R is equal to F S divided by cos phi plus lambda minus alpha. And this one F C is equal to R cos lambda minus alpha. So, I can get a expression for F C. We get a nice expression for F C, F C is equal to w t 1 tau 0 tau 0 is the ultimate shear stress when there is no normal stress sigma on the plane. So, w t 1 tau 0 cos lambda minus alpha that is sin phi and then we are putting these values cos phi that this value has been put. So, F S is equal to cos phi plus lambda minus alpha here it will be multiplied and then you will get minus k 1 sin phi plus lambda minus alpha like this.

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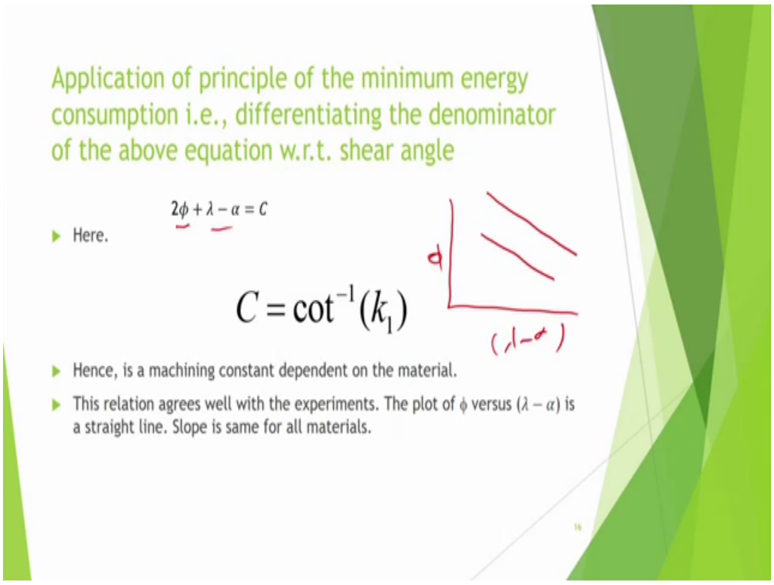

So, application of principle of minimum energy consumption that is differentiating the denominator of the above expression with respect to shear angle because on the top everything is constant, so basically in order to minimize F C, I want to maximize this expression in the, that bottom side this expression has to be this one minimized. And so denominator has to be minimized, then you get if you differentiate that with respect to phi and make it equal to 0, you get 2 phi plus lambda minus alpha is equal to C, where C is equal to cot inverse k 1. So, you no longer get phi by 2; only if k 1 is equal to 0, then C cot inverse 0 will be phi by 2 then it will be same as the Merchant's first solution

So, this is Merchant's second solution in which two phi plus lambda minus alpha is equal to c and C is a material dependent constant because k one is dependent on the material. So, you get different type relations for different materials. Here it is a machining constant dependent on the material this relation agrees well with the experiments, if we plot phi versus lambda minus alpha it will be straight line and slope will be same for all materials, but intercept will be different. This equation you can see 2 phi plus lambda minus alpha

So, in this case, this is phi and lambda minus alpha, they are linearly related no doubt, but this C will be different here, so that type of thing you can get for different materials you can get different type of relation. Here this may be phi, lambda minus alpha, for one material it may be like this, for other material it may be like this. This type of things you can do. In fact, two ways conducted around 50s lot of experiments and in that those experiments he has plotted these types of relations for various materials.

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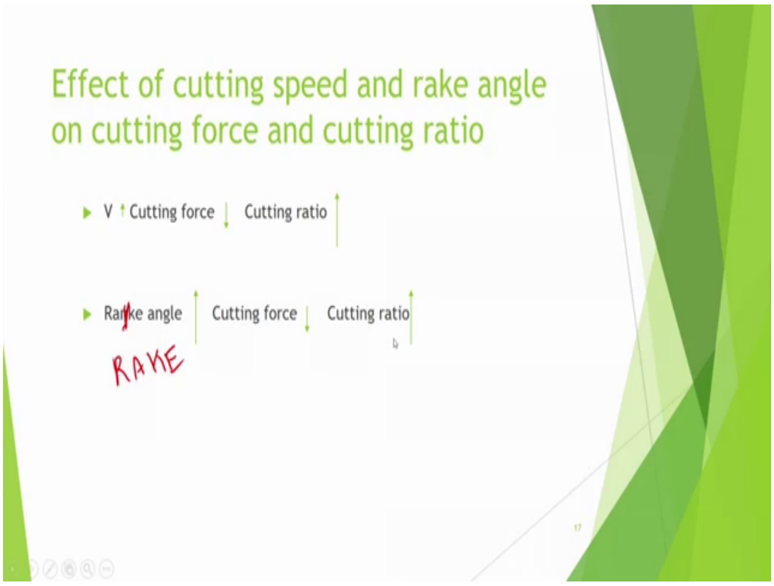

Now, effect of cutting speed and rake angle on cutting force and cutting ratio, we know that if the cutting we have observe that physically that if V increases, then the cutting force actually increases this is not coming from this simple analysis, this is our observation may be that when v is increasing then the friction force is getting reduced even coefficient of friction that is why that way it can come. And cutting ratio is basically is increasing ok. Maximum it can go up to 1 only, but it is increasing.

