Mechanics of Machining
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Lecture – 02
Plastic Deformation

Hello students, welcome to the course and Mechanics of Machining. Now, yesterday we told you some basics of mechanics that means, what is meant by this course. So, here we discussed this one mechanics of machining first lecture, you have already gone through that we told what is the machining [FL].

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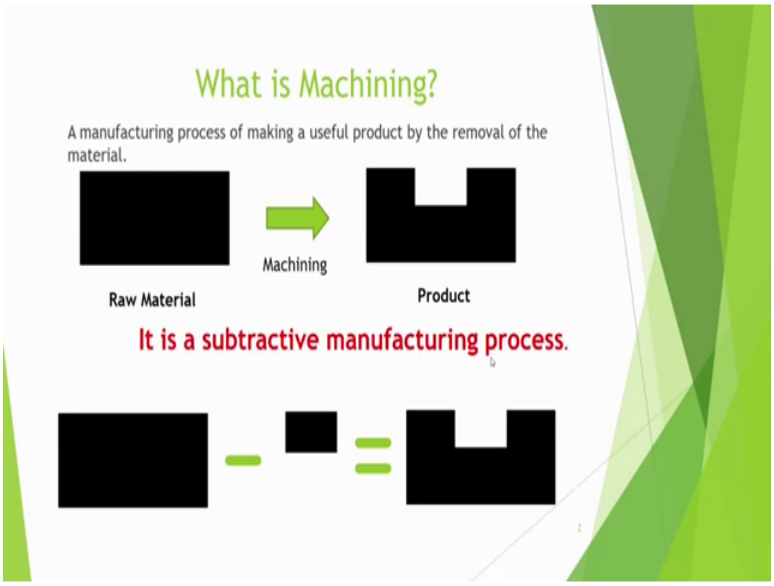

It is a subtractive machining process that point we established.

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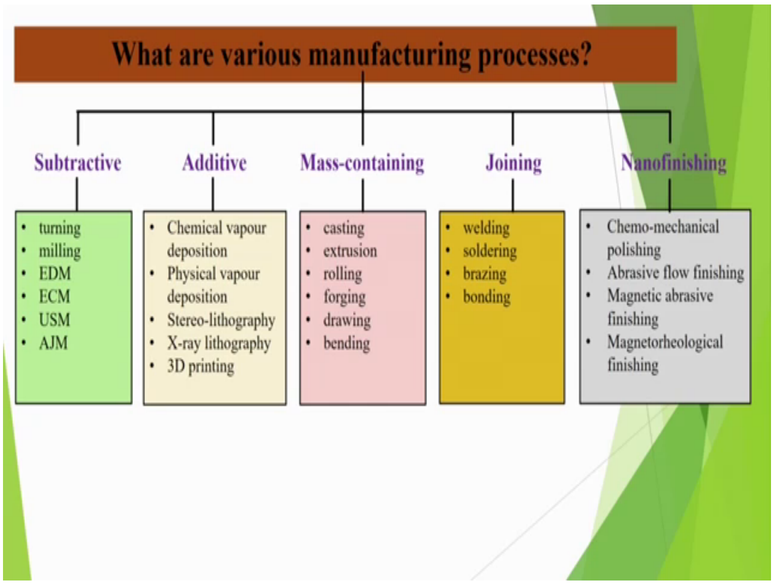

There are other type of processes also like additive manufacturing processes, mass containing processes, joining processes and nanofinishing although it is also subtractive machining process. But the focus here each the small removal of material just for improving the surface finish.

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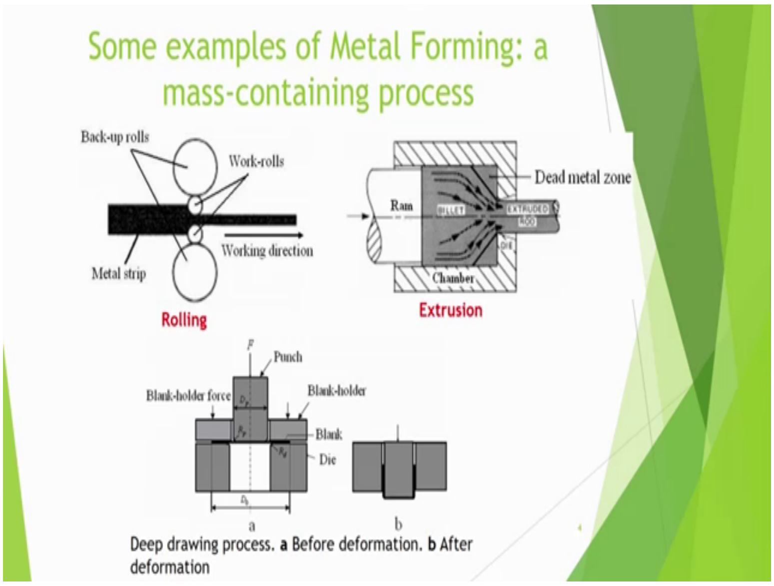

And then after that we give some examples of the other type of manufacturing processes, but finally, our focus is on the machining.

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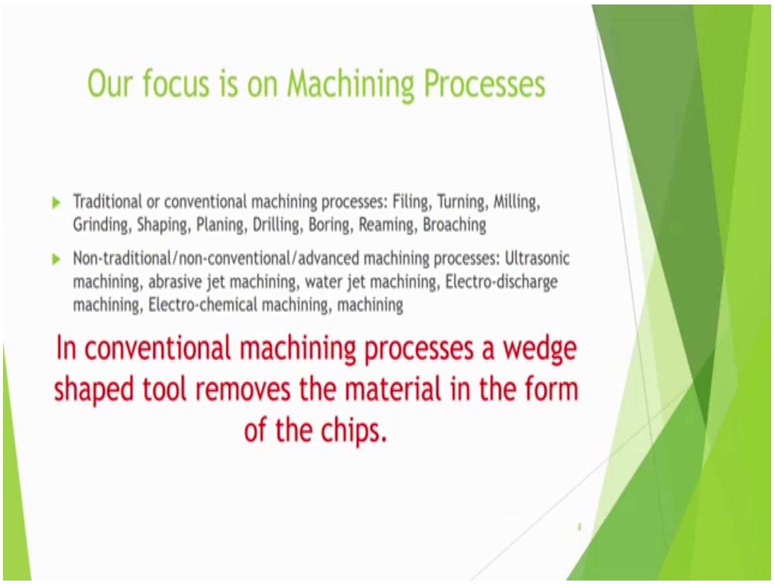

So, first we discuss conventional machining process. In conventional machining process a wedge shape tool removes the material in the form of the chips and we also analyzed that why do we use a wedge shaped tool because it acts as an incline application device. We apply some force, but the normal force is amplified.

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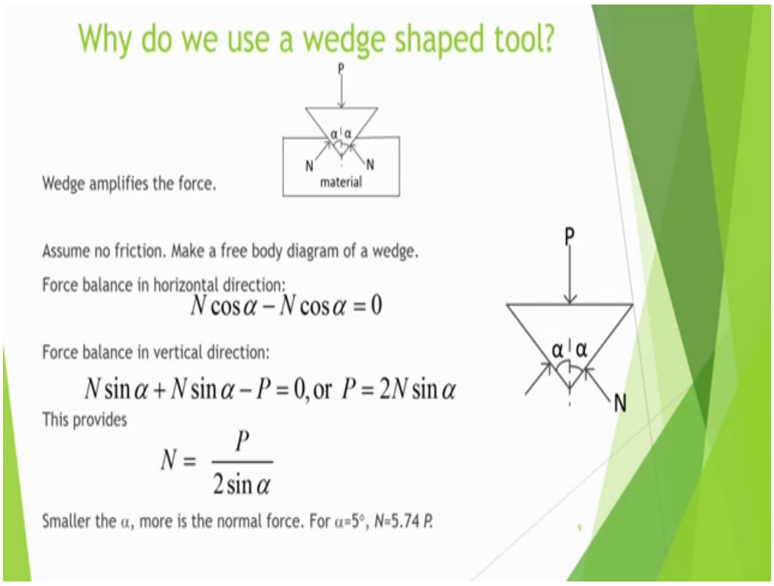

Material is removed in the form of the chips that point we told, and after that we told what is the mechanics and what is the motivation in studying the mechanics of machine.

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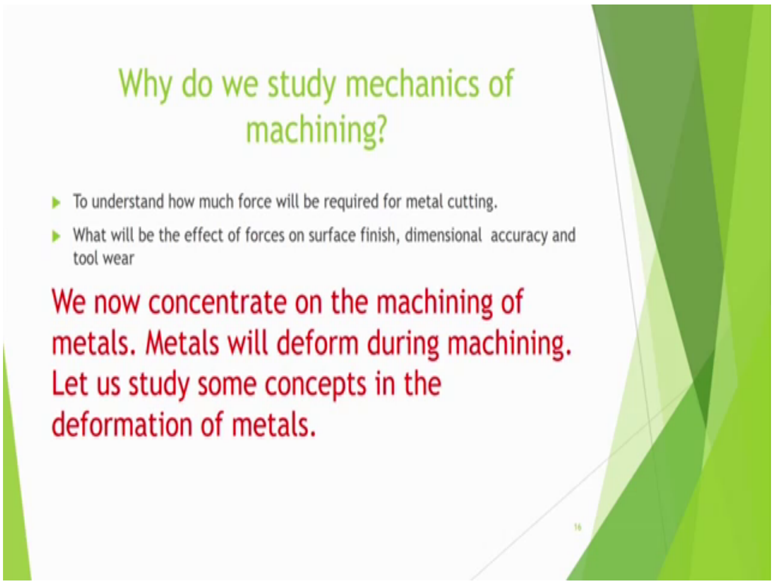

Then we started our discussion on the deformation of metals because ultimately we are going to study the machining of the metals. So, we must understand the behavior of the metals.

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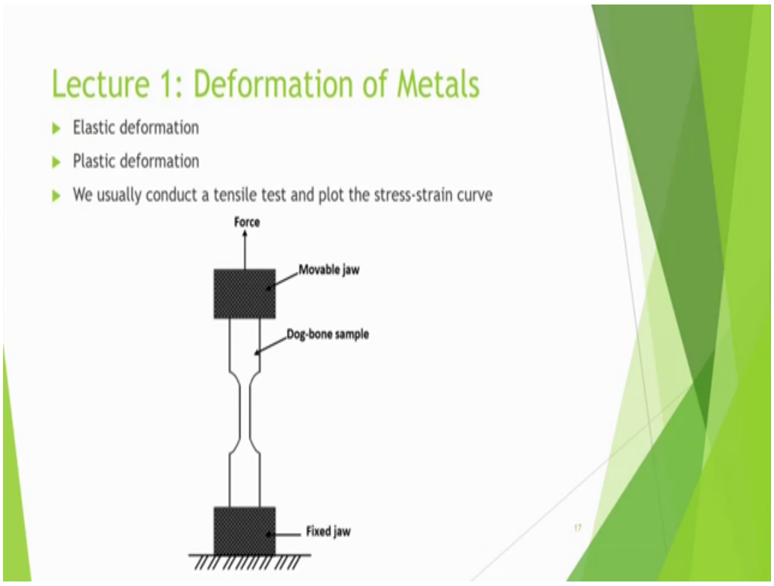

One common test is the uniaxial tensile test in which we make a dog bone sample and we fix it between one movable jaw and one fixed jaw and then after that we apply the force.

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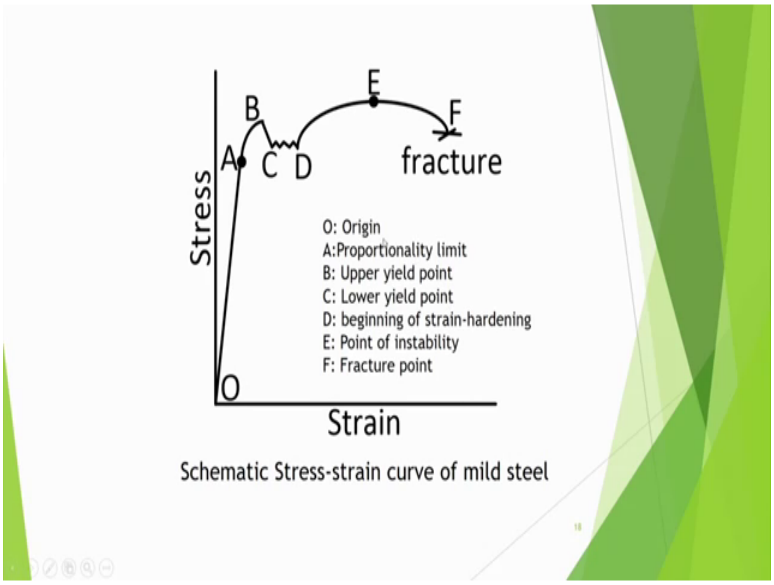

So, we get this type of stress-strain diagram which tells us many things. For example, from this diagram you can find this slope up to A direct will be the Young's modulus of elasticity E. That point can be obtained and then after that you can find out the yield stress of the material. B is the upper yield point, C is the lower yield point you can also find out the Ultimate Tensile Stress of the material UTS or sometimes it is called Ultimate Tensile Strength. Here this point E is there, this is expressed if we find out the stress in terms of the nominal stress that means engineering stress then this is UTS. UTS is generally expressed in terms of the engineering stress not in terms of the true stress.

