

Introduction to Machining and Machining Fluids
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Lecture – 05
Forces in Machining

So, now we are moving to forces in machining; normally we have seen the introduction to machining process. So, we deal with only the orthogonal.

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Need of studying “Forces in Machining”

Knowledge of cutting forces needed for:

- Estimation of power requirements ($F_c \cdot V$)
- Machine tool design e.g. static/dynamic stiffness
- Part accuracy e.g. tool-workpiece deflections

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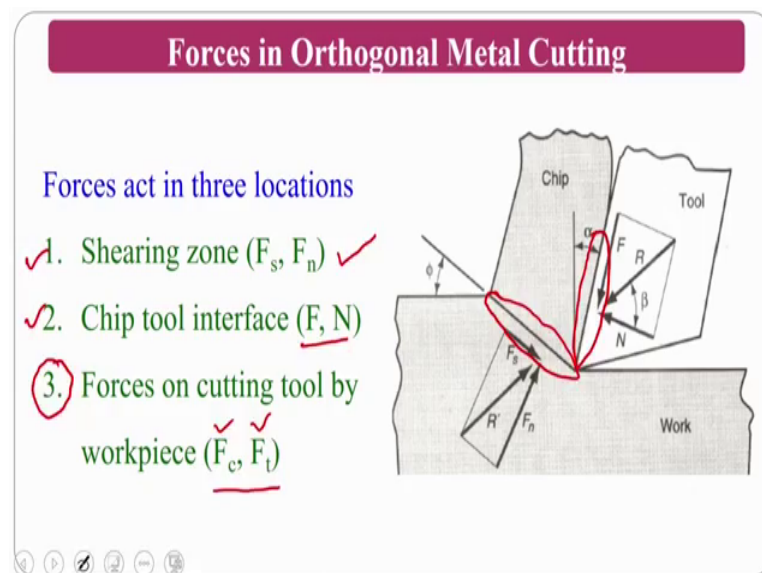
So first we should know what is the need of studying the forces in machining operation? If you see the knowledge of cutting forces is required to estimation of the power requirements. Normally power requirements, if you see it is F_c into V ; so, if you can calculate the cutting force and you know the velocity cutting velocity. So, you can calculate the what is the power required or the energy required.

For the machine tool design that is the static and dynamic stiffness; what is the stiffness required and based on that you can design the tool. Normally the tool machine tool are designed like a lathe bed and all those things are designed with a material cast iron because the cast iron has a graphite in it. So, graphite will have the damping effect ok; so, you in order to have the damping and all those things for that purpose also it will be useful what are the forces that the lathe bed or the cutting tool at the same time tool holder, tool post these are all to be designed.

So, if the forces exceeds what will happen? So, in order to be safe the machine tools are designed according to that for that purpose one has to calculate the forces required part accuracy and tool work deflections. So, the part accuracy is also important if I can calculate what are the forces that is experienced during the machining by the tool. And if there is any deflection of the tool if the deflection is there then there will be problem; so, the part accuracy goes back.

So, for that purpose one has to calculate the forces ok. So, to maintain the no deflection or there is minimum deflection one has to do calculate the forces in the machining operation.

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So, there are the forces in orthogonal metal cutting that is a machining processes; force act in three locations one is shearing zone if you see the shearing zone that is nothing, but the shear force and normal to the shear force; this is a shearing zone. So, the chip tool interface the other forces that one observes in this region is chip tool interface.

Normally there are two forces one is the frictional force and the normal to the frictional force; these two are there in this. Forces on the cutting tool by the work piece normally F_c and F_t are what one can experimentally measure is this are the F_c and F_t ok. These two forces are the forces experienced by the cutting tool; normally this cutting tool is placed on the dynamometer. So, the dynamometer will give the forces.

So, this the third one this is what the forces that one can measure and the 1 and 2 are calculated from the experimental measured forces which we can see in the upcoming slides.

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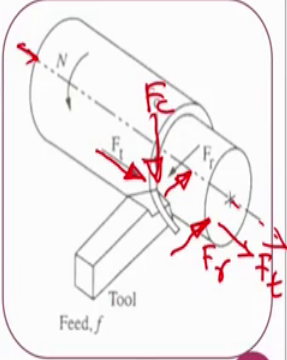
Force Components in Metal Cutting

There are three force components in metal cutting are:

Cutting force (F_c) acts in tangential direction. It is also called power component as it being acting along and being multiplied by cutting speed (V_c) decides cutting power ($F_c \cdot V_c$) consumption.

Thrust Force (F_t) acts in the direction of feed (Axial direction). Generally, this force is small in magnitude but is responsible for causing dimensional inaccuracy and vibration.

Radial Force (F_r) acts in radial direction. This force is least harmful and hence least significant.



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So, if you see there are three force components in the metal cutting operation; one is cutting force, second is the thrust force and the radial force; these are the three forces. The normally the cutting forces tangential to direction it is also called as a power component, normally this is as I said in a beginning of this forces F_c into V gives me the power requirement.

That is why it is called power component the and the cutting force normally acts on the tool by the chip ok. So, I will explain once I explain thrust force and radial force, I will come back to the cutting force how the cutting force will act. So, thrust force it acts in the direction of the feed of feed that is axial direction.

That means in this picture if you see this is the thrust force ok. So, my feeding direction is this that is nothing, but the work piece axial direction also in this direction. So, this is the feed direction, this is the radial component which is also you can see which has a radial direction; that means, as per the depth of cut is concern it will move in this direction.

So, this is called F_R and this is called F_t and this is called F_c ; that means, whenever I am cutting with the cutting tool my chip is moving on top of it if that is this my chip will kick down the tool that is nothing, but the cutting force; that means, that it will be in the this direction.

So, the cutting chip whatever the chip is coming out; it will kick the tool downside that is the cutting force. And along the direction or the axial direction of the work piece that is called the thrust force and or the sometimes some people they call as a feed force also and the radial force is nothing, but as per the depth of cut is concerned it will be radially inside into the work piece.

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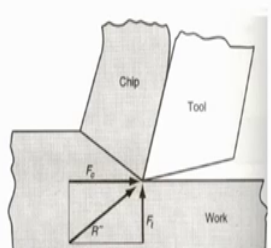
Cutting Forces and Thrust Forces

- Forces F , N , F_s , and F_n (or N_s) can not be directly measured.
- Forces acting on the tool that can be measured:
 - Cutting force F_c and Thrust force F_t

How F_c and F_t measured?

$F_c \cdot F_t \propto \alpha, \beta, \phi \Rightarrow F_c \cdot F$

Measurable $\xrightarrow{MCR} F_s, F$



Forces F , N , F_s , and F_n can be written in terms of Cutting force F_c and Thrust force F_t .

So, the forces F that is frictional force normal to the friction force shear force and normal to the shear force. These are all not calculated directly ok, these are all measure from a relation forced relation that is which we will see in the upcoming slides which is nothing, but the Merchant circular relation. The forces acting on the tool that is nothing, but the cutting force and thrust force also other forces if you have a dynamometer of which two components you can measure F_c and F_t .