So, what happens that suppose the friction is suppose reducing, in that case phi will increase and increasing phi may cause to increasing cutting ratio. And effect of rake angle that even this is rake angle, this is not Ranke angle, it is rake angle. rake angle has been shown that if rake angle is increases then the cutting force basically reduces and cutting ratio increases ok. So, rake angle and cutting velocity have got similar type of effect. rake angle this can be seen also here from those relations also that means, earlier which we have do not force ration. If the rake angle is increasing, then the cutting force is decreasing.

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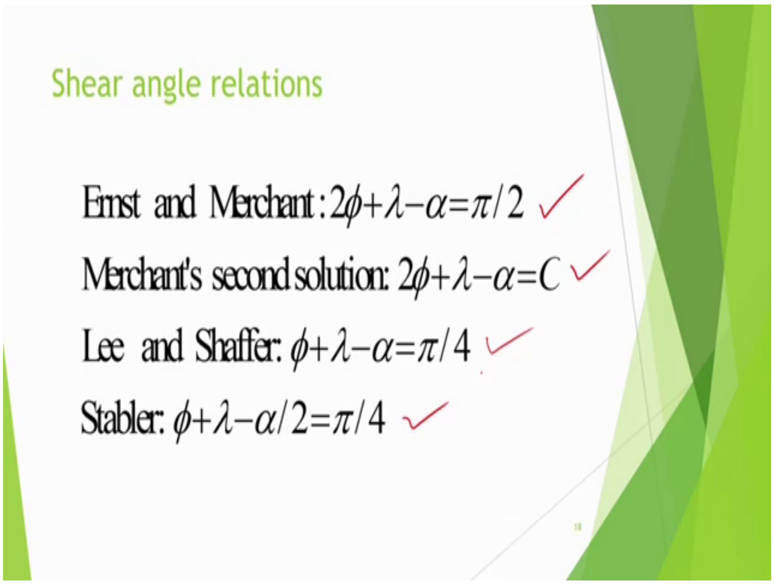

Now, there are various type of other relations also people have done different, different type of analysis. So, in those analysis, these type of relations have come, first you have seen the shear angle relation of Ernst and Merchant. These two authors, those two researchers have given that is first solution that means, 2 phi plus lambda minus alpha is equal to pi by 2.

And Merchant's second solution has given you 2 phi plus lambda minus alpha is equal to C. These two relations you have seen that is one relation this, other relation is this. And I have not derive this one. May be later on I will explain you Lee and Shaffer also have has given this relation based on the shift line field solution. Lee and Shaffer and they have even phi plus lambda minus alpha is equal to pi by 4 that is the relation. And Stabler has given phi plus lambda minus alpha by 2 is equal to pi by 4. So, there are different type of relations people have proposed.

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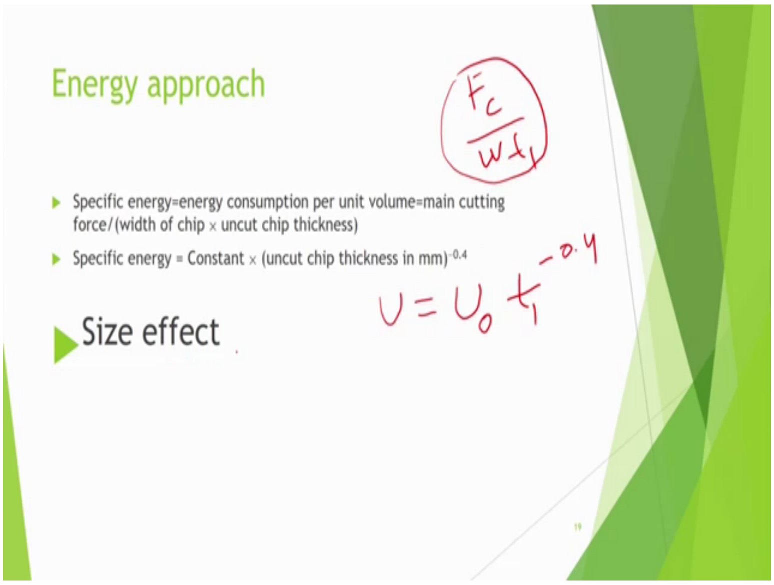

Now, we can find out the cutting energy, cutting force by energy approach. So, we give the concept of a specific energy that is energy consumption per unit volume that can be written as main cutting force divided by width of chip in two uncut chip thickness that will be energy consumption per unit volume. Why it is energy consumption, because suppose on the top side, I put main cutting position two velocity that will you power and width of the chip into uncut chip thickness two velocity that will give material removal rate.