And this is you can say that point F denotes the breaking strength of the material we can also find out the total elongation we because we can find out the total strain, we can find out the relative elongation, how much it is elongating that is total elongation. But if you want to know that what is the permanent elongation then you can unload it. So, from this point F you can actually unload start unloading. So, you can move a in a direction parallel to O A. So, for example, like this because I told that unloading is elastic it moves in this one. So, you come up in this point and then you say that this distance from O to this point obtained this indicates the permanent strain.

This one will of course, will be almost same as the total strain because elastic strain is very small, but that is what has been told.

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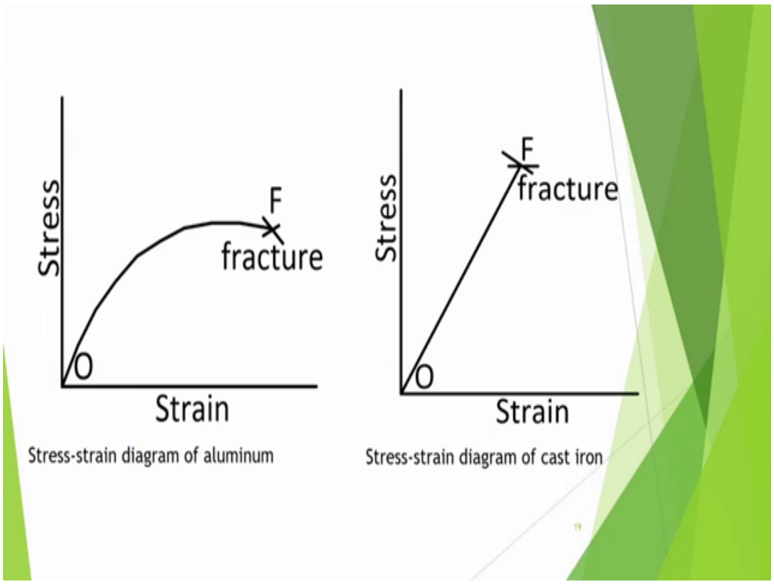

And now coming to the next one we told that stress strain diagram for aluminum, stress strain diagram for cast iron which is a brittle material. We told that definition of nominal stress and true stress and we also told that how they are related.

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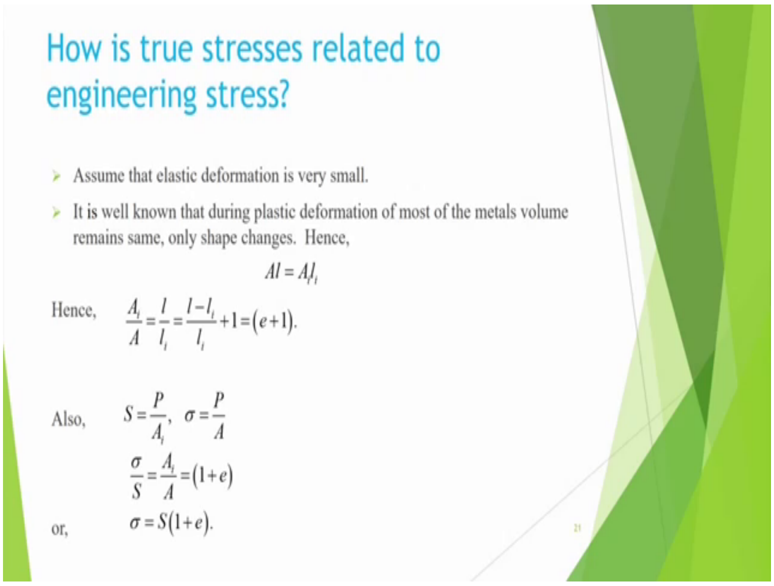

We also told that nominal strain or engineering strain and true strain that is logarithmic strain and how they are related logarithmic strain is also called naturally strain we expressed those type of relation and then we were discussing the behavior of the material in the plastic region. In the plastic region we get the phenomena of strain hardening direct will be expressed by these expressions.

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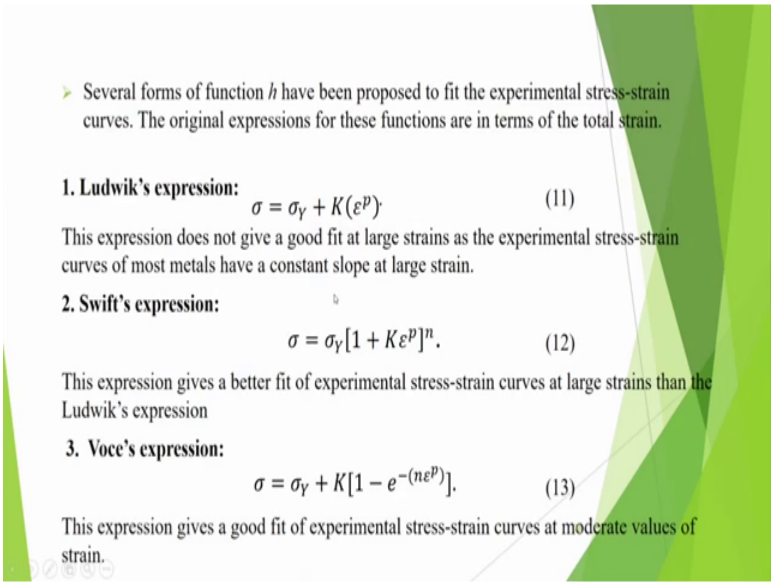

Ludwik's expression, Swift's expression, Voce's expression or you can make another type of expression this way we expressed that.

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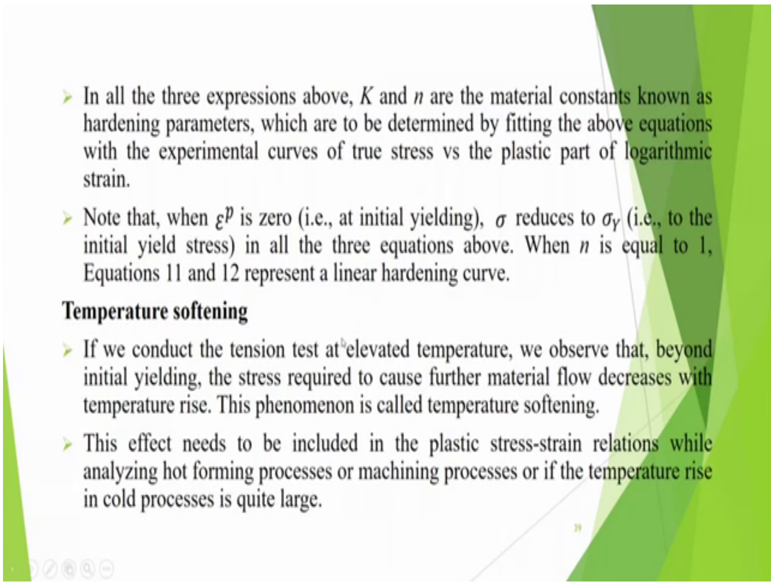

Now, we have we also get temperature softening phenomena. When we increase the temperature of the material it softens, it is a yield the stress decreases that phenomena is also there. So, temperature effect is also prominent and strain rate effect is also there ok.

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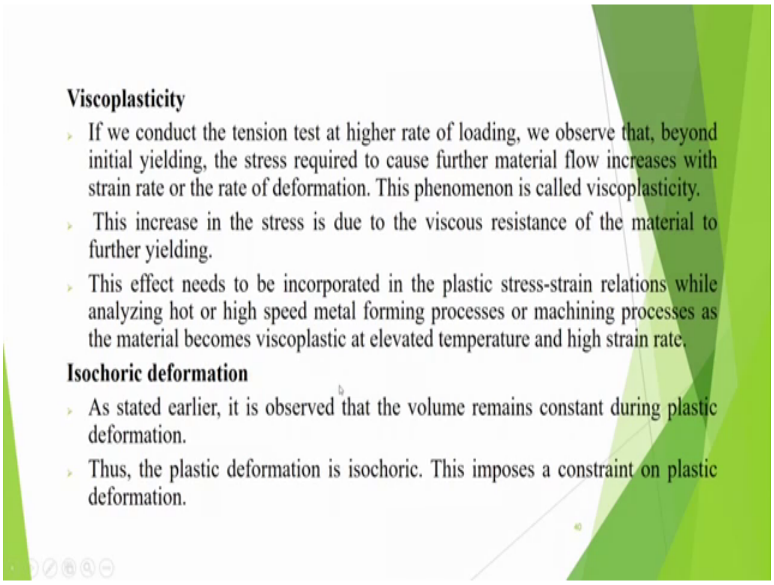

So now, we discuss that what is vicsoplasticity? If we conduct the tension test at higher rate of loading we observe that beyond initial yielding the stress required to cause further material flow increases with strain rate or the rate of deformation. That means it is dependent on the rate of deformation. If you conduct the test at slow strain rate you will have different observation, if you conduct the same test at high strain rate you will have different observation.

Usually in the normal life we do that strain a test means tensile test at strain rate of something 1 into 10 to the power minus 3 per second 10 to the power minus 3 per second which is very small. But most of other cases we can go suppose 10 per second 100 per second even we can have 10 to the power 5, 10 to the power 6, 10 to the power 7 suppose bullet is hitting one target then the strain rate can be very high it may be of the order of 10 to the power 7 or 10 to the power 8 also per second like that depending on what is the speed of the bullet.

So, if at high strain rate naturally the material will appear to be stronger. So, this dependence of the flow stress on the strain rate or the rate of deformation is known as vicsoplasticity, we say material is viscoplastic. Increase in the stress is due to the viscous resistance of the material to further yielding, this effect needs to be incorporated in the plastic stress strain relations while analyzing hot or high speed metal forming processes or in machining in machine in also the strain rate can be very high particularly in the high speed machining. And this effect is more prominent if simultaneously the temperature is also present.

So, when you have elevated temperature and also high strain rate then these effects have to be incorporated. Then there is one term called isochoric deformation. Isochoric means volume preserving we already told that there are metals which remain, they are they are volume remain same during the plastic deformation. Volume remains constant during plastic deformation, so plastic deformation can be called as an isochoric process; that means it is a volume preserving process volume does not change this imposes a constraint on plastic deformation ok, that can put in constraint on the plastic deformation.

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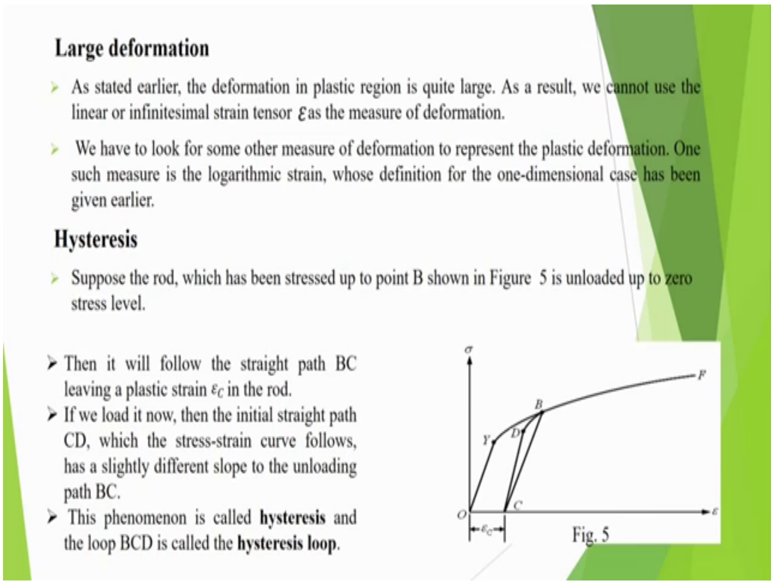

Now, we large deformation, in machining we will be dealing with large amount of deformation it is not a small deformation day today's life materials also get deformed, but they mostly undergo elastic deformation and those deformations are very small I told that the strains are of the order of 10 to the power minus 3, 1 by 1000 But in the machining process strains can be very high it may be of the order of one or more. So, in that case our normal infinitesimal strain tensile measure is not valid that the way we have been defining in the elasticity that is strain that is not proper. So, we have to use some different measure like suppose logarithmic strain measure, how much is the logarithmic strain or how much is the true strain how much is the natural strain. So, this is this point has to be kept in mind.

