Fundamentals of Nuclear Power Generation
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Lecture – 08
Theory of Elastic Scattering

Hello friends, welcome back to our MOOCS course on the topic of Fundamentals of Nuclear Power Generation. In our last lecture, we discussed about the topic of nuclear fission which is a part of our third module and today we are targeting to finish this particular module; that is in this second lecture we are looking to finish our third module on nuclear fission. As a brief recap of whatever we have done in the last lecture you were introduced to the topic of the nuclear fission.

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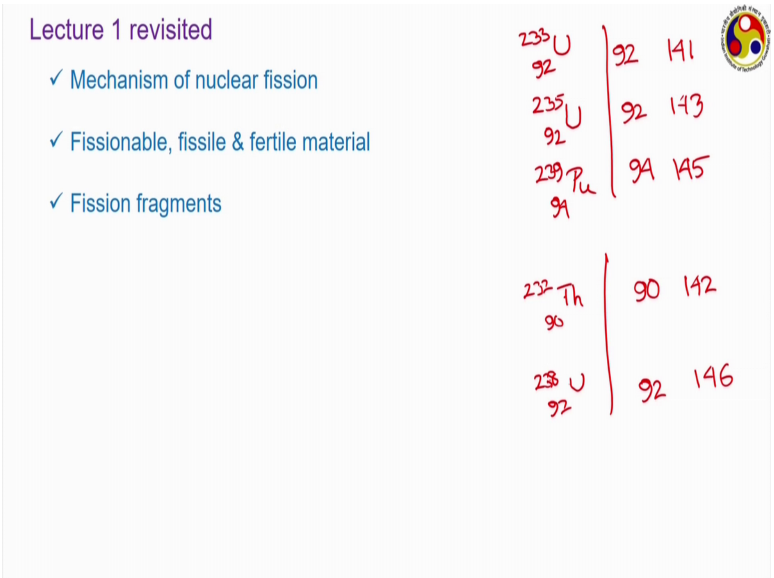

We now know that whenever a particular nucleus, a fissile nucleus that is when it absorbs a neutron; generally a thermal neutron, it is a excited to very high energy level and accordingly it can go through two different kinds of phenomenon. One possibility is that it can release that energy in the form of gamma photon and come back to the ground state in the form of another isotope of the parent nuclear itself. But the more probable option for the fissile nucleus is for this to undergo the fission process; that is to get splinted into two lighter nucleus or lighter or some other kind of isotropes.

So, according to the nature of this material you are introduced to this definition of fissionable fissile and fertile materials, where the term fissionable is associated with any particular nucleus which can be fissioned by striking it either by thermal neutron or fast neutron, but fissile the term is used exclusively for the nucleus, which can undergo fission only after absorbing a thermal neutron also can sustained a chain reaction following that fission.

But as fertile as those which cannot be fissioned through a thermal neutron, but can be converted to a fissile isotope. Like the example as we have seen uranium that is 92 U 2; 35 is the only naturally occurring fissile isotope, but there can be couple more which are generally artificially produced, but are very commonly used; one is uranium 233 and other is the (Refer Time: 02:42) isotope which is plutonium that is 94 Pu 239. Whereas an example of fertile material there are several, but very common example is generally thorium; thorium 232, thorium is actually having an atomic number of 90.

And other is uranium 238 that is 92 U 238, where this uranium 238 can be converted to plutonium 239 and thorium 232 can be converted to uranium 233; just one odd coincidence you can look at if you go through these examples say first for uranium 233; it is having 92 numbers of protons and how many neutrons it has? It has 141 neutrons in its isotope. Now come to uranium 235 again it is having 92 protons and 143 numbers of neutrons.

And in case of plutonium 239; it is having 94 protons and if I calculate it is 145 I think number of neutrons it has. Whereas now if you take a look for this two fertile isotopes, for thorium 232 it is having 90 number of protons and 142 number of neutrons and for uranium 238 it is having 92 number of protons and 146 number of neutrons; can you see this coincidence? It is quite odd I do not have any explanation for this.

But generally for the fissile isotopes you will get to see there is a odd even or even odd kind of combination in nucleus; that is even number of protons and odd number of neutrons. Whereas for the fertile isotopes, you have even even combination that is both even number of protons and even number of neutrons; again I repeat I do not have any explanation for this anything at all exist; nuclear physics make it some idea or it may be just a coincidence, but this is a quite interesting thing to observe. Now if we move further once you are introduced to this topic of fissile and fertile isotopes.

Then the next thing that we discussed is the fission fragments; fission fragments refer to the two daughter isotopes which are produced because of the fission of some kind of fissile isotopes. Fission fragments are generally radioactive in nature and therefore, they go through several steps of decay, but another interesting fact is that fission fragments generally vary in their mass numbers it is highly unlikely that both the fission fragments really having the same mass number.

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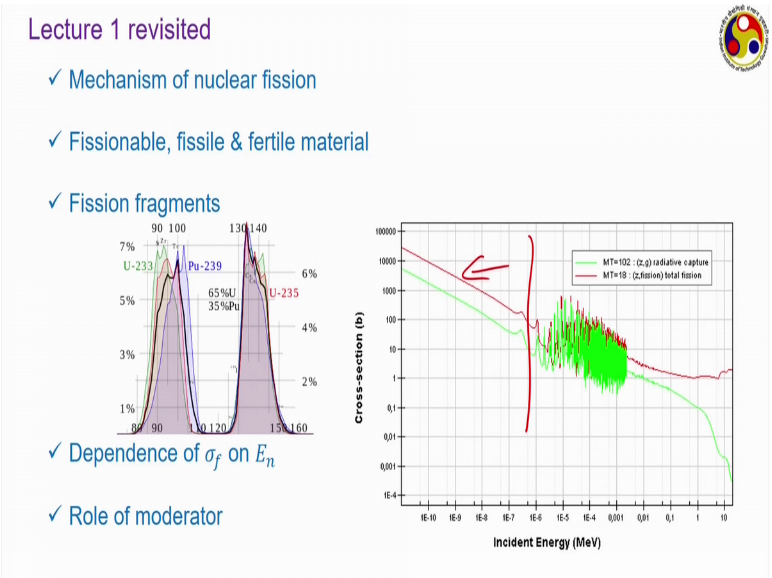

Rather they vary in the hm magnitude of their mass numbers and the difference between their mass numbers keeps on reducing with increasing the mass number of the parent isotope. That is the difference between the mass number of two fission fragments produced by uranium 233 fission will be higher than the same for plutonium 239. Then we discussed about the effect of energy of the neutron on on the fission on the cross sections.

Actually I should have written here as the dependence of E n on sigma f and not what is written here. We now know that we can get generally three kinds of regions where the most interesting one is the initial 1 by v region which refers to this particular part, where the both fission and non-fission capture cross section almost linearly reduces with the energy of the neutron.