So, when we I divide power by material removal rate, I get energy consumption per unit volume. So, basically it is like that. So, we can say that basically F C divided by w into t 1 ok. This is moon mean for each material roughly effect of the velocity and effect of the rake angle etcetera. If we discard more unless depending on per each material this quantity may be different, but so that way, if we know that for each material this figure, then I can estimate my F C.

And this specific energy, but what may happen that actually this cutting energy total cutting energy U may be function of U 0 multiplied by t to the power minus 0.4. Where t t 1 to the power t 1 is the uncut chip thickness, that means, we can take some base value U 0 some constant. And we can say of uncut chip thickness in millimeter, this is a empirical type of relation. So, do not try to convert the units and to the power minus 0.4.

If this relation is showing what that means, if uncut chip thickness is decreasing, then the specific energy in increasing. That is why, it is more difficult to cut a small chip thickness material this one, by if we increase remove this small chip thickness, my cutting force is going to a increase means because of the chip thickness itself is reduced. So, cutting force we reduce also, but not in that proportion, because this specific energy basically increases. So, this thing is called size effect that means, micro cutting and micro cutting behavior is different, so that is size effect.

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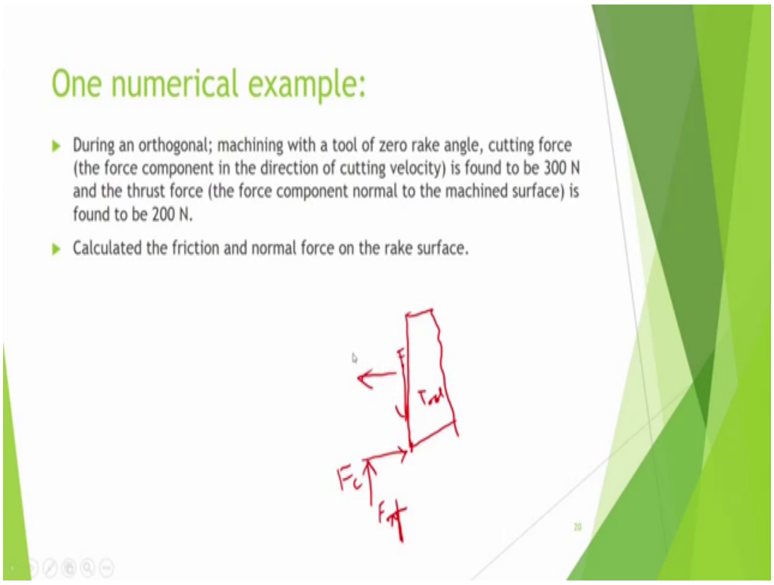

Now, having told you this thing, let us discuss one numerical example. During an orthogonal; machining with a tool of zero rake angle, cutting force that means, force component in the direction of cutting velocity is found to be 300 Newton. These are some realistic figures. And the thrust force the force component normal to the machine surface is found to be 200 Newton. Then calculate the friction and normal force on the rake surface that is the question that he is asking this question.

So, what we can do that as the rake angle is zero machining with a zero rake angle. So, rake angle is already 0, so that means, cutting is basically being done. Like this, this is the say tool and this is the tool ok, this is this is the tool that means, rake angle is 0. So, this is suppose the cutting force is F C. And this is F N is also there thrust force, but the same thing that means, shear on this tool.

The friction on the chip, we will also be in this direction F, and the normal force is N. So, you can see the cutting friction force itself is nothing but the thrust force here this is T, so that is what, this for so by even observation we do not have to do anything. Even if there is a relation in the relation also, we can we had a relation between friction force between F C this one. Suppose, suppose F is equal to F C sin alpha plus F T cos alpha.

If sin alpha is 0, then F becomes F T that means, thrust force itself is same as the friction as the friction force on the tool. And a normal reaction offered by tool is nothing but it will directs the cutting force F C, so that is, but is when physically it is understood, that it is a orthogonal cutting. So, as I have made a figure from this figure it should be clear to you that here this is F C is equal to basically normal reaction of the tool.