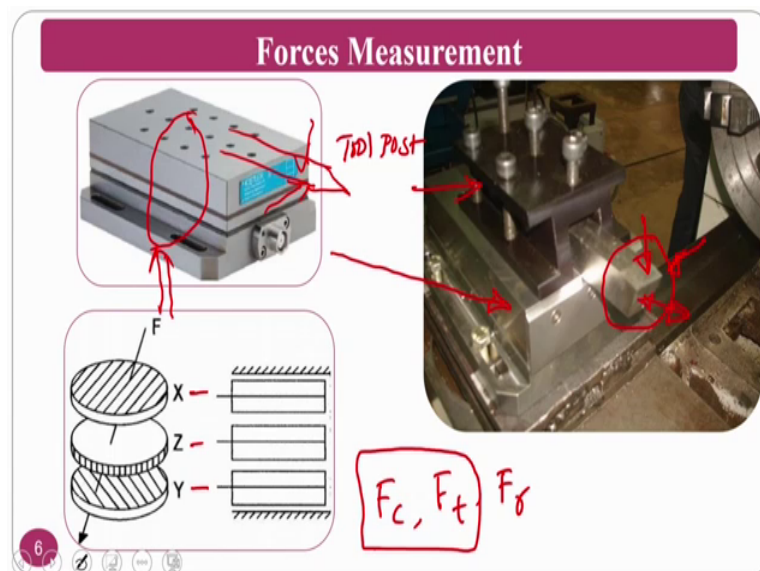
Nowadays you also get the 3 dimensional measurement of the cutting forces. So, where you can get the third component also that is called radial component also; so, normally if you see how to measure F_c and F_t ok. So, that I will discuss in the next slide these are all can be written in the cutting force. So, only that we can measure is in the

dynamometer is F_c and F_t . So, if we know two forces and different angles tool angle that is a rake angle shearing direction all those things.

That is α the friction angle β if you know the shearing angle ϕ . So, with this relations you can calculate the frictional forces shear force, frictional force normal to the shear force, normal to the frictional force this all ok.

That means these are all measurable ok; from the Merchant relation Merchant circle relation you can calculate all this F_s ; frictional force, shear force other things. Now the question is how to measure and how one can measure this cutting force thrust force. Just we are seeing in the orthogonal that is why we are only dealing with the two forces.

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So, the force measurement if you see the force measurement normally this is the dynamometer you can see which is located beneath the tool. This is the tool this is a tool post, this is the tool post or the tool holder you can say this the dynamometer.

Dynamometer is the beneath the tool. So, this dynamometer inside the dynamometer if you see the anatomy of the dynamometer; normally to explain you there are three piezoelectric sensors will be there. Normally piezoelectric sensors works on if there is a deformation, it will give certain voltage or EMF.

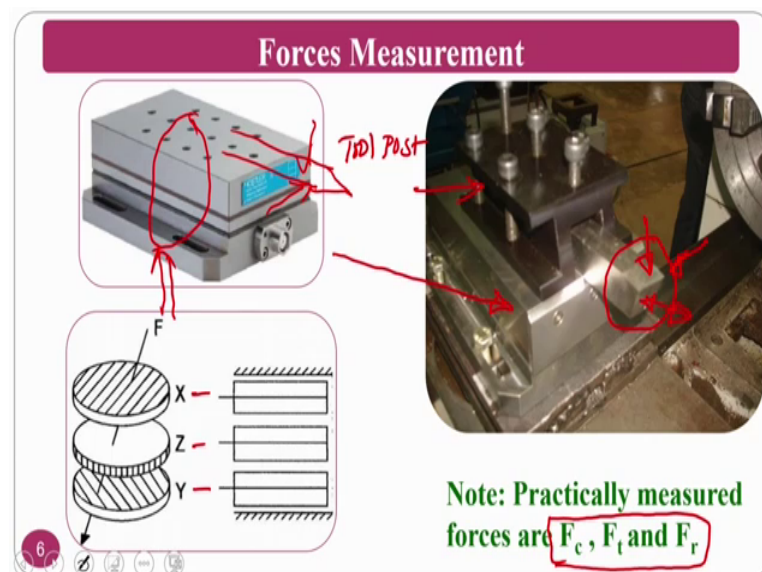
If you give the EMF; normally it will default that is the principle of the piezoelectric materials. So, piezoelectric materials are oriented as per the X, Y and Z axes. So,

whenever if you mount a tool on top of it normally a tool is mounted and if you are giving some forces in different directions; what will happen? The piezoelectric materials that are there inside it this are the inside what will happen? This will deform; whenever this deform this will give certain voltage or the micro volts or nano volts or volts some volts it will give depend on the forces.

So these forces if you are measuring in the three directions on the cutting tool. So, the forces that is acting on the cutting tool you are measuring that is means you are going to get only cutting force thrust force and radial force. But normally we see only two forces; however, if you have a three sensors you can measure the three forces; this is about the machining process the same thing can be represented here also.

Normally machining takes place on the work piece like this and the chip will work on this one and the radial force will be in this direction. So, normally it will be in the opposite direction also ok.

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As you can see clearly note that practically measured forces are F_c and F_t and F_r ; these are the practically measured forces. However from here using the Merchant circle relation you can calculate other values ok. So, now, we will go for the Merchant relation; how to calculate because how much shear force is required to shear the material and all those things is required.

Because we are only seeing the cutting force, but we want to calculate what is the frictional force, what is the shear force and all those things. So, that we can say how much is useful energy that we are imparting to the machine and how much is going as a waste and all those things can be calculated.

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Forces in Orthogonal Metal Cutting

Assumptions

1. Cutting edge is too sharp (Sharpness radius = 0)
2. Cutting edge is perpendicular to cutting velocity
3. Deformation is in two dimensions (No side flow)
4. Continuous chips without BUE
5. Workpiece material is rigid, perfectly plastic
6. Coefficient of friction is constant along chip-tool interface
7. Resultant force on the chip (R') applied at the shear plane is equal to opposite and collinear to the resultant force (R) applied to the chip at chip tool interface.



So, to have the Merchant circle relation; we are going to have some of the assumptions to measure this forces. This assumptions are cutting edge is too sharp; that means, that sharpness radius is 0.

Normally slightly some people will have doubt what is the sharpness radius? And what is the see some people they understand that sharpness radius is nothing, but the nose radius, but there is a slight deviation between the sharpness radius and nose radius ok. As I already explained normally if you see a 3 dimensional tool the 3 dimension tool looks like this.

So, the nose radius is this one this is called nose radius which is represented by R this is nothing, but if assume that this is my principle cutting edge; just for understanding purpose I just erased the nose radius hope you understood. So, now, what I mean to say this is my flank face, this is my rake face which are meeting at the principle cutting edge this is the principle cutting edge.

Every cutting edge will have certain radius that is nothing, but the sharpness radius how sharp this is; that means, this is if you meet this is the radius this is nothing, but the sharpness radius. So, nothing in this world is 100 percent or 0; that means, the sharpness radius is practically it cannot be 0, but; however, we are assuming that this is 0 ok.