Now, we come to the other phenomena of the material that is called hysteresis. You see these this figure in this they are stress strain diagram of a hypothetical material has been plotted material is elastic up to point Y. From O you load and reach to the point Y at Y there is a yielding and after that there is a plastic deformation there is a hardening region also. So, YF is a hardening YF need not be straight line, if it is a straight line then it can be called linearly strained hardening otherwise there is a hardening here phenomena is there. So, this is YF and suppose you load the material is starting from 0 stress increases reached to Y plastic deformation is starts and you reach at point B then you remove the load that means, there is a unloading.

So, unloading is elastic that means, during unloading the material will follow this path BC from B to C where BC is parallel to OY you come to this point this point C which is on the strain axis this point indicates the plastic deformation permanent deformation. So that means, OC is what it is a permanent set it is a permanent strain and I am indicating it by epsilon C then from C. Suppose you again load it means material has been loaded and after that unloaded remove the load again.

I am putting the load in that case material ideally should follow this path it should go from C to B. But it will not go from C to B instead it will follow a path that is CDB that means, the there is a small loop that means, there is a small energy loss; so that means, the energy contained in the loop CDB is basically lost and this phenomena is called hysteresis and loop BCD is called hysteresis loop. So that means there is some amount of hysteresis is there.

Now, for deforming the material you are requiring here for the same total strain you are requiring. Now, whole stress because of the hysteresis effect. However, this value is very small that is why in most of the cases it is neglected. So, we do not talk much about the hysteresis here, but if we are studying the vibration behavior hysteresis actually causes the dumping, in those cases this becomes very significant how the vibrations are decaying how this one. But since here strains are very high so that is why here the hysteresis not important.

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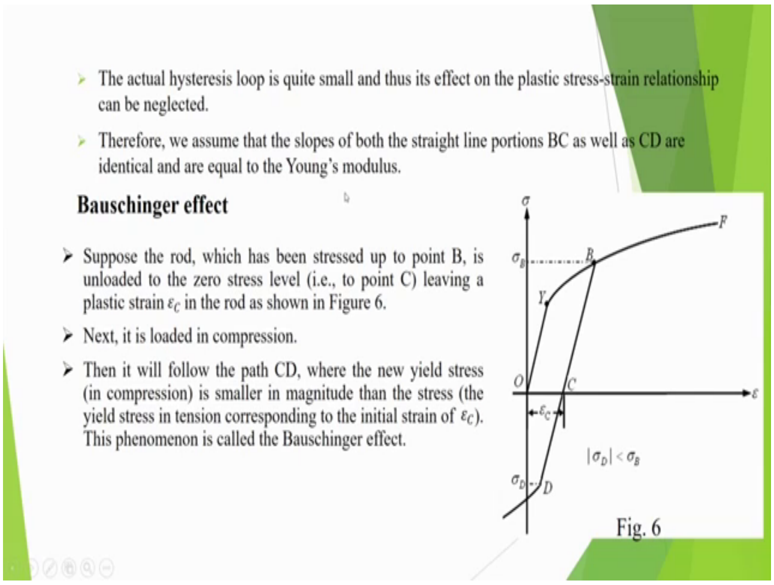

So, actual hysteresis loop is quite small and thus its effect on the plastic stress strain relationship can be neglected therefore, we assume that this slopes of both the straight line portion BC as well as CD are identical and they are equal to Young's modulus. I told that unloading is also elastic. Now, we are going to discuss another effect that is called Bauschinger effect.

Now, in the Bauschinger effect what happens that suppose you have stress strain diagram and now, you load the material stress keeps on increasing from 0 point to Y. At Y the yielding is starts you may go up to say let us say point B means material has been plastically deformed there is a strain hardening also then after that you are unloading the material. So, after you unloaded you have reach pint C there is a permanent strain epsilon C. Now, again you in load, but this time if you load in the reverse direction that means you are loading in the reverse direction. So, the yielding will be start at some point D.

Ideally we say that strength in tension and compression is same. So, that is why sigma B should be equal to sigma D, but you will be notice that sigma D will not be equal to sigma B in its sigma D will be less than sigma B that needs the compressive yielding will be start a bit earlier. You expected that it will we start at some stress a 200 megapascal may be it may be start at 195 megapascal. So, this is due to Bauschinger effect.

Now, it is this phenomena that which is called Bauschinger effect is mostly due to some a small amount of residual stresses left. What we when we are load the material at the microscopically will some small amount of stresses are left and they are already present is small amount of compressive stresses. So, when we put the load these stresses add up to apply these stresses and therefore, they cause early yielding so that means, this. But many times even we neglect the Bauschinger effect also that is point has to be kept, but in certain situations this effect may be very significant particularly if you are having fluctuating type of loading. That means, material is going to tension then compression then tension like that if it is going in that direction then the strengths in both the direction may keep on reducing, ok so, this phenomena is there.

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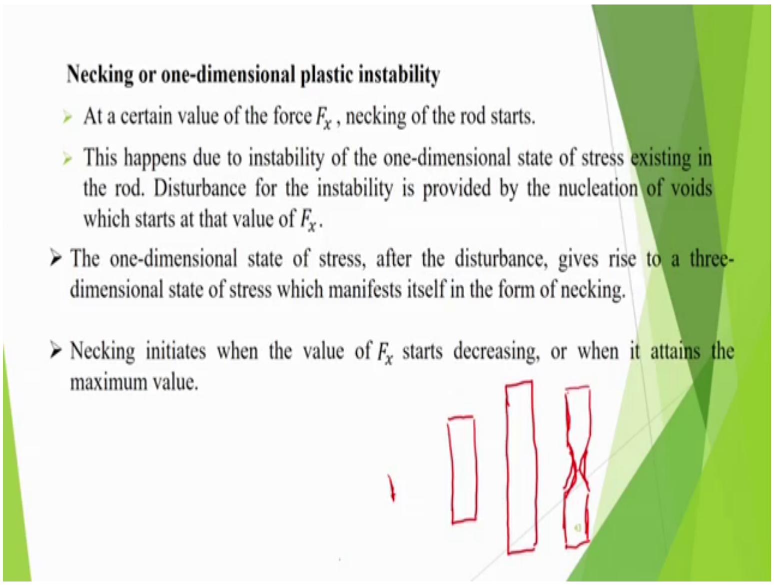

Now, we come to the point about necking or one-dimensional plastic instability. When there is a specimen and it is loaded intension then you might have observe that it is elongates and there is a slight reduction in the class section also that is Poisson's effect. You keep on reducing then there is a significant amount of elongation, but more or less the class section remains same this is called the uniform elongation type of thing you have got uniform elongation. Then there are comes a pointed that cross sectional area rapidly starts decreasing and you got a type of neck type of formation, ok.

So, you might have observed in your; this one test that there will be a neck type of observation suppose here we all having this one, it is necking will be observed like this let me draw some sketch. Here this is suppose a uniform this is a material and it is elongating it got elongated here right. It got elongated here. But if the necking has is started it may be like this that material may start you know that forming a neck type of thing here that means, here material will deform and it becomes the state of stress becomes the triaxial and ultimately it will lead to the fractural. So, necking phenomena occurs this one due to this.

So, this we have already showed that one typical stress strain diagram of the mild steel here you had a dog bone type of sample see this cross sectional area is there this cross sectional area will remain uniform. But when the necking will you start then that portion it will become radial type of thing here that radial group type of thing and the state of stress will no longer be uniaxial it will become triaxial and this is indicated in the stress strain diagram by this point E that means, after that point because the cross sectional area suddenly will reduces so, engineering stress suddenly drops.

So, this point E is actually the highest engineering stress point or it is the highest load. So, at this point the necking has the started after that the material has become unstable and this one has become this material has become unstable this point E at this point E, DP will be equal to 0 ok, that means, condition is that DP equal to 0 because it is the point of maxima P is the apply load; so, DP equal to 0.

So, let us develop some expressions for these necking phenomena, that is plastic instability we call because the material becomes instability. So, at a certain value of the force F x necking of the rod starts F x is the maximum load. This happens due to instability of the one-dimensional state of stress existing in the rod disturbance for the instability is provided by the nucleation of voids which is start at that value of F x that means, inside the material voids is start nucleating means the it start generating and they you know glow that is why the necking is occurring.

So, the one-dimensional state of stress after the disturbance gives rise to a three-dimensional state of stress which manifests itself in the form of necking. Necking initiates when the values of F x starts decreasing that means, after that you observe that in UTA machine also that load automatically started decreasing you know because we are doing the displacement controlled thing that means, only jaw is moving load comes as result of that. Firstly, load is increasing, but then the load is starts decreasing when it attains the maximum value.

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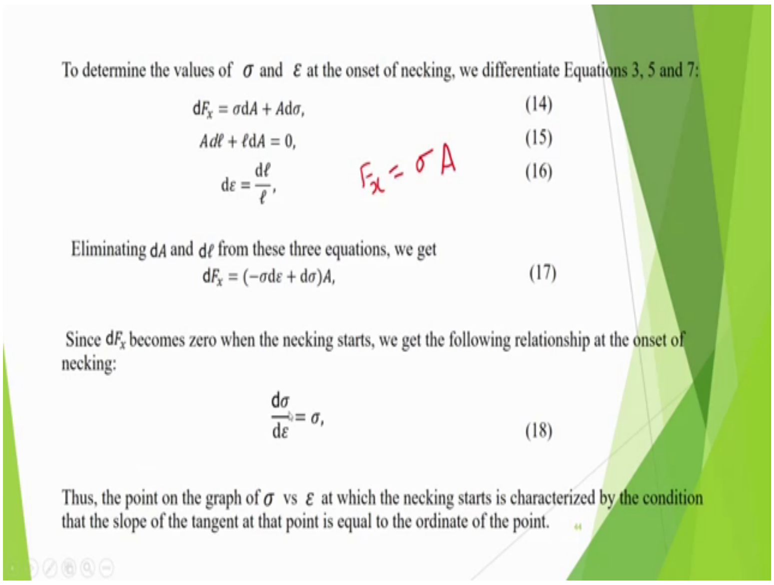

So, to determine the value of sigma and epsilon that means, true stress and true strain at the onset of the necking that means, when the necking just it starts we differentiate equations, this one earlier equations that of the F x. So, F x is equal to sigma times A, F x is equal to sigma times A was the l sigma times A, that is the general one because sigma is the true stress and A is the true area. So, sigma times A will do the two force and D we differentiate we get equation 14, you how to apply the product you see d F x is equal to sigma d A plus A times sigma d A is it not first you only differentiate with respect to a keeps sigma constant then you differentiate with respect to sigma keep a constant and you put these values.

Now, you know that you also have another equations area is equal to constant so that means, you can do d A l equal to 0 and this gives you a times dl plus l times d A equal to 0, but true strain infinitesimal true strain or incremental true strain is defined as D epsilon is equal to dl by l. Now, if you eliminate d A and dl from these 3 equations we get basically this type of equation d F x is equal to minus sigma d A plus D sigma into A, you are observing this equation.

Now, since d F x becomes 0 when the necking starts why it becomes d F x becomes 0? Because that is the peak at the peak there is no slope you know it is a continuous curve you have each the peak there is no slope we get the following relationship at the onset of the necking. That means, we get D sigma by D epsilon is equal to sigma that is the criterion for instability. Thus the point on the graph of sigma versus epsilon at which the necking starts each characterized by the condition that the slope of the tangent at that point is equal to the ordinate of the point that means, D sigma by D epsilon is equal to sigma, but remember this that this is expressed in terms of the true stress and true strain. If you express in terms of true engineering stress and engineering strain then it may be somewhat different, ok.