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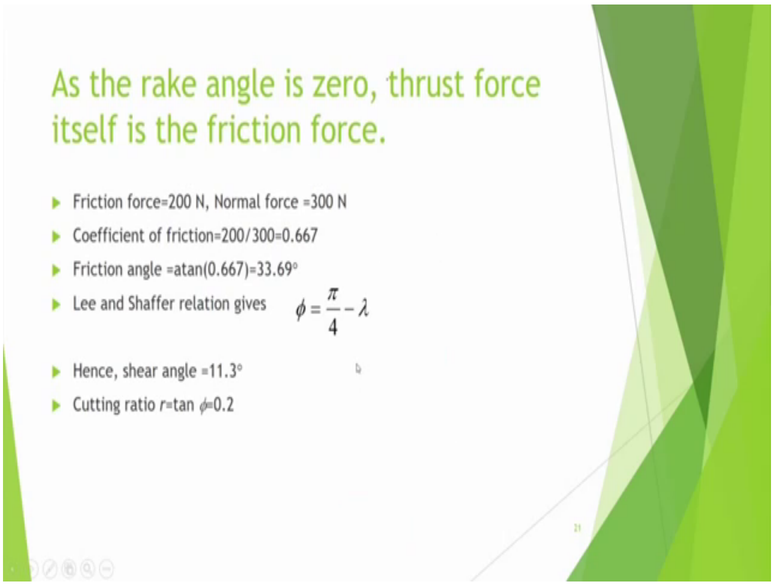

So that means, friction force is 200 newton, and normal force comes out to be 300 newton. So, coefficient of friction is now 200 by 300 that means 0.667. And friction angle is tan inverse 0.667 that comes out to be 33.69 degree. And if I use Lee and Shaffer relation that is phi is equal to pi, pi by 4 minus lambda, because alpha is 0. Lee and Shaffer relation was phi plus lambda minus alpha is equal to pi by 4.

So, phi is equal to pi by 4 minus lambda. And that is giving you shear angle is giving you as 11.3. And the cutting ratio r is coming tan phi is equal to 0.2, because alpha is equal to 0. So, we have cutting that relation is also there you are seeing here, r is equal to sin phi cos phi. So, r is equal to tan phi, where phi is equal to 0. So, for alpha is equal to 0 case, you are now getting, cutting ratio also cutting ratio r is equal to 0.2. And this has completed this part.

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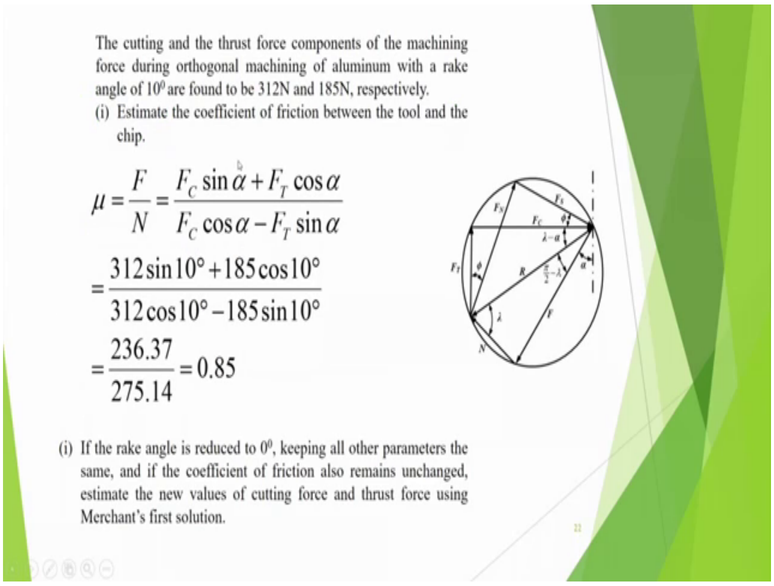

Now, let us do the enable problem. Cutting and the see thrust force component of the machining force during orthogonal machining of aluminum with a rake angle of 10 degree are find to be 312 Newton and 185 Newton, respectively. Estimate the coefficient of friction between the tool and the chip that is first part of the question.

So, in this case mu is equal to F by N. And this will be F C sin alpha plus F, because alpha F C cos alpha minus F T sin alpha this relation. You need not remember this relation just simply make a Merchants circle diagram. You can show here that F is shown here and this one. From this diagram, it will be clear that F is equal to basically F C sin alpha plus F T cos alpha. Like that N is equal to F C cos alpha minus F T sin alpha.

So, we put these values alpha is equal to 10 degree. So, 312 sin 10 degree plus 185 cos 10 degree. And this will be equal to this. So, this becomes 236.37 this 275.14, and this is equal to 0.85. So, we get coefficient of friction as 0.85. Now, the next part is that if the rake angle is reduced to 0, keeping all other parameters be same, and if the coefficient of friction also remains unchanged, then estimate the new values of cutting force and thrust force using Merchant's relation. Earlier you are getting this one, now I am just reducing it to 0 degree, earlier positive rake angle was 10 degree here.