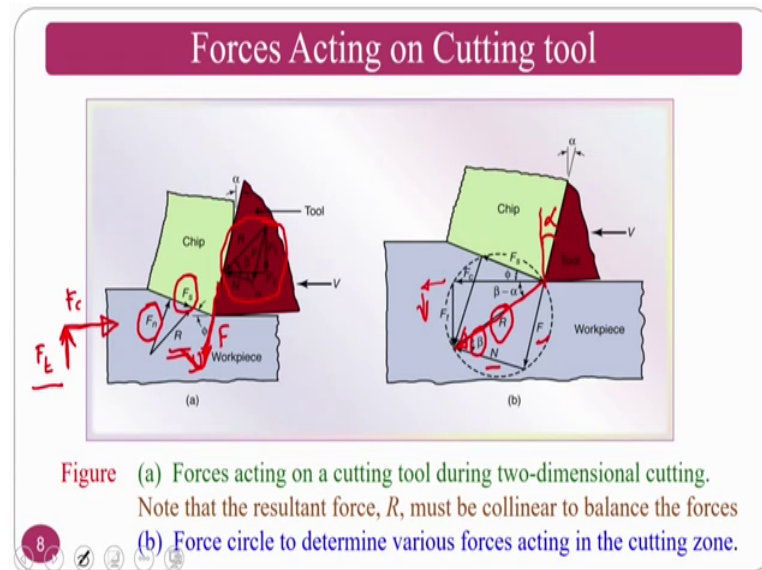
So, hope you understood the difference the difference is this is nothing, but sharpness radius whatever the radius that is generated on the cutting edge, this is my nose radius. So, assuming the this one; so, the second point is cutting edge is perpendicular to the cutting velocity; that means, that my I am doing orthogonal machining. So, deformation in two dimensions no side flow; that means, that only the deformation is taking place in the plane where we are machining. So, there is no side flow or deformation and all those things.

Continuous chip without BUE; so, the chip is flowing on the rake surface is without BUE. So; that means, that is a continuous and smooth cutting operation, work piece material is rigid perfectly plastic material; that means, that the material is removed perfectly.

So, there is no slip part something the coefficient of the friction is constant along the chip tool interface; that means, the there is no variation in COF; that is cutting friction is constant if some friction is x ; that means, that throughout the chip tool interface it is constant because we have the two zones one is the sticking zone, another one is the sliding zone. So, practically speaking there will be a slight difference will be there in the coefficient of friction between the sticking region as well as the sliding region; however, we are assuming it is constant for the this model.

So, resulting force on the chip that is R_1 is applied on the shear plane is equal to equal to opposite and collinear to the resultant force; that means, that two forces that is acting in the directions; both are opposite in direction and collinear in the same direction.

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So, now we will move to the forces acting in the cutting tool since we have already seen in the previous to previous slide. So, there the forces if you see this is the shear and this is the normal to the shear force; the resultant is this one.

Similarly if you see here it is in the red color here; so, you may not see properly. So, that this is the force act parallel to the my rake face is nothing, but the frictional force perpendicular two will be the normal to the frictional force. At the same time you have the two forces that is nothing, but one is the cutting force, another one is perpendicular to the cutting force that is the thrust force ok; so, this called F_t as you can see; this it is red in red. So, you can see the frictional force normally follow as per along the direction of the rake surface or the chip flow direction interface of the chip flow and the tool rake surface and perpendicular two will be the normal force normally.

So, at the same time two forces if you see that cutting force and thrust force; these are the two forces that acts on the tool from the work piece. So, this two can be measured; so, you can see all the forces here the frictional force and a normal to the frictional force and this is called the frictional angle.

The shearing angle along the direction of the shear and I will explain u how to draw this Merchant circle; this is the cutting force and the thrust force. So, you can see the resultant and normally this is shearing angel and if you drop this is called alpha that is called rake angle.

This is the forces acting on the cutting tool during the 2 dimensional cutting note that the all the resultants must be R ; the resultant force is R which is equivalent to the resultant of all the forces that is frictional force, normal to the frictional force, cutting force, thrust force, shear force, normal to the shear force all boils to be resultant is only R that is what the assumption is ok.

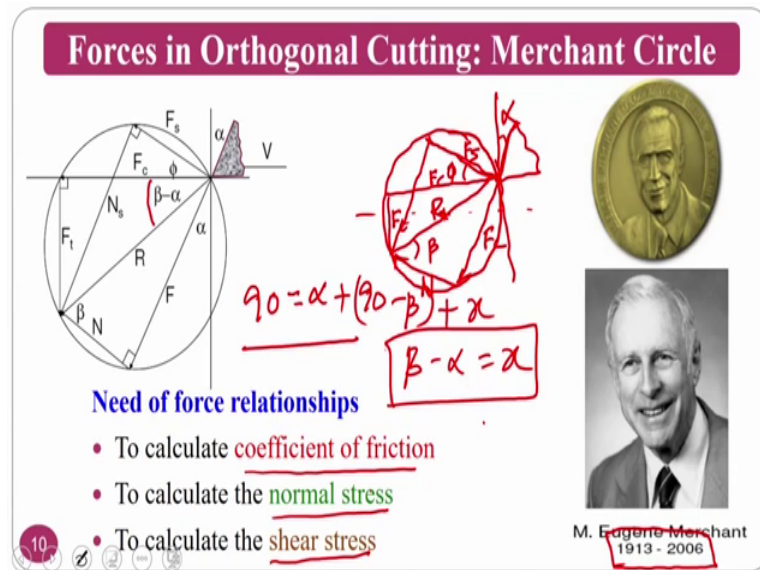
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RESULTANT FORCE

- Vector addition of F and N = resultant R
- Vector addition of F_s and F_n = resultant R'
- Forces acting on the chip must be in balance:
- ✓ R' must be equal in magnitude to R .
- ✓ R' must be opposite in direction to R .
- ✓ R' must be collinear with R .

Vector addition of the frictional force normal to the frictional force resulting; in R that is the resultant force. So, vector addition of shear force normal to the shear force gives rest to the again R and the force act acting on the chip must be balance for that purpose R is equivalent to R ; R must be in opposite direction to the R ; so, R dash must be collinear with R .

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So, this is what you can see; so, now, you can see the same picture here the Merchant circle, where Merchant is the person invented it and all those things. The need for force relations to calculate the coefficient of friction the normal stress and the shear stress as I said in the previous one. So, I will let you know how to draw simply ok.

So, the Merchant circle is a circle; so, Merchant circle normally just first you draw a circle. So, normally it should be on the upside; so, normally the tool should be above the centre point ok. So, you can take any position you can draw the tool; so, this is the point. Now your frictional force is parallel to this rake surface and your cutting force is parallel to the cutting direction that is the cutting velocity direction and your shearing in this direction ok.

So, this is the direction; so, I know frictional force the cutting force and shear force just three directions. So, I can choose any now normally the thrust force is perpendicular to it. So, the resultant is R which you have seen this is nothing, but R this is my thrust force; so, R will connect to R .

So, this is nothing, but my N ; so, this is my frictional angle β , this is the shearing angle that is ϕ , this is α that is rake angle. So, from the geometry or the trigonometry and all those things, you can calculate other angles. If you see like this this is also become α as you can see here this two alternative angles and this is the ϕ ;

that is the shearing angle this is beta which I am already explained you know that is the frictional angle.