And the most interesting of that is generally the thermal neutron part because they are the in a fission cross section is found to be quite high. And we finished the last class discussing or introducing the term moderator. Moderator refers to the material which is used in a nuclear reactor to reduce the kinetic energy of the neutrons so that it can be converted from initial fast neutron levels to the thermal neutron level..

And thereby increasing the possibility of fission because at we always have to remember that at lower energy level, the fission cross section is much higher, possible reason can be lower energy level means lower velocity of the neutron and therefore the neutron has much longer time to spend in close proximity of the nucleus and therefore, much bigger chance of having any kind of interaction..

So, today we shall be focusing primarily on this topic of moderator and see we shall be seeing how we can reduce the energy or at least we shall be doing lots of mathematics here to see how can we calculate; what is the energy the neutron we shall be having or neutron will be having after undergoing collision with any kind of moderator nucleus. So, I am starting with a slide with which I finish the last lecture; I have to do lots of mathematics here, but its quite odd to present that on slides. But I do not have the option of using board and that is why I am trying to keep it as much interactive as much step by step as I can please try to follow the discussions.

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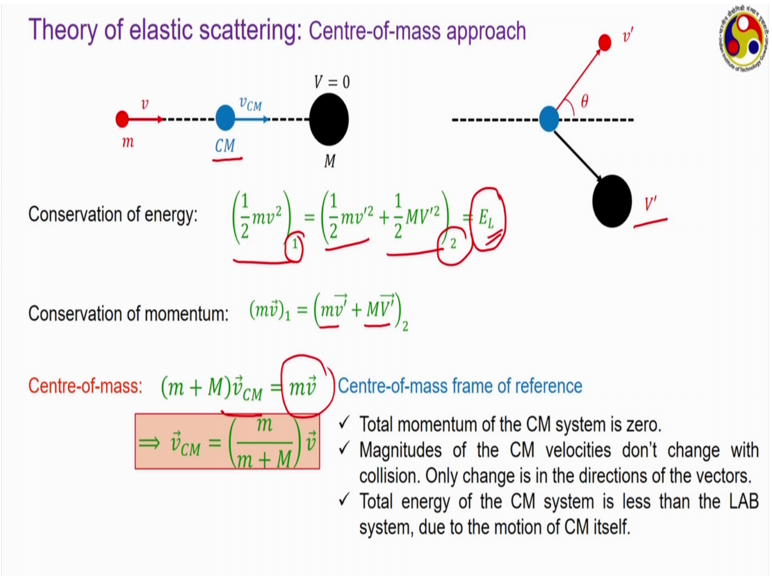

Here we have one neutron shown by that red dot of mass m which is moving with a velocity v and it is approaching a nucleus which is initially stationary capital V is the velocity of this nucleus, which is equal to 0 and its mass is m. And once this neutron collides with this nucleus, then it transfers a part of its momentum and kinetic energy to the nucleus and accordingly both of them gets deflected to certain angles. Like here theta refers to the angle with which the neutron gets deflected from its original path of motion v prime; small v prime is the velocity of the neutron after collision whereas, capital V prime is the velocity of the nucleus.

Now, if we write the conservation of energy then the as this being elastic scattering or elastic collision both kinetic energy and momentum will be conserved. So, as per the conservation of energy initial before collision the energy is half mv square; this part where 1 refers to that state before collision, 2 refers to the state after collision.

So, after collision total energy is the summation of the kinetic energy of the neutron which is this and the kinetic energy of the nucleus. Both being equal let us use this term EL to denote the in a total energy available in the system before and after collision. And as per the conservation of momentum initial momentum is m v or the momentum of the neutron alone and after collision the momentum is the summation of momentum plus for neutron and momentum also nucleus both should be conserved. While we can continue with this particular approach, but this is generally term the lab scale approach; here this subscript EL refers to the lab scale ah, but generally it is quite convenient to do the rest of this mathematical procedure following something called the centre of mass approach.

Centre of mass is an hypothetical object which is having the total mass of the system; like here this CM refers to the centre of mass. This hypothetical object is having the total mass of the system that is its mass will be small m capital M that is mass of neutron plus mass of nucleus. And also it is moving with such a velocity that its momentum is equal to the total momentum of the system..

Then for this centre of mass we can write its total momentum will be small m plus capital M; which is its mass into some velocity with which it is moving, this you should be equal to the momentum of the system; which is the momentum of the neutron alone before collision the centre of accordingly we can calculate the velocity of this centre of mass..

Now this centre of mass approach has certain advantage, if we choose this one as the frame of reference accordingly we can calculate the relative velocities for both neutron and nucleus. And as we shall be seeing shortly, it will provide you three different advantages. Firstly, total momentum of the CM system or as per the centre of mass system total momentum will be equal to 0..

The magnitude of the centre of mass velocities that is magnitude of the velocities of neutron and nucleus as per this new approach; that will not change after collision only change will be in their directions. And thirdly total energy of the CM system will be less than the lab system because the centre of mass itself is moving yes we shall be proving all of them.

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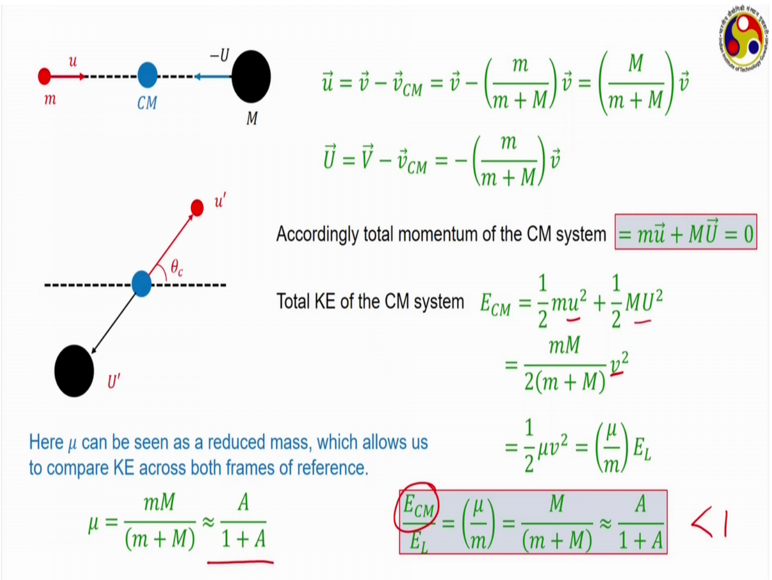

Now this is the modified situation before and after collision as per the centre of mass system here small u refers to the relative velocity of the neutron.

As with respect to the centre of mass and this capital U or minus capital U refers to the relative velocity of the centre of relative velocity of the nucleus with respect to again the centre of mass minus sign indicates that it is in the yeah. We have taken the direction for motion of neutron to be positive accordingly capital U is negative..

And after collision they gets its some velocity as a small u prime and capital u prime and theta c refers to the angle as per the centre of mass system. So, now, small u can be this being relative velocity of the neutron with respect to the CM. So, it will be of course, velocity v minus v of CM and putting the expression for v of CM we get this. Similarly capital U will be equal to v capital V minus v CM capital V is 0; remember the nucleus we have assumed to be stationary. So, capital V this is equal to 0 and accordingly we get the expression for capital U as well.