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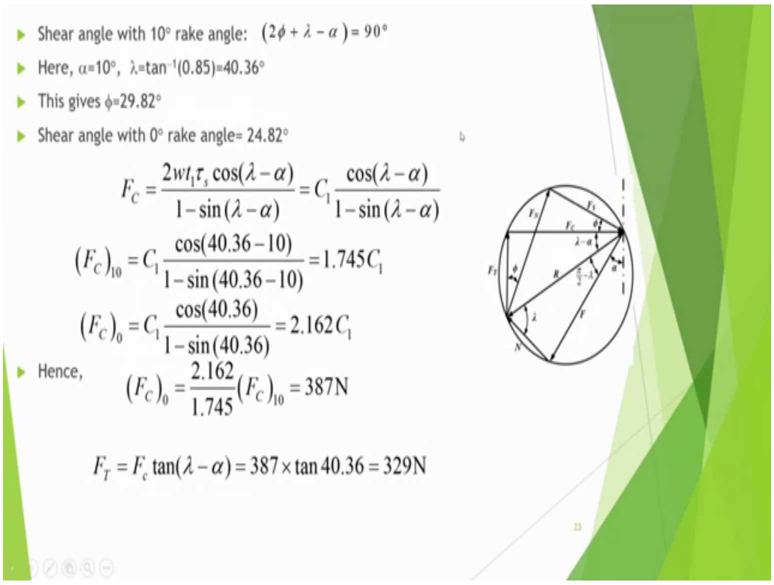

So, what we have we can do, we can find out shear angle be 10 degree rake angle, we can find by this relation 2 phi plus lambda minus alpha is equal to 90 degree. Merchants first solution if you use and alpha is equal to 10 degree, then lambda is equal to tan inverse 0.85 that is 40.36, because friction is 0.85 coefficient of friction. So, you get lambda is equal to tan inverse 0.85 that means, 40.36 degree, this gives phi is equal to by this relation 29.82 degree. Now, shear angle with 0 degree rake angle comes out to be 24.82 degree, because this one, 2 phi plus lambda minus alpha. So, lambda now alpha is equal to if we remove that 10 degree part, then it will come 24.82 degree.

Now, F C is equal to this relation is already there, I told in the previous slide. And so, here you relating which constant that is C 1 times this much. So, for the 10 degree case, F C 10 is equal to C 1, and I put cos 40.36 minus 10 and 1 minus sin 40.36 minus 10 that comes out to be 1.745C 1. And F C at 0 is coming like this, 2.162C1.

So, F C 0 is equal to 2.162 divided by 1.745 is F C at 10 that means, the cutting force at 0 rake angle is this much times, cutting force at 10, so that means, it is coming 387 newton. So, you see that this has increased, either it was 312, but it is increased. And F T is equal to F c tan lambda minus alpha simple relation, put that we find out that F T is equal to 329 Newton. So, this also has significantly increased. Here it is one 185, but there it is coming 329 Newton.

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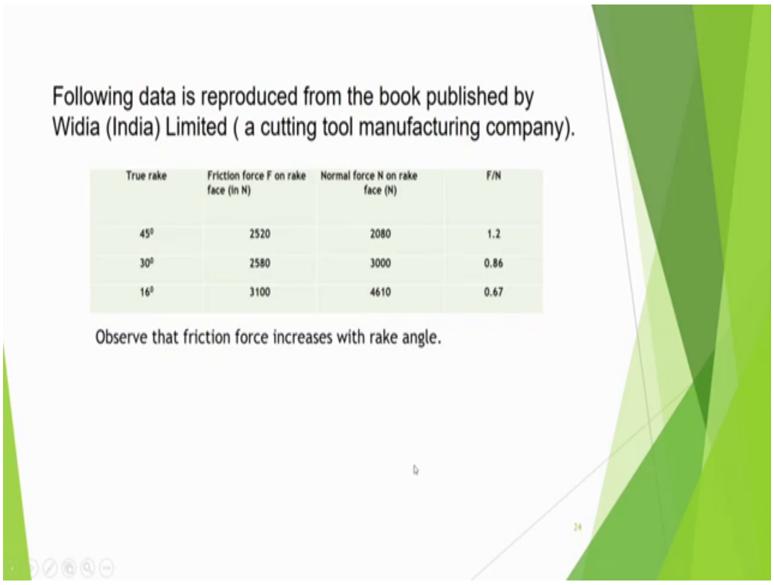

That way, we have done this problem also ok. This data once I took it from a book published by Widia India limited about the cutting tool. Widia is a cutting tool manufacturing company, and in that they have given this data practical data, this is a true rake, and this is friction force F on the rake face, and normal force N on the rake face N. This data is given 45, 30, 16, and we find out F by N that is the coefficient of friction. So, we see the coefficient of friction also is dependent on the rake angle that how much rake angle is there, because that surface inclination angle we think will changed, and the way even the surface effective area, so that is what that it is coming like this. You are observe that friction force increases with rake angle.