If you take this is 90 degrees; so, 90 degrees equal to, so alpha plus some other things which you can boils out to be beta minus alpha here ok. So, and this will be 90 minus beta ok; so, if alpha plus 90 minus beta this angle is then you can calculate this angle plus x assume that x

If you can calculate now your x will be like beta minus alpha is equal to x so, that is nothing, but this angle ok. So, this is how you can calculate all the things; so, that how my x which I want to calculate is beta minus alpha.

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Coefficient of Friction (μ)

$\tan \beta = \mu = F/N$

Normal stress (σ) = N_s / A_s

Shear stress (τ) = F_s / A_s

Total work done,

$$F_c \cdot V = F_s \cdot V_s + F \cdot V_c$$

What is the need of measuring coefficient of friction...?

The coefficient of friction; the first thing what we are going to calculate from this one is the co efficient of friction. The co efficient of friction is nothing, but tan beta; so, this is my beta angle, tan beta equal to F by N that is mu this is coefficient of friction that I am going to calculate.

So, the normal stress is N s by A s which is N s is nothing, but the this normal to the shear force to the area of shearing. So, shear stress is nothing, but my shear force by the area of shear. So, this is how you can calculate we are going to calculate this three from the known sources.

So, F_c into V equal to shear force into shearing velocity and the chip velocity into the frictional force; these are this is useful energy, this is un useful energy assume that I can say it is a energy going as a waste ok. So, now we will calculate all these things.

If I can calculate coefficient of friction normally coefficient of friction is friction forced by normal to frictional force that is proportional directly proportional to my frictional force this one. If my coefficient of friction is very high; that means, that my un useful energy is very high. So, the useful energy goes down for the same input energy that is why we want to calculate the coefficient of friction.

Coefficient of Friction (μ)

$\tan \beta = \mu = F/N$

$F = AB = AC + BC$
 $F = \underline{F_c \sin \alpha} + \underline{F_t \cos \alpha}$

$N = BD = NC = MC - MN$
 $N = \underline{F_c \cos \alpha} - \underline{F_t \sin \alpha}$

$\tan \beta = \mu = F/N$

$\mu = \frac{F_c \sin \alpha + F_t \cos \alpha}{F_c \cos \alpha - F_t \sin \alpha}$ ✓

$\uparrow \mu$
 $\downarrow F, \downarrow \alpha$

Now, if you see from the geometry $\tan \beta$ that is nothing, but F by $N \tan \beta$ this is the β ; $\tan \beta$ is F by N this is opposite side by adjacent side. Now I want to calculate

what is the F ; see what I always said in the previous slides also I do not know what is the F value, what is the N value, but the from the dynamometer; what can I measure is cutting force and thrust force.

So, whatever I want to calculate; I have to form the triangular notation to the joining to the F_c or F_t . For that purpose what I have did is F equal to AB normally this is A , this is B ; F equal to AB , if it is AB which is I am dividing into AC plus BC AC plus BC .

Now, AC I am connecting like this; this is a triangle that is called ACM triangle; from this triangle I can say that $F_c \sin \alpha$ that is called this is α from the trigonometry you can get this is α its comes as α ok.

So, this is $F_c \sin \alpha$ and $F_t \cos \alpha$; BC is $F_t \cos \alpha$ now see BC is equal to DN DN this is α , this is the triangle that is called MND or DM , this is the triangle which in this triangle this is my α . So, it is called $F_c \cos \alpha$; so, at the same time in I have to calculate again normal to the frictional force that is called N , this DB which is called BD or DB which is equivalent to NC this is called NC here is N just let me erase.

So, that I will come back again now NC equal to MC minus MN ; MC is nothing, but this one, MC is this one MC minus MN this is MN . So, now MC is connected to my cutting force this force; so, $F_c \cos \alpha$ $F_c \cos \alpha$ which is adjacent to this one that is why it is $F_c \cos \alpha$ minus $F_t \sin \alpha$ ok.

Now, $F_t \sin \alpha$ because I am talking about MN which is opposite side; so $F_t \sin \alpha$. So, the now we have known values that is called F_c is known to us rake angle is known to us F_t is known to us because F_t and F_c are measured and α is a rake angle; here also F_c and F_t are measured and α is a rake angle of the tool.

So, now I can calculate the frictional force as well as normal to the frictional force; if I know the frictional force and normal to the frictional force I can calculate the coefficient of friction this is the final term that is called $F_c \sin \alpha$ plus $F_t \cos \alpha$ by $F_c \cos \alpha$ minus $F_t \sin \alpha$; this is the final equation where all are known to me and I can calculate coefficient of friction.

As in said if my μ is high my useful energy will goes down that for the same input energy; that means, that my frictional force into chip velocity will goes up so; that means, that unwanted or waste of energy will be more.

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Normal Stress on the chip

Normal stress (σ) = N_s / A_s

Where,

$N_s = DQ + QP = F_t \cos \phi + F_c \sin \phi$

$A_s = \text{Area of shear plane} = b(t_c)$

$A = \text{Area of uncut chip thickness} = b \cdot t_0$

$t_0 = t_c \sin \phi$ (Orthogonal Cutting)

$\sigma = (F_c \sin \phi + F_t \cos \phi) \sin \phi / A$

Since we know, depth of cut and feed Area of uncut chip thickness can be easily calculated

So, now we will see the normal stresses on the chip normal stress is equivalent to N_s by A_s . So, N_s if you see the N_s N_s is nothing, but this is my shear F_s is shear force and perpendicular to is N_s this is N_s .

Now, I am dividing N_s that is nothing, but the $D P$ is nothing, but normal to shear force. So, I will divide into two things; one is already I have one Q is there if you see in the picture. $D P$ I am dividing into DQ plus QP . So, DQ which is relevant to as I said earlier also I have unknown things I have to correlate with the known things.

That is F_t and F_c ; so, so that I have to form my triangle triangles, if I can make the triangles that is integral part of one should be know that is F_c and F_t so that I can calculate. So, now I am going to say DQ is equal to now I say DQ , this is DQ and I know this is a ϕ . So, D ; this is a adjacent side to this triangle $D M Q$; so, that it can become $\cos \phi$; so, $F_t \cos \phi$. So, plus at the same time QP now my QP is there, QP is equivalent to MO ; if you say the MO ; then I am forming a triangle $A M O$; where I know the ϕ is the shearing angle in this triangle for that purpose MO is equivalent to my QP .

So, this is opposite side to this triangle AOM to the phi; that means, $F_c \sin \phi$. So, area of shear plane if you see b into t c; now if you see the bottom picture this is nothing, but my t c which is not known to me because I have not measured it, but considering the two dimensional metal cutting that is orthogonal metal cutting t naught that is called uncut thickness and chip thickness for the after cutting and before cutting.

So, after cutting t naught is equal to my feed if I know feed normally I will give whenever the cutting operation is going to start I have to give feed to the machine tool. So, this t naught is known to us; t c not known us; that means, that this is unknown to me and b is nothing, but my width of the chip or depth of cut. So, area of thickness is nothing, but b into t naught and now we have to find the relation between t naught and t c that is a uncut thickness to the thickness.

