Fundamentals of Nuclear Power Generation
Dr. Dipankar N. Basu
Department of Mechanical Engineering
Indian institute of Technology, Guwahati

Lecture – 09
Neutron multiplication factor

Hello everyone. Welcome back to our MOOC course on the topic of fundamentals of nuclear power generation. And today we are going to start our 4th module, which is on chain reactions in reactors. As you can remember in the previous module we discussed about the fusion reactions, which is from coordination point of view probably the most important interaction that we can have between a neutron and nucleus. And whenever a nucleus is absorbing one neutron, then it can lead to either a non-fission capture or a fission reactions, and accordingly to their probability we generally classify materials into different categories. The one of our interest is a fissile material, which can absorb a thermal neutron and can go for fission.

But if you remember the definition of a fissile material, actually just getting fission by thermal neutron was not the only criteria; rather the definition of fissile material requires 2 criterion to be satisfied. The first is it can or it should undergo fission reaction after observing a thermal neutron. But the second one is that it should also sustain a chain reaction; that means, after absorbing one neutron it undergoes a fission, and after observing a thermal neutron; that is, it can undergo fission, but if nothing happens afterwards then also you should not call that a fissile material. Like a fertile materials are those which does not satisfy either of the criterion. Like, it can not undergo fission through the it can not undergo fission by absorbing thermal neutrons generally, and even if it can undergo fission by absorbing a thermal neutron. It can not sustain a chain reaction following that; that means, once one reaction is done that is it nothing happens afterwards.

So, the chain reaction is a very important one to have from reactor point of view that which of course, as the term suggests.

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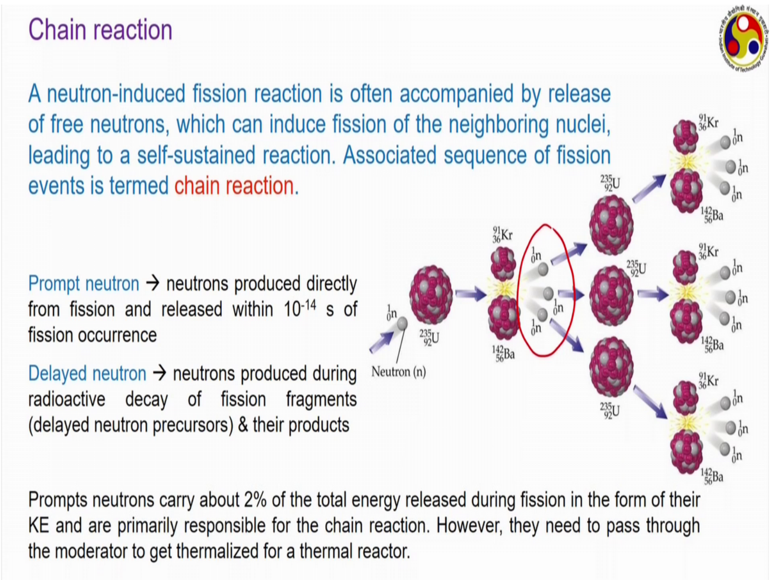

And I think most of you have the idea, it is quite analogous to the chemical chain reaction that we can have, chain reaction refers to the continuous fission reaction; that is, once suppose you have some quantity of well you have several fission shell material nuclei given to you.

Now, you have a beam of neutron, say thermal neutrons and you strike the nucleus with that neutron. Now once a neutron is captured by a nucleus and it undergoes a fission reaction, then that fission reaction is associated with the parent nucleus being spitted into 2 daughters. And also, you will always find that it is emitting neutrons. The number of neutrons emitted during a fission reaction can vary it can be anything between one to 7, but most commonly it is 2 or 3.

Now, those 2 or 3 neutrons that has been released by the fission reaction, if they are given proper condition, then they can also strike another nucleus, and that nucleus can also undergo fission reaction; that is what we call as chain reaction. Just like shown in this diagram like to start with here one uranium 235 isotope is being struck by this particular neutron which is a thermal neutron, let us assume this to a thermal neutron now uranium 235 is has a very high fission cross section, when it is subjected to thermal neutrons. So, accordingly it is very likely to undergo fission, by producing 2 isotopes, 2 daughter isotopes; that is and they are generally krypton or barium it can be anything else, but our interest here is these 3 neutrons which are just got produced because of this fission.

This 3 neutrons if we can allow proper condition to them they should also strike 3 different nucleus, and each of those nucleus will again produce their common daughters or their daughters. And each subsequent reaction there to a or again likely to produce 2 or 3 more neutrons; that means, just in 2 steps you can see here we have started with just a single neutron. And after 2 steps here we are having 9 neutrons to deal with. And these 9 neutrons again are likely to strike 9 more nucleus and those nucleus again will undergo fission reaction to produce 9 into 3 that is 27 neutrons, this is what we call a chain reaction; however, this is a very, very ideal diagram that is shown to you, because the neutrons that has been emitted or produced by a fission reaction, it is not that all of them will induce fission to some subsequent nucleus, some of them will do some of them would not it is also possible that none of them also undergo none of them are able to induce any kind of fission reaction.

Because after getting emitted from this fission reaction here, and before striking the nucleus here in between this portion the neutron can undergo several phenomenon; which we shall of course, be discussing in later sections or later part of this lecture itself. But probably one you can immediately recognize at this moment. The neutrons which are being produced by the fission nutri action here, that is a this neutrons that we are talking about, there innovatively they are fast neutrons. And here I would like to also mention neutrons depending upon when they are getting produced we can classify into 2 categories. One is prompt neutron which refers to this particular neutrons; which are direct product of a fission reaction and they are generally released within 10 to the power minus 14 seconds of fission occurrence. Basically, that is the timescale 10 to power minus 14 second which is associated fission reaction. And so, this neutrons this prompt neutrons they are immediately available as soon as we have a fission reaction.

But there can be some further neutrons that can come later on into the sequence. Like here the products of fission that you can see after this first type of fission reaction you have this krypton 91 and barium 142 as the products as per this diagram. Generally, both of them are radioactive and they will undergo some more fissions, and accordingly they will produce I am sorry they will not undergo fission rather they will go through radioactive decay of their own forming their own radioactive chains till the formation or appearance of some kind of stable nucleus.

Now, during the radioactive decay of this fission fragments, we can also have some further neutrons produced, and those neutrons are called delayed neutrons. It is not that every fission fragment decay will lead to the formation of delayed neutrons, but some of the fission fragments may be quite rich in terms of neutron and they can emit neutrons. So, corresponding fission fragments which are neutron rich and therefore, are capable of emitting neutrons are called delayed neutron precursors, we shall be discussing about that in a later chapter. But the neutrons which are emitted accordingly they are delayed neutrons. Now while prompt neutrons are emitted which is integral minus 14 seconds of fission occurring, when the delayed neutrons will be released that is a difficult to say because that depends upon the decay rate of these fission fragments, that can be released immediately we in may be just a few nanoseconds or even less times, or it may require minutes or hours or sometimes even longer for this delayed neutrons to appear following any particular fission.

Now, the prompt neutrons which are produced following the fission reactions or immediately after a fission reaction they are innovatively very high energy neutrons. Because the prompt neutrons carry about 2 percent of the total energy, energy emitted during the fission reactions. And this energy they acquire in form of their own kinetic energy therefore, they are always ultra-fast velocity levels, and they are having high amount of kinetic energy. But as we have seen in the previous module that the kinetic energy when kinetic energy of a neutron is high it is absorption cross section is extremely low. It is must it must slow down so that it can reach to somewhere near the thermal neutron level so that it can attain some significant value of this absorption cross section or fission cross section so that it can induce fission to a nucleus.

That means this group of 3 neutrons which has just been produced following the fission of the first uranium 235 nucleus. They are prompt neutrons and therefore, they are also fast neutrons; that means, they must undergo through the moderator, through the moderation process where because of repeated elastic scattering they can get thermalized, and then only they will be able to induce fission in this subsequent nucleus like here. And when it is undergoing in this moderation process, it is there is every possibility that they may get absorbed by the moderator itself or they may get absorbed by some other elements, that may be present inside the nucleus inside the nuclear reactor core, generally a nuclear reactor core can have several kinds of elements some of them used for controlling the rate of reaction, some of them just as the fission products.

Like the fission products that are getting produced because of the radioactive decay of the fission fragments some of them may also eat up some of the neutrons there by the it is not that all these 3 neutrons produced here will be available to cause subsequent fission. Rather only some of them or maybe a very, very small number that may be available for a subsequent fission.

That means chain reaction well while it may seem that it is a quite natural process, but actually a nuclear reactor point of view a chain reaction somehow, we have to ensure the appearance of chain reactions because only because of the chain reaction we can sustain the energy production inside reactor. Rather if the chain reaction is not able to sustain then after passing this beam of neutron, we can get one fission reaction done and we are able to harness corresponding energy, but that is it if there is no for chain reaction then there is no further fission. So, no further energy release somehow, we have to sustain this chain reaction. And we have to sustain this chain reaction by then what if we want to sustain the chain reaction then what we have to ensure. We must ensure that this neutrons which are getting produced following a fission reaction at least some of them are capable of inducing fission to some other fission (Refer Time: 11:16) nuclei.

And that is quantified in terms of something called an effective multiplication factor or just multiplication factor in order to ensure a stable sustained chain reaction. We must have at least one maybe more number of neutrons produced during a fission to and to cause another fission reaction. And accordingly, this effective multiplication factor is defined as the ratio of number of neutrons.