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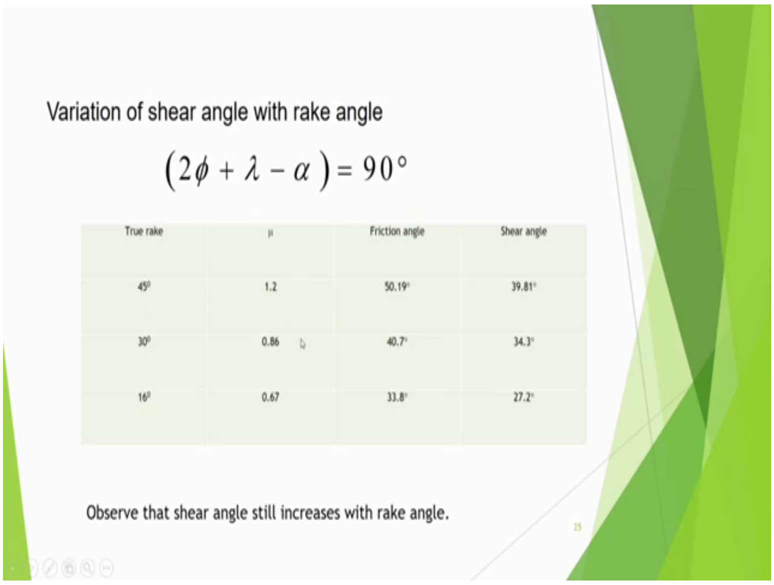

So, they have given, and then the variation of shear angle with rake angle, we want to study. Suppose, question is that how the shear angle is increasing with rake angle. So, we use Merchants first solution only, and we say 2 phi plus lambda minus alpha is equal to 90 degree. So, in this case, for each case I am knowing lambda; how, because we are for each case I know mu, and then I can take tan inverse of that mu, and I can find out the friction angle. So, lambda is known; alpha is also known for each case, we can find out phi. So, we find out the phi like this.

So, we observe that is still the shear angle is increasing with increasing rake angle, you know increase of the shear angle means, it is better that means, it is how tendency to reduce the force. So, here rake angle of 45 degree, then your shear angle is 39.81 degree, but with rake angle of 16 degree. If the rake angle of 16 degree, then the shear angle is 27.2 degree, this one.

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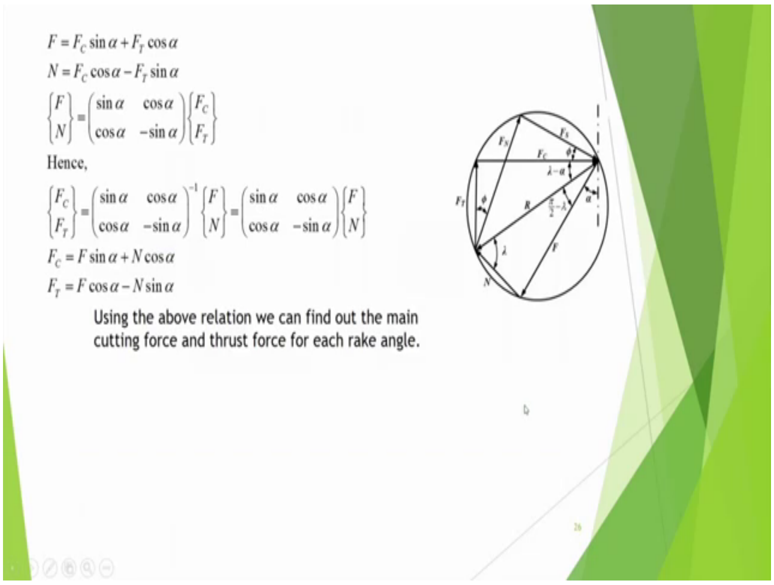

So, now we have done this part. Now, we can also find out the cutting and thrust force in each case. So, we have an relation you see F is equal to F C sin alpha plus F T cos alpha N is equal to F C cos alpha minus F T sin alpha. We can write this thing in a nice, vector and matrix form, this is F N is equal to sin alpha cos alpha and cos alpha minus sin alpha and F C F T.

Then I can invert it, so that I get F C F T in terms of F and N. So, F C into F T is equal to sin alpha cos alpha cos alpha minus sin alpha and we take it inverse, and this is F N, so this will be equal to sin alpha cos alpha and cos alpha minus sin alpha and this becomes F N. So, we have got, now we can again multiply this one the, this matrix multiplication. So, F C is equal to F sin alpha plus N cos alpha and F T is equal to F cos alpha minus N sin alpha.