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Normal Stress on the chip

Normal stress (σ) = N_s / A_s

Where,

$N_s = DQ + QP = F_t \cos \phi + F_c \sin \phi$

A_s = Area of shear plane = $b(t_c) \rightarrow$ ①

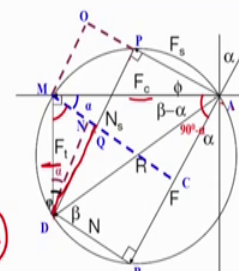
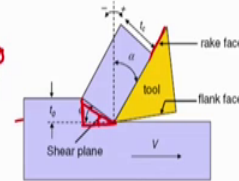
A = Area of uncut chip thickness = $b \cdot t_0 \rightarrow$ ②

$t_0 = t_c \sin \phi$ (Orthogonal Cutting) \rightarrow ③

$\sigma = (F_c \sin \phi + F_t \cos \phi) \sin \phi / A$

$\leftarrow b \cdot t_0$

Since we know, depth of cut and feed Area of uncut chip thickness can be easily calculated

As if you form a triangle; so, t naught is this whatever I have drawn here; if you take this as a triangle this is the phi. So, you can calculate from this triangle t naught equal to t c into sin phi. From that now we do not know what is the t c and if you put this equation 1 and 2; if you put equation 2 in the 1; normally you will get equation 3 ok.

So, now you can put this into the above equation that is N s equal to this equation ok. So, now you can calculate sigma equal to this value that is called $F_c \sin \phi$ plus $F_t \cos \phi$ by multiplied by sin phi into a where this is a is nothing, but your depth of cut multiplied

by uncut thickness that is nothing, but feed. So, since we know the depth of cut and feed area of uncut thickness can be easily measured or calculated.

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Shear Stress on the chip

Shear stress (τ) = F_s / A_s

Where,

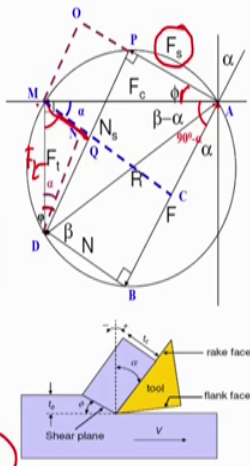
$$F_s = AO - OP = F_c \cos \phi - F_t \sin \phi$$

$A_s = \text{Area of shear plane} = b \cdot t_c$

$A = \text{Area of uncut chip thickness} = b \cdot t_0$

$t_0 = t_c \sin \phi \quad A_s = A / \sin \phi$

$(\tau) = (F_c \cos \phi - F_t \sin \phi) \sin \phi / A$



So, now we are going to the shear stress on the chip now the shear stress is nothing, but the F_s by A_s ; I already seen what is A_s that is the area of the shear plane ok, now we have to calculate F_s . So, F_s is nothing, but this one; so, this is about AP this is nothing, but my F_s .

Considering this I will I am making a extension to the O and I am saying now considering the $AO M$ triangle; if you see that one AO minus OP gives me F_s that is PA AO minus a p . So, now AO which is a shearing angle that is adjacent side, so, you can say $F_c \cos \phi$ minus OP . Now OP is equivalent to this MQ MQ I know this ϕ ; so, that is opposite side so; that means, that if this if this is F_t this is opposite side $\sin \phi$; so, $F_t \sin \phi$.

So, we can from this again in the from the previous slide these are all same, where area of the shear plane is nothing, but b into t_c which is where the t_c is not known and area of uncut thickness is nothing, but b into t_0 naught, where t_0 naught is a feed b is the width that is depth of cut.

So, depth of cut is known and feed is known; so, you can know this value you know this value ok. So, now from this one you can say A_s equal to A by $\sin \phi$ whenever you put

back this into the equation that is a shear stress equation; you will get the shear stress is nothing, but this one; that is $F_c \cos \phi$ minus $F_t \sin \phi$ multiplied by $\sin \phi$ by area of undeformed chip; that is nothing, but which is nothing, but your depth of cut multiplied by feed.

So, from the orthogonal metal cutting operation; if it is oblique metal cutting normally there will be a some other terms also will come. Because there will be a slightly inclination will be there and all those things will come into the picture.

Since we are studying only the orthogonal metal cutting operation, this is how you can calculate the shear stress. If you know the shear stress normally you have to apply the more energy compared to the shear stress then only the material will shear. So; that means, that whenever I want to shear a material I have to put more stress than the required stress then only the plastic deformation will takes place severe plastic deformation will takes place.

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Work done and specific cutting energy

Total work done $= F_c \cdot V$

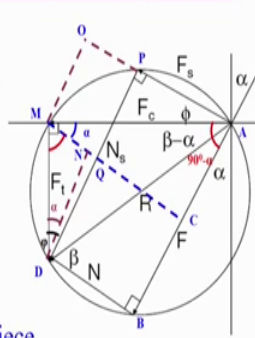
Work done in shear (Useful) $= F_s \cdot V_s$

Work done in Friction (Un useful) $= F \cdot V_c$

Total work done,
 $F_c \cdot V = F_s \cdot V_s + F \cdot V_c$

SPECIFIC CUTTING ENERGY (SCE):
 $SCE = \text{Total work done} / \text{Unit volume of workpiece material removed}$
 $= F_c \cdot V / MRR$

Where, $MRR = \text{Material removal rate} = V \cdot b \cdot t$



Force relationships

Work done in in specific cutting energy; so, now, work done total work done is F_c into V . As I already said the cutting force multiplied by the cutting velocity is the total work that is done during the machining operation or that am imparting that is the energy.

Work done in shear that is called useful and work done in friction, as I said coefficient of friction is calculated or frictional force is calculated and all those things. So, now total

work is F_c into V and which is divided into useful and as well as waste energy this is the main equation.

Now, coming to the if my friction is more what will happen? My un useful energy or the waste energy will goes up. So, my whatever the input I am giving; the most of the energy goes as a waste, so I always should think in the sense where how to reduce the friction and for that purpose only the people uses cooling fluids that is what our course also deals about is machining fluids.

At the same time tool coatings are done, lubrications is done, different types of lubrication is done like minimum quantity lubrication, flood lubrication or cryogenic lubrication and all those things. So, that my this factor F ; F into V_c will go down. So, that my useful energy if I am giving F_c into V ; so, I have my main aim is to increase this F_s into V_s .

So, these are the alternatives the cooling and all those things. So, that my useful energy goes increases for the same input F_c into V ok. So, that is about the useful now coming to the specific cutting energy. So, specific cutting energy is nothing, but the total work done on the total work done or the energy given to the volume of the work this material removed that is nothing, but F_c into V .