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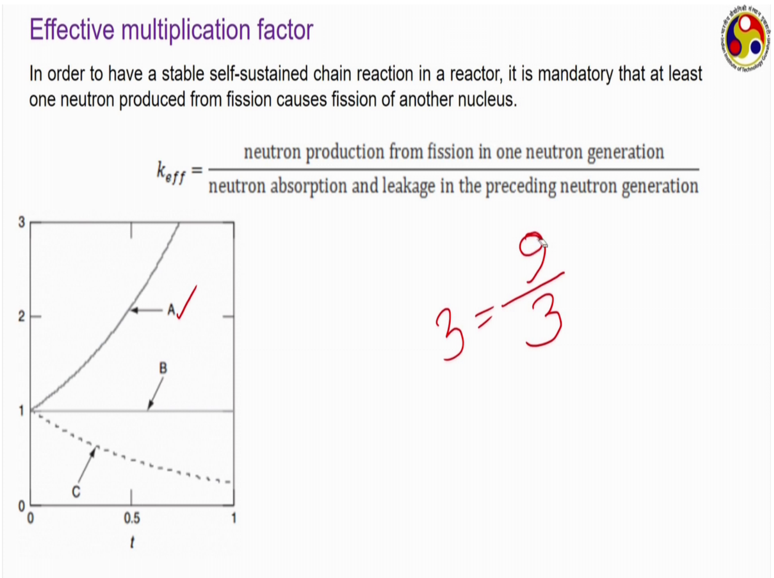

Which are produced from fission of one neutron generation divided by the number of neutrons absorbed and leaking in the preceding neutron generation. Neutron generation what it refers we shall be seeing shortly, but you can think about this like if you go back to the previous diagram, here we have this 3 neutrons which are getting produced from one fission, and they are being fast neutrons they are immediately absorbed by the reactor core. Absorb or I should not use the term absorb they are immediately available to the reactor core for any subsequent action, which may be fission may be non-fission capture maybe just moderation etcetera or maybe just leaking out of the core. So, this 3 neutrons that are available for the reactor to cause any subsequent steps.

And then some of them they will go through the moderation process they will go through different other activities. And accordingly, some of them will be kept you know will be able to induce some further fission, and say out of these 3 only this one is capable of inducing fission other 2 are either getting absorbed in different steps of the moderation process or leaking out of this. So, the out of these 3 only one is capable of inducing another fission of this particular nucleus, and because of that it is again producing 3 more neutrons. So, this is what is termed as one generation. That is, this is one generation a starting from here and that is finishing before the next set of neutron appears following a fission. Now at the beginning of this generation we have this 3 neutrons available we have this 3 neutrons available for the reactor to act upon, and at the end of generation once this new generation starts that is also starting with this 3 neutrons. Or maybe some other number depending upon exactly how many number of a neutrons are participating in the subsequent fission. So, the ratio of the neutrons which are available in this generation, and the neutrons which are available in this generation that is what we term as the effective multiplication factor. Look at this diagram here horizontal axis refers to time and vertical axis represents this effective multiplication factor.

Now, look at case a this one, this refers to a situation when this effective multiplication factor is continuously increasing with time. What does that refer? That means, the number of neutrons produced in the subsequent generation is more compared to the number of neutrons which are available to the previous generation, like referring to the previous diagram again where 3 neutrons were available at the beginning of one generation; and say if all 3 of them were capable of inducing fission, then in the next step we shall be having 9 neutrons available or at the beginning of the next generation we shall be having a 9 neutrons available.

So, corresponding effective multiplication factor we are getting to be equal to 3. And this situation is shown by this case a here; that is, the effective multiplication factor is greater than 1 and therefore, it keeps on increasing with time, and hence the total number of neutrons that is available inside the reactor code that also keeps on increasing with time. Now what does that imply? If the number of neutrons available inside a reactor core that keeps on increasing with time, then we can expect the rate of fission also to go up, and hence that will keep on increasing the amount of energy released during fission. So, the implication of having an effective multiplication factor value greater than 1 is that the rate of energy released with time keeps on increasing continuously.

Look at case C now, it is the other extreme. Here this effective multiplier factor is less than 1; that means, again referring to the previous example where 3 neutrons are available at the beginning of one generation.

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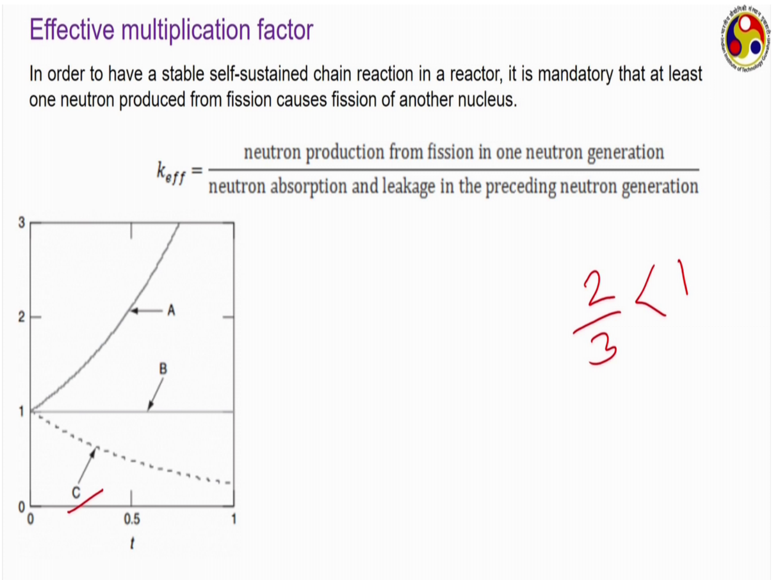

And in the next generation see we have only 2 neutrons available. Then effective multiplication factor is 2 by 3 which is of course, 0.67; that is less than 1. So, while 3 neutrons are available for getting captured or acted upon by the reactor in the next generation we have only 2 available. So, we can always expect in the subsequent generation the number of neutrons will be less than 2, because there are much lesser probability of any further fission happening. And so, the rate of fission reaction keeps on coming down accordingly the rate of energy release from the reactor also continues to come down.

The third case the case B shown in this diagram it is a very interesting one. Here the multiplication factor is equal to 1, and that is maintained constant with respect to time. Then what is the significance of this it refers that whatever may be the number of neutron that is getting absorbed, or that is available to a reactor at the beginning of a generation, and the beginning of the next generation we have exactly the same number of neutrons available. So, the total number of neutrons available inside the reactor, that is being maintained constant over the over a long period of time, and hence that implication has to be a constant rate of fission, and hence a constant rate of energy release a very ideal situation.

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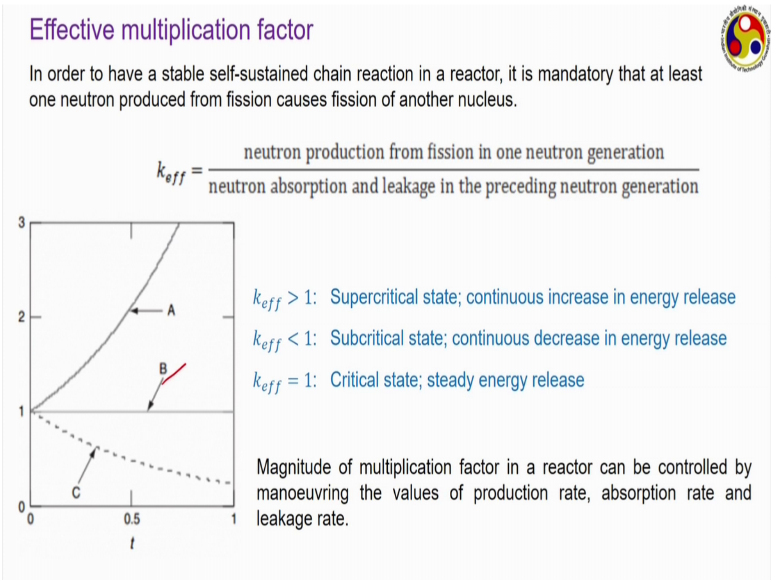

In terms of reactor nomenclature point of view, this case B is called critical. It is called a reactor has gone critical here I am summering the 3 situations first is case A which is effective multiplication greater than 1. This refers to a supercritical state a supercritical reactor which refers to a continuous increase in the energy release case C is the effective multiplication factor less than 1 which is referred to as a subcritical reactor. And it is associated with a continuous decline in the energy release. And third case that is our case B is the critical state, where we have a steady rate of energy release, which is an ideal situation which generally we would like to maintain in a commercial power plant commercial nuclear power plant that is.

So, magnitude of multiplication factor in the reactor can be controlled by manoeuvring the values of different factors, generally the rate of production rate of absorption and also the leakage rate. Because we can always write the change in the number of neutrons or rate of change in a number of neutrons inside the reactor should be equal to the rate of production, minus the rate of absorption minus the rate of leakage, thereby balancing these 3 quantities we can maintain, or we can manoeuvring the value of this effective multiplication factor inside the core.

Now, can you guess out of this 3 which is the most ideal situation; that has to be k effective equal to 1, if we can maintain a effective multiplication factor equal to 1; that means, we are in a steady state situation the number of neutrons is maintained constant across all the generations. And hence we are able to produce a fixed amount of power continuously over a time period time, but that does not mean that other 2 are not at all required. Like suppose the system is producing a fixed amount of power, now there is a sudden increase in the demand. Then we have to increase the rate of energy release, then what we should like to do we would like to control the reactor such that the effective multiplication factor at least for some period of time becomes greater than 1. So, that the rate of reactions and rate of energy lease increases, and once the rate of energy release matches the new demand, then we can again control the reactor related parameter such that, this situation reverse back from this k equal to 1; that is, it is able to maintain this news at newly attained steady state.

Then at what situation we may desire to be the k effective less than 1. Of course, k effective less than 1 is the other situation when we have to reduce the energy production, then we would like to reduce the effective multiplication factor by using some controls. So that the rate of energy is decreases. And once we are able to attain the reduced energy level, then we can again go back to this k effective equal to 1. There can be other situations also like you have to shut down a reactor a running reactor which is running for some period of time now we have to shut it down. Then you have to maintain this effective multiplication factor less than 1. So, that the rate of energy release dies down and finally, it becomes subcritical and finally, after some reasonable period of time the multiplication factor becomes 0 or almost equal to 0 and there is no further reaction inside.

Similarly, just or I should say on the contrary, we would like to have this situation when you are starting a fresh reactor, or something that was shut down for quite some period of time. Here there was no generation to start with we have to provide a higher effective multiplication factor. So, that the rate of reaction keeps on increasing rate of energy release keeps on increasing till we attain some kind of rated capacity.

So, if you are asked about what is the most important parameter from reactor control point of view, you have to mention the name of this effective multiplication factor. Because just by controlling the value of this one, we can have a critical reactor a subcritical reactor or a supercritical reactor. And we can control the rate of energy release of course, it is not that the controlling this effective multiplication factor is a just a very easy one, there are several factors associated with this, but it can be done.

Also, another interesting situation is presented when this supercritical reactor we are dealing with like this case A. You can clearly see the rate of energy release is continuously increasing. What that can be synonymous to? That has to be a nuclear weapon or a nuclear bomb, a nuclear weapon is something where we do not have any control on the rate of reaction or we may have some control, but somehow, we make it highly supercritical. So, that the rate of reaction keeps on increasing, and it is able to release large amount of energy within a very short duration time. Now we come to something called a neutron life cycle. Life cycle deals with the life history of a neutron or the events through which a neutron undergoes from the beginning of one generation to the maybe you can say just before the beginning of the next generation. That is what we refer to as a life cycle.

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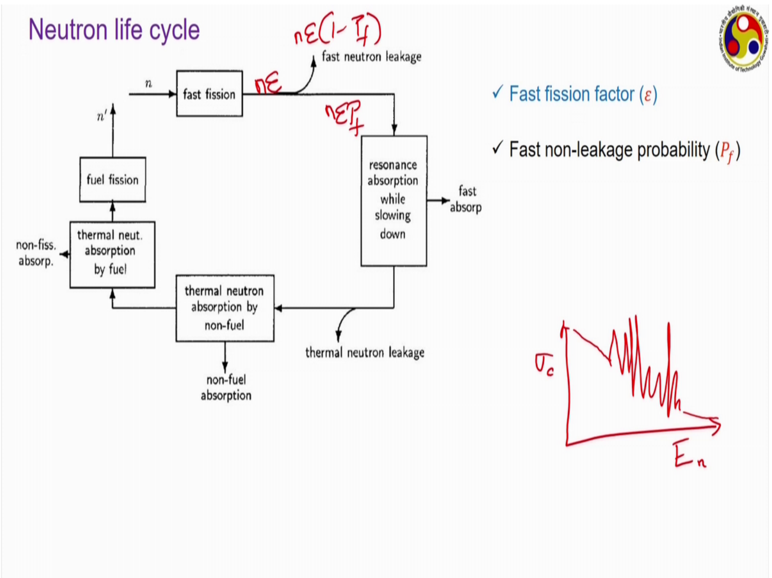

Here we have n number of fast neutrons available at the beginning of a lifes a generation. So, this neutrons may have been produced by a fission reaction or by some other means. So, they are always fast neutrons. Then what they are most likely to undergo to start with. Of course, fast fission probability is quite low or the fission cross section for common isotopes like uranium 235 or 238 are very, very low, but still small amount of fast fission can always happen. And so, some among some of these neutrons will participate in a fast fission process thereby producing some more neutrons adding that to this. And that is characterized by something called the fast fission factor. This fast fission factor refers to the ratio of the number of neutrons after the fast fission divided by the number of neutrons before the fast fission. Then the value of this fast fission factor should be greater than 1, and less than 1 it has to be greater than 1 because whatever will be the number of neutrons initially available total number will increase may be by very small fraction, but it will increase because of the neutrons fresh neutrons which has been released during this fast fission. So, at the end of this fast fission process, you have n into epsilon number of neutrons available in the framework.

Then some of this fast neutrons may leak out through the reactor. So, next factor that we have to introduce is called fast fission or fast non-leakage probability or fast neutron non-leakage probability. Remember the term here we are defining this as a non-leakage probability; that means, we are trying to capture the number of neutrons which will survive this leakage of fast neutrons, then how many neutrons will leak out from as a first neutron level? And how many neutrons will survive? Of course, n into epsilon was the total number of neutrons available this fast neutron non-leakage probability is defined as number of neutrons surviving this leakage divided by the total number of fast neutrons available. So, number of neutrons surviving this leakage has to be n epsilon into p F and n epsilon into 1 minus p F is the fraction; that is, leaking out from the reactor core as a fast neutron or as fast neutrons.

What is the next step? These are being fast neutrons their fission cross section general is quite low like in they can induce fast fission, but their probability is very, very small. So, they go through the moderation process. They go through leaving the moderation process they are energy keeps on coming down. Now I am just very roughly drawing the curve that we have discussed earlier the energy level here, and the absorption or capture cross section here. If you remember the curve for low energy we have a 1 by E region where the capture cross section continuously decreases for very high energy also we have a decreasing trend that is with the increasing neutron energy the capture cross section or absorption cross section comes down, but in between we have a resonance region. In this resonance region you can see lots of peaks. Something like this that is at certain values of this energy, the absorption cross section can be extremely high. These are of course, non-fission capture; that is observing these neutrons the corresponding element will only create another new isotope of it is own and may release a large amount of gamma radiation.

So, particularly uranium being the most common fuel we generally use uranium 235 and 238 has a mixture in a reactor in a thermal reactor. Now uranium 235 has a very high thermal fission cross section, but generally a very low fast fission cross section. Whereas, uranium 235 is a fertile material; that means, it requires only fast neutrons to cause any kind of reaction. Therefore, in this fast fission stage both uranium 235 and 238 can participate. Or rather any fissionable material can participate, but generally the cross section is very low.

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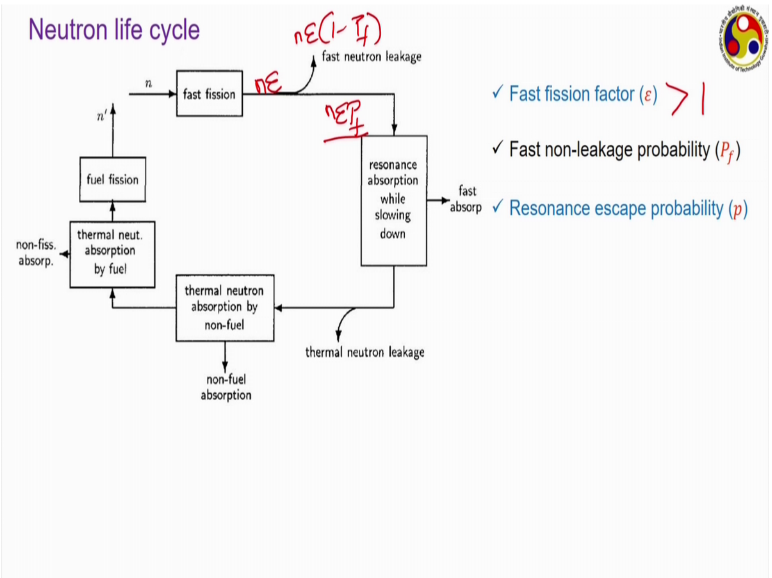

Therefore, this epsilon is greater than 1, but just only greater than 1. And when it is undergoing this resonance situation now, the it is every there is a large possibility that some of them may get absorbed or may get captured during this resonance peaks. Particularly as was shown in the previous module uranium 238 has very high resonance absorption cross section, there some of their peaks can be 100 or maybe 1000 times larger compared to the base values. So, this resonance absorption is something which will consume a good amount of this fast neutrons which is n epsilon p f, and which while it is passing through this moderator and also a fuel.

So, accordingly we define something called the resonance escape probability, resonance escape probability is defined as the amount of neutrons which are able to survive across the resonance divided by total number of neutrons which are coming to get moderated in the form of fast neutrons. So, the number of neutron coming is this one only, and the ratio of number of neutrons available after the resonance section divided by the neutrons available before this is to call this is called the resonance escape probability. The value of this resonance escape probability is strongly depend upon the percentage of uranium 238 which is which can be present there and also may depend upon the moderator. Now this if the resonance escape probability is defined by small p, then how can you compute the total number of neutrons available after this resonance absorption.

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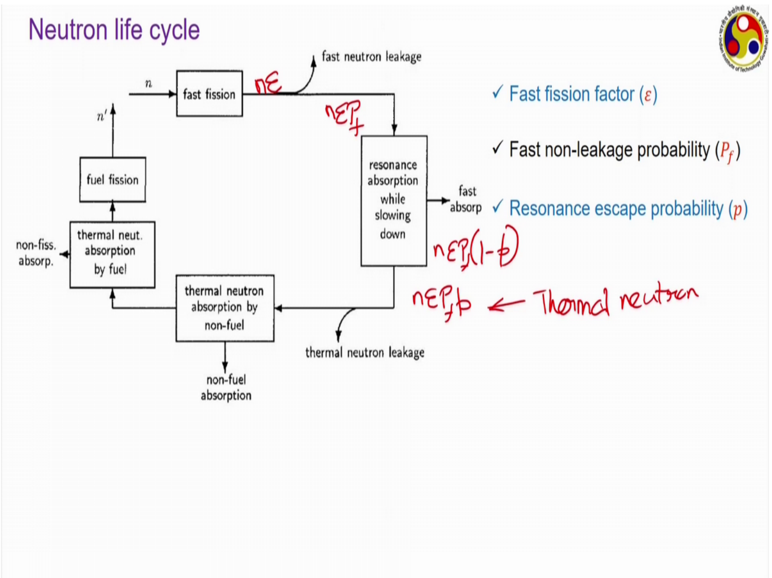

N epsilon into p F was the number of neutrons available before this, if you multiply it with small p which is this resonance escape probability we have a number of neutrons available after resonance. And what should be the nature of them? They has to be thermal neutrons, because his resonance obsession happens when it is energy is coming down through that intermediate level. And number of neutrons which are getting absorbed during resonance absorption that has to be n epsilon p F into 1 minus small p. These are number of neutrons which are getting absorbed during resonance and n epsilon p F into small p is the number of neutrons which are available at the end of resonance. And as they have passed through the resonance region so, they must be thermal neutron.

So, we have now thermal neutrons available after this resonance absorption step is over. And now once this resonance absorption is done, we have this n epsilon p F into small p number of thermal neutrons available, which you are ready to get absorbed by the fuel.

But before that some of the thermal neutrons can also leak out of the reactor similar to the first neutron.

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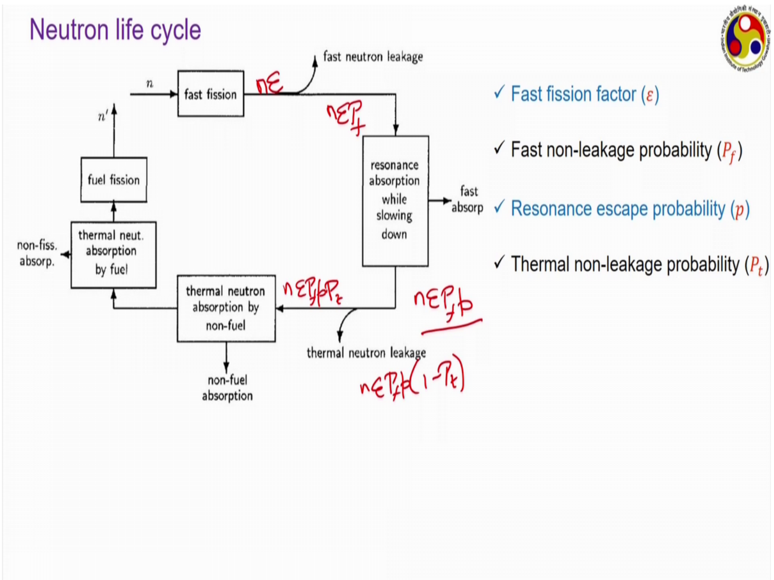

And accordingly, we can define a thermal neutron or leakage probability or just thermal non-leakage probability. These thermal non-leakage probability refers to the number of neutrons which are able to survive this leakage that is number of neutrons available here, divided by the number of neutrons available after the resonance absorption zone is done. So, it is basically the ratio of the number of neutrons at this particular point and number of neutrons available at this particular point. Then how many neutrons will be available after this thermal neutron leakage is done. We have n epsilon p F into small p number of neutrons available as a thermal neutron. So, out of that a portion will get leaked out that portion is into 1 minus p t because we have defined here as non-leakage probability quite similar to the fast neutrons. And this total number of neutrons available here, available after this leakage is this now this. Thermal neutrons of course, they also have to get absorbed in the reactor core they being sorry, I should have kept this one maybe let me keep this. Now this thermal neutron will get absorbed in the fuel, but along with fuel there can be some other kind of materials also. Like, we can some examples will be given later on, but along with fuel inside the reactor core we may have certain other materials also, which again can participate or contributing in a non-fission capture kind of reaction. That is characterised by this thermal utilization factor.

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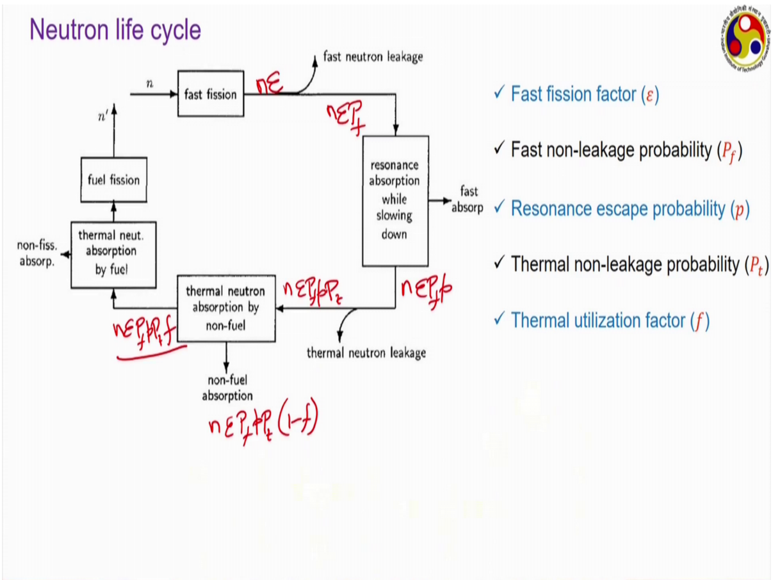

Thermal utilization factor refers to the number of neutrons available, number of neutrons absorbed by the fuel divided by the total number of neutrons absorbed in the core. Remember, thermal utilization factor is number of neutrons absorbed in the fuel divided by total number of neutrons absorbed in the core. So, in the core fuel itself is present, but there can be other components also like the moderator and a few examples I shall be giving shortly. So, number of neutrons that are being absorbed in the fuel, thermal neutrons that is that should be n epsilon p F small p into p t into this f. These are the number of neutrons that has been captured by the fuel, and number of neutrons that has been absorbed by other materials like moderator etcetera; that is, the 1 minus F fraction of whatever is coming in. So now, this is the number which has got absorbed in the fuel. So, they will participate in the reaction, and some of them of course, will get captured some of them may undergo a fission non-fission capture reaction, like whatever may get captured in the you if you think about the earlier examples that are given uranium 235 whenever it captures a neutron there is 85 percent probability of having a fission reaction. But there is also 15 percent probability of having a non-fission capture. So, it is not that this exactly this number of fission reactions we are going to have rather we are going to have only a percentage of that like maybe 85 percent of this, in case of uranium 235 and each fission reaction will produce a variable number of neutrons which can also range between one to 7, but on average between 2 to 3 and finally, after the fission is done here again we are going to get back a good number of fast neutrons. Fast neutrons because these neutrons which will be appearing here they are all products of this new fission.

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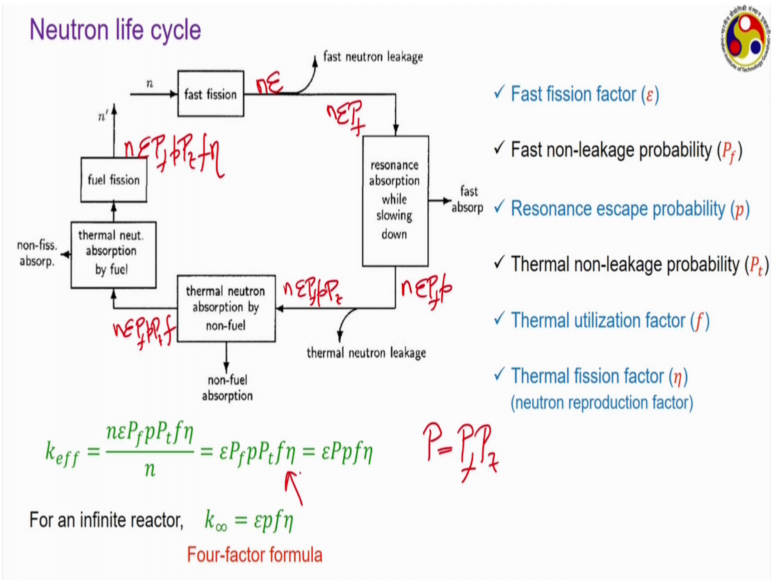

And it is characterized by this thermal fission factor also called neutron reproduction factor or thermal reproduction factor. It is a ratio of fast neutrons produced by fission here, and the number of thermal neutrons that is absorbed by the fuel that is this quantity.

So, the total thermal fission factor how many neutrons will be available after this. This is the number of neutrons that has got absorbed. So, total number of neutrons which will be available here, these are the number of neutrons that has got absorbed in the fuel. And eta fraction of that let me write it properly eta is the fraction, total number of neutrons that will get produced with this. So, these are the number of neutrons which will be available at the end of this generation, and at the beginning of the new generation. This completes one neutron life cycle.

And now if we want to calculate or define the effective multiplication factor here in terms of this 6 terminologies or 6 definitions that we have provided, then effective multiplication factor of course, is the number of neutrons available at the beginning of a generation, divided by the number of neutrons available at the beginning of the previous generation. So, at the beginning of this new generation, these are the number of neutrons available which is in the numerator. And at the beginning of the this previous generation these are the number of neutrons that was available, which we have in the denominator.

So, accordingly we get this effective multiplication factor as the product of all this 6. And quite occasionally these 2 non-leakage probabilities are coupled into one capital P which is the product of this first non-leakage probability, and the thermal non-leakage probability. So, this 6 factors or generally 4 factors associated with the reactor characteristics, and or the fuel and other characteristics plus 2 leakage non-leakage probabilities to leakage general characteristics. Their product defines the effective multiplication factor.

A simplified or idealized version of this can be defined for an infinite reactor infinite reactor means something that does not have any leakage. It is in finite size and therefore, there is no neighbour and no had to for the neutron to leak through. So, what should be this non-leakage probability for a an infinite reactor? It has to be one because there is no leakage. So, accordingly the effective multiplication factor in that case we substitute this suffix if epsilon as this infinite, it is just a product of this 4 factors; that is, fast fission factor epsilon the resonance escape probability small p the thermal utilisation factor small F and finally, thermal fission factor eta.

This infinite it is called the infinite multiplication factor, it is found to be a product of this factors the 4 factors and according it is very popularly known as this 4-factor formula. Quite occasionally this particular one is term as a 6-factor formula, but that is a less used terminology, but this 4-factor formula is a classical term that we use to characterize any neutron life cycle and also the characteristics of a nuclear reactor in general.

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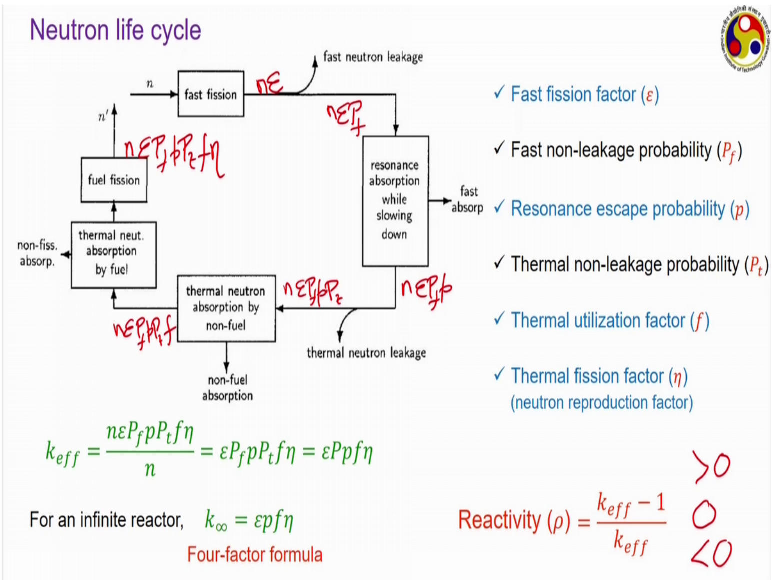

Another term very popularly used that is reactivity, denoted by the symbol rho and it is defined as the k effective minus 1 by k effective. So, why one here can you guess effective multiplication factor equal to 1, what should be the value of this reactivity for a critical reactor. We know that a critical reactor refers to something where the rate of power generation is constant, because effective multiplication factor is equal to 1. So, for a critical reactor, effective multiplication factor has to be 1, and here is the value of reactivity equal to 0 whereas, for a supercritical reactor k effective is greater than 1. So, reactivity is greater than 0, for a subcritical reactor effective multiplication factor is less than 1.

So, reactivity is less than 0. And so, we can visualize the rate of reaction of the nature of the reaction that is going on inside a nuclear reactor, in terms of either the multiplication factor or the reactivity. Here we have an example. But before going to the example, out of this let us forget the non-leakage probabilities for the let us keep them in picture also. Out of this 6 factors, can you say which one will be greater than 1 and which one will be less than 1? What about the fast fission factor? Because of the fast fission total number of neutrons in the reactor, that should increase because say we have 100 neutrons present here, this small n equal 100. Out of that maybe just one or 2 are participating in the first fission. But every such fast fission will lead 2 new 2 or 3 neutrons getting added to this. So, total number of neutrons has to increase.

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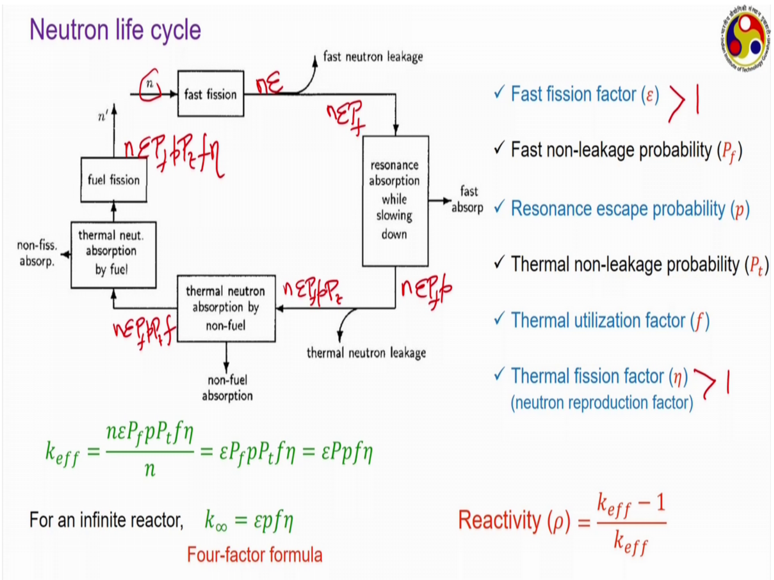

And therefore, this fast fission factor is greater than 1. What about thermal fission factor? Quite similarly it is also greater than 1. Of course, we know that a part of the number of neutrons that is getting absorbed in fuel will participate in non-fission capture. But generally, that fraction is quite low, and every fission reaction will produce a large number of neutrons. So, generally this thermal fission factor is also greater than 1 generally much higher than 1; depending upon the composition of the fuel.

What about the others? What about the 2 non-leakage probabilities? They are being non-leakage probabilities they always refer to the number of neutrons which are surviving the leakage. So, both the non-leakage probabilities has to be less than 1, what about the resonance escape probability?

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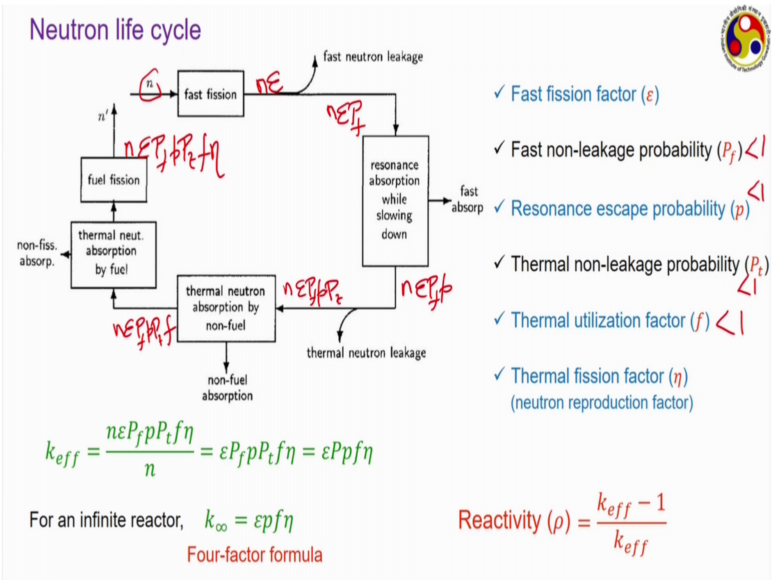

That is the ratio of number of neutrons surviving the resonance absorption, divided by the number of neutrons absorbed or available for resonance absorption. So, that is also less than 1. And finally, the thermal utilization factor is a ratio of number of neutrons absorbed in fuel. And number of neutrons absorb in the core; that is, in fuel moderator plus everything else. So, that also has to be less than 1. So, out of the 6 factors 4 of them are less than 1, but the 2 fission factors are greater than 1.

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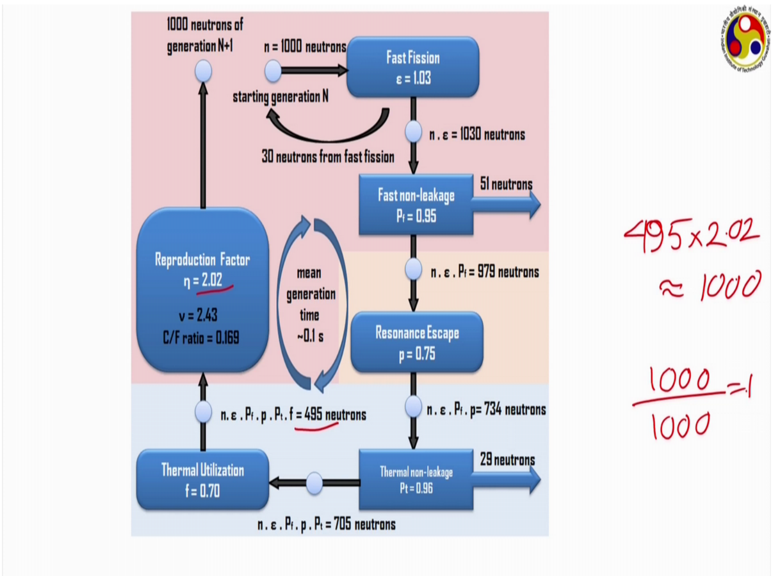

Here is one example. Here we are having small n equal to 1000 we are starting with 1000 neutrons. And first we are having a fast fission of 1.03. So, after the fast fission is done, we are having 1030 number of neutrons available. These neutrons will go through the leakage phenomenon. 30 neutrons are getting generated because the fast fission. Fast fission and so 1030 number of neutrons are available.

Now, let us take a non-leakage probability of 0.95. So, 90 5 percent of this 1030 will survive this, but 5 percent will leak out and that 5 percent is 51 and after surveying this we have 979 number of fast neutrons that are available for moderation. So, this 979 number of neutrons go through the moderation process during which they encounter the resonance absorption. We have resonances capability of 0.75, then number of neutrons available after the resonance or number of neutrons which are able to thermalized will be 75 percent of whatever we started with, and we started with this 979. So, once this resonance absorption step is over, we are having 734 number of thermal neutrons available.

These thermal neutrons some of them may leak out, again we are having a leakage non-leakage probability of 0.96 or 4 percent of this 734 that are leaking out, which are 29 neutrons and 705 number of neutrons are available for participating in the fission or any other thermal capture. So now, we have a thermal utilization factor of 0.7; that means, 30 percent of this 705 neutrons are getting absorbed in the moderator and other elements inside the reactor. But 70 percent are getting absorbed in the fuel. So, 70 percent of this 705, which is 495 that is getting captured in the fuel.

Now, here we have a typical reproduction factor or thermal fission factor of 2.02, which are generally corresponds to a capture to fission ratio 0.069 this diagram actually shown for uranium 235. You may not be remembering, but if you can go back to our previous the slides for the previous module, they does mention that for thermal neutrons uranium 235 has a fission cross section of around 580 something and capture cross section of around 100. Or rather 85 percent probability of fission, and 15 percent probability of capture, and correspondingly we get a capture to fission ratio 0.069. That is about that is a basically the ratio of this fission and capture cross section. Or this or not fission capture rather using fission and capture cross section we can calculate this number.

And each fission reaction is producing on an average 2.43 number of neutrons. So, we are getting a reproduction factor of 2.02, and hence we need total number of neutrons available, after this thermal fission is done that will be equal to the number of neutrons absorbed in fuel that is 495, multiply by the thermal fission factor or reproduction factor that is 2.02 which is approximately 1,000.

So, we started with 1,000 number of neutrons, and we have finished up at or we have reached another generation with again 1,000 neutrons. Then what is the multiplication factor here? Here the effective multiplication factor is number of neutrons at the beginning of this new generation, divided by the number of neutrons available at the beginning of the previous generation, here both are equal and hence the effective multiplication factor is equal to 1; that means, here the all the numbers that are given for this factors they all corresponds to a critical reactor.

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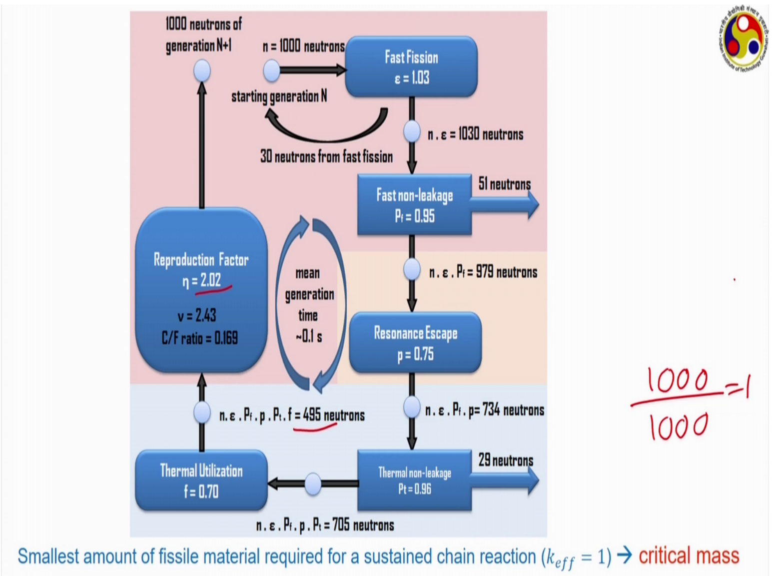

That is not very easy rather several factors that need to be adjusted to get a reactor during the into the critical condition, and the minimum mass of any fissile material, that is required to have a sustained chain reaction; that is to attain this reactivity of 0 or multiplication factor of one is called critical mass. Life of uranium 235 the critical mass is something around of the order of 51 k g something it is a very rough information just to give you some idea if the total mass of the fuel is less than this critical mass, then the reactor will become subcritical, if it is much higher than this critical mass then it will become supercritical. So, balancing the critical mass or the value of the critical mass depends on all these factors.

We have to see how we can control all these factors. First let us discuss about the leakage. Any leakage means the neutrons are going out of the system.

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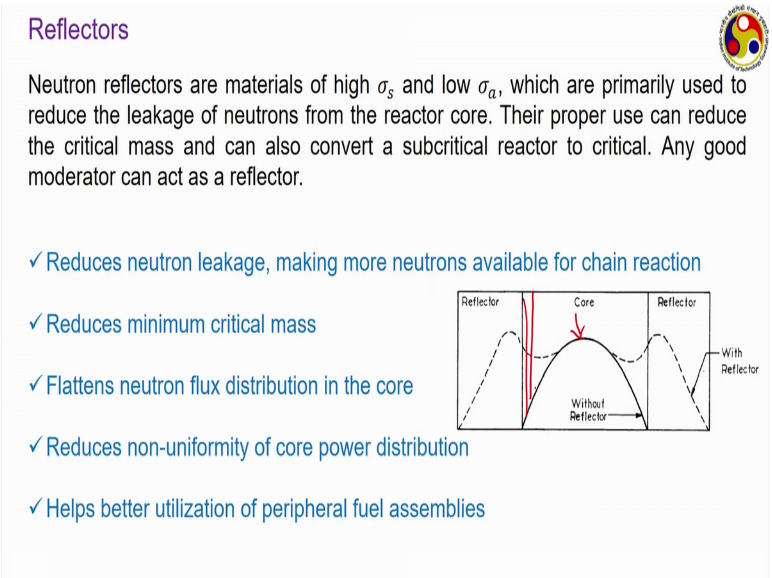

And so, they are loss reflectors are the elements which are used to reduce the neutron leakage, or just as the name suggests to reflect the neutrons back into the reactor core. So, what should be the properties of a reflector? Reflector should be materials which has a high scattering cross section, and also very low absorption cross section. Because if the reflector starts to absorb neutrons on their own. Then definitely that will lead to a net loss. Rather, they should primarily focus on scattering the neutrons back into the reactor core. And therefore, reflectors of the materials which are having high scattering cross section, but low absorption cross section.

Now does this sound familiar? Because in the last module we have discussed about the moderator, which are also likely to have similar properties high absorption sorry, high scattering cross section and low absorption cross section. Therefore, any good moderator material can also act as reflectors. Like, use of a like it was mentioned water normal water is a very good moderator, because it has a very high scattering cross section and reasonably low absorption cross section. So, moderator it is a the water can itself act as both moderator. And reflector and it is quite common for boiling water reactors to immerse the entire core into a pool of water. So, that neutrons will not be able to come out of the reactor whenever it is trying to coming out trying to come out, it will get reflected back by the moderating element or the reflector which is water in that case.

Now, reflector offers several advantages. Firstly, it reduces a neutron leakage making more neutrons available for the chain reaction. And as more neutrons are available for chain reaction. So, the total rate of reaction that keeps on increasing and hence that reduces the total on fuel requirement, hence it corresponds to a lower critical mass or it minimizes the critical mass.

It also another advantage I would not like to describe the details of that, but it flattens the neutron flux distribution inside the core. Normally in a reactor core you will get this kind of hump kind of profile. That is reactor neutron density is high at the centre, and quite low at the edges because from the sides they will neutrons will keep on leaking outside. If we use the reflector, then those neutrons will be reflected back and accordingly we get more or less an uniform kind of profile like shown by the dotted line here. We will get quite uniform profile of neutron. And hence that as the rate of fission reaction of course, depends upon the neutron density here like if the neutron density or neutron distribution is more here, there is more likely to have fission reaction and hence higher energy release.

So, the flatter profile of this neutron flux distribution reduces the non-uniformity of power generation. And also, it utilize helps in utilizing the peripheral fuel assemblies like the fuel assemblies if we allow this hump kind of profile, then the fuel assemblies which are mounted somewhere here, they will receive very low amount of neutrons to have any kind of interaction. And hence they will not get utilized properly. Therefore, there use of reflectors helps nucleation of this peripheral assemblies as well.

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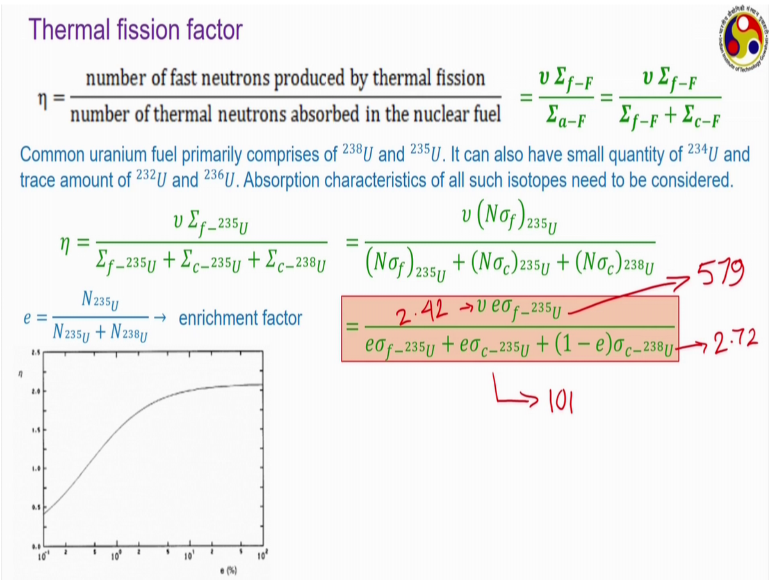

Now, we have to discuss about all this 4 factors. First is the thermal fission factor. Thermal fission factor as has already been defined is equal to the number of fast neutrons produced by thermal fission divided by the number of thermal neutrons absorbed in the fuel itself. So, mathematical we can say, in the denominator it is the scattering it is the absorption cross section of the fuel, total absorption or macroscopic absorption cross section of the fuel which is fission plus capture because once the neutron gets absorbed in a fuel it can partition either in fission reaction or capture reaction. So, corresponding macroscopic cross sections can be added together to get the denominator. In the numerator we have the macroscopic fission cross section multiplied by this mu this mu indicates the average number of neutrons produced per fission.

And hence this ratio or this thermal fission factor can be defined by this particular ratio. Like, a commonly we have uranium as the fuel in thermal reactors which comprises of a several isotopes primary uranium 238 which is a more than 99 percent. In case of natural uranium some quantity like in 0.7 percent in natural uranium is uranium 235, and we can also have small quantity of uranium 235. And we can also has small quantity of uranium 234 or 232 etcetera.

So, absorption characteristics of all this need to be taken into consideration and calculation of this thermal fission factor, then it should be if we neglect the other isotopes let us just consider uranium 238 and 235, then in the denominator it should be the total absorption cross section of uranium 238 plus total absorption cross section of uranium 235. Uranium 235 for thermal neutrons has both fission and capture cross section, but uranium 238 it is fission cross section can be neglected. So, what we have in the denominator is the capture cross section microscopic capture cross section of uranium 238 and macroscopic capture plus fission cross section of uranium 235. Whereas, in a numerator it is the fission cross section of uranium 235 multiplied by the mu for uranium 235 uranium 238. It does not participate in thermal fission, because it is a fertile material and hence in the numerator we have the fission cross section of uranium 235 only.

If you are dealing with some kind of fuel, which has 2 different fissile isotopes then of course, in the numerator you need to take the summation of mu into the fission cross section for both those materials. And also remember that this mu depends on the isotope here is the example concerns only one fissile isotopes I have not used any subscript, but suppose you are using a fuel which contains both uranium 235 and plutonium 239, then their mu values will be different, and accordingly you need to take the summation of mu into sigma F for both of them.

Now, macroscopic cross section is the multiplication or is the product of nuclear density into the microscopic cross section. So, accordingly we can express this and now dividing the equation by the n and nuclear density for uranium 235 we get this where this small E is nuclear density of uranium 235 divided by the total nuclear density, which is called an enrichment factor like for natural uranium it is about 0.7 percent because only 0.7 percent is the uranium 235. But higher the enrichment factor of course, more will be the value of this thermal fission factor, and accordingly we can adopt some kind of mechanism of increasing the fissile isotope in the fuel so, that we can get higher value of this thermal fission factor. This is a graph which shows the variation of this thermal fission factor with a enrichment.

This here it has been considered to be mixture of uranium 235 and 238, and therefore, the standard values of uranium 235 fission and 238 cross sections were taken like for uranium 235 this sigma F is approximately 579 bonds or some let us suggest our 585 bonds. This sigma C for this is approximately 101 bonds, and sigma capture cross section for uranium 235 is to 0.72. So, if we and mu this number of average number of neutrons produced by fission of uranium 235 is approximately 2.42, this one. So, if we put all these numbers here, then we can of course, calculate the thermal fission factor for different values of E. Which is shown in this graph you can you can try this calculation on your own just put the value of put these numbers here, and put different values of E to get the corresponding variation in this thermal fission factor. Like for natural uranium the enrichment factor is just about 0.7. So, that should lie somewhere here, somewhere here.

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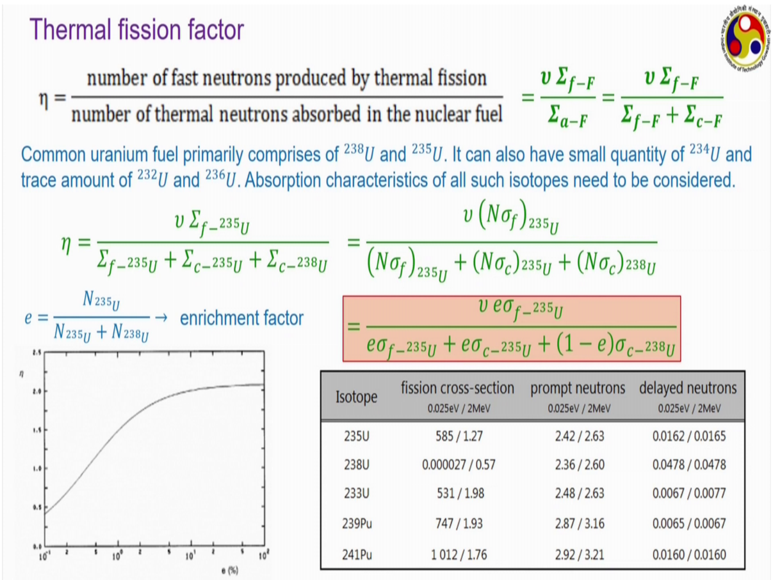

And you can see for this value is quite less than 1 whereas, the maximum value you are getting something around this 2. Around 10 beyond an enrichment level of 10 there is hardly any increase in the value of this enrichment factor or rather this thermal fission factors. So, it is not required also to enrich the uranium 235 percentage more than 10. And here we have a chart which you can use while doing this calculation for fission cross section values for the different isotopes of uranium 235 and 238 and a few other materials are given and also, the prompt neutron values that has produced here. The first data corresponds to thermal neutrons second corresponds to the first neutrons delayed neutrons we shall be coming back later on next we have the thermal utilization factor.

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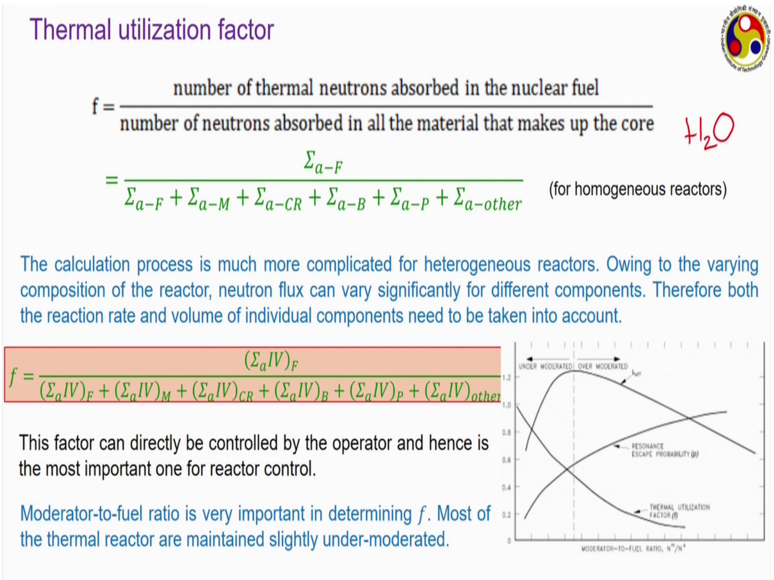

Thermal utilization factor refers to the number of thermal neutrons absorbed in a nuclear fuel divided by a total number of neutrons absorbed in the core by fuel and all other materials. So, in all other materials we have to consider moderator and any other kind of material that can be present. Like numerator is the macroscopic absorption cross section for the fuel in the denominator we can have several substances like this is of course, the fuel here this corresponds to the moderator and something else like CR can be control rods B refers to the boric acid which is commonly present both control rod, and boric acid we shall be discussing about them in the their role in the later module on reactor control.

But they are inevitably present in any reactor p refers to poison some of the fission products can act as neutron poison basically they just have very high absorption cross section and therefore, they can eat up lots of neutrons. So, they are often termed as poison and there can be some other materials also. This is how we define for a homogeneous reactor, but when you are dealing with a heterogeneous reactor actually homogeneous refers to a reactor, where in every part of the reactor we have the same composition of fuel and reactor fuel and moderator etcetera, but heterogeneous means there is a variation in the composition.

And in so, if like in some part we have only the fuel in some other part we have only the moderator, accordingly the neutron density also varies. Like if we are having a fuel in one part whatever may be the neutron flux there, if we are having moderator in other part then neutron flux will be much lower. Corresponding cross sections also will be different. So, initially with the heterogeneous reactor we also have to take the reaction rate into consideration. That is, every terms need to be multiplied or every cross-section values need to be multiplied, with I which is the corresponding neutron flux, or beam intensity and v that is the volume of the corresponding component. And accordingly, we get this particular definition.

This factor can this is actually factor which can directly be controlled in a reactor by the operator. Like say if our objective is to increase the if our objective is to increase the multiplication factor then we have to increase the thermal utilization factor. So, accordingly we can control any of them if we pick up say, this particular one control rods are elements which can be inserted or moved away or taken away from the reactor and then generally comprised of materials like boron or cadmium which have high absorption cross section. So, if we want to increase the utilization factor, then we can take these control rods out of it.

So, their corresponding cross section whereas, reduces and the value of utilization factor increases. If our objective is the opposite, then we can insert the control rods a bit more. So, that this particular quantity increases. And accordingly, the value of this utilization factor also decreases and in multiplication factor decreases the fuel to moderator or moderator to fuel ratio is also very important in determining this value like a graph is shown here where we have moderator to fuel ratio on the horizontal axis. And different others on axis is our interest is this particular graph. As the amount of moderator inside the reactor core that if increases, moderator has a significant contribution in the denominator here and accordingly the value of F continuously decreases. But generally, for a certain ratio of this moderator to fuel we get multiplication factor equal to 1 just shown here.

If the amount of moderator is less than that we call it under modulated under moderated, and that corresponds to a lower value of effective multiplication factor. Similarly, if the amount of moderator is increased beyond level again effective multiplication factor reduces. Actually, I should not say one this is something the optimum level that we get. So, any quantity moderator quantity less than that refers to under moderator anything greater than that refers to over moderated.

Again, by neglecting all other factor in the denominator of this particular equation by considering only a mixture of fuel and moderator and taking suitable cross section values, you can always calculate this effect of moderator to fuel ratio just assume a homogeneous reactor. So, that this I and v values goes off you take uranium as the fuel uranium 235 plus 238 and you take any moderator say if you take h 2 o as the moderator, you already have the values of it is absorption characteristics, and cross section values. And so, put the value of corresponding cross sections into this particular reaction, into this expression for F and you can always see how the ratio of moderator to fuel affects this particular k effective.

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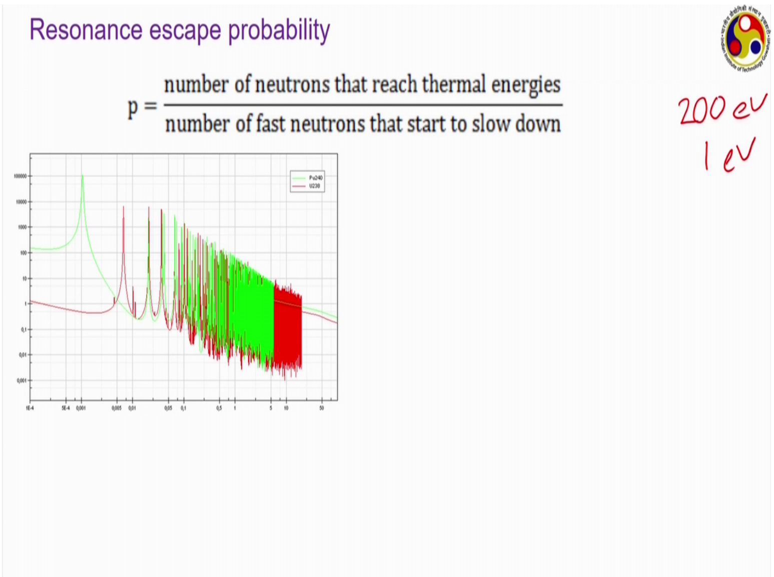

Third is resonances escape probability, it is something very difficult to predict, but the definition is number of neutrons that can reach thermal energy level divided by the number of fast neutrons that start to slow down in the moderator. Resonances escape probability like here this graphs already you have seen, but I am showing again for comparison purpose the resonance region resonance absorption region for uranium 238 and plutonium 240. You can see both of them have extremely high absorption cross section at certain energy levels. Like whereas, at certain energy levels their cross-section levels is somewhere here of the order of 0.1 at certain level that can be 1000s also. And in most of the cases that is in the order of tens.

So, at certain energy level the absorption cross section can have 100 to 1000 time increase. And hence there is a large small probability of good number of neutrons this intermediate energy neutrons that can get captured. Generally, for uranium 235, this resonance absorption zone expands from something like 200 electron volt to something about 1 electron volt. So, which is a large energy band to have and if the resonance absorption of course, this value of this probability depends strongly on the reactor structure and orientation and geometry. But if it is not carefully designed large number of neutrons may get absorbed here.

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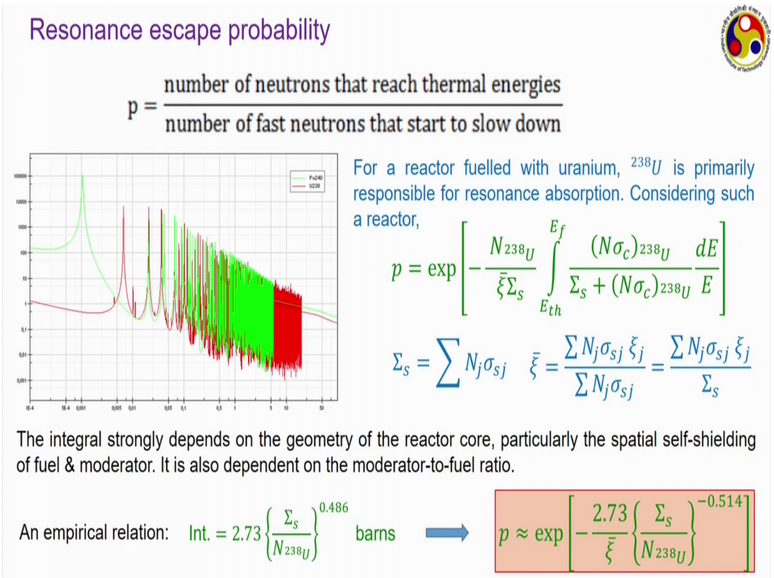

It is quite difficult to predict just assuming a reactor filled only with uranium, uranium 238 is the one which goes through this resonance absorption. Because uranium 235 does not have any significant resonance absorption characteristics. Then I am not going to the detailed of this calculation, it can be expressed in a form like this. Where in the numerator we have it is an exponential function here we have nuclei density of uranium 238 zeta bar and sigma s in the denominator are 2 combination values, zeta sigma s refers to the total scattering cross section. Actually, it is quite confusing because of similar symbols we are using this one for summation. And this one to represent the microscopic cross section. So, this sigma s here refers to the n into sigma microscopic scattering cross section for all elements like it can be uranium 235 and 238 in present example.

And zeta bar is again an weighted average logarithmic energy decrement which is defined like this again taking the contribution from all possible elements that can be present. Actually, the contributions in both sigma s and zeta were can come from uranium 235 uranium 238 the moderator etcetera. And then we have an integrand here E thermal is the lowest energy level where it is trying to reach to if F is the first energy level from here it is starting. Like I have mentioned E thermal commonly is around one to 10 electron volt for uranium 238, E f is about 200 electron volt. This evaluation of this integral is extremely difficult and depends strongly on the spatial self-shielding of fuel and moderator. That is where moderator and fuels are kept separately or not accordingly that is value depends. This is one empirical relation, where this integrand can be expressed like this. And if we put it back, then this is some kind of approximate relation we can get which we shall be using later on for solving numerical problems.

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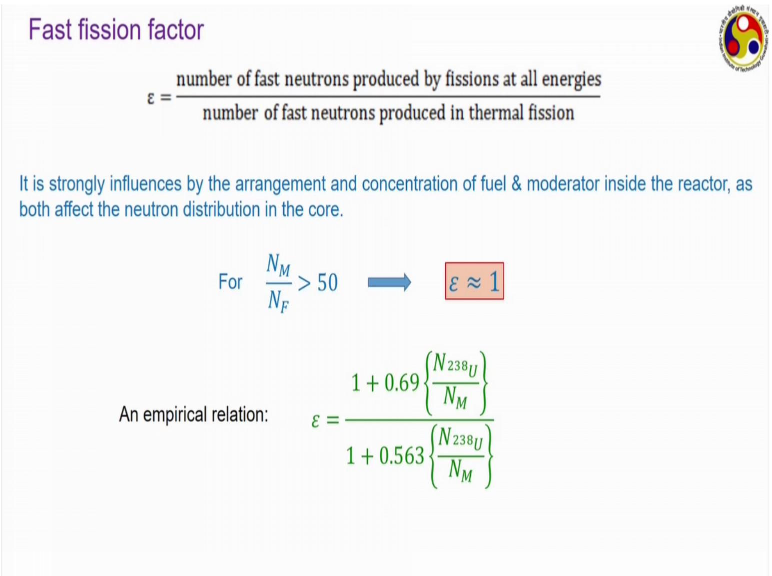

And finally, the fast fission factor in it can invariably be neglected for most of the thermal reactor, because it represents the ratio of number of fast neutrons produced by fission at all energies divided by number of fast neutrons produced in thermal fission. Or number of neutrons available at the beginning of energy generation. When the number of moderated nucleus is more like if you think about a homogeneous reactor, every filled nucleus is likely to be surrounded by moderator nucleus. And hence there is every chance that fast fission may not at all happen.

Like when this some moderator to fuel ratio is more than 50 fast fission factor is nearly one it can invariably neglected. But there are certain situations when it is less than 50, and we have to consider this factor again another empirical relation which can occasionally be used which again depends. On the F well to moderator ratio to have and uranium 235, it is fast fission can be negligible, uranium 238, it is something which can undergo fast fission. And therefore, this ratio contains nuclear density of uranium 238 and the nuclear density of the moderator.

So, these are the way we can calculate the 4 factors to compile the value of this infinite multiplication factor and reactivity of any reactor. Today I will I would like to keep it up to this. In the next lecture to start with we shall be seeing a sample numerical problem, where we shall be calculating the values of all these factors for a real reactor situation and trying to get an idea about this multiplication factor.

So, thanks for now.