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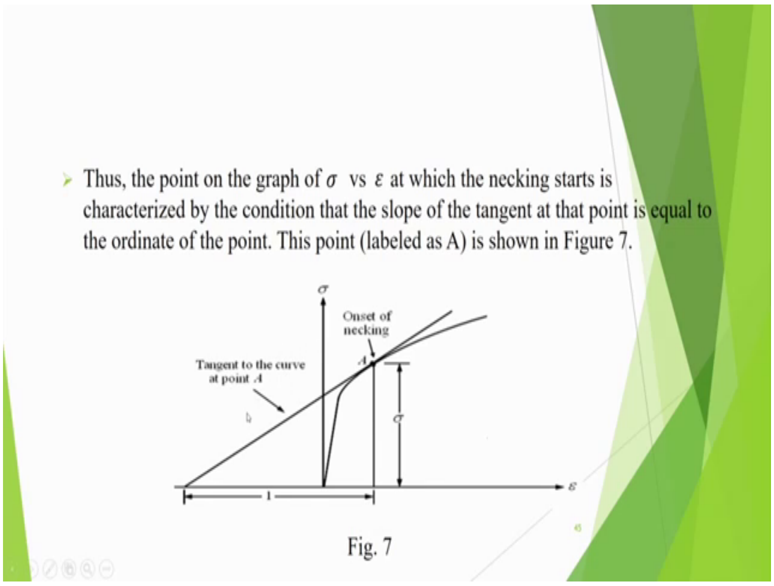

So, here it is shown graphically it is true stress and true strain diagram. You see there is no point of dip here the true stress is always increasing it is going up to fracture it is increasing movably. However, like point A which actually corresponds to ultimate tensile strength point in the engineering stress strain diagram in that point if you take this slope, ok, then that slope will be equal to nothing, but this height that is sigma. So, this point is the onset of the necking. So, we can understand this necking behavior we should.

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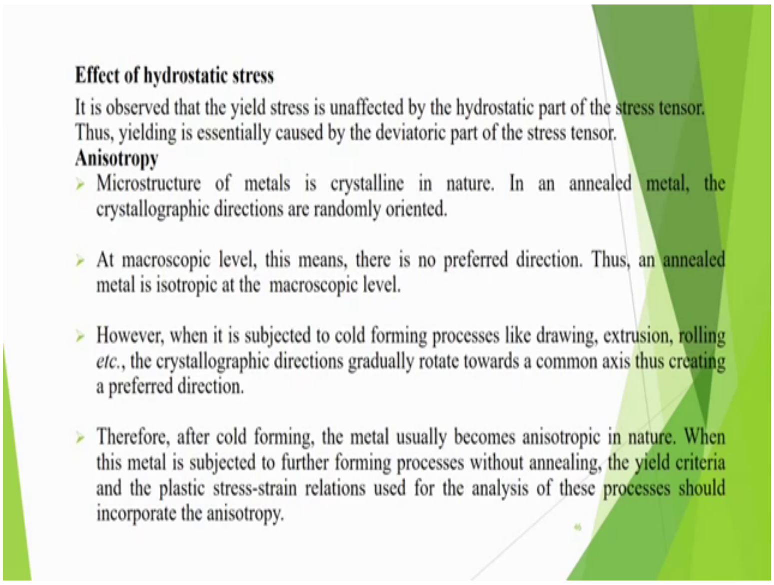

Now, I discuss that what is the effect of hydrostatic stress. Before that I must tell you what is the hydrostatic stress; as you know that stress has got basically 9 components ok. And there may be so, stress usually is a symmetric so you can say there are 6 components of stress out of that there are 3 stresses which are normal, normal means direct stress and you know that there are shearing stress shearing stress this one suppose I am having this block I am trying to pull it. Now, this is called this is normal stress that means, I take the perpendicular cross sectional area into and the load divided by perpendicular cross sectional area that will be normally stress.

So, normally stress in x direction, similarly I can find out normally stress in y direction similarly we can find out normally stress in z direction. So, we have got a 3 dimensional state of stress. We also have shearing stress that means material is getting sheared in which we find out the stress is the force divided by the area and which that force is acting that means, parallel area that means, if I apply a force here. Now, this force and this tangential area that will gives shear stress. But right now, just concentrate that what is normally stress. So, normally stress means that like we have conducted uniaxial tensile test in that what you are is studying that that sigma which who was plotting is basically a normally stress that is not a shearing stress is it not, that is why you are applying a force in the axial direction and dividing it by the normal area that means, cross sectional area.

So, this was a normal stress, but it was in x direction only similarly you can have in y direction you can imagine in actual cases when we material is in countering different type of forces and similarly you can have z axis. So, sigma x plus sigma y plus sigma z will be divided by 3 will be called the mean stress. So, mean stress you can say average stress of the 3 normally stresses. So, that mean stress is basically called the hydrostatic stress. So, that is called the hydrostatic part of this stress.

And if you write the 9 component of the stresses in the form of a matrix and you subtract from them that hydrostatic part that means, hydrostatic matrix then will have mean stresses 3 mean stresses in the 3 diagonal places and you subtract it you get a deviator only part of this stresses. So, it has been observed that hydrostatic part of the stress tensor does not cause any yielding, it is only causing may be some volume change, some elastic deformation, but it is not causing the plastic deformation and essentially the yielding is caused by means of the deviatoric part of the stress tensor.

That means, you can imagine that lot of experiments have been done and you can understand if we take some suppose solid bal solid bal or solid object and we submerged it under the deep water where is lot of hydrostatic pressure is there, from all sites if pressure is equal in that case that material there will be some deformation of the material. But as soon as you pull it out from the water it will regain its safe that means there is no permanent deformation that deformation was only the elastic deformation.

So, hydrostatic stress does not cause plastic deformation that is one concept most of the time it is assumed, ok. This may not be necessarily to in many cases. For example, in powder metallurgy this cases and also in the study of the fractural mechanics significance of hydrostatic stress is there, but we will discuss that point later on.

Now, another phenomena is anisotropy you see that in the material if the properties are same in all directions, then the material is called isotropic material in isotropic material properties are same in all direction that means, if you pull it in this direction or you pull it in this direction behavior will be same it will fracture at the same point load also, but otherwise the materials may be anisotropic also.

So, at microscopic level anisotropy means there is no proper direction. Generally an annealed metal is considered isotropy, if you annealed the material heat in the furnace and then slowly cool that becomes an isotropic. However, when it is subjected to some cold forming process suppose you have done drawing, rolling, exclusion, if you have rolled that sheet then grains will be elongated in that particular direction and the material may becoming stronger in that direction and it will be weak in other direction. So, you will not no longer have in a isotropy and material will become anisotropic and isotropic.

So, after cold forming the metal usually becomes anisotropic in nature. When this metal is subjected to further following processes without annealing the yield criteria and the plastic stress strain relations used for the analysis of these process should incorporate the anisotropy. So, there are some anisotropic yield criteria are also there for example, yields criteria, but that point we are not going to discuss now.

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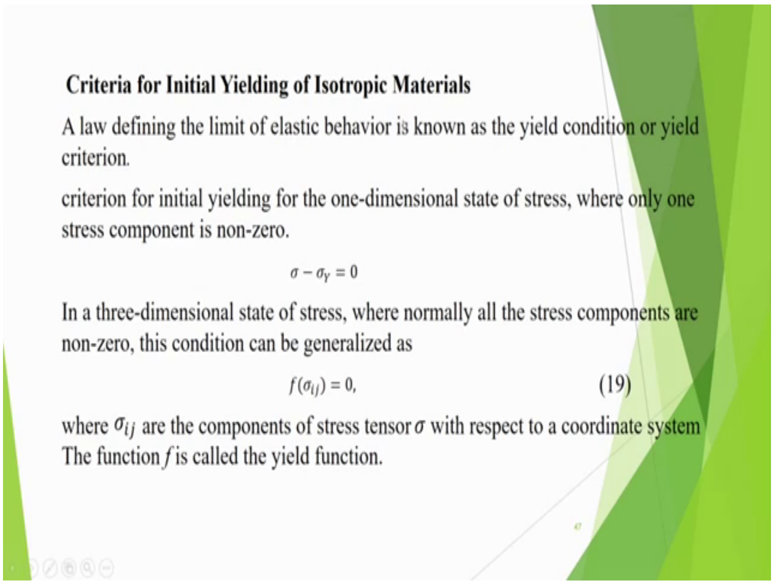

So, here now, criteria for initial yielding of isotropic material a we can define a law which tells that limit of elastic behavior that is called yield condition or yield criteria.

So, for one-dimensional case yield criteria is very simple sigma minus sigma Y equal to 0 that means, when the yielding is starts then sigma minus sigma Y becomes equal to 0. If the yielding is not starting then sigma minus sigma Y will be negative, ok, it will be less than 0, but when the yielding is starts then sigma minus sigma Y is equal to 0. In a 3 dimensional state of stress where normally all the stress components are nonzero that means, we have basically 9 components of this one this condition can be written like this. There is some function it is a function of sigma ij, sigma ij is denoting what and stress component here I can take an value from one to 3 and j can take the value from one to 3. So, there are 9 components. So, sigma ij means 9 components.

And so function each the function of are these 9 components and that will become 0 so that becomes a the yield criteria and this function is called yield function.

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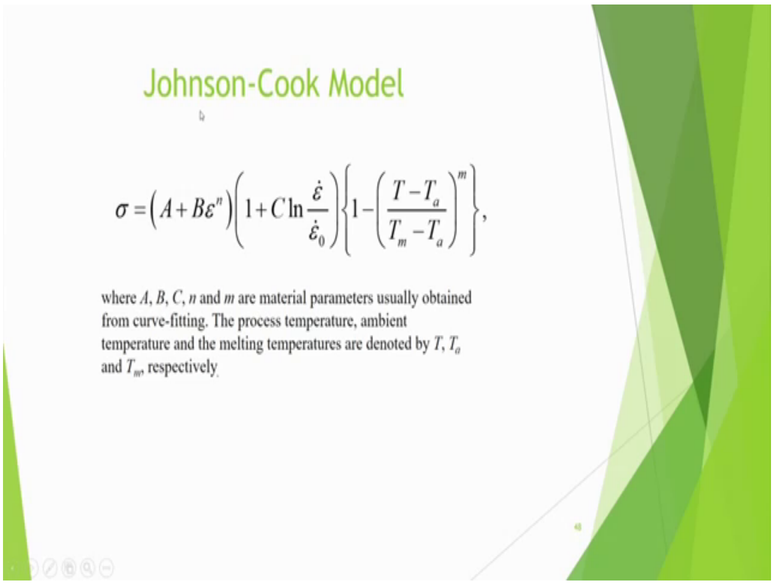

Now, I am telling you in this slide that what is what do we have one model that it is called Johnson cook model that tells the behavior earlier I told some strain hardening phenomena and then we told there are some empirical expressions, but those were taking only the effect of the strain and the flow stress. But here Johnson cook model takes the effect of the strain also the strain rate and also the temperature, that is your then these model is very popular it is used in modeling of the machining processes here sigma is the flow stress of the material.

Then we have got this part is the strain hardening part, this part is the strain rate effect part that means, viscoplasticity. Here if you see that if epsilon dot is increasing then this function will increase sigma will increase, like a epsilon increases then also sigma increases epsilon dot increases then also it increases, but here one logarithmic term is there natural log is there that means, it will increase, but that increase effect will be significant if you do large variations.

Suppose you are conducting one test on 1 into 10 to the power minus 3, other you are doing a 2 into 10 to the power minus 3, this difference is not that significant because it is not that strength will become double in this case. It will increase as per logarithmic rule so that means, if you increase suppose you are having 10 to the power 3 then epsilon 10 to the power of 3 is 3, and if you have 10 to the power 4 then epsilon 10 to the power 4 is 4. Now, so that means, effect is logarithmic way know it that effect is little bit slow down if you if you do not use logarithmic then it is linear you double it that means, strength doubles, but we are having logarithmic term and then this is the temperature dependency. Here you see if the temperature T is increasing then the flow stress will actually decrease.

So, this is softening behavior; material is softened due to the temperature that point is there and tm is the melting temperature. And T a is the ambient temperature or you can take some preference temperature and this we you fit and these A, B, C, n and m these are the material parameters they have to be obtained by curve fitting you how to do data fix few events. And then you how to shield which one use will give the good results you can do in a systematic way or you can do in a heated to trial whatever way we do, but this has to be obtained by fitting.

So, process temperature ambient temperature and the melting temperatures are denoted by T, T a and T m respectively here.