So, now, we know the relative velocities or the velocities for both neutron and nucleus as per the centre of mass system. So, the total momentum we can calculate according to your CM system should be equal to small m into small u multi plus capital M into capital U and if you put the expression it is equal to 0; which is the first point I have mentioned. As per the centre of mass system total momentum will be equal to 0 both before collision and after collision. Now total kinetic energy as per the CM system should be equal to the same for neutron plus the same for the nucleus..

Here of course, we are using small u and capital U because we are doing everything with respect to the CM system. And accordingly we can get a simplified expression and you can relate that to the original velocity of this neutron which is the velocity as per the lab scale and here we rearrange the term so, that we get finally, mu by small m into EL; remember EL was the kinetic energy as for the lab system which was half mv square small v square.

Here mu can be seen as a reduced mass which allows us to compare the kinetic energy across both frames of reference. And it is given as small m into capital M by small m plus capital M. Now one important you can say this approximation, but this is quite it is not a bad one actually. We know that mass of a neutron is slightly above 1 AMU, where as the mass of the nucleus is it will be quite close to the atomic mass of this one expressed in terms of AMU. So, it is very much likely that if we get this quantity this is most likely to be equal to the atomic; this most likely to be equal to the mass number of that particular nucleus, accordingly we do this substitutions to get mu equal to A by 1 plus A.

So, putting this back to the original expression we get the energy as per the centre of mass system divided by the energy of the with respect to the lab scale system is approximately equal to A by 1 plus A. Now whatever may be value of A surely this E CM will be less than EL that is this ratio always has to be less than 1 because the denominator always have the numerator..

So, that proves the third point that I have mentioned earlier; the energy at with respect to the CM system should be equal to or should be less than rather with respect to the for the compared to the energy with respect to the lab scale system. And the difference being the centre of mass itself is moving so, itself is having some kinetic energy..

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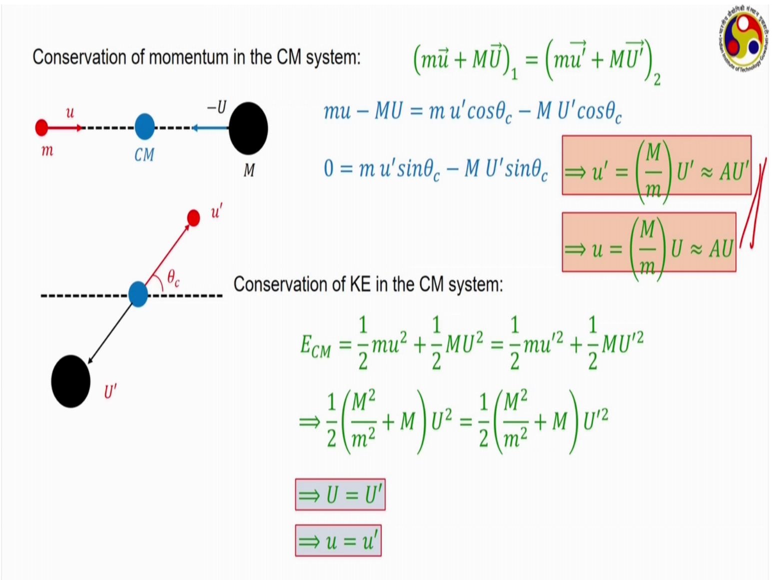

Let us move forward now the same diagrams reproduced here and we are writing the conservation of momentum with respect to the CM system. So, small u prime and capital u prime refers to the velocities of neutron and nucleus respectively with respect to the CM system. So, we write the conservation of momentum in this form; now we this being a vector equation, we can always separate that into as per the components and sticking to our 2 dimensional representation; this is according to let us say this is our x direction and this is our y direction..

So, accordingly before collision the momentum of the system is equal to m into U contributed by the neutron and this one is contributed by nucleus, the direction being opposite we are having this minus sign in between. And after collision this u prime is having the cos theta c component in this direction. And again this is also having the cos theta c component this direction; so we get this particular expression.

And similarly we can also represent we can also get the y component of the same equation. Before collision both the components that is neutron and nucleus are moving in the x direction. So, the y component is 0, but after collision we can have an y component. So, from the second part we get u prime is nearly equal to A into capital U prime. And putting that back into the first expression; we get small u is equal to nearly equal to A into capital U.

Now conservation of kinetic energy as per the CM system, conservation of kinetic energy or energy as per the CM system can be calculated as this is the kinetic energy before collision, this is the kinetic energy after collision small u and capital U are the velocities with respect to the CM system before collision; small u prime and capital u prime are the velocities after collision with respect to the CM system. So, by kinetic energy has to be conserved similar to momentum.

We can we can equate them and by using this substitutions we finally, attended this which proves capital U and u prime are equal and same applies to small u and small u prime; that means, we can clearly see that the magnitude of velocities for both neutron and nucleus remains the same before collision and after collision. However, before collision both of them are moving only along the x direction, but after collision there is a change in their directions we while magnitude remains the same. So, these are the three points that we have; three certain characteristics for the CM system that we have mentioned earlier which are proved here.

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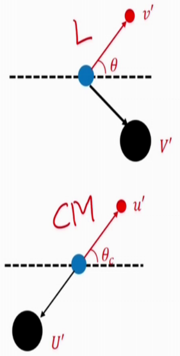
Ratio of KE of neutron after & before collision = $\frac{E'}{E} = \frac{\frac{1}{2}mv'^2}{\frac{1}{2}mv^2} = \frac{v'^2}{v^2}$



Now, let us move to what we are trying to find. We know that the objective of having this elastic scattering is to reduce the kinetic energy of the neutron. So, let us calculate the ratio of this energy before collision and after collision; before collision the energy was this half mv square, after collision if v prime represents the actual velocity, then velocity respect to the lab scale that is then this is the kinetic energy of this and correspondingly we get this to be equal to the velocity square ratios..

Here on the left; I have the diagram this is as per the lab scale system and this is as per the CM system. Theta is the angle at which the neutron gets scattered as per the lab scale where as theta c is the angle at which it gets scattered as per the CM scale..