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Work done and specific cutting energy

Total work done $= F_c \cdot V$

Work done in shear (Useful) $= F_s \cdot V_s$

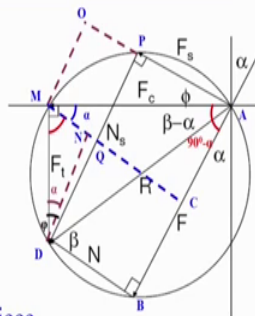
Work done in Friction (Un useful) $= F_t \cdot V_c$

Total work done,
 $F_c \cdot V = F_s \cdot V_s + F_t \cdot V_c$

SPECIFIC CUTTING ENERGY (SCE):
 $SCE = \text{Total work done} / \text{Unit volume of workpiece}$

$U = F_c \cdot V / MRR$

Where, $MRR = \text{Material removal rate} = V \cdot b \cdot t$

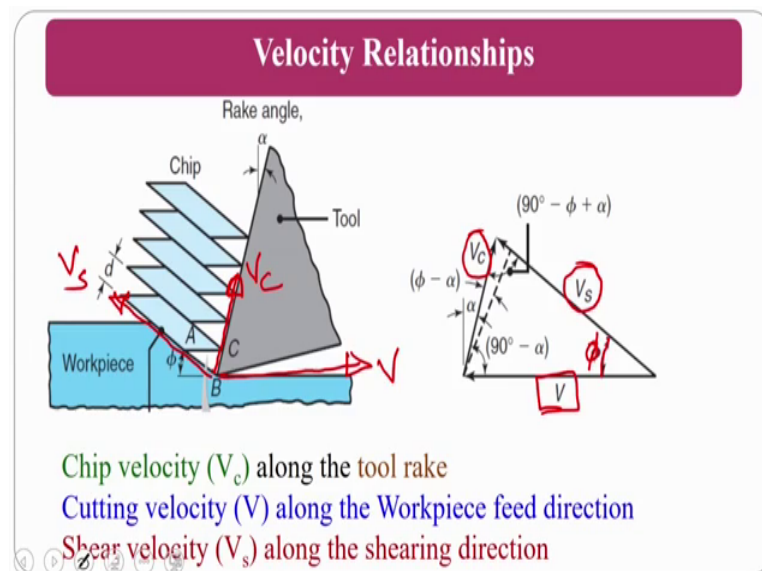


Force relationships

Specific cutting energy normally represent in terms of U basically. So, specific cutting energy is nothing, but energy given to the material removal rate ok; how much is removed per unit time volume that is nothing, but my chip thickness; that means, uncut thickness into the this is called my feed, this is called my depth of cut and this is called my cutting velocity.

So, if I use this is $M\ M$; this is also $M\ M$ and this is $M\ M$ per minute or second. So, now, you will get is material removal rate that is about the specific cutting energy some people will calculate in other way also.

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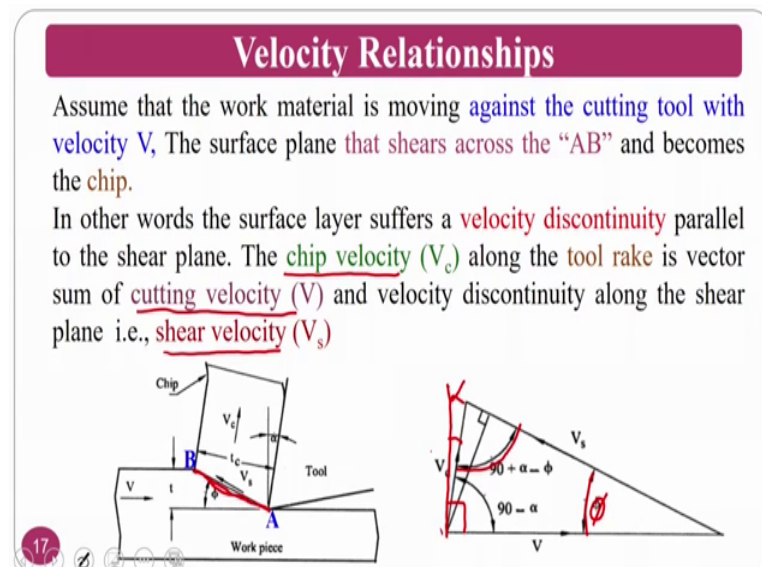


Now, we are moving to the velocity relationships; velocity relationship as we know there are three velocities in the machining one is chip velocity. Now you can see the chip for the betterment of understanding, we have the chips are there assume my chip velocity will be the direction of my frictional force. At the same time cutting velocity, cutting velocity will be like along the direction of this one that is nothing, but V this is nothing, but V_c and the shear velocity; shear velocity will be in the direction of shearing ok.

So, now you can see this velocity V is cutting direction just I am telling you the directions only because my machining is taking place. And the shearing velocity you can see here also shearing velocity is taking place and my chip velocity is moving in this direction.

So, shearing velocity with respect to cutting velocity will have ϕ ; this is known. So, rest all the things we will see in the next slide.

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Assume that the work material is moving against the cutting tool with the velocity V that is the cutting velocity. The surface plane that shears is AB; that is what I am saying this is the plane shearing plane which is nothing, but the V_s .

In other words, as I already said that chip velocity is there; cutting velocity and shear velocity. The chip velocity long the tool rake is the vector sum of the cutting velocity and the velocity discontinuity along the shear plane. As I said velocity discontinuity what I mean to say is that whenever there is a discontinued chips that this is the that is what they are specifying ok.

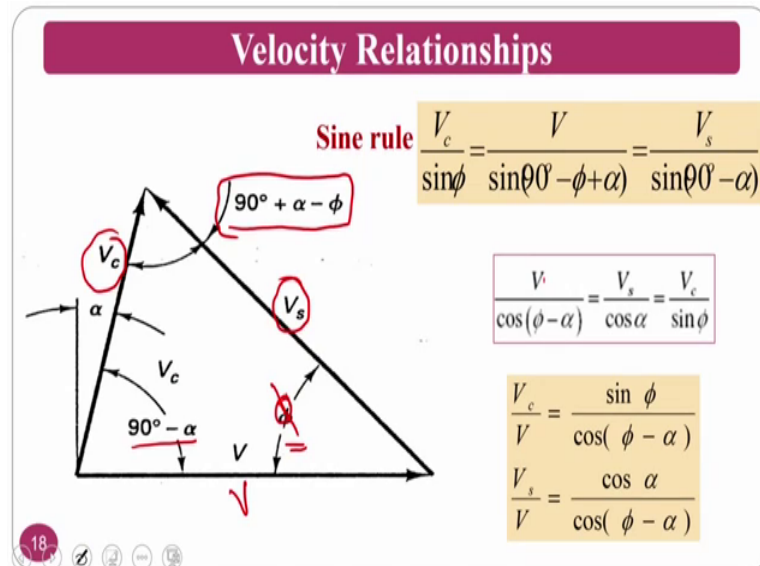
So, the bottom line of this slide what I want to say is the chip velocity that is nothing, but the V_c ; this V_c along the tool rake vector sum of the cutting velocity and shear velocity. This is the cutting velocity and this is the shearing velocity; what they say is sum of, sum of V_s is equal to V_c .

From the cutting, if you see from the geometry of the machining we have already seen the angles. This is shearing angle and we know this is 90 minus because this is a rake; if I see the rake angle for that purpose I have to draw like this is α . So, this my 90

degrees; so, 90 minus alpha; so, this is if I know this is 90 minus alpha this is phi now I can calculate this as a 90 plus alpha minus phi.

So, this is how we have divided to that one.

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So, if you can see clearly for the for understanding purpose; this is the phi, this is 90 minus alpha, this is 90 plus, this phi minus alpha minus phi ok.

So, from the sin rule what we can understand is V the side of this one to the opposite sin of that angle that is nothing, but V_c by sin phi and V that is nothing, but this one cutting velocity by sin 90 minus phi plus alpha. So, another one is V_s that is nothing, but V_s by 90 minus alpha sin 90 minus alpha.

So, it gives raise to V by cos phi minus alpha because 90 is there so, here cos there and here also 90 minus alpha is there. So, V_s by cos alpha equal to V_c by sin alpha sin phi; this is the relation. And from the relations you can calculate in terms of this V_c by; normally V_c by V some people in some textbooks you can also see is say as chip thickness, it is correlated to chip thickness also like chip reduction ratio or chip thickness ratio they can be calculated like this also.

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Shear strain

The **shear strain** (i.e. deformation relative to original size) that the material undergoes can be expressed as

$$\gamma = \frac{AB}{OC} = \frac{AO}{OC} + \frac{OB}{OC} \Rightarrow \gamma = \cot \phi + \tan(\phi - \alpha) = \gamma$$

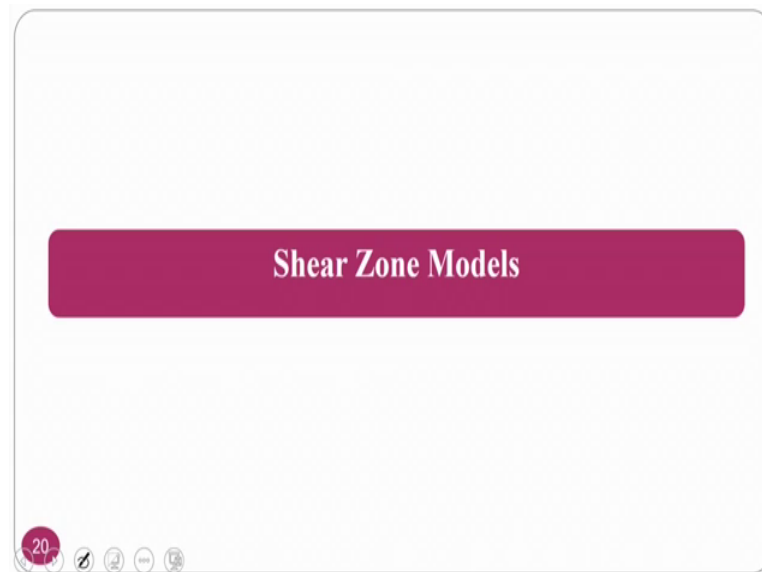
Now, we are going to calculate what is the shear strain; the shear strain that is the deformation related to the original size; so, that is the material undergoes as explained as possible that is nothing, but the gamma, so gamma is nothing, but AB by OC.

So, this is the triangle A B C and where we know the phi this is the one. So, now, taking the triangle enlarging here now A B C is the triangle; I am just dropping a perpendicular to the AB. So, that I can this angle is phi minus alpha from the geometry just it is the simple geometry you can calculate as we have also seen this is the phi; so, alternate angles.

So, that will be also phi; now we know the angles, this is phi this is phi minus alpha. So, if you see AB by OC is nothing, but my shear strain that is nothing, but AB this is AB AB by OC; this is the what will now AB you divide into O; AO into OB. So, divide by AO by OC plus OB by OC which gives rest to cot phi plus tan phi minus alpha this is the final equation which is equal to gamma that is shear strain. So, this is about the shear strain.

Now, we will move on to the shear zone models.

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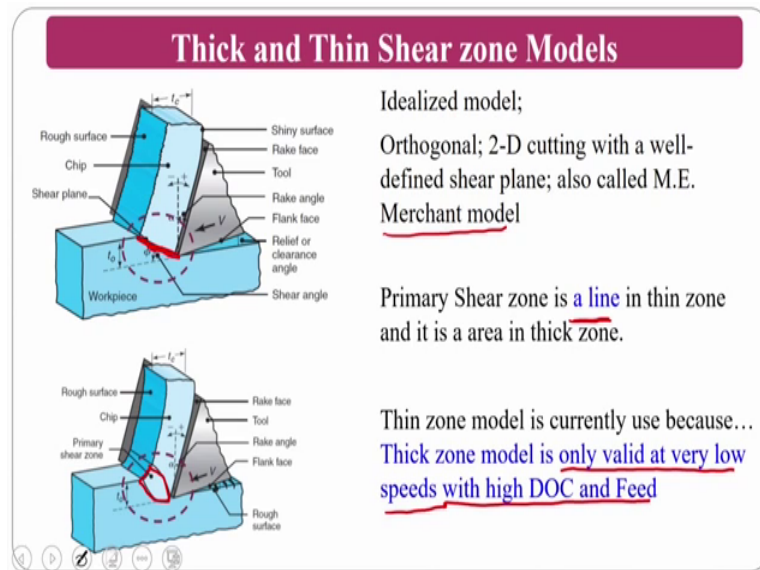


So, what are the shear zone models; so, there are many shear zone models like thin zone model, thick zone models and all those things; however, since this is the introduction course on metal cutting and machining and machining fluids. So, since we are there in the machining; so, we only deal with very primitive model.

For understanding better models you can go through the text books or you can register for the next upcoming course which is on mechanics of machining by some other professor from IIT only. So, they are going to teach you the mechanics of machining where they will take you to the complete into the mathematics spectrum of this machining process.

Anyhow I will come to that slides also and all those things.

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So, there are thin zone models and thick zone models. So, the thin and thick zone models; so, normally idealized model; the orthogonal 2 D cutting with the well defined shear plane also called as a Merchant model this is called thin zone model.

Primary shear zone is a line basically, you can see in a thin zone model this shearing is a line ok, but in the thick zone model thick zone model it is valid for only high depth of cut and feed; than the low speeds this is thick zone models are valid. Normally thick zone model you can see there is a area ok. So, that is the difference between thin zone model and thick zone model.

Practically speaking thick zone model is not considered because if you see it is only valid when there is a low speeds with the high depth of cut and high feed. Normally the cutting operations may be done using these input conditions; however in the general practice basically this may not be used. That is why for a better understanding purpose we are going for the thin zone models.

As I already said you thin zone model let me summarize this thin zone model is a line and the thick zone model is a plane or it is an area. So, that is a difference between a thin zone model and a thick zone model. So, we are going to see only one thin zone model that is where the shear angle relationship that can be drawn.