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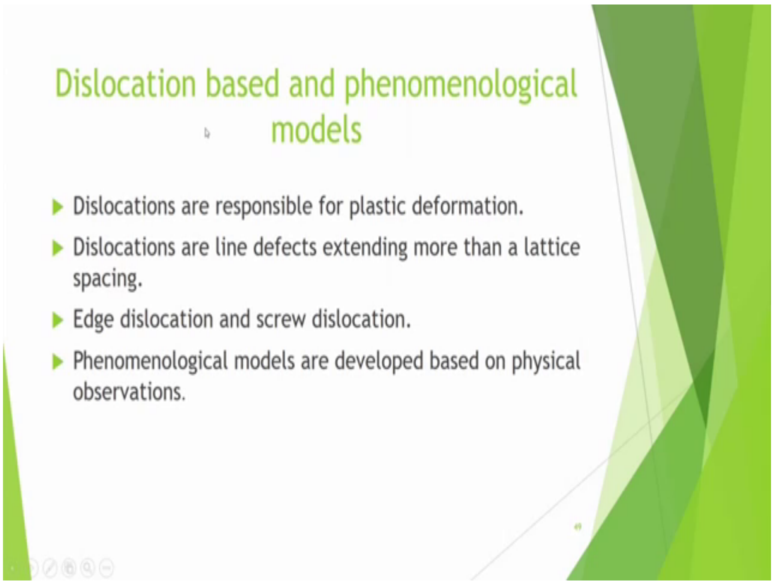

Now, we have two terms dislocation based and phenomenological models. Till now, whatever we have told that suppose Johnson cook behavior and this one that is phenomenological models that means, based on some phenomena we just make some rule, but other is dislocation based you know dislocations are the imperfections in the material. Dislocations they cause the plastic deformation. Very soon I am going to tell you about the dislocations.

So, dislocations are basically line defects in the material. What do we mean by line defects in the material in metal? Atoms are arranged in a very systematic way periodic arrangement of metals is there. So, you know that is cordial crystal and that crystal is not a perfect. In a diagram we may say crystal that where nicely if I make a two-dimensional diagram we will just show nice type of diagram maybe we can show this grid type of thing, and we can say yes there are atoms at the edged like that we can make, but actual material will not be like that there will be lot of defects.

So, some defects can be coin defects that means, there will be void in the material some cases there may be some other atom may inter inside occupy that side that means, it may be it may substitutes the materials, so substitutional defect is there. That may that means, inclusion of some foreign material may be there are that material main materials atom may be removed from that these are called point defect because they all localized at one point.

But there are some defects which extend only lying; that means there is a line of atoms which is much more longer than the spacing between two atoms, that lengthy line. In that there is some defect is there that is called line defect and dislocations are that type of defects we will discuss that. There are two types of defects in dislocation that is edge dislocation and screw dislocation. So, dislocation phenomena is actually is possible for causing the plastic deformation, but we will study phenomenological models are also which are developed based on physical observations. They do not see that what is happening inside the material, they are very useful for example, you will one Mises criteria or Tresca criteria how the material is yielding these can be called as phenomenological materials models.

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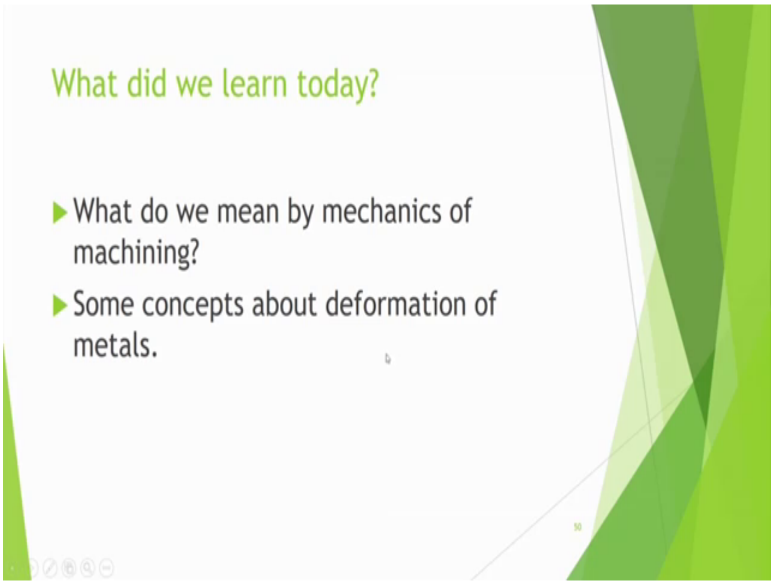

So, here what did we learn here, what do we mean by mechanics of machining that concept we have understood and we also understood some concepts about deformation of metals that we have studied. And now, we are going to tell something more about the deformation of material. So, let me just open the other PPT, so that we can get to know about other things. So, here I am opening other PPT this is lecture two that is about the Plastic Deformation. So, let us focus on the plastic deformation.

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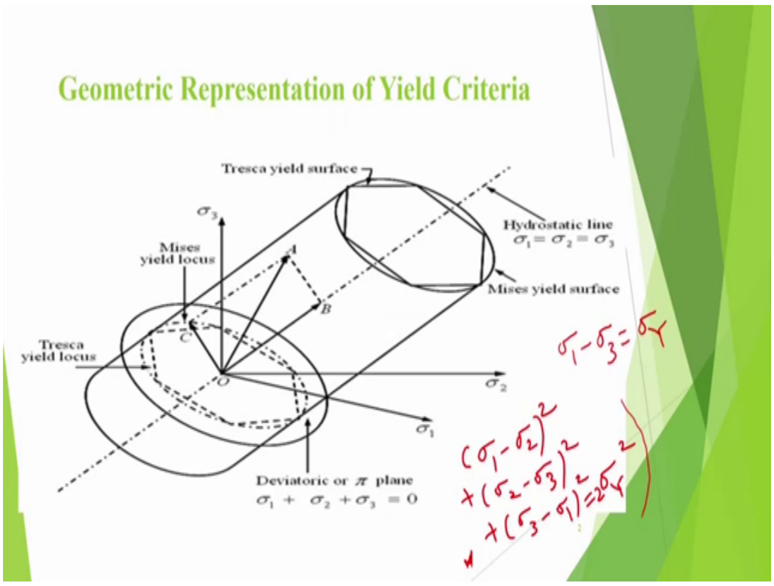

And in this first slide is showing this one 3 dimensional figure. Do not get frighten by this it is just I am going to explain you about the various type of yield criteria. Yield criteria means we understood that in uniaxial tensile test if your sigma is increasing in exceeding sigma y yielding it starts, but how will know that if there are number of stress is acting. For example, yield strength of the material may be 300 megapascal. So, you may think that if sigma x is 300 megapascal, sigma y is 300 megapascal, sigma z is 300 megapascal from all sides and shear stress is are not there then also the material will fail, but no in that case the material will not yield because that means. So, this is because this is hydrostatic state of stress.

So, then we how to make some criteria in which we can express this in terms of the stresses acting on that or we can express in terms of the principal stresses. That means principal stresses are those stresses if we consider 3 orthogonal planes in a material that we find in those 3 orthogonal planes it is possible to find out 3 orthogonal planes in which there will not be any shear stress and there will be only direct stress. So, those are called the principal stresses and they are indicated by sigma 1, sigma 2, sigma 3. You can write it increasing order sigma 1 is, sigma 1 is the biggest and sigma 3 is the lowest if we have that type of situation then you can of course, discuss you will yield criteria in terms of those principal stresses also it is very easy to transform any state of stress is the if it is known to us I can get the principal stresses.

So, for example, here that is in this case I can have sigma 1 minus sigma 2 whole square plus sigma 2 minus sigma 3 whole square plus sigma 3 minus sigma 1 whole square and this is equal to sigma Y square 2 sigma Y square. So, this is well known one Mises criteria, one Mises criteria is known like that if this condition is satisfied then the yielding will occur otherwise no. For example, if sigma 1 is equal to sigma 2 is equal to sigma 3 all are equal to 300 megapascal then it becomes 0 plus 0 plus 0 your left hand side is 0 only and right hand side is some things 2 sigma Y square because sigma Ys the yield strength, so this material is not going to deform plastically. So, this is suppose you have got one criteria that is called one Mises criteria similarly you can have Tresca criteria means suppose we say sigma 1 minus sigma 3 ok, highest minus lowest is equal to sigma Y.

So, we can say if that condition is satisfied then according to Tresca criteria it fails, here also if we can say if sigma 1 is 300 sigma 3 is equal to 300 material will not fail. So, these two scientist they have given the criteria. Now, what I am showing here that I have made a diagram state of the stress in principal coordinate system. So, I have got sigma 1, then I have got sigma 2, then I have got sigma 3 here ok.

Now, here if I make one cylinder; so, if I write the equation of this that means, one Mises criteria, this I have written up this equation if I put it in this one if you know 3D geometry very well you can know that this will be in this is nothing but the equation of I circular cylinder. That means there is a cylinder that cross sectional area is circled until a cylinder that has been shown here, but in a plane surface I cannot know 3D. So, that is why in isometric view it has been shown I think you are understanding that this is basically cylinder and this is called basically Mises yield surface that whole thing is this one. That means, if any stress point state of stress is inside this one cylinder then it is not failing, but once it is on the surface it is failing.

And axis of this cylinder will be hydrostatic line that is sigma 1 is equal to sigma 2 is equal to sigma 3 at this point this is. So, if I am at hydrostatic line there is no yielding because im inside the cylinder. Tresca yield surface will also be shown like a like a prism, but it will be hexagonal cross sectional will be this is hexagonal all this.

Now, if I cut this cylinder by a deviatoric or pi plane that in which hydrostatic stress is not there that means, if I cut it by a plane sigma 1 plus sigma 2 plus sigma 3 equal to 0 that we are calling as a pi plane. You know sigma 1 plus sigma 2 plus sigma 3 equal to 0 in 3D it is the equation of the plane, in 2D if we say x plus Y equal to 0 that is a straight line, but in 3D x plus Y equal to 0 is a plane X plus Y plus Z equal to 0 that is also a plane. So, this plane has been cut then you will see that we will get a yield locus and that yield locus will be circled for one Mises criteria and it will be regular hexagon in case of the Tresca criteria.

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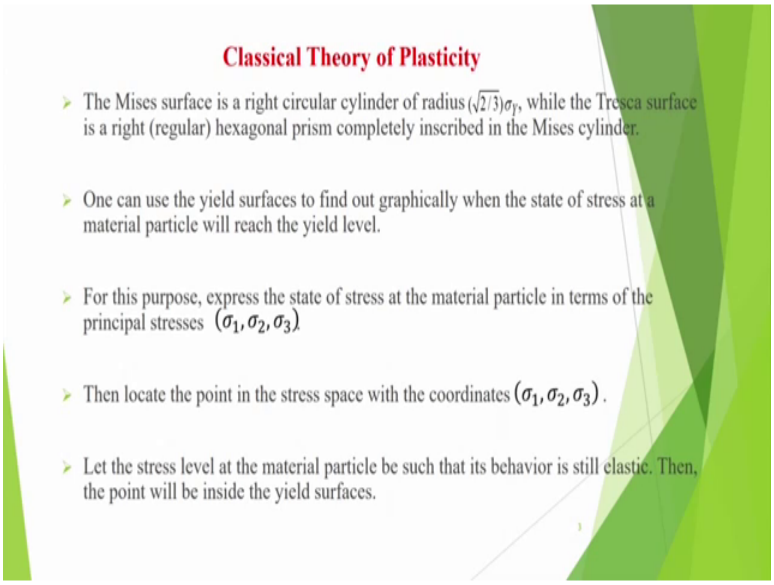

So, now, you all observing like this here. So, Mises surface is basically a right circular cylinder of radius under root 2 by 3 sigma Y; while the Tresca surface is a right regular hexagonal prism completely inscribed in the Mises cylinder. So, Tresca remains inside the one Mises that means Tresca is conservative, that means if the material is failing as per one Mises it will definitely fail as per Tresca.

One can use the yield surfaces to find out graphically when the state of stress at the material particle will be reach the yield reveal. For this purpose express the state of stress at the material particle in terms of the principal stresses that is sigma 1, sigma 2, sigma 3 then locate the point in the stress space with the coordinate sigma 1, sigma 2, sigma 3, let the stress reveal at the material particle be such that its behavior is still elastic then the point will be inside the yield surface, it has to be inside the yield surface, ok. It will be inside like suppose A it is inside the yield surface suppose, B is inside the yield surface, ok.