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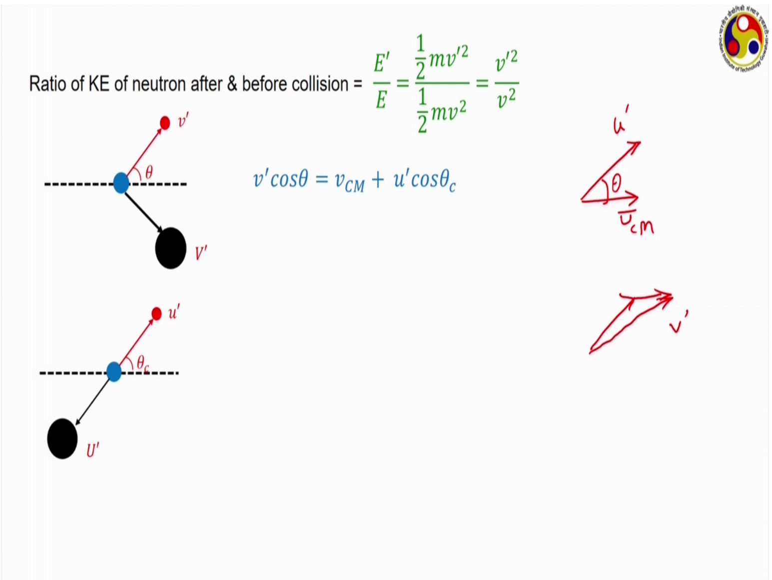

Now we can velocity being an vector; we can get as per their components that is v prime cos theta has to be equal to v CM plus u prime cos theta C. Basically here just seeing this; this is the v prime this is the actual velocity and if we take their components; this should be, this is the u prime and then let me rewrite say relative velocity; this is u prime and this is the v CM and this is u prime or.

I should say then if we follow the triangular law of this vector addition, then we can add them together and this resultant should be the actual velocity v prime, where theta represents that an intruded angle. This angle is the theta and accordingly we can write this particular expression that is v prime cos theta c should be equal to the v CM plus u prime cos theta.

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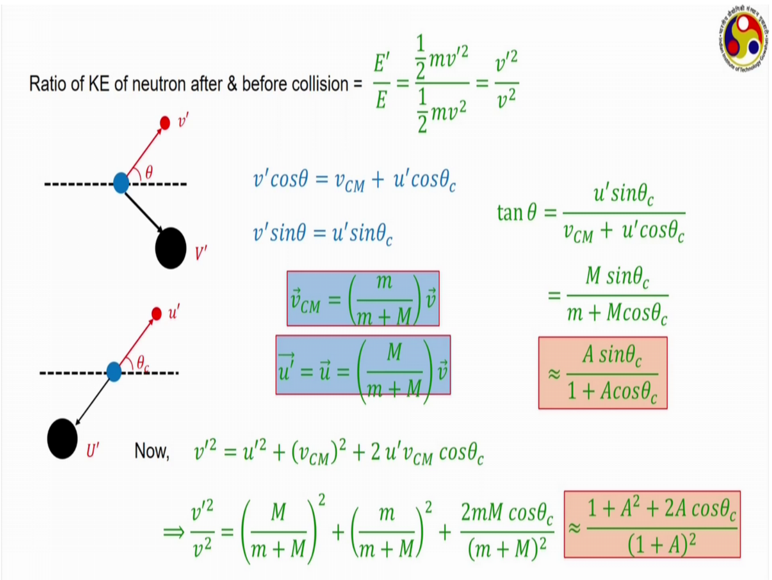

v prime cos theta is equal to v CM plus u prime cos theta c and their sine counterpart taking the ratio the angles as per the lab scale which is theta and angle as per the CM scale is theta c, we can get the relation between them. And then we substitute couple of relations which we have derived earlier you. Remember if you go back couple of slides; the v CM was expressed earlier in terms of v and u prime was also expressed in terms of v.

So, we put this back and we arrive at this particular expression or assuming capital M by small m to be equal to A, we get this tan theta to be equal to A sine theta c divided by 1 plus A cos theta c. So, using this vector combination or the vector components we can find the relation between the angle of scattering as per the lab scale, which is theta and as per the as per the CM scale, which is theta c.

Now we square the both the equations and add them together then this is what we get and why is changing the sides or divide this entire equation by v square, we arrived at this particular form and putting capital M by small m nearly equal to A; this is what we end up to, this is of what our interest what we were looking for. collision.

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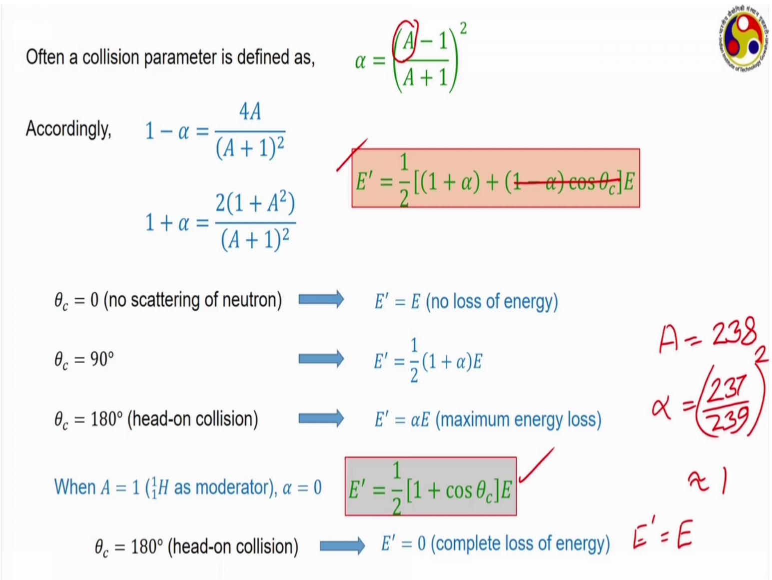

Often we define a operator as alpha equal to A minus 1 by A plus 1 whole square, then 1 minus alpha is equal to 4 A by A plus 1 whole square and 1 plus alpha is equal to 2 into A plus 1 plus A square by A plus 1 whole square.

If we put this back to the equation that we have derived here this one; we relate to this 1 minus alpha and 1 plus alpha is the final expression that is E prime is equal to half into 1 plus alpha plus 1 minus alpha cos theta c into v; is an extremely important relation. You remember E is the; this E is the energy of the neutron before collision and E prime is the energy of the neutron after collision; cos theta c is the angle of deflection or angle of scattering as per the CM system and alpha is this is the definition..

So, alpha is a function of only the mass number of the moderator nucleus. So, it is a property of the moderator; for different moderator value of A will be different, but once a moderator is given alpha is a constant; E is the kinetic energy before collision. So, suppose I give you a pair of a pair of neutron and nucleus; then both E and alpha specified..

Then the energy after collision is governed solely by the angle at which it can get deflected. There can be several situation; so theta c is equal to 0 that is no scattering of neutron and neutron is not at all changing any direction that basically signifies. That there is no collision the neutron has not at all collided with the neutron sorry nucleus. And if we put this equation then you will get E prime equal to E; there is no loss of energy because there has not been any collision..

If we put theta equal to 90 degree, then cos theta equal to 0, E prime is equal to half into 1 plus alpha into E. More interestingly if you put theta equal to 180 degree that is the neutron is having a complete change in its direction and is going back from where ever it has come from; which is often referred as a head on collision. For theta c equal to 180 degree, cos theta c equal to minus 1 and accordingly we get E prime is equal to alpha E.