But, we could have obtained this relation by simple that Merchants circuit diagram also directory, that F C is equal to take the projection of F, here F sin alpha, because this is called pi by 2 minus alpha, and N cos alpha ok, that is this one F C and F T will be F cos alpha minus N sin alpha, so that is F cos alpha minus N sin alpha. So, we could have also find out. Now, using this relation, we can find out the main cutting force and first force for each angle. For each angle, we can find out you will see that the although the friction coefficient is increasing with the rake angle, even then the forces will in general decrease with increasing rake angle, that is why rake angle is increased somewhat.

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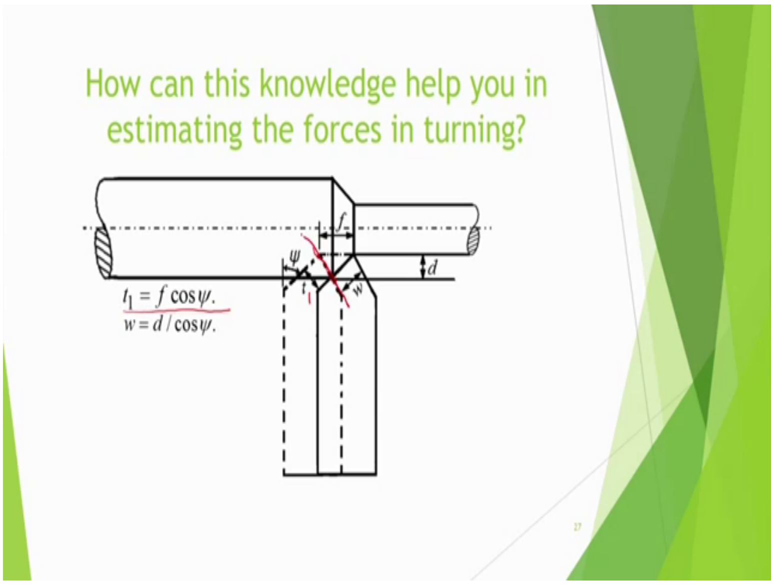

Now, this is a simplified orthogonal analysis by assuming a single shear plane. Now, we have to see that how can we make use of this knowledge in our actual machining process say for example, turning. Now, you observe that thing in the turning operation, suppose I am doing the turning, actually it is no longer orthogonal cutting, but sometimes we can approximate it like a orthogonal cutting data idea. And in that case, by cutting edge is this, and this cutting edge is moving in this direction, it is inclined a bit. And this one is I have taken here, so this t is the thickness here.

So, this t 1 is the thickness, and this is basically t 1. And t 1 is equal to f divided by cos psi, where cos psi is the side cutting edge angle in ASI system. So, we got t 1 is equal to f cos psi, and we got w is equal to d divided by cos psi. So, we are having d divided by cos psi, so (Refer Time: 57:10) width is this much. So, do not confuse in this thing, that t 1 is equal to f cos psi.

Many people even when I was a student, then we used to think that may be the depth of cut is what we have to do, because we get confused with this earlier figure, this figure we may get confused. Here usually many books are shown, call it depth that tool is giving this much depth, but depth means, we have to see cutting edge is perpendicular to that. So, here in the turning, if you approximate that thing, then here you have to see that t 1 is equal to f cos psi. And w is equal to width is actually really this, because this is the cutting edge, and cutting edge actually taking placed like this like this, so that is why w is equal to d cos psi.

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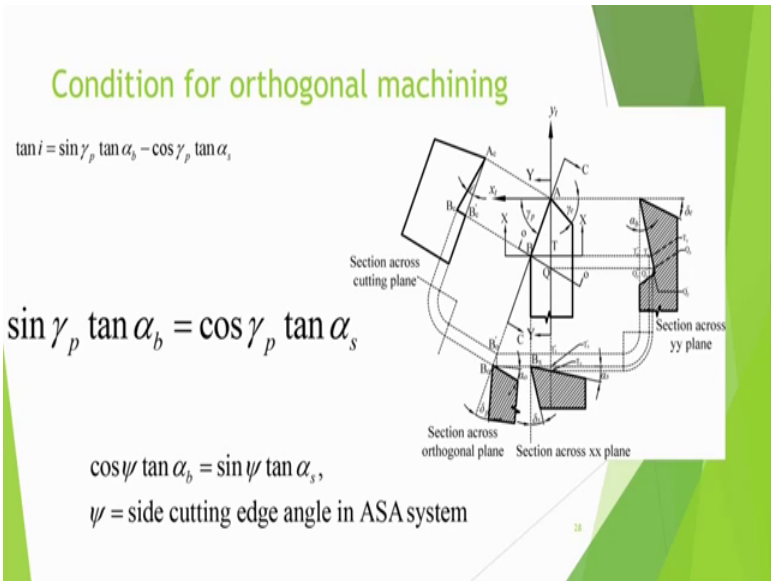

And but, since analysis is valid only for orthogonal. So, condition for orthogonal machining is actually tan i is equal to sin gamma p tan alpha b minus cos gamma p into tan alpha s. In orthogonal system, gamma p is the principal cutting edge angle, and gamma i this is gamma p, and this relation was derive derived early earlier alpha b is back rake angle, alpha s is the side rake angle. So, if the process is orthogonal, then i is 0, and you will get this condition. So, we can check this condition, how far the condition of orthogonality is true. And in terms of the side cutting edge angle, you get this type of relation. So, you can check it.

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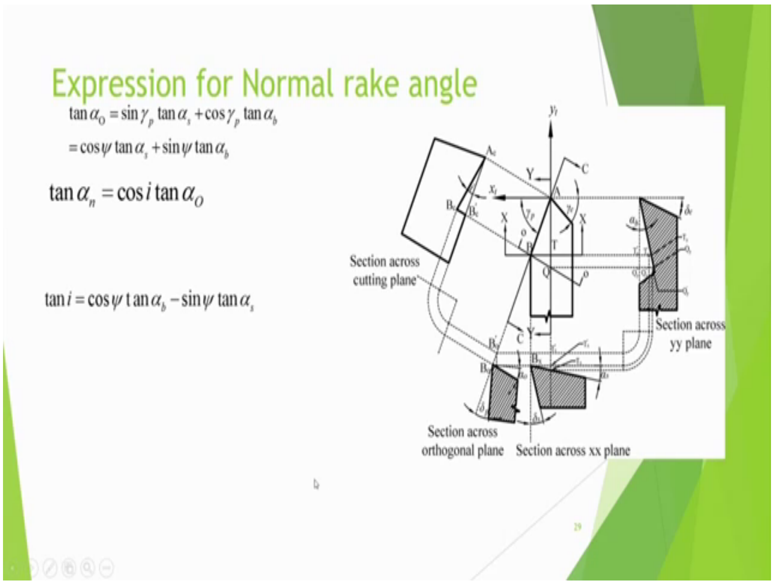

And expression for normal rake angle, what type of rake angle I should choose in the normal, well it can be used usually people say two rake angle, which I discussed later on. But, some people some books sometimes they just take normal rake angle, and they think that cutting is taking place. In that way, so therefore already there is a relation for orthogonal rake angle, that relation we can use, and we can also find out the normal rake angle tan alpha n is equal to cos i, i is the inclination angle tan alpha 0, as I have told in the previous lecture, tan i is given like this. So, I can find out the normal rake angle.

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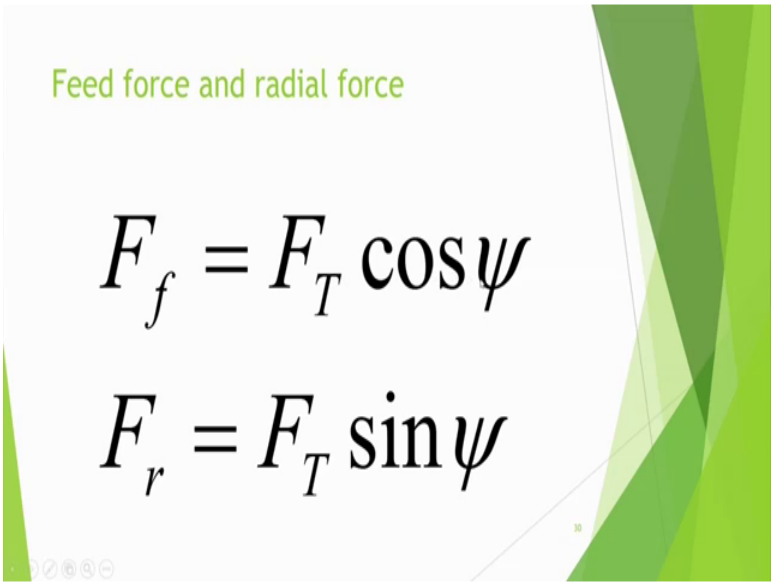

And if I put this thing here, then I can find out in the same way, I can find out the cutting force and thrust force. So, cutting force really will be in the vertical direction, means perpendicular to your screen. But, the trust force is actually in this plane perpendicular to the cutting edge like this. So, this thrust force in this case is having two components; one is the in the feed direction that is F T into cos psi, and another is the radial direction that is F T into sin psi.

So, these by this relation you can find out the feed and radial component also. You can see that if side cutting edge angle is increasing, then your radial force component is increasing, and this way it has completed. So, this a way, I have told you that how can this in a simple manner; we can estimate forces in the cutting operation ok.

Thank you.