Now, let us denote this point by A then the vector OA represents the state of stress at the material particle. This vector can be decomposed into two components the component OB along the hydrostatic line and I can have another component OC along the deviatoric plane. So, component OB of that one represents the hydrostatic part of the stress while the component OC represents the deviatoric part of stress. Now, let there be an increase in the stress you increase the stress, but for let increase should be in direction OB that means, hydrostatic part increases you will see in that case a point will never reach to the surface. It will only remain inside and then it cannot cause any yielding, so because hydrostatic part of stress has no effect on the yielding, ok.

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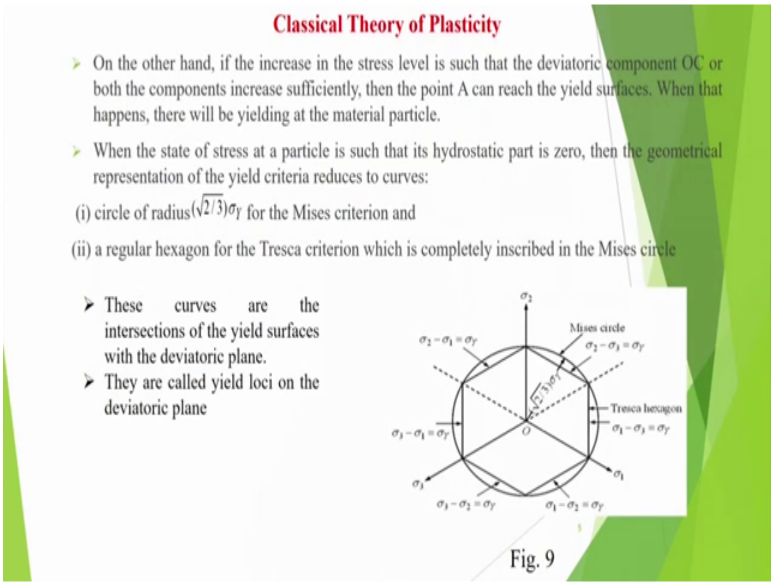

So, now, but on the other hand if the increase in the stress level is such that the deviatoric component OC or both the components increase sufficiently then the point A can reach the yield surface when this happens there will be yielding at the material particle. So, when the state of stress at a particle is such that its hydrostatic part 0, then the geometrical representation of the yield criteria reduces to curves then we can make yield curve. So, suppose we have made a yield curve here.

So, circle of radius under root 2 by 3 sigma Y is for Mises criteria and a regular hexagon for the Tresca criteria which is completely inscribed in the Mises circle. You see this is having hexagon it is consisting of this one each 6 line is representing one type of equation here sigma 2 minus sigma 3 is equal to sigma Y sigma 1 minus sigma 3 equal to sigma Y sigma 1 minus sigma 2 is equal to sigma Y. That means, any of these 6 conditions is satisfied that means, yielding will start, but you can see intersection of the yield surface with the deviatoric part is called yield loci we say this is basically yield locus. So, this is one is the yield locus and we can say one Mises yield locus is a smooth where as the Tresca is not smooth actually. So, that is what here you are getting parallel points also.

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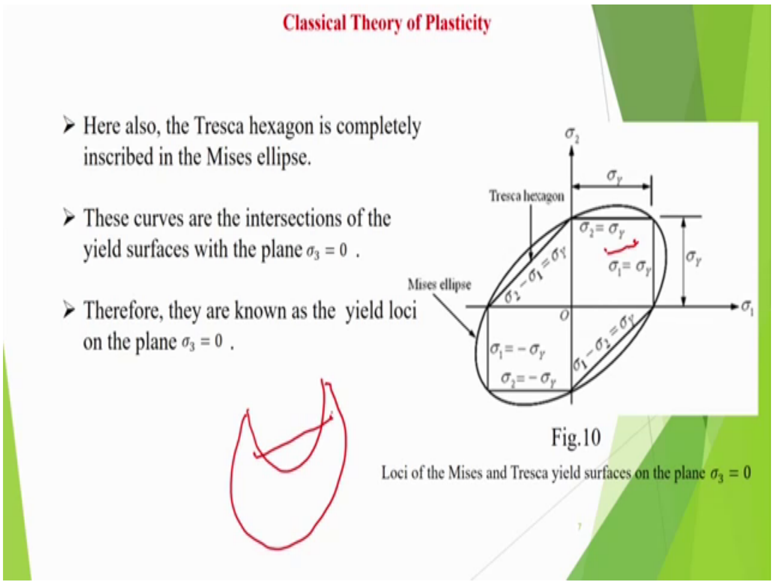

So, now, it is on the other hand if the state of stress at particle is of plane stress type we can have plane stress type of thing in which only in X Y plane stresses are acting you have got sigma X, sigma Y and tau X Y shear stress is there. If one of the three principal stresses is 0 at the particle then the geometrical representation of the yield criteria will become too different curve that means, we can put in the same expression sigma 3 is equal to 0. So, we get, one equation we get this is Mises criteria this is basically the equation of the ellipse in represent and we get for Tresca criteria we get this type of expression.

So, this is for one Mises criteria you are getting ellipse in plane stress case sigma 1 and sigma 2, but for Tresca criteria also you are getting hexagon, but it is no longer regular hexagon are the sides are different you can see here these are this one. So, this is this one.

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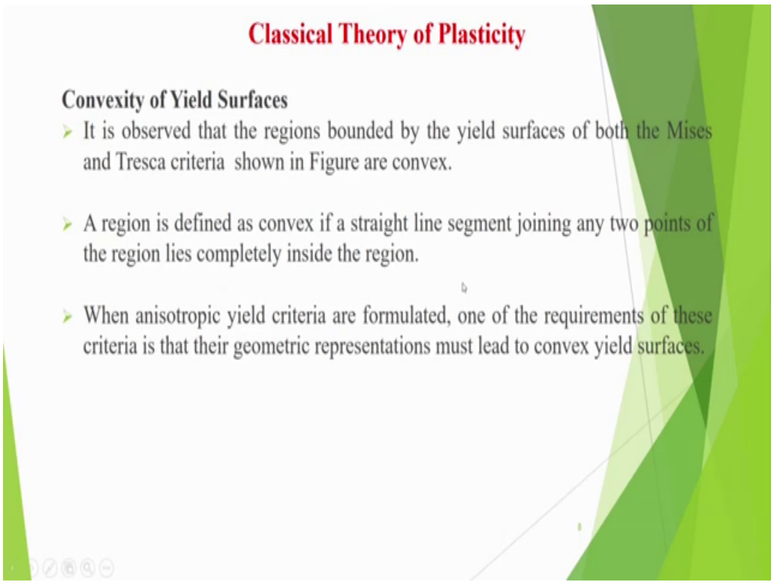

So, in general now, a one point is there you have observed that these yield surfaces are convex. How do we define mathematically, what is a convex? A region is defined as convex if a straight line segment joining any two points of the region lies completely inside the region then it is constant and this is importance. For example, this is why I am calling it is convex because if I take a point here and I take a point here I join them that point is also convex, but if I make this type of some profile this one and if I take a point here inside I take a point here and joint it then you see that something is going out that means, this is not convex, ok.

So, what happens that here, when anisotropic yield criteria are formulated one of the requirement of these criteria is that their geometric representation must lead to convex yield surface that means, we should always have the convex type of yield surface.

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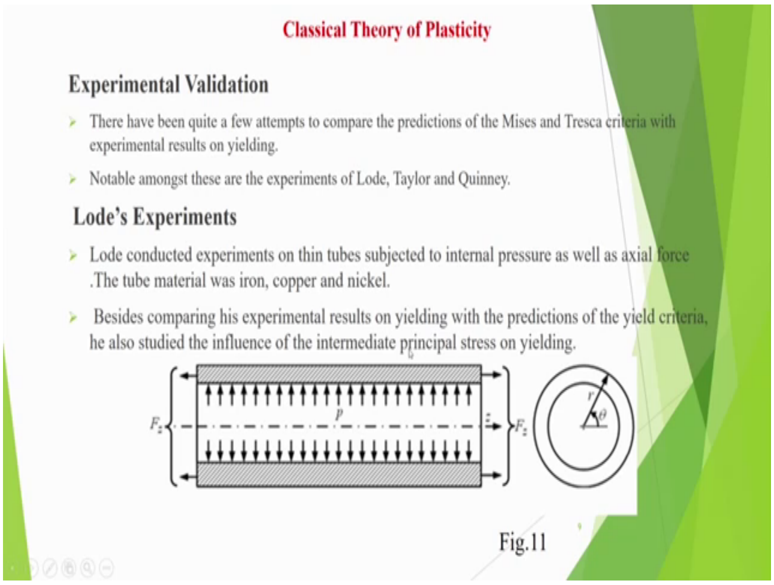

Now, I am coming to the some comments about the this is called classical theory of plasticity in which you have in studied for isotropic material, one Mises and Tresca criteria are actually important. So, they are have been quite a few attempts to compare the predictions by the Mises and Tresca criteria with experimental results on yielding. And notable experiments are that of lode and another set is by Taylor and Quinney.

Now, lodes experiments let us discuss what lode did what he did that he took a thin cylinder this type of thin cylinder applied the internal pressure and also simultaneously the axial type of load so that means, and it is he represented in the form of r theta coordinate system here.

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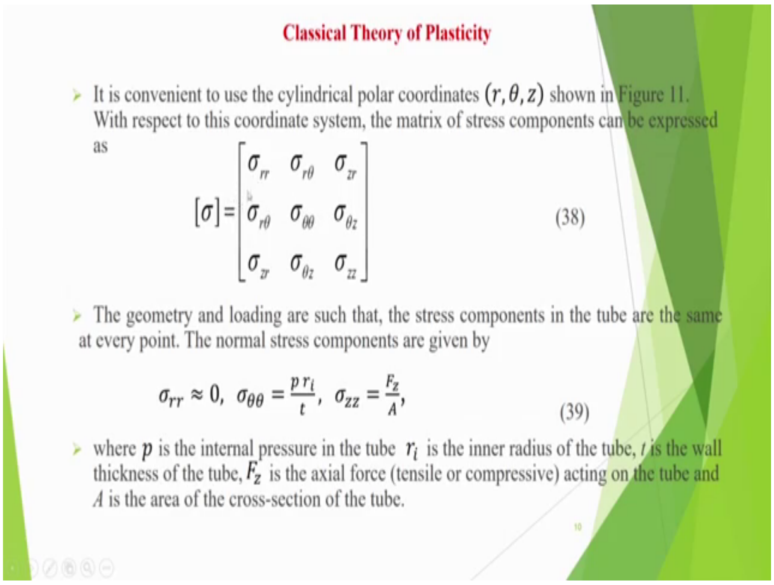

And if you do that then you can express the stress matrix in terms of r theta z components. So, you are getting sigma rr then you are getting sigma r theta, then we are getting sigma zr, then we are getting sigma r theta and all these things we are.

Now, geometry and loading are such that these stress components are like that because it is a thin cylinder so sigma rr is 0, radially stresses almost 0. In fact, dominating is the who the stress. So, sigma theta theta is pri by t and sigma zz is equal to axial stress that is F z by A. So, p is the internal pressure r i is the inner radius and t is the wall thickness of the tube F z is the axial force tensile or compressive acting on the tube and A is the cross sectional area of the tube, ok.

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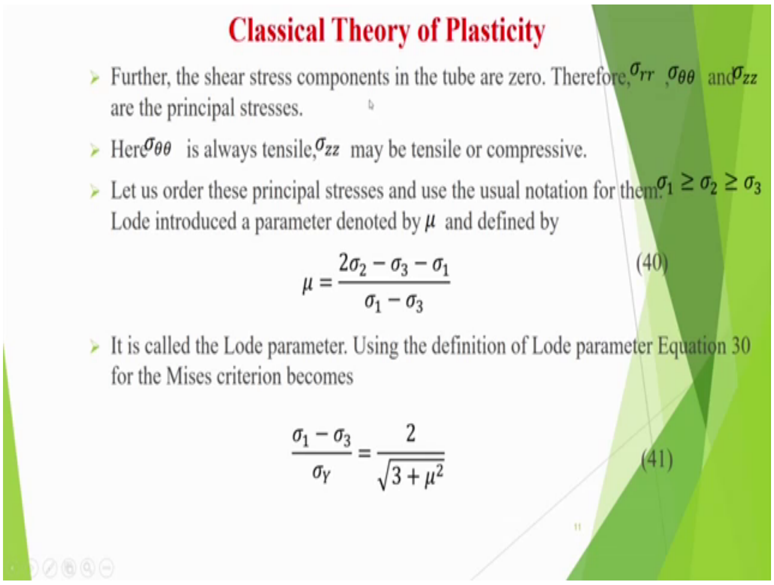

Now, further the shear stress components in the tube are 0 because it is axis symmetric tube there is a symmetry sigma rr sigma theta theta and sigma zz will be the principal stresses in that case sigma rr is 0. Here sigma theta theta is always tensile, and sigma zz may be tensile or compressive I may stress longitudinally that tube are I my may compress. So, let us order these principal stresses and use the usual notation suppose we say sigma 1 is greater than sigma 2, and sigma 2 is greater or equal to sigma 3 then lode introduced a parameter denoted by mu and he defined mu like this 2 sigma 2 minus sigma 3 minus sigma 1 divided by sigma 1 minus sigma 3. It is called the lode parameter.