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Shear angle relationship: Ernst-Merchant Theory

Assumptions:

1. Shear stress is maximum at shear plane and it remains constant
2. Shearing action takes place in the direction in which the required shear energy is minimum ($F_c \cdot V = 0$)

$$F_c = R \cos(\beta - \alpha) \quad \rightarrow 1$$

$$F_s = R \cos(\phi + \beta - \alpha)$$

$$F_c = F_s \left(\frac{\cos(\beta - \alpha)}{\cos(\phi + \beta - \alpha)} \right)$$

Since, $F_s = \frac{\tau b t_0}{\sin \phi} \quad \gamma \cdot A_s$

$$F_c = \frac{\tau b t_0}{\sin \phi} \left(\frac{\cos(\beta - \alpha)}{\cos(\phi + \beta - \alpha)} \right) \quad \rightarrow 2$$

That is Ernst Merchant theory; so, the first assumption is shear stress is maximum at the shear plane and it remains constant that is nothing, but the shear stress is maximum and it is constant ok.

The second important point if which I want to emphasize is the second point that is the shearing action takes place in the direction in which the required shearing energy is minimum that is nothing, but not this one; F_c into V is not the 0, here it is the derivate of F_c into V is should be 0 ok. So, our F_c both are approximately similar.

So, now the F_c and F_s these are all are measured; so, from the F_c F_c is nothing, but $\cos \beta$ minus α ; now F_c is this one. So, F_c you can say now in terms of R ; this is the resultant one; $\cos \beta$ minus α this is adjacent side to the $A D M$ triangle; for that purpose this is this one and now F_s equal to $R \cos \phi$ plus β minus α .

So, now, F_s is this one; so now you can say I can have to include another ϕ ; so, that it will give. Now F_c you are writing in terms of F_s ; so, I can write; now F_s since we know F_s is nothing, but shear stress shear stress is nothing, but the shear force sorry shear force is nothing, but the shear stress multiplied by the area.

So, in that circumstances you can see the shear stress multiplied by area shearing area. So, the shearing area is already given here from that from that we can say; now F_c

equation if you incorporate this equation in the first equation; equation 1, what will happen? You will get this equation 2 ok.

As from the point number 2 or the assumption number 2; the shearing action takes place in the direction in which required shearing energy is minimum ok.

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Shear angle relationship: Ernst-Merchant Theory

Shearing action takes place in the direction in which the required shear energy is minimum So, derivative of shearing energy along the shearing direction is zero)

$$\frac{d(F_c)}{d\phi} = \frac{-\tau b t_0 \cos(\beta - \alpha)}{\sin \phi} \left(\frac{\cos \phi \cdot \cos(\phi + \beta - \alpha) - \sin \phi \cdot \sin(\phi + \beta - \alpha)}{(\sin \phi \cdot \cos(\phi + \beta - \alpha))^2} \right) = 0$$

$$\cos \phi \cdot \cos(\phi + \beta - \alpha) - \sin \phi \cdot \sin(\phi + \beta - \alpha) = 0$$

$$\cos(\phi + \phi + \beta - \alpha) = 0 = \frac{\pi}{2}$$

$$2\phi + \beta - \alpha = \frac{\pi}{2}$$

$$2\phi = \frac{\pi}{2} - (\beta - \alpha) = \frac{\pi}{4} - \frac{1}{2}(\beta - \alpha)$$

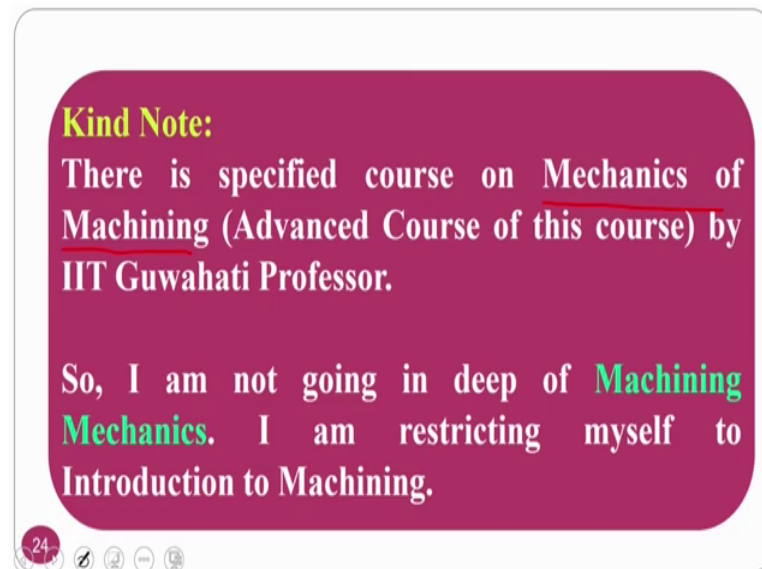
Shear angle (ϕ) = $\frac{\pi}{4} - \frac{1}{2}(\beta - \alpha)$... (Ernst - Merchant Shear angle relation)

From this point; if you understand that is nothing, but shearing action. So, the derivative of shearing energy along the shearing direction should be 0; that means, that derivative of F_c with respect to the shearing angle should be 0 ok. Now you saw if you do the derivation of this one, you will get a this is the big equation that you can see here which you will get. From that you just say is equivalent to 0; from that from the cos equation you will get this cos phi plus phi plus beta minus alpha equal to 0.

Normally when cos angle is at certain angle what angle it is 0? Normally phi by 2 so; that means, that 2 phi plus beta minus alpha equal to pi by 2; so, cos. So, now, 2 phi equal to plus alpha minus beta or if you take the minus outside what will happen pi by 2 minus beta minus alpha V; this 2 if you are sending this direction. So, it will gives raise to pi by 4 minus 1 by 2 beta minus alpha. So, that is what the shearing angle relation is here ok.

So, this is about the one model only; now the question is why I am I am not touching many models as I already said that there is a separate course delivered in the MOOCS on just I want to give the glimpse.

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Because though there is a specified course on mechanics of machining which is a advanced course of pertaining to this course by IIT Guwahati professor.

So, you will see in this semester in this one or the next one you may be ok. So, I am not going in deep mechanics that is not going into deep of mathematics dealing with all these things. So, I am restricted to myself because this is the introductory course to the machining.

So, I will deal only certain part of the machining as well as the machining fluids ok. So, if at all you are interested to go into the deep mathematics, the conversion of the angles NRA system; some of the things that I have not discussed is like NRA system, MRA system, angle conversions.

At the same time thick zone models, thin zone models, what is shear deformation and what is the severe plastic deformation and all those things. And mathematically other professor is going to deal; so, you can register to that one that course is called mechanics of machining.

That is courtesy to IIT Guwahati; so, IIT Guwahati centre for education is going to bring you this course in the upcoming semester.

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Summary on Forces in Machining

- Cutting forces in machining process ✓
- Cutting force measurement ✓
- Merchant Circle to calculate various forces $F_c, F_t \rightarrow F_s, F_N$ ✓
- Coefficient of friction, shear stress, normal stress etc
- Velocity relationships ✓
- Shear strain
- Thin zone models ✓

So, summary of this is what the cutting forces in the machining process is done. The cutting force measurement how using dynamometer and all those things and the Merchant circle to the calculate various forces.

We know the F_c and F_t from there we have calculated F_s ; F_N and the normal to the shear force and all those things coefficient of friction that is we have calculated shear stress, normal shear stress and shear rate also we have calculate that is shear strain rate. The velocity relationships that we have and thin zone models we have done even though the thick zone models are there, which you can see in the upcoming one.