It is the maximum energy loss that a neutron may suffer; like we can also calculate this say this expression for E prime is given to us. Now and as I have mentioned for a given neutron nucleus pair capital E and alpha both are constant and therefore, value of a prime depends solely on theta c. So, we can always calculate what is the value of theta c; for which the E prime becomes a minima. And if we do the calculation as d prime d theta c this is differentiate with respect to theta c and equate that to 0; then this should give you theta c is equal to pi or 180 degrees.

And you can also check that this actually refers to a minimum value for this E prime; that is when there is a head on collision between neutron and the stationary nucleus, then the neutron is expected to suffer maximum amount of energy loss or maximum amount of loss in its kinetic energy. And its magnitude is alpha into E; that is it is again depending on alpha; now alpha intern depends on A as per this relation..

So, for different value of A ah, we can get different values of alpha say for example, what is the lightest possible or smallest possible value for A we can have or the lightest possible event? Of course, that is a equal to 1 that is that refers to hydrogen; if we put A equal to 1 here; then alpha equal to 0. So, if we use that is 1 H 1 that is proton as a moderator, then alpha equal to 0.

And E prime is actually half into half into 1 plus cos theta c into E, which is the maximum possible or smallest possible value of kinetic energy after collision that we can have using any moderator. And again this depends on the value of theta c; the actual value of E prime. So, if we now use the head on collision that is theta equal to 180 degree then E prime equal to 0.

That means, it refers to a complete loss of energy or you can say if a neutron of whatever may be the initial energy level of the neutron, it comes and collides with a proton;

collides with hydrogen nucleus and suffers a head on collision. Or you can say a neutron of any energy coming and and having a head on collision with the hydrogen nucleus; head on collision means it is having an 180 degree change in its direction, then it is expected to loss entire of its energy or its kinetic energy after the collision will become 0..

So, then what is the gist that we can get from this entire discussion? Of course, theta c is something not in our control, E is the energy that we have from the incoming beam of neutron and our target is to reduce this E to the thermal neutron level at the earliest. Of course, a in a moderator generally the neutron requires to go through repeated collisions before it can reach to the thermal neutron level.

ah But if you are talking about a sink col energy reduction in a single collision then E is the energy with which it is coming and we have to choose a proper moderation moderator material. Theta c something that we do not that is not in our control rather the it is more a probabilistic definition, a neutron can get deflected to any possible angle after having a collision with this.

Then what we can control? That is of course, this alpha or rather we can control this E. Then what conclusion can we draw in terms of the preferable value of A so, that we can have a larger reduction in the energy? We should ideal cases having A equal to 1 because for any given value of theta that gives you the lowest possible value of E prime. And hence we should always want A to be as low as possible. A smaller value of A will give you a much larger reduction energy..

Just compare this particular case; situation with something that is having very high value of A; like say if we take uranium 238 that is a equal to 238; then alpha is equal to 237 by 239 whole square which is virtually equal to 1 and if you put the value of alpha here; in this orginal expression, then correspondingly E prime is equal to; alpha equal to 1 for very that is a maximum limit of alpha that we can have, what is approximately equal to 1 or tends to 1.

And if you put alpha equal to 1; in this expression then this term goes to 0 and we get E prime nearly equal to E that is there is hardly an reduction in the energy of the neutron. So, while when theta equal to 0 we that is no collisions situation we cannot have a reduction in energy also either moderator nucleus is having very high mass number then also the energy reduction is extremely small or almost insignificant.

So, it is always preferable to use a smaller value of A..

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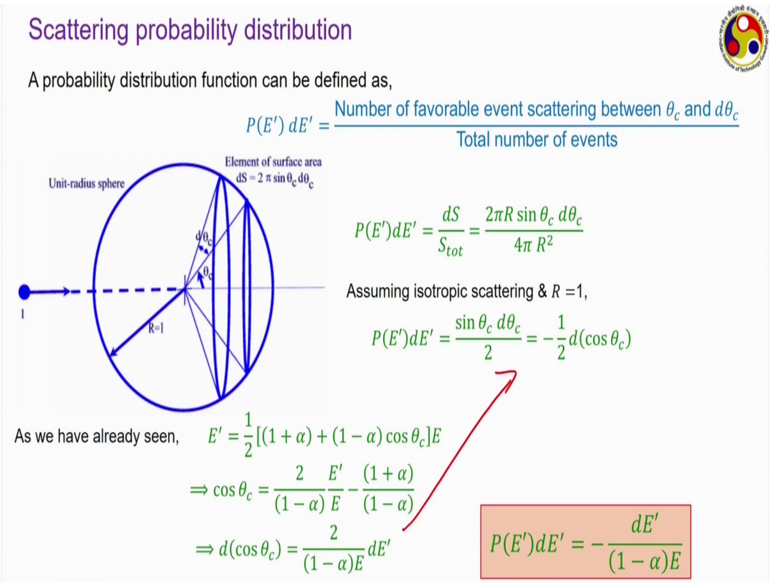

But practically the change or the in the direction or the value of theta is not something that is in our hand. Rather it is possible to have any value of deflection or any scattering angle. Therefore, we need to go to some kind of statistical averaging or some probability distribution. A probability distribution function can be defined as number of favorable event scattering between some angle theta c and d theta c divided by a total number of events.

ah Let us discuss this with respect to this diagram, here a neutron is coming whose mass number or its mass is assumed to be nearly equal to 1. And it is striking a nucleus which is situated here, initially nucleus is here that is at the centre of a circle; after the collision the neutron can get scattered into any possible direction.

Then this probability distribution function says what is the probability of finding a neutron in a very small angle around this theta c or probability at angle this theta c if we take very small (Refer Time: 30:52) angle this d theta c it is an (Refer Time: 03:54) small solid angle you can say. Then this function should be equal to the amount of area available over this (Refer Time: 31:03) portion through a neutron can pass through divided by the total area of the sphere.

That is the neutron can pass through any possible direction of a sphere or as per the any of the race of the sphere. So, if capital R represents the radius of this sphere then total area available is 4 pi R square and the area of this (Refer Time: 31:25) small portion, then how much is this? Here we are talking about this particular segment basically; here 2 pi R sine theta is the corresponding thing multiplied by d theta c that represents the thickness of this particular conical portion.

So, this ratio represents your probability distribution function; assuming isotropic scattering because isotropic scattering refers to in all possible direction; the intensity of scattering remains the same and also taking r equal to 1, this reduces to sine theta d theta c by 2 or you can also write this as minus half of d of cos theta c. Now we have already seen E prime is equal to half into 1 plus alpha plus 1 minus alpha cos theta c into v; so, cos theta c can be represented in this form and d of cos theta c can be represented like this..

So, take it back here then what we are getting is v prime d prime is equal to or this probability distribution function can be represented as minus d prime divided by 1 minus alpha into e.

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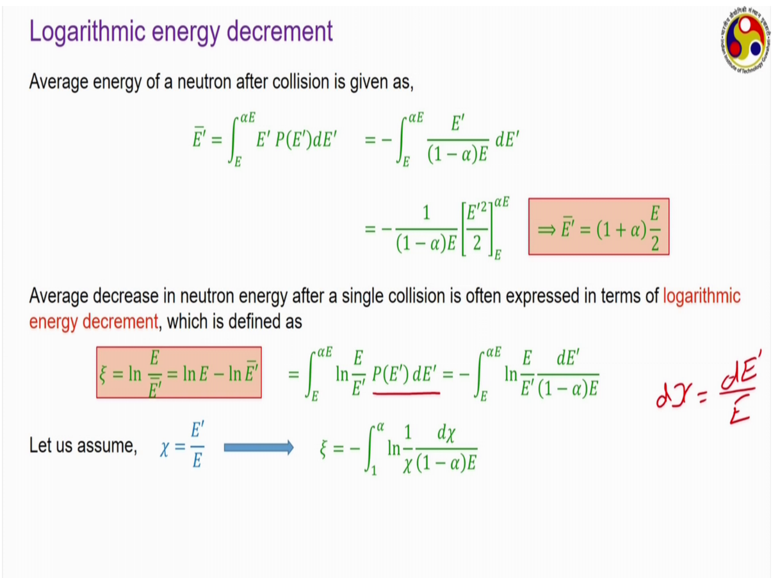

Finally, a very important concept that is quite often used to characterize this moderator; firstly, just waiting here in this light. So, far we have discussed if one neutron collides with nucleus then how much decrement energy it can have? That we have seen that depend to depend on the kinetic energy that is; depending on the kinetic energy of the neutron the pairing was dependent on the mass number of the nucleus; moderator nucleus and also depending on the angle of scattering.

But whenever a beam of neutrons strikes a nucleus we now know from our previous modulus that it is not a job of a single neutron, rather we need to supply large number of neutrons to have any significant rate of interaction. Because most part inside the nucleus is hollow or void; now whenever such a beam of neutron is approaching a nucleus, it is expected to have all possible scattering angles. That is was some neutron make it scattered by say an angle of 10 degrees, some other make it scattered by 120 degrees maybe just 1 in a million is suffering some kind of hidden collision.

So, this probability distribution function that allows us to get an idea about the probabilistic energy distribution; over a sphere around the around the nucleus initial position of the nucleus that is. And accordingly we can calculate the average energy of the neutron after the collision as this thing. We know E is the initial energy; so, if after collision the maximum possible value of energy it can have is E. And similarly their lowest possible value it can have is equal to alpha in two E which happens when theta equal to 180 degree. So, we can perform this integration now P E prime d prime al already seen in the; previous slide here we have seen that P E prime d prime is equal to minus d prime by 1 minus alpha into E. So, putting into back here now E is the initial energy of a neutron and alpha is a property of the moderator. So, for a given neutron nucleus sphere; we have this term to be constant of this denominator is a constant.

So, we can perform this integration only over E prime and we are getting that average energy after collision; average energy of the neutron after collision is just 1 plus alpha into E by 2. But the more important definition of a logarithmic energy recruitment, which is basically the ratio of energy before collision divided by kinetic energy; average kinetic energy that is is after collision..

Here we are not at all dealing with a single neutron rather we are dealing more with a beam of neutrons. As a different neutrons in a single beam can undergo different amount of scattering. So, the energy of the neutron after collision also will be varying a lot, but this E prime bar that is this quantity; average is that over all possible directions. And now logarithmic energy recruitment is represented as energy before collision; divided by this average kinetic energy after collision and take a log of that so, that we have log E minus log E prime bar.

If we put the expression for E prime bar; then this is what we end up to and putting the expression this P E prime d prime; this is what we are getting that is integration limits remains the same because E is the largest possible energy after collision and alpha is E is the smallest possible energy after collision; for any given value of alpha.

Now how to perform this integration? This looks a bit complicated because we are having a log of E by E prime that is available and E prime is the independent variable also. Let us assume some xi is equal to E prime by E; so, putting that here d xi will be equal to d prime because capital E is a constant (Refer Time: 36:55) you know this prime will be equal to d prime by E..

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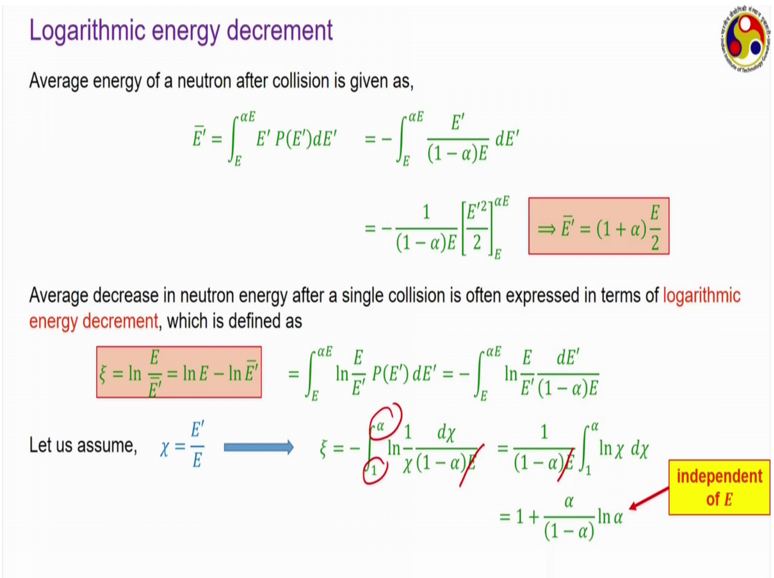

So, that can be put back here and actually I think I have a; I had made a typing error I had made a typing error here. This E should not be a part of this expression it is because d prime by E is equal to this particular term. So, 1 minus alpha being a constant comes out of the integration and another important change is the limit of the integration.

Now, when E prime is equal to E; I am struggling to write this particular symbol xi is equal to 1. So, lower limit becomes 1 and upper limit for E prime is equal to alpha into E; accordingly this is equal to alpha, so this is the upper limit. Now putting this expressions and also as I mentioned; this E goes off here this is not present this, then we have log In x dx; we can integrate this to over the given limits of 1 to alpha to get 1 plus alpha by 1 minus alpha log alpha..

So, logarithmic energy recruitment you can see the final expression that we are getting that includes only alpha or it is a sole function of the moderator; it does not depend upon the energy of the or a neutron or initial energy of the neutron at all.

So, average decrease in the energy level of the of a neutron beam that very interestingly is not at all dependent upon the energy of the neutron with which it is coming; rather it depends only on the moderator this is sole function of the moderator nucleus. Now we know alpha is equal to A minus 1 by A plus 1 whole square.

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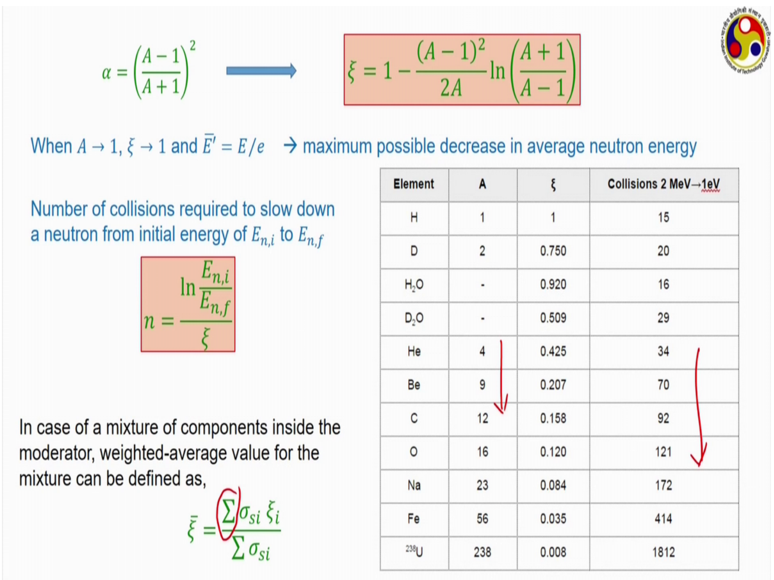

See if you put this back we get zeta is equal to this excellent expression which is again a sole function of capital A. And for any value of capital we can calculate the value of corresponding energy decrement or average decrease in the energy. Like when A tends to 1, zeta tends to 1 and E prime bar is equal to E by small e; which is the maximum possible decrease in the average neutron energy..

When number of; generally we need to have several collision for the neutron so, that this energy can be reduced from some given initial level to some given final level. This logarithmic energy decrement also allows us to calculate the total number of collisions required to slow down the neutron from an initial level to a final level. And this can be calculated using the definition of zeta; it can be calculated as log of E n initial by final divided by zeta.

These are some sample values; let us say when H is equal to 1; that is hydrogen A equal to 1, if we put that then is directly getting zeta equal to 1 and corresponding collision to reduce say if E n i is equal to 2 MeV and final energy is equal to 1 electron volt. Then we are getting the corresponding 15 collisions will be required, this all numbers are actually skilled up that is if we are getting 14 point something that has been converted to 15 say getting 15 collisions for this. Similarly when d equal to 2; you can calculate the value of zeta from the source I have taken this does not seem to be correct. If we put the numbers, it should be 0.725, but it will get 20 or collisions to reduce a neutron energy from two MeV to 1 electron volt.

Similarly, you can see like if we take a very heavy isotope like uranium 238 value of zeta is extremely small 0.008 and more than 1800 collisions are required to have this amount of reduction; just to compare this number for hydrogen it is only 15; whereas here it is 1812 is an very large number of collisions will be required. And we can clearly see a number of collisions or as the mass number of the moderator nucleus goes on; again this direction, the corresponding number of collisions also increasing continuously as a value of zeta is decreasing.

Quite often the moderator material comprises of several kinds of isotopes or several kinds of elements. In that case we need to define an average value of zeta or weighted average and that is defined as summation of scattering cross section; summation of this scattering cross section into the logarithmic energy decrement for any component; divided by the summation of the scattering cross section for all components. This is slightly confusing here, this symbol here refers to summation; however, for macroscopic cross section also we are using a same symbol; now where here it refers to the summation only..

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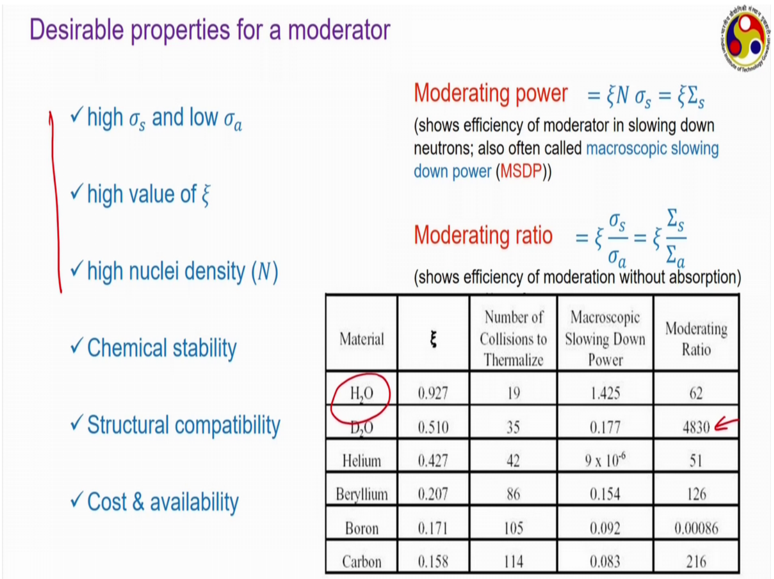

Material	ξ	Number of Collisions to Thermalize	Macroscopic Slowing Down Power	Moderating Ratio
H_2O	0.927	19	1.425	62
D_2O	0.510	35	0.177	4830
Helium	0.427	42	9×10^{-6}	51
Beryllium	0.207	86	0.154	126
Boron	0.171	105	0.092	0.00086
Carbon	0.158	114	0.083	216

Then what should be the desirable properties moderators should have these are something you have mentioned earlier. A moderator must have large scattering cross section and low absorption cross section. If the scattering cross section is high, it will be able to capture a good amount of energy reduction in just single collision. Whereas it should have a very low absorption capacity if the moderator itself starts to absorb the neutrons; then there will be neutron dot that is creating inside the nucleus and the reaction cannot sustain the chain reaction.

But now we are also seeing that having high sigma alone is not sufficient rather the logarithmic energy decrement also needs to be high. So, that in a small number of a attempts; we can reduce the energy of neutrons from some very high value to some reasonably low value generally at the thermal level. But what else the energy decrement can depend on? That should depend on the number of nucleus moderator nucleus that is that is present in the system.

If the number of the moderator nucleus is more; then there will be more number of interactions or the chances of interactions scattering that is that has to be directly proportional to the number of moderating nucleus present in the system; so we want a high nuclear intensity for the moderator. Now this three terms quite often are combined to these two definitions moderating power and moderating ratio. Moderating power is defined as zeta N into sigma S that is the it is the product of this three quantities logarithmic energy, decrement into the nuclear density of the moderator into the scattering cross section; here this capital sigma S refers to the microscopic scattering cross section.

Because we know this scattering cross section or any macroscopic cross section is sigma N into the nuclear density into the microscopic cross section. And we can use say s substitute for both macroscopic and microscopic cross sections. In the previous slide of course, the same symbol was used to denote summation, but here it is the macroscopic cross section. This moderating power shows the efficiency of moderator in slowing down the neutrons, it is quite often called the macroscopic slowing down or MSDP.

Other is moderating ratio which is may the energy decrement logarithmic energy decrement multiplied by the ratio of scattering to absorption cross section, which we can write as if we multiply with both numerator and denominator with N; then we can convert that to the macroscopic cross sections. So, it shows the efficiency of the moderation without have absorption. So, what kind of values for this moderating power and moderating ratio you should prefer? Of course, both should be high; we should prefer a higher value of moderating power because in a single collision larger amount of energy reduction is possible.

Similarly moderate ratios should also be high because for given amount of absorption or given amount of scattering the absorption; should be very low. This is some numbers or some of the isotopes that are present earlier. Like we have seen helium oh sorry we have seen that hydrogen and deuterium both are excellent as a moderator. Hydrogen having a mass number of 1 is the best possible material that we can have a as a moderator. But the problem for hydrogen deuterium is that they it is gaseous in nature. And therefore, the nuclear density or number of nuclear present per unit volume should be very low. That is why those materials are generally not used rather they are converted to their compounds in the form of common water or the heavy water and those are used as a moderator.

These two are the most preferred moderator that we can have in any kind of nucleus whereas, the common neutron nucleus or common water is the most preferred choice heavy water is generally used in candu types reactors or reactors where the neutron is highly enriched we shall be discussing about neutron enrichment in the next module. And you can see to reduce the energy level from say 1 mega electron volt to the thermal neutron level. Whereas, the common water requires 19 number of collisions; heavy water requires 35. There if you other materials also quite often used like helium and beryllium has been tried in the few reactors recently; boron not best choice because its moderating ratio is very low. Boron has a high as the high absorption cross section; so, while its slowing down power is not that bad, you can see helium has an extremely low slowing down power something like this.

Whereas the; it is the lowest among this and this is the highest. So, as per macroscopic slowing down power or moderating ratio definitely common water is the best choice whereas the helium is extremely poor from that point of view. But as of you compare from moderating ratio point of view this not quite bad, but this is extremely high. Because deuterium has an extremely small absorption cross section; it hardly absorbs a neutron that whether whatever neutrons comes that gets scattered.

And therefore, it is an extremely high moderating ratio on the contrary as I just mentioned boron has extremely small absorption cross section because it absorbs a good amount of neutron therefore, it is not at all a good choice as a moderator. Carbon that is graphite that is that is also used in certain kinds of reactors, it is also has been quiet high moderating ratio because it does not absorbs too much, but it is a moderating power is lower even lower than boron. So, if U see only from moderating power point of view this looks to be the best one; in our moderating ratio point of view this is extremely good; the deuterium or heavy water.

But this three parameters alone are not sufficient to justify the choice of a moderator rather there are several associated issue that also we must look into. Like the chemical stability as a nuclear reactor we expect to have high temperature involvement; quite often the certain materials may become soft at the higher higher temperature or may start to go through some kind of chemical reactions..

So, it should be chemically compatible with the other materials present inside also should be strong enough to sustain higher temperature; that structural compatibility is the issue it should not get deformed or should not just buckled down at higher temperatures. That is another reason of not using gaseous material like carbon or graphite while a carbon we can use in the form of gaseous natures like a carbon dioxide etcetera graphite is always a much better choice. And that is generally used not in the reactor where we have water present rather it is used gaseous use as the coolant. So, more in gas cool reactors graphite is used as the moderator whereas, beryllium has been tried in certain reactors.

Another the cost and availability; I have mentioned heavy water is having an very high moderating ratio and its moderating power is also not that bad. So, it is probably the best choice among the six that those are listed here, but it is extremely costly on contrary; this is cheap very very easily available and handling this one is also not at all difficult and that is why common water is generally the most preferred moderator that you will find everywhere. So, that brings us to end of this particular module.

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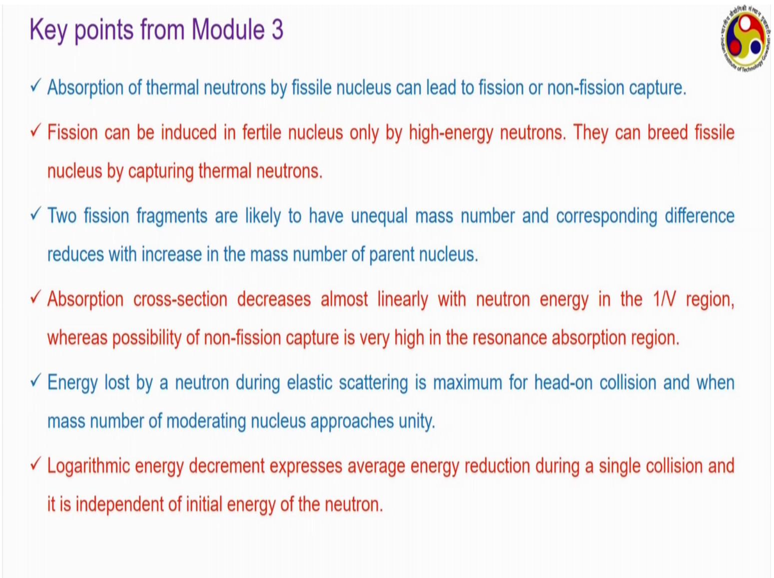

Let us try to summarize whatever we have learned in our third module. We have seen that absorption of thermal neutrons by fission nucleus can lead to fission and non fission capture; depending upon the relative fraction of fission cross section or absorption cross section. Uranium 235 is the only natural occurring fission fissile isotope which is which can undergo fission only by absorbing thermal neutrons, but plutonium 239 uranium 233 can also be easily produced. There are isotopes like uranium 238 thorium 232 which are fertile isotopes, they cannot undergo fission after absorbing thermal neutrons.

But can get converted to some kind of fissile nucleus the process which is called breeding. We have also seen that the two fission fragments which have produced because of fission reactions are not likely to have equal mass number rather their mass number will vary. And the difference between their value of mass number that reduces with increase in the mass number of the parent nucleus.

Absorption cross section varies almost linearly that is decreases linearly with the kinetic energy or velocity of the neutron in the 1 by V region ah, but there is also resonance capture region in between; while the resonance absorption cross sections are quite low for the fissile materials, but the materials like uranium 238 can have extremely high resonance absorption cross section. And therefore, it is a zone that better to be avoided or at least we should try to make the neutron flow through this zone at the earliest; in order to avoid the absorb cross section; avoid the resonance absorption. Energy lost by the neutron during a single elastic collision or single elastic scattering is found to be maximum in case head on collision and also when the mass number of moderating nucleus approaches unity.

Therefore, it is always preferable to use some kind of moderating material whose mass number is quite low; preferably quite close to 1. And finally, logarithmic decrement was defined to express the average energy deduction of a beam of neutrons during a single collision. And it is found to be very interestingly it was found to be independent of the initial energy of the neutron; depending whether on the at mass number of the moderating nucleus.

And finally, it is desirable to have high moderating power and moderating ratio for the selected moderator along with several commercial factors like cost availability and other compatibility. So, that is the end of our third module; there are several other topics which are related to fission reaction, but I thought about keeping that as a part of our fourth module where we shall be discussing about the chain reactions and the neutron diffusion equation. Thanks for your attention, you shall be having access to the assignment, so please solve the assignment and if you have any doubt any where you face an issue; do not hesitate to mail us back.

Thanks.