If we substitute in the one Mises criteria then one Mises criteria becomes like this sigma 1 minus sigma 3 by sigma Y is equal to this much. And for Tresca criteria this sigma 1 minus sigma 3 by sigma Y will become equal to 1 so that means, both criteria can be compared that is why lode defined this type of parameter.

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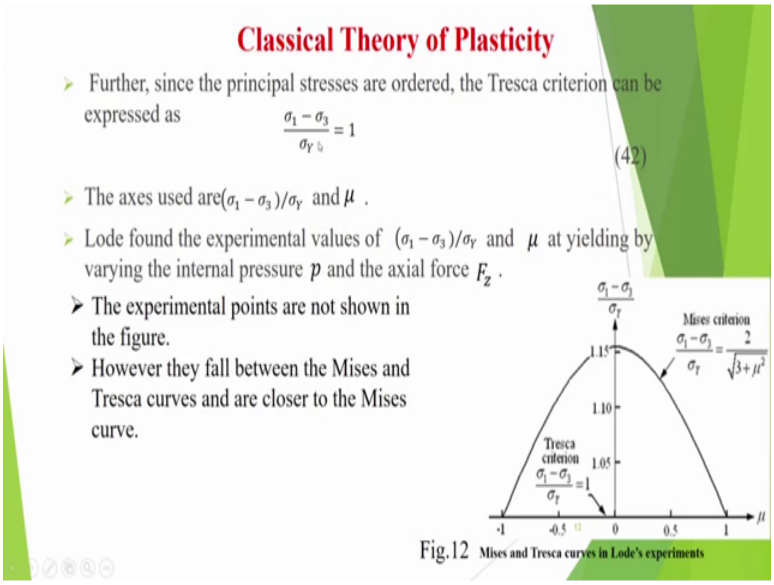

And here for Tresca you have got sigma 1 minus sigma 3 is equal to sigma Y equal to 1 and axes used are sigma 1 minus sigma 3 by sigma Y that is one axis on the top and this is equal to mu. So, in the Tresca criteria now I am starting the vertical axes from 1. So, that ok, horizontally this is mu 0 to this one 1 this side, 0 to minus 1 this side so that means, this is the point 0 1.

So, as per Tresca criteria this is the point and Mises criteria is giving this type of curve. So, now, what happens? Then experiment points are not show in this figure, but all the experiments he conducted they fall between the Mises and Tresca curve. So that means, Mises and Tresca curve it is this one so that means, both the criteria are able to predict, but Mises criteria is providing a type of upper bound.

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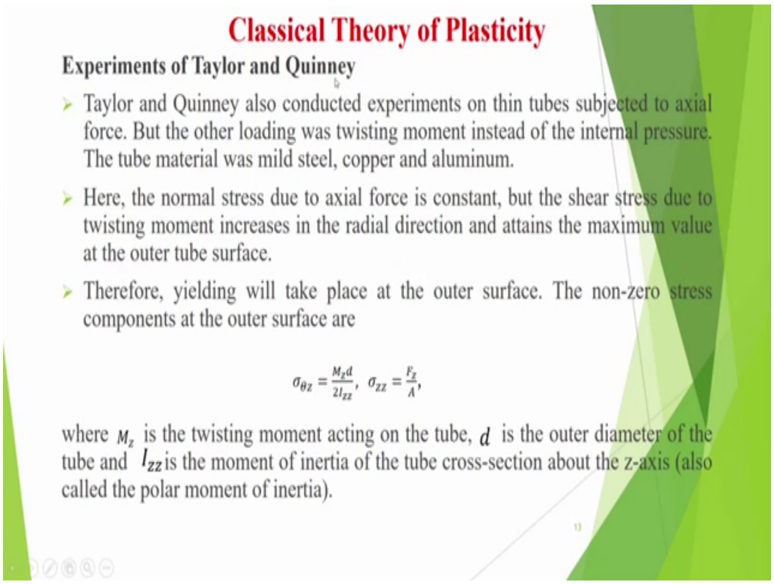

Now, then similarly experiments of Taylor and Quinney they conducted the experiments on this tubes, but subjected to axial force, but instead of applying any pressure they applied the twisting moment.

So, when they applied the twisting moment then in that case nonzero stresses was sigma theta z that was the shear stress this is given by this expression M z is the twisting moment and I zz is the moment of inertia and sigma zz was F z by A.

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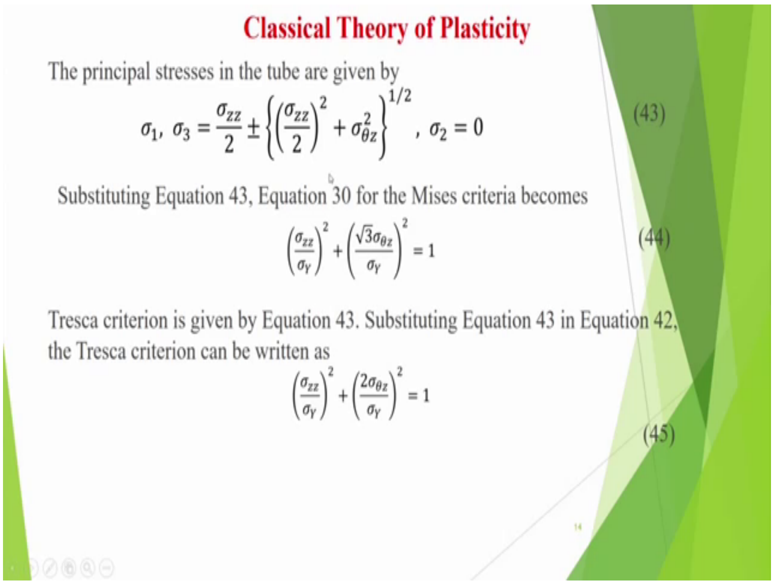

And then the principal stresses in the tube are given by sigma 1 sigma 3 and sigma 2 equal to 0, then the one Mises criteria becomes like this and the Tresca become like this.

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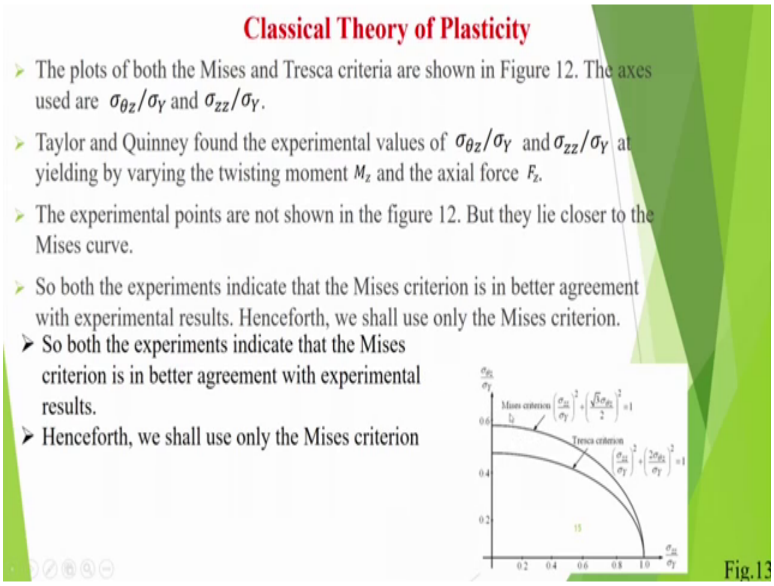

So, both the criteria are here plotted nicely in this expression, this is Mises criteria this is Tresca criteria. So, most of the experimental points they lie between these two criteria, so we can see, but Mises criteria predicts more stresses compare to the Tresca criteria.

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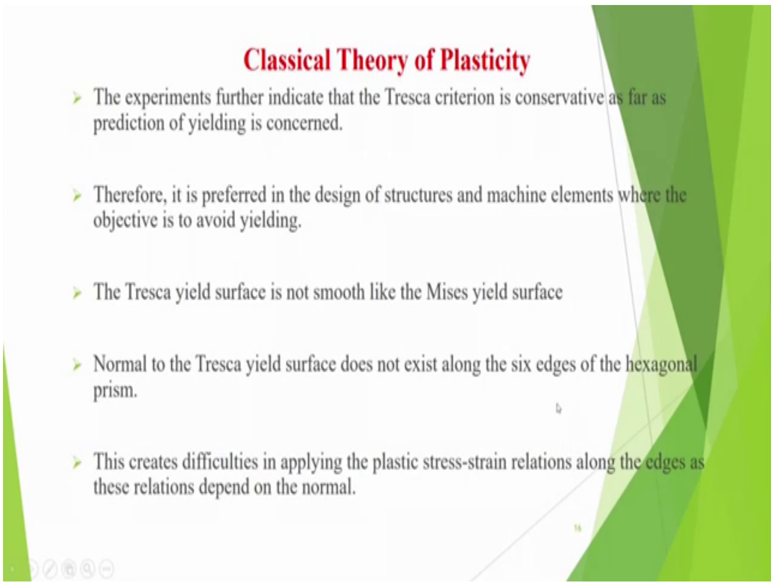

So, the experiments indicate that the Tresca criteria is conservative as for as the prediction of yielding is concern that means, according to Tresca criteria yielding will start first.

So, if you are designing some component for failure; better use Tresca criteria that means, assume the failure will occur, but if you are designing a machine tool then assume that one Mises criteria because if otherwise if material will deform as per Tresca criteria, but actual practice it may not deform because one Mises stress will give more amount of the stresses that is why this was happened. So, therefore, it is preferred to the design of structure and machine elements where the objective is to avoid yielding that means, Tresca criteria. Tresca yield surface is not smooth like the Mises yield surface and normal to the Tresca yield surface does not exist along the 6 edges of the hexagonal prism. So, this creates difficulty in applying the plastic stress strain relation along the edges as these relations depend on the normal. So, most of the time we use this one Tresca criteria, we do not use because of this one difficulty it is not continuous.

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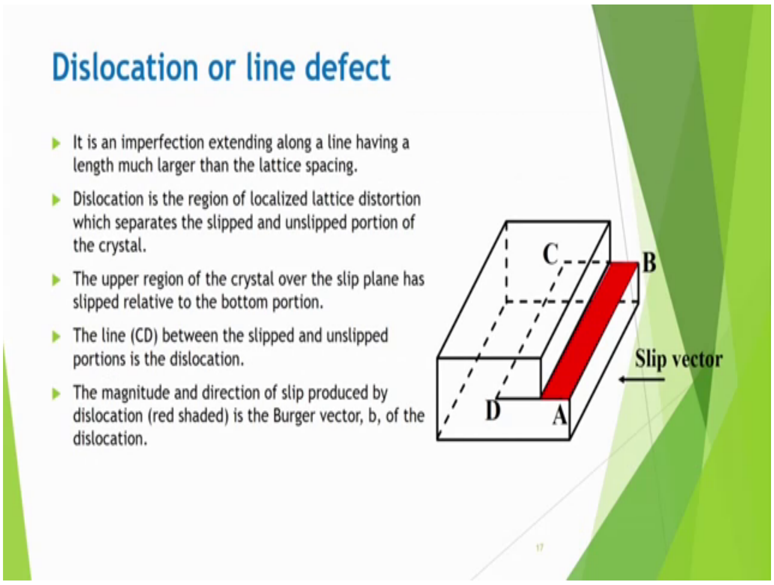

Now, we come to these one phenomenological model. Now, I am going to explain something about dislocation base thing. So, while just give the basic concept about that. In the course on the material size you will here study about dislocations in detail; dislocations are line defective. So, basically dislocation is an imperfection extending along a line having a length much larger than the lattice spacing you know that one unit cell that is called arrangement of the crystal that is lattice.

So, dislocation is the region of localized lattice distortion which separates the slipped and unslipped portion of the crystal. Schematic diagram is shown, here it has slipped and here this upper portion has slightly slipped and this one lower is this one. So, for example, that it is like this here it is this portion has slipped and this is bottom portion is here. So, you are seeing this type of thing this can be called as a slip vector, this is called slip vector. So, the upper region of the crystal over the slip plane has slip relative to the bottom portion line CD between the slipped and unslipped portion is the dislocation. So, this may be called a dislocation.

The magnitude and direction of slip reduced by dislocation is the burger vector B of the dislocation. So, we obtain the burger vector.

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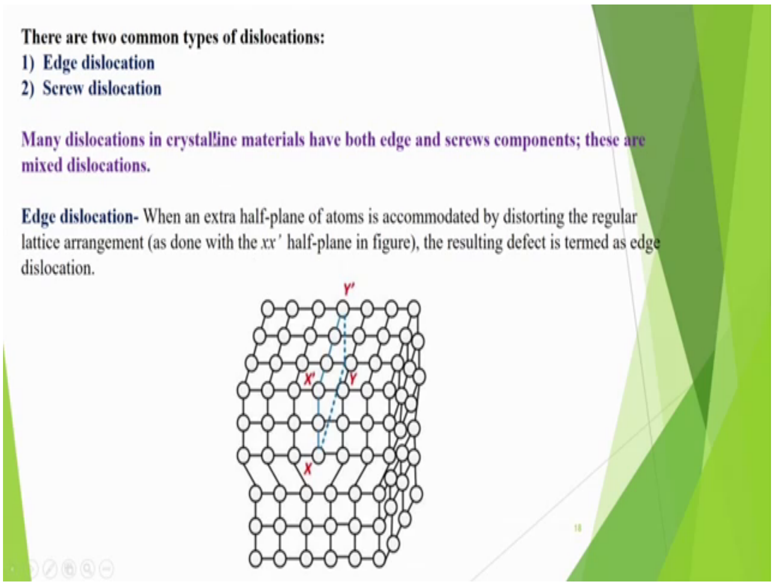

And here there are two common types of dislocations one is called edge dislocation other is called screw dislocation. Many dislocations in crystalline materials have both edge and screw components these are mixed dislocations.

So, edge dislocations that means, there is an extra half plane of atoms in the crystally structure it is accommodated by distorting the regular lattice arrangement you are seeing this type of phenomena this effect is called edge dislocation you see that here on the top you have 1 2 3 4 5 6 7, 7 planes are there in the bottom 1 2 3 4 5 6 only are there that means, one extra half plane is there that is X prime, Y prime, XY. So, this one is there that is why there is some distortion here so this is called edge dislocation.

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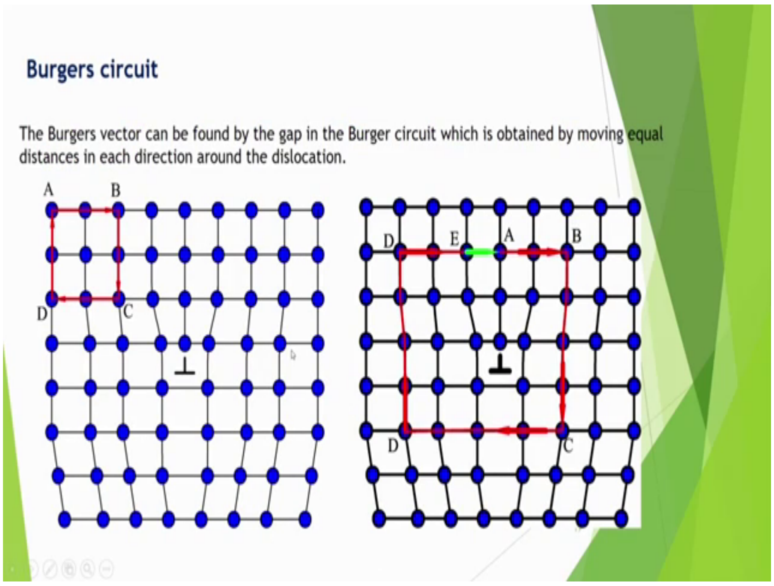

And then so how do we quantify edge dislocation? This edge dislocation is quantified by burgers circuit. We make burger circuit and we obtain burger vector. Suppose we start from a point called A move in the clock wise direction go to the point B, then we go to the point C, then we go to the point D and then finally, we go to the point A if we do this type of thing then we are reaching to the same position ok, it is regular arrangement that. But suppose you see it on the right hand side figure I am starting point A going to B to step movement then I went 1 2 3 4, 4 is step down went to C, then I am moving 4 step in that direction 1 2 3 4, then I go to up direction 4 step, 1 2 3 4. So, upper side we moved 4 step down, 4 step is step up concealed.

Now, here in the horizontal side already we had move two step towards right hand side and 4 step left hand side. So, we should move 2 step again. So, we moved 2 step again to the right side, but we reached at point E we did not get the previous point we reach the point E. So, difference between E and A each called burgers vector. So, EA is a burgers vector its length will give the magnitude of that dislocation and that is what you are bring.

So, this is burgers vector that which characterizes the dislocation. Now, I am we have also a term called dislocation density. What is the dislocation density? It is basically the length of the dislocation because it is edge dislocation. So, how much length of the atom is affected that length divided by the volume. So, it will have what unit? The length unit is centimeter and volume is centimeter. So, was centimeter cube volume is centimeter cube. So, therefore, dislocation density will have unit of 1 by centimeter square. Do not say that it has say kg per meter cube or something like that; it is 1 by meter square because dislocation density means dislocation length per unit area and this is this one.

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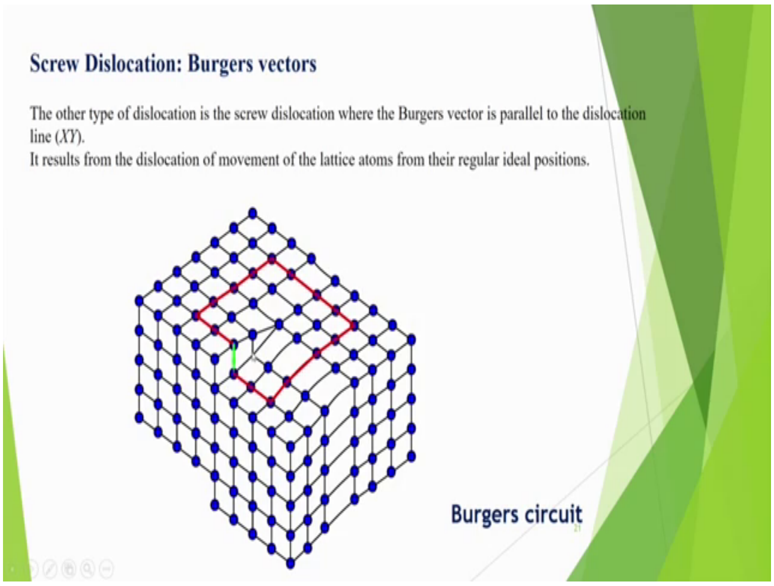

See what is a screw dislocation? Screw dislocation is like this that here there is a short of that shearing. You see that here this half plane has sheared in this direction and you this is a dislocation line XY, behind that there is a perfect crystal. In this case what you are observing that you can observe that by means of the burgers vector here also that you move in this one direction.

Let us say that we are starting from this point let us say that we are going 1 2 3, then you go 1 2 3 4, 1 2 3 4, 4 steps again we go 1 2 3 4 step, again we go 4 step here, we are moving again 2 step and we are coming to this point and then we draw a burger vector this time. But now, this time your burger vector is actually parallel to the dislocation line. So, this is called screw type of dislocation, this it is a screw type of thing. It is something like there is a type of shearing that means, this metal this surface is just sheared that means, one block just got sheared like this. So, you are seeing that here it has moved little bit more and here it has moved a bit less that means, atomic distance movement is less. So, you are getting this screwed dislocation.

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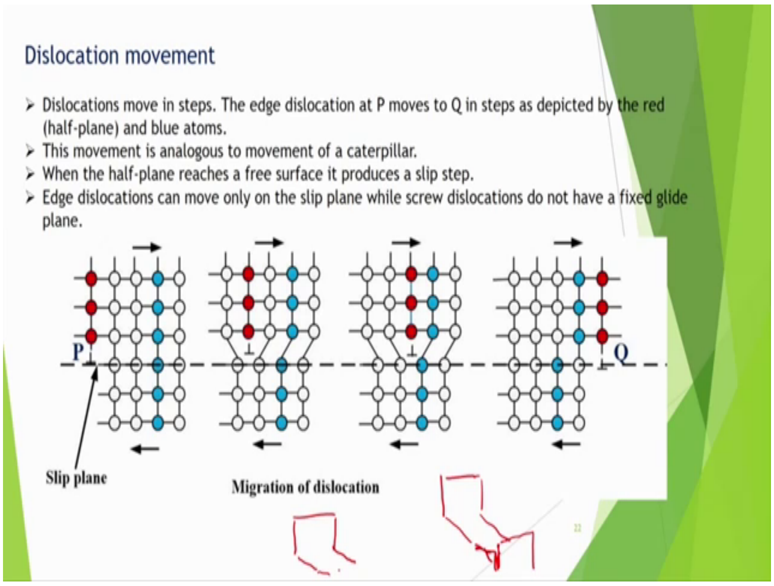

Now the in this one dislocation movements actually causes the plastic deformation; dislocation usually there can be two phenomena for plastic deformation one is the slipping. What is slip? Slip means the dislocation it is due to dislocation movement mostly there is a slip plane and which the dislocation will move here suppose this is the point P here, this dislocation then it migrates to the another point. It comes in this direction when force is applied then further force application migrates it to this point and finally, it has come to this one. So, this type of migration of dislocation it is causing this one.

So, this dislocation we give some time the analogy that metal deforms that is why your you remains constant because metal deforms like a card of dig, you can take playing card I do not play cards. So, I have brought this one visiting cards, my visiting cards you see, my visiting cards are there I have put stack this one you consider this is the you know atoms are arranged here then I am pushing it and you see that here. It is like this it is going in this way that means, there is a movement, but the top surface (Refer Time: 64:38) moving almost the same distance, but still that from bottom to top you see large difference in this for shape large this length is only this much you see large amount of shearing, but not that individual atom is not undergoing.

So, this is like that. So, this is how this I am showing a bit exaggerated you see this is how that it is slipping that means, each one is slipping little bit another is slipping somewhat distance this one. Suppose 1 percent slips 1 mm, on top of that another slips 1 mm, on top of another that slips 1 mm. So, so total will be 3 mm, but not that 3 mm has been stretch that one go otherwise you will require huge amount of force may be 100 times or may be more that much more than 100 to 1000 times more force will be required, but because of this movement less amount of force is required you see that this thing. I am not stretching into like this rather I am causing the slipping of this. So, that is what that you know it has been shown by the slipping behavior and this one will be card of technology.

Another analogy given that movement is analog us to the movement of a caterpillar or I would say the carpet analogy. What is a carpet analogy? You see I have brought a carpet for you here this a small carpet. Now, if I want to pull this carpet here I am requiring huge amount of force it is not easy. So, what we can do that I just make a small repel type of here and then I propagate I just make some repel here then I keep moving it like this, keep folding its slightly like this, keep moving like this and keep moving like this. And finally, it has moved so that means, I am moving repel by repel that means, this is my carpet analogy that is also called like this.

So, movement is analogous to movement of caterpillar when the half plane reaches a free surface is produces a slip step edge dislocations move only on the slip plane, while screw dislocations do not have a fixed glide plane is screw dislocations they can cause twinning also twining means you make the mirror image of thing that means, one portion of the atom gets a twisted a bit and it makes this one. But the twinning is generally observed in some materials which have hexagonal close materials most of the easily machine dye material they deform by means of the slip deformation under the favorable circumstances.

So, twinning is not that prominent phenomena and this will be like this. So, what happens that here it is in the twinning you may get some type of that type of situation, that here you it may twin along that is side and it may be I mean something like that some sort of twisting, some sort of this one. Here I am making may be yes may be like this, and this is this one. So, some sort of twisting is there and this will be this will be this one twinning. So, twinning phenomena will not discuss much most of the discussion we will assume that the material is deforming by slipping. If there is a twinning phenomena really present then this becomes a very difficult to model, it is having more complexity in the modeling so, this much for today. We will continue in the next